

Optimal Portfolio Selection Using the Sharpe Ratio and the Tangency Portfolio Approach: A Long-Term Case Study on the Istanbul Stock Exchange (BIST)

Mustafa Fatih Boz

Assistant Professor, Department of Energy Management, Economics, Administrative and Social Sciences, KTO Karatay University, PO box 42020, Konya, Türkiye

E-mail: fatih.boz@karatay.edu.tr

Abstract

This paper investigates the practical application of the mean-variance optimization framework and the Sharpe ratio in the Istanbul Stock Exchange (BIST) over a twenty-five-year period from July 2000 to June 2025. An initial portfolio was constructed with ten equally weighted stocks drawn from diverse sectors to achieve effective diversification. Using the Markowitz model, a set of portfolios was generated that minimized variance for specified expected returns, collectively forming the efficient frontier, which illustrates combinations of return and risk that dominate all other alternatives. Among these, the optimal portfolio—identified through the tangency portfolio approach and the Sharpe ratio—yielded returns approximately thirty-five percent higher than those of an equally weighted portfolio of the same ten stocks.

Keywords: Sharpe ratio, Capital Market Line, mean-variance approach, optimal portfolio, tangency portfolio, efficient frontier

DOI: 10.7176/RJFA/16-7-04

Publication date: August 31st 2025

1. Introduction

The primary objective of investors operating in financial markets is to maximize returns. However, significant price fluctuations in the assets they invest in often lead to irregular and unpredictable performance. This price volatility is typically measured by the standard deviation. As the standard deviation increases, the associated risk of asset returns also rises. For investors aiming to construct an efficient portfolio, achieving an appropriate balance between risk and return largely depends on diversifying the assets within their portfolios. Through diversification, the adverse effects of large-scale negative price movements in certain assets can be mitigated. Historically, traditional investors sought to reduce portfolio risk by including assets with lower variance, often neglecting the correlations among them. This approach assumed that selecting low-variance assets alone would suffice to minimize overall portfolio risk. Nevertheless, later empirical research challenged this perspective, emphasizing the critical role of correlations among financial assets in portfolio construction.

By 1952, this conventional view was fundamentally challenged by Harry Markowitz through his seminal study, in which he mathematically analyzed the relationship between risk and return in investment portfolios. This work, authored by Markowitz, is widely recognized as the first academic contribution to the field of Modern Portfolio Theory. In his study, he argued that assets should be evaluated not only based on their expected returns but also in terms of their variances (risk) and covariances (interrelationships with other assets). The mean-variance model developed by Markowitz provides a structured framework for identifying portfolios that either offer the lowest possible risk for a given level of expected return or deliver the highest possible return for an acceptable level of risk. All such optimal portfolios lie along a graphical representation known as the efficient frontier, which illustrates the set of portfolios offering the highest expected return for a given level of risk. Among these, the tangency portfolio occupies a unique position: it is the point where the Capital Market Line is tangent to the efficient frontier. While every portfolio on the frontier is considered efficient, the tangency portfolio is particularly significant because it maximizes the Sharpe ratio and thus represents the most favorable risk-adjusted return. The ultimate portfolio selection, however, depends on the investor's individual risk tolerance and investment objectives.

This paper analyzes the performance of portfolios composed of stocks from various sectors traded on the Istanbul Stock Exchange (BIST), applying the fundamental framework described above. The remainder of the article is structured as follows: Section 2 provides an overview of the Markowitz Model and the Sharpe Ratio; Section 3 presents the empirical analysis of BIST data using the Mean-Variance Model and the Sharpe Ratio; and Section 4 discusses the key research findings.

2. A Review of Optimal Portfolio Selection and the Sharpe Ratio in the Literature

A portfolio consists of a combination of financial instruments—including stocks, bonds, currencies, and other assets—selected to achieve a targeted balance between risk and return. The central question of portfolio construction revolves around which configuration delivers the highest expected return for a given level of risk or, conversely, the lowest risk for a predetermined return goal.

The conceptual foundations were laid by Harry Markowitz in the 1950s, who introduced the mean-variance framework. In this model, the expected returns and the variances of individual assets, along with their covariances, are the primary inputs used to determine the so-called efficient frontier. This frontier delineates the optimal portfolios that cannot be improved in terms of risk-return trade-offs without sacrificing one dimension for the other (Markowitz, 1952: 77–91). While Markowitz's framework remains central, it was further refined by Sharpe (1964), Lintner (1965), and Mossin (1966), who showed that, in the presence of a risk-free asset, investors should combine it with a single tangency portfolio that maximizes the Sharpe ratio, defined as the excess return per unit of risk (standard deviation).

The Markowitz model is built upon several key assumptions, which are outlined as follows (Vanini & Vignola, 2001: 6): (1) the investment environment comprises solely N risky assets, with no inclusion of a risk-free asset; (2) the model operates under a single-period framework; (3) uncertainty within the market is formalized using a complete probability space; (4) the model assumes the absence of transaction costs; (5) all financial markets are assumed to be perfectly liquid; (6) assets are infinitely divisible; (7) the full investment constraint is enforced, with no allowance for borrowing or short-selling; and (8) portfolio selection is guided exclusively by the mean-variance optimization criterion.

Since its introduction, the Sharpe ratio has become a standard criterion in both academic studies and professional asset management for comparing and selecting portfolios (Elton & Gruber, 1997; Estrada, 2010). A review of various studies on the Sharpe ratio reveals several significant findings. Bilir (2016) selected the optimal portfolio among those located on the efficient frontier by using the tangency portfolio and the Sharpe ratio, reporting that the resulting portfolio generated returns approximately three times greater than those of the initial portfolio. Vuković, Vykylyuk, Masiuk, and Maiti (2020) demonstrated that a neural network-based approach provides a practical and effective decision-support tool for Sharpe ratio-focused portfolio selection. Similarly, Dong (2022) found that, within a given set of assets, the combination of

those with the lowest pairwise correlations and the highest individual risk–return ratios resulted in the highest Sharpe ratio.

3. Empirical Analysis

This section presents the empirical application of the Markowitz mean–variance model and the Sharpe ratio, based on Borsa Istanbul (BIST) data covering 6,249 trading days from July 2000 to June 2025. The analysis includes the construction of portfolios, the evaluation of their performance, and the identification of the optimal portfolio configuration in accordance with modern portfolio theory.

3.1 Data and Formulas

Markowitz asserts that investing in a large number of securities with low individual variances alone is not sufficient for effective diversification. True diversification cannot be achieved merely by increasing the number of assets in a portfolio; the degree of correlation among assets must also be taken into account. In particular, assets with high covariances should be avoided. From a portfolio management perspective, selecting assets from different industries is advisable, as firms operating in distinct sectors typically exhibit lower interdependencies due to varying economic structures.

In this study, a hypothetical portfolio is constructed using ten stocks traded on Borsa Istanbul (BIST). Although the selection process was random, care was taken to ensure that the stocks represent a broad range of industries to enhance diversification. Moreover, firms were chosen based on the availability of uninterrupted long-term price data (6,249 trading days) to ensure consistency in the empirical analysis. The selected sectors range from automobile manufacturing and banking to oil refining and pharmaceutical production (Table 1).

Table 1. Company Information

| | Assets | Trade Name of Companies | Industry |
|----|--------|---|--------------------------|
| 1 | TUPRS | Türkiye Petrol Rafinerileri A.Ş. | Oil refining |
| 2 | THYAO | Türk Hava Yolları A.O. | Transportation |
| 3 | ISCTR | Türkiye İş Bankası A.Ş. | Banking |
| 4 | MGROS | Migros Ticaret A.Ş. | Retail |
| 5 | TOASO | Tofaş Türk Otomobil Fabrikası A.Ş. | Automobile manufacturing |
| 6 | CEMTS | Çemtaş Çelik Makina Sanayi ve Ticaret A.Ş. | Steel production |
| 7 | ISGYO | İş Gayrimenkul Yatırım Ortaklığı A.Ş. | Real estate investment |
| 8 | TATGD | Tat Gıda Sanayi A.Ş. | Food processing |
| 9 | YATAS | Yataş Yatak ve Yorgan Sanayi Ticaret A.Ş. | Home textiles |
| 10 | ECILC | Eis Eczacıbaşı İlaç Sınai ve Finansal Yatırımlar Sanayi ve Ticaret A.Ş. | Pharmaceutical |

In this study, the market returns of the selected companies are estimated using daily return data, calculated based on prices adjusted to US dollars. These data were obtained from the İş Yatırım database.¹ The analysis covers a continuous period of 6,249 trading days from July 2000 to June 2025. The mathematical formulations used throughout the study are presented in the following section.

- daily return

$$R_s = \frac{R_{st} - R_{st-1}}{R_{st-1}} \quad (1)$$

where “ R_s ” is a daily return of stock s , “ R_{st} ” is a closing price of stock s in t date and “ R_{st-1} ” is a closing price of stock s in $t - 1$ date

- average return

$$E(R_s) = \frac{1}{n} \sum_{t=1}^n R_s \quad (2)$$

where “ $E(R_s)$ ” is a average return for stock s , “ R_s ” is a market return in t date, “ n ” is a number of dates

- abnormal return

$$R_{ab} = R_s - E(R_s) \quad (3)$$

where “ R_{ab} ” is a abnormal return for stock s , “ R_s ” is a daily return of stock s , “ $E(R_s)$ ” is a average return for stock s

- expected portfolio return

$$E(R_p) = \sum_{i=1}^n w_s \cdot E(R_s) \quad (4)$$

where “ $E(R_p)$ ” is the expected value of the portfolio return, “ n ” is the number of stocks, “ w_s ” is the proportion of the funds invested in stock s , “ $E(R_s)$ ” is a average return for stock s , “ $\sum w_s = 1$ ”

- covariance

¹ İş Yatırım (n.d.) Tarihsel fiyat bilgileri [Historical price information]. Available at: <https://www.isyatinir.com.tr/tr-tr/analiz/hisse/Sayfalar/Tarihsel-Fiyat-Bilgileri.aspx> (Accessed: 4 July 2025).

$$Cov(x, y) = \frac{\sum_{t=1}^n (x_t - \bar{x}) \cdot (y_t - \bar{y})}{n - 1} \quad (5)$$

where “ x_t ” is a daily return of stock x, “ \bar{x} ” is a mean of stock x, “ y_t ” is a daily return of stock y, “ \bar{y} ” is a mean of stock y and n is the number of samples

- variance (risk) of portfolio

$$\sigma_p^2 = \sum_{x=1}^n \sum_{y=1}^n w_x w_y \dots Cov(xy) \quad (6)$$

where “ σ_p^2 ” is a variance of portfolio, “w” is the weight of each stock in the portfolio, “Cov(xy)” is the covariance between the stocks in the portfolio

- standard deviation of portfolio

$$\sigma_p = \sqrt{\sum_{x=1}^n \sum_{y=1}^n w_x w_y \dots Cov(xy)} \quad (7)$$

or

$$\sigma_p = \sqrt{\sigma_p^2}$$

where “ σ_p ” is a standard deviation of portfolio, “w” is the weight of each stock in the portfolio, “Cov(xy)” is the covariance between the stocks in the portfolio

- variance–covariance matrix

$$M_{vc} = \begin{bmatrix} Var R_1 & Cov(R_1, R_2) & \dots & Cov(R_1, R_n) \\ \vdots & \ddots & & \vdots \\ Cov(R_n, R_1) & \dots & & Var R_n \end{bmatrix} \quad (8)$$

where “ M_{vc} ” is a symmetric matrix that represents the relationships between stock returns, “R” is a daily return of stock, “Var R_1 ” is a variance of stock, “Cov ($R_1 R_2$)” is the covariance between the stocks, n is the number of stocks (or refers to the n th stock)

3.2. Empirical Results

As explained by Bodie, Kane, and Marcus (2014), constructing an optimal portfolio under the Markowitz mean–variance approach involves two essential steps. First, investors must estimate each asset’s expected return and standard deviation, which represent the asset’s individual performance and risk. Second, they need to assess how assets interact with one another by calculating pairwise covariances or correlation coefficients. This step is crucial for understanding diversification effects and for minimizing the total risk of the portfolio by combining assets that do not move perfectly together (Bodie, Kane, & Marcus, 2014: 220–225).

In this study, all computations were performed using Microsoft Excel, specifically its built-in functions and the Solver add-in. The results presented in the subsequent tables were derived by applying formulas (1) through (8) outlined in the methodology section. The computation of the variance–covariance matrix began with the calculation of abnormal returns. As presented in Table 3, abnormal returns are defined as the deviation of a stock’s daily return from its mean return. Once the abnormal returns for all ten stocks were determined, these values were used to construct the variance–covariance matrix, as shown in Table 4.

Table 2. Risk ve return stocks (daily)

| Date | TUPRS | THYAO | ISCTR | MGROS | TOASO | CEMTS | ISGYO | TATGD | YATAS | ECILC |
|----------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| July 2000 | -0,00465 | -0,07389 | -0,06320 | -0,04872 | -0,02780 | -0,00465 | -0,00465 | -0,00465 | -0,00465 | -0,00465 |
| August 2000 | -0,00128 | -0,01995 | -0,00128 | 0,04499 | -0,04884 | -0,00128 | -0,03969 | -0,02506 | -0,04123 | -0,00128 |
| September 2000 | 0,00241 | 0,05014 | 0,00241 | 0,00241 | 0,05253 | 0,00241 | 0,08260 | 0,02686 | 0,04417 | 0,00241 |
| October 2000 | -0,00016 | -0,04561 | -0,00016 | -0,00016 | -0,02397 | -0,00016 | -0,07422 | -0,02397 | -0,08015 | -0,00016 |
| November 2000 | -0,00384 | -0,00384 | -0,00384 | -0,00384 | 0,04476 | -0,00384 | -0,00384 | -0,00384 | 0,03948 | -0,00384 |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| February 2025 | -0,07253 | 0,08764 | 0,03513 | 0,05400 | 0,03455 | 0,02906 | 0,04167 | 0,02323 | 0,05030 | 0,09475 |
| March 2025 | -0,02332 | -0,01315 | -0,00480 | -0,02041 | 0,00319 | -0,01562 | 0,01870 | -0,00317 | -0,01008 | -0,01400 |
| April 2025 | -0,00671 | -0,02449 | -0,00236 | -0,01579 | 0,00822 | -0,03059 | 0,00212 | 0,00864 | -0,00783 | -0,01800 |
| May 2025 | 0,01609 | 0,00832 | 0,03910 | 0,00857 | -0,01020 | 0,00290 | 0,00131 | -0,01023 | -0,00109 | -0,00575 |
| June 2025 | 0,02945 | 0,05887 | 0,09881 | 0,05382 | 0,06173 | 0,02195 | 0,05773 | 0,03328 | 0,04068 | 0,02347 |
| Average | 0,00083 | 0,00073 | 0,00070 | 0,00061 | 0,00085 | 0,00155 | 0,00055 | 0,00032 | 0,00082 | 0,00102 |
| Variance | 0,00089 | 0,00102 | 0,00133 | 0,00089 | 0,00108 | 0,00300 | 0,00108 | 0,00092 | 0,00148 | 0,00137 |
| Std.Dev | 0,02988 | 0,03198 | 0,03650 | 0,02989 | 0,03280 | 0,05475 | 0,03290 | 0,03041 | 0,03852 | 0,03706 |

Table 3. Abnormal return (daily)

| Date | TUPRS | THYAO | ISCTR | MGROS | TOASO | CEMTS | ISGYO | TATGD | YATAS | ECILC |
|----------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| July 2000 | -0,00548 | -0,07462 | -0,06389 | -0,04934 | -0,02865 | -0,00620 | -0,00520 | -0,00497 | -0,00547 | -0,00567 |
| August 2000 | -0,00211 | -0,02067 | -0,00198 | 0,04438 | -0,04969 | -0,00283 | -0,04024 | -0,02538 | -0,04205 | -0,00230 |
| September 2000 | 0,00158 | 0,04941 | 0,00171 | 0,00179 | 0,05167 | 0,00085 | 0,08205 | 0,02653 | 0,04335 | 0,00138 |
| October 2000 | -0,00099 | -0,04633 | -0,00086 | -0,00078 | -0,02482 | -0,00171 | -0,07477 | -0,02429 | -0,08097 | -0,00118 |
| November 2000 | -0,00466 | -0,00456 | -0,00453 | -0,00445 | 0,04391 | -0,00539 | -0,00439 | -0,00416 | 0,03865 | -0,00486 |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| February 2025 | -0,07336 | 0,08691 | 0,03443 | 0,05338 | 0,03370 | 0,02751 | 0,04112 | 0,02291 | 0,04948 | 0,09373 |
| March 2025 | -0,02415 | -0,01387 | -0,00549 | -0,02103 | 0,00234 | -0,01718 | 0,01815 | -0,00349 | -0,01090 | -0,01503 |
| April 2025 | -0,00754 | -0,02522 | -0,00306 | -0,01641 | 0,00737 | -0,03214 | 0,00156 | 0,00832 | -0,00865 | -0,01902 |
| May 2025 | 0,01526 | 0,00760 | 0,03841 | 0,00796 | -0,01105 | 0,00135 | 0,00076 | -0,01055 | -0,00191 | -0,00677 |
| June 2025 | 0,02862 | 0,05814 | 0,09811 | 0,05321 | 0,06088 | 0,02040 | 0,05718 | 0,03296 | 0,03986 | 0,02245 |

Table 4. Variance – covariance matrix

| | TUPRS | THYAO | ISCTR | MGROS | TOASO | CEMTS | ISGYO | TATGD | YATAS | ECILC |
|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| TUPRS | 0,00089 | 0,00054 | 0,00062 | 0,00047 | 0,00056 | 0,00046 | 0,00051 | 0,00046 | 0,00047 | 0,00048 |
| THYAO | 0,00054 | 0,00102 | 0,00069 | 0,00053 | 0,00060 | 0,00055 | 0,00058 | 0,00051 | 0,00055 | 0,00054 |
| ISCTR | 0,00062 | 0,00069 | 0,00133 | 0,00061 | 0,00068 | 0,00056 | 0,00069 | 0,00057 | 0,00065 | 0,00063 |
| MGROS | 0,00047 | 0,00053 | 0,00061 | 0,00089 | 0,00054 | 0,00055 | 0,00052 | 0,00049 | 0,00050 | 0,00051 |
| TOASO | 0,00056 | 0,00060 | 0,00068 | 0,00054 | 0,00108 | 0,00059 | 0,00058 | 0,00053 | 0,00057 | 0,00055 |
| CEMTS | 0,00046 | 0,00055 | 0,00056 | 0,00055 | 0,00059 | 0,00300 | 0,00060 | 0,00054 | 0,00057 | 0,00074 |
| ISGYO | 0,00051 | 0,00058 | 0,00069 | 0,00052 | 0,00058 | 0,00060 | 0,00108 | 0,00051 | 0,00055 | 0,00056 |
| TATGD | 0,00046 | 0,00051 | 0,00057 | 0,00049 | 0,00053 | 0,00054 | 0,00051 | 0,00092 | 0,00052 | 0,00052 |
| YATAS | 0,00047 | 0,00055 | 0,00065 | 0,00050 | 0,00057 | 0,00057 | 0,00055 | 0,00052 | 0,00148 | 0,00055 |
| ECILC | 0,00048 | 0,00054 | 0,00063 | 0,00051 | 0,00055 | 0,00074 | 0,00056 | 0,00052 | 0,00055 | 0,00137 |

As shown in Table 5, the initial portfolio consists of ten equally weighted stocks. The annual return of this portfolio is calculated as 19.97%, while its associated risk, measured by the standard deviation, is 39.79%.

Table 5. Portfolio risk-return with equal weight

| A | B | C | D | | E | F | G | H | I | J | K | L | M | N |
|----|-----------------------------------|--------|------------------------|-------------------------|----------------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1 | Weight | Stocks | Average return (daily) | Average return (yearly) | Variance-covariance matrix | | | | | | | | | |
| | | | | | TUPRS | THYAO | ISCTR | MGROS | TOASO | CEMTS | ISGYO | TATGD | YATAS | ECILC |
| 2 | 0,1 | TUPRS | 0,08% | 20,71% | 0,00089 | 0,00054 | 0,00062 | 0,00047 | 0,00056 | 0,00046 | 0,00051 | 0,00046 | 0,00047 | 0,00048 |
| 3 | 0,1 | THYAO | 0,07% | 18,16% | 0,00054 | 0,00102 | 0,00069 | 0,00053 | 0,00060 | 0,00055 | 0,00058 | 0,00051 | 0,00055 | 0,00054 |
| 4 | 0,1 | ISCTR | 0,07% | 17,41% | 0,00062 | 0,00069 | 0,00133 | 0,00061 | 0,00068 | 0,00056 | 0,00069 | 0,00057 | 0,00065 | 0,00063 |
| 5 | 0,1 | MGROS | 0,06% | 15,37% | 0,00047 | 0,00053 | 0,00061 | 0,00089 | 0,00054 | 0,00055 | 0,00052 | 0,00049 | 0,00050 | 0,00051 |
| 6 | 0,1 | TOASO | 0,09% | 21,32% | 0,00056 | 0,00060 | 0,00068 | 0,00054 | 0,00108 | 0,00059 | 0,00058 | 0,00053 | 0,00057 | 0,00055 |
| 7 | 0,1 | CEMTS | 0,16% | 38,80% | 0,00046 | 0,00055 | 0,00056 | 0,00055 | 0,00059 | 0,00300 | 0,00060 | 0,00054 | 0,00057 | 0,00074 |
| 8 | 0,1 | ISGYO | 0,06% | 13,78% | 0,00051 | 0,00058 | 0,00069 | 0,00052 | 0,00058 | 0,00060 | 0,00108 | 0,00051 | 0,00055 | 0,00056 |
| 9 | 0,1 | TATGD | 0,03% | 8,04% | 0,00046 | 0,00051 | 0,00057 | 0,00049 | 0,00053 | 0,00054 | 0,00051 | 0,00092 | 0,00052 | 0,00052 |
| 10 | 0,1 | YATAS | 0,08% | 20,54% | 0,00047 | 0,00055 | 0,00065 | 0,00050 | 0,00057 | 0,00057 | 0,00055 | 0,00052 | 0,00148 | 0,00055 |
| 11 | 0,1 | ECILC | 0,10% | 25,58% | 0,00048 | 0,00054 | 0,00063 | 0,00051 | 0,00055 | 0,00074 | 0,00056 | 0,00052 | 0,00055 | 0,00137 |
| 12 | | | | | Daily | Yearly | | | | | | | | |
| 13 | Return of portfolio = | | | | 0,0008 | | 0,19972 | | | | | | | |
| 14 | Variance of portfolio = | | | | 0,0006 | | 0,1583 | | | | | | | |
| 15 | Standard deviation of portfolio = | | | | 0,0252 | | 0,39788 | | | | | | | |

In the context of Markowitz portfolio analysis, the primary objectives are to maximize returns for a given level of risk or to minimize risk for a specified level of expected return. In this study, the tangency portfolio was first derived by maximizing the Sharpe ratio, as defined in Equation (9). Subsequently, the minimum-variance portfolios were calculated using the optimization function described in Equation (10), subject to the constraints specified in Equations (11), (12), and (13). The resulting portfolios are presented in Tables 8 and 9.

The objective function for the tangency portfolio (sharpe ratio) is presented below:

$$\text{Max } \alpha_t = \frac{R_p - R_f}{\sigma_p} \quad (9)$$

Where “ R_p ” is expected return of portfolio, “ R_f ” is risk free rate and σ_p is standard deviation of portfolio.

The objective function for the Markowitz minimum variance model is defined as follows:

$$\text{Min } \sigma_p^2 = \sum_{x=1}^n \sum_{y=1}^n w_x w_y \dots \text{Cov}(xy) \quad (10)$$

Where “ w_x ” and “ w_y ” are weights of stocks in the portfolio and “ $\text{Cov}(xy)$ ” is covariance value between stocks x and y .

This study applies three main constraints commonly used in the Markowitz mean–variance framework. These are mathematically expressed in Equations (11), (12), and (13).

$$\sum_{t=1}^n w_s \cdot E(R_s) \geq \bar{E} \quad (11)$$

Where “ \bar{E} ” the target expected return, “ $E(R_s)$ ” is an expected return and “ w_s ” is a weight of each stock.

$$\sum_{t=1}^n w_s = 1 \quad (12)$$

$$w_s \geq 0 \quad t = 1 \dots \dots \dots n \quad (13)$$

The first constraint (Equation 11) ensures that the portfolio’s expected return is equal to the target return. The second constraint (Equation 12) requires that the sum of all asset weights in the portfolio equals one. The third constraint (Equation 13) prohibits short selling, meaning that no asset can have a negative weight or be included in the portfolio without actually being held.

Table 6. Parameters of Excel Solver

| | |
|-------------------|---|
| Target cell | H 17 (tangency portfolio) |
| | Equal to maximum (Tangency portfolio) |
| By changing cells | \$C\$2 : \$C\$11 |
| Constraints | \$C\$2 : \$C\$11 ≥ 0 (short sale restrictions) |
| | \$C\$12 = 1 |
| | \$H\$13 = 0,27 (yearly target return) |

In this study, all three constraints were implemented using the Excel Solver tool. The tangency portfolio was obtained through Solver’s optimization functions. In solving the minimum-variance problem, the target cell representing the tangency portfolio (H17) was set to be maximized. Additionally, the cell representing the portfolio variance (G14 or H14) was configured to be minimized using the Excel Solver tool. Both approaches yielded identical results in terms of portfolio composition, expected returns, and risk levels. Detailed information regarding the constructed portfolios is presented in Tables 8 and 9.

In order to accurately calculate the Sharpe ratio for identifying the optimal portfolio—given that the risk-free rate is annual—it is first necessary to convert the daily returns and standard deviation into annual terms. Since there are approximately 250 trading days in a year, the daily return of the portfolio is multiplied by 250, and the daily standard deviation is multiplied by the square root of 250. In this way, the annualized expected return and annualized standard deviation of the portfolio are obtained.

Table 7. Portfolio risk-return with different weight

| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O |
|----|--------|---------|------------------------|-------------------------|----------------------------|-----------------------------------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1 | Stocks | Weight | Average return (daily) | Average return (yearly) | Variance-covariance matrix | | | | | | | | | |
| | | | | | TUPRS | THYAO | ISCTR | MGROS | TOASO | CEMTS | ISGYO | TATGD | YATAS | ECILC |
| 2 | TUPRS | 29,34% | 0,00083 | 0,21 | 0,00089 | 0,00054 | 0,00062 | 0,00047 | 0,00056 | 0,00046 | 0,00051 | 0,00046 | 0,00047 | 0,00048 |
| 3 | THYAO | 0,00% | 0,00073 | 0,18 | 0,00054 | 0,00102 | 0,00069 | 0,00053 | 0,00060 | 0,00055 | 0,00058 | 0,00051 | 0,00055 | 0,00054 |
| 4 | ISCTR | 0,00% | 0,00070 | 0,17 | 0,00062 | 0,00069 | 0,00133 | 0,00061 | 0,00068 | 0,00056 | 0,00069 | 0,00057 | 0,00065 | 0,00063 |
| 5 | MGROS | 0,00% | 0,00061 | 0,15 | 0,00047 | 0,00053 | 0,00061 | 0,00089 | 0,00054 | 0,00055 | 0,00052 | 0,00049 | 0,00050 | 0,00051 |
| 6 | TOASO | 11,86% | 0,00085 | 0,21 | 0,00056 | 0,00060 | 0,00068 | 0,00054 | 0,00108 | 0,00059 | 0,00058 | 0,00053 | 0,00057 | 0,00055 |
| 7 | CEMTS | 28,02% | 0,00155 | 0,39 | 0,00046 | 0,00055 | 0,00056 | 0,00055 | 0,00059 | 0,00300 | 0,00060 | 0,00054 | 0,00057 | 0,00074 |
| 8 | ISGYO | 0,00% | 0,00055 | 0,14 | 0,00051 | 0,00058 | 0,00069 | 0,00052 | 0,00058 | 0,00060 | 0,00108 | 0,00051 | 0,00055 | 0,00056 |
| 9 | TATGD | 0,00% | 0,00032 | 0,08 | 0,00046 | 0,00051 | 0,00057 | 0,00049 | 0,00053 | 0,00054 | 0,00051 | 0,00092 | 0,00052 | 0,00052 |
| 10 | YATAS | 6,97% | 0,00082 | 0,21 | 0,00047 | 0,00055 | 0,00065 | 0,00050 | 0,00057 | 0,00057 | 0,00055 | 0,00052 | 0,00148 | 0,00055 |
| 11 | ECILC | 23,82% | 0,00102 | 0,26 | 0,00048 | 0,00054 | 0,00063 | 0,00051 | 0,00055 | 0,00074 | 0,00056 | 0,00052 | 0,00055 | 0,00137 |
| 12 | | 100,00% | | | | Daily | Yearly | | | | | | | |
| 13 | | | | | | Return of portfolio = | | 0,00108 | 0,27 | | | | | |
| 14 | | | | | | Variance of portfolio = | | 0,0008 | 0,20815 | | | | | |
| 15 | | | | | | Standard deviation of portfolio = | | 0,0289 | 0,45624 | | | | | |
| 16 | | | | | | Risk free rate = | | 0,0426 | | | | | | |
| 17 | | | | | | Tangency portfolio = | | 0,49842 | | | | | | |

The tangency portfolio was derived by maximizing the objective function defined in Equation (9). For a given target return level, it is possible to construct multiple efficient portfolios. In this study, a total of seventeen portfolios were generated under various target return scenarios, with their details presented in Tables 8 and 9. However, no feasible portfolio could be formed for a return level exceeding 38%. The optimized portfolio consists of five stocks with the following weights: 29.34% TUPRS, 11.86% TOASO, 28.02% CEMTS, 6.97% YATAS, and 23.82% ECILC.

Table 8. Stock Weights in Optimized Portfolios

| Portfolios | TUPRS | THYAO | ISCTR | MGROS | TOASO | CEMTS | ISGYO | TATGD | YATAS | ECILC |
|--------------------|---------------|--------------|--------------|--------------|---------------|---------------|--------------|--------------|--------------|---------------|
| Portfolio 1 | 20,01% | 7,75% | 0,00% | 22,20% | 1,49% | 0,00% | 12,31% | 28,41% | 5,24% | 2,58% |
| Portfolio 2 | 24,69% | 8,26% | 0,00% | 19,41% | 6,27% | 3,03% | 7,25% | 15,33% | 7,42% | 8,35% |
| Portfolio 3 | 27,43% | 8,47% | 0,00% | 17,56% | 8,87% | 6,05% | 4,12% | 7,57% | 8,58% | 11,35% |
| Portfolio 4 | 30,15% | 8,63% | 0,00% | 15,65% | 11,44% | 9,16% | 0,92% | 0,00% | 9,74% | 14,32% |
| Portfolio 5 | 31,15% | 7,10% | 0,00% | 10,89% | 12,52% | 12,14% | 0,00% | 0,00% | 9,85% | 16,34% |
| Portfolio 6 | 32,06% | 5,31% | 0,00% | 5,64% | 13,48% | 15,29% | 0,00% | 0,00% | 9,88% | 18,35% |
| Portfolio 7 | 32,97% | 3,52% | 0,00% | 0,38% | 14,42% | 18,44% | 0,00% | 0,00% | 9,90% | 20,37% |
| Portfolio 8 | 32,12% | 0,00% | 0,00% | 0,00% | 13,88% | 22,86% | 0,00% | 0,00% | 8,89% | 22,24% |
| Portfolio 9 | 29,34% | 0,00% | 0,00% | 0,00% | 11,86% | 28,02% | 0,00% | 0,00% | 6,97% | 23,82% |
| Portfolio 10 | 26,56% | 0,00% | 0,00% | 0,00% | 9,83% | 33,17% | 0,00% | 0,00% | 5,04% | 25,40% |
| Portfolio 11 | 23,79% | 0,00% | 0,00% | 0,00% | 7,80% | 38,32% | 0,00% | 0,00% | 3,11% | 26,98% |
| Portfolio 12 | 21,03% | 0,00% | 0,00% | 0,00% | 5,77% | 43,47% | 0,00% | 0,00% | 1,17% | 28,55% |
| Portfolio 13 | 14,27% | 0,00% | 0,00% | 0,00% | 0,55% | 53,98% | 0,00% | 0,00% | 0,00% | 31,20% |
| Portfolio 14 | 2,90% | 0,00% | 0,00% | 0,00% | 0,00% | 64,74% | 0,00% | 0,00% | 0,00% | 32,35% |
| Portfolio 15 | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 71,24% | 0,00% | 0,00% | 0,00% | 28,76% |
| Portfolio 16 | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 86,36% | 0,00% | 0,00% | 0,00% | 13,64% |
| Portfolio 17 | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 93,92% | 0,00% | 0,00% | 0,00% | 6,08% |

All portfolios located on the minimum-variance frontier, from the global minimum-variance portfolio upwards, offer the most favorable risk-return trade-offs and are therefore considered potential optimal portfolios. The segment of this frontier above the global minimum-variance portfolio is referred to as the efficient frontier of risky assets. For any portfolio that lies below this point on the frontier, there exists another portfolio with the same level of risk (standard deviation) but a higher expected return positioned directly above it. Consequently, the lower portion of the minimum-variance frontier is deemed inefficient (Bodie, Kane, & Marcus, 2014: 220). Investors may select any portfolio along the efficient frontier depending on their individual risk preferences.

However, the optimal portfolio is located at the point where the efficient frontier is tangent to the Capital Market Line, known as the tangency portfolio. This portfolio offers the highest expected return per unit of risk. The point at which the efficient frontier is tangent to the Capital Market Line is shown in Figure 1.

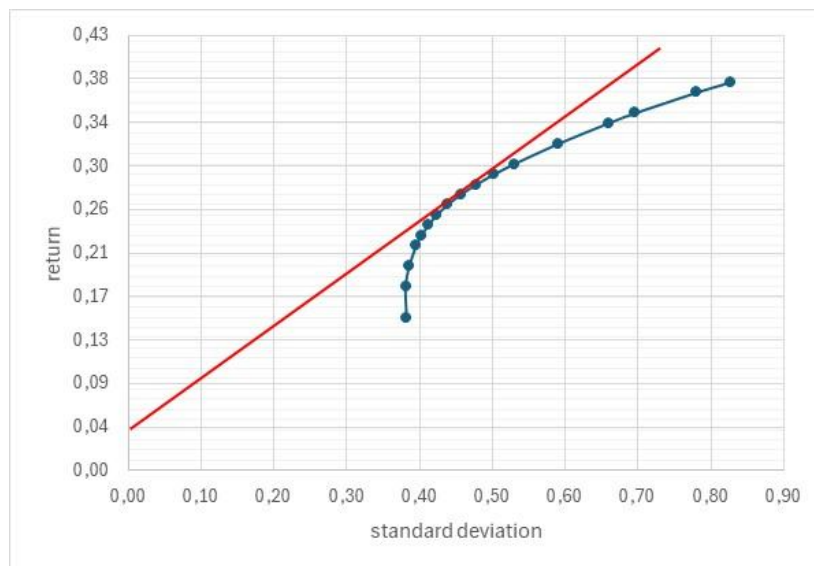


Figure 1. Tangency portfolio and efficient frontier

Table 9 presents all efficient portfolios, which were obtained by solving the minimization problem defined in Equation (10). The Sharpe ratio, given in Equation (9), was employed to identify the optimal portfolio. In the calculations, the 10-year U.S. Treasury bond yield² was used as the risk-free rate. As shown in Table 9, Portfolio 9 was determined to be the optimal portfolio, as it yielded the highest Sharpe ratio. This value was calculated by dividing the portfolio's risk premium by its standard deviation.

Table 9. Determination of Optimal Portfolio

| Portfolios | Return | Standard Deviation | Risk Free Rate | Sharpe Ratio |
|--------------------|-------------|--------------------|----------------|---------------|
| Portfolio 1 | 0,15 | 0,3829 | 4,26% | 0,2805 |
| Portfolio 2 | 0,18 | 0,3814 | 4,26% | 0,3602 |
| Portfolio 3 | 0,20 | 0,3864 | 4,26% | 0,4074 |
| Portfolio 4 | 0,22 | 0,3958 | 4,26% | 0,4482 |
| Portfolio 5 | 0,23 | 0,4027 | 4,26% | 0,4653 |
| Portfolio 6 | 0,24 | 0,4121 | 4,26% | 0,4790 |
| Portfolio 7 | 0,25 | 0,4238 | 4,26% | 0,4894 |
| Portfolio 8 | 0,26 | 0,4382 | 4,26% | 0,4961 |
| Portfolio 9 | 0,27 | 0,4562 | 4,26% | 0,4984 |
| Portfolio 10 | 0,28 | 0,4778 | 4,26% | 0,4969 |
| Portfolio 11 | 0,29 | 0,5024 | 4,26% | 0,4924 |
| Portfolio 12 | 0,30 | 0,5297 | 4,26% | 0,4860 |
| Portfolio 13 | 0,32 | 0,5908 | 4,26% | 0,4696 |
| Portfolio 14 | 0,34 | 0,6594 | 4,26% | 0,4510 |
| Portfolio 15 | 0,35 | 0,6964 | 4,26% | 0,4414 |
| Portfolio 16 | 0,37 | 0,7805 | 4,26% | 0,4195 |
| Portfolio 17 | 0,38 | 0,8267 | 4,26% | 0,4081 |

The comparison of the returns and risks of the original portfolio and Portfolio 9, which is identified as the optimal portfolio, is presented in Table 10. As shown in Table 10, compared to the original portfolio, the risk of the optimal portfolio increases by 15%, while its expected annual return rises by 35%.

² Federal Reserve Bank of St. Louis (n.d.) 10-year Treasury constant maturity rate [Data set]. FRED, Federal Reserve Bank of St. Louis. Available at: <https://fred.stlouisfed.org/series/DGS10> (Accessed: 4 July 2025).

Table 10. Comparison of Portfolio Returns and Risks

| Portfolios | Annual Return (%) | Standard Deviation (%) |
|---------------------|-------------------|------------------------|
| Original Portfolio | 19,97% | 39,79% |
| Optimal Portfolio 9 | 27,00% | 45,62% |
| Increase (%) | 35% | 15% |

4. Conclusion

This study aims to evaluate portfolio performance by applying the Markowitz mean–variance model and the Sharpe ratio using long-term data from Borsa Istanbul. While there are similar studies in this field, the distinct contribution of this research lies in its use of an extended dataset covering twenty-five years and the conversion of daily stock returns into U.S. dollars. These methodological choices are expected to enhance the reliability and robustness of the results. The empirical findings highlight several key conclusions.

First, merely diversifying across a variety of sectors or assets—as done in the equally weighted portfolio—does not necessarily lead to optimal outcomes. Without accounting for the correlations (covariances) among assets and their individual risk–return profiles, such naïve strategies may yield suboptimal portfolios. Second, the use of quantitative optimization models, such as the Sharpe ratio and variance minimization, enables investors to construct portfolios that are both theoretically and empirically more efficient. The tangency portfolio derived from Sharpe ratio maximization significantly outperformed the equally weighted portfolio, offering greater returns at reduced levels of risk. Third, this study demonstrates that modern portfolio theory, although based on simplifying assumptions, provides valuable insights and practical tools for investors seeking to improve risk-adjusted performance. Even with limited asset selection and basic software tools, both individual and institutional investors can apply these methods to enhance portfolio construction.

In summary, the integration of statistical methods and financial theory through the Markowitz framework and the Sharpe ratio offers a robust foundation for investment decision-making. This approach encourages a more systematic evaluation of return–risk trade-offs and supports more rational, evidence-based portfolio strategies.

References

- Bilir, H. (2016) ‘Determination of optimal portfolio by using tangency portfolio and Sharpe ratio’, *Research Journal of Finance and Accounting*, 7(5), 53–59. Available at: <https://www.iiste.org/Journals/index.php/RJFA/article/view/29327>
- Bodie, Z., Kane, A. and Marcus, A.J. (2014) *Investments*. 10th edn. New York: McGraw-Hill Education, 220–225.
- Bodnar, T. and Zabolotsky, T. (2022) ‘How risky is the optimal portfolio which maximizes the Sharpe ratio?’, *ASTA Advances in Statistical Analysis*, 101(1), 1–28. <https://doi.org/10.1007/s10182-021-00425-7>
- Dong, Z. (2022) ‘Sharpe-ratio related portfolio selection’, *BCP Business & Management*, 24, 316–321. <https://doi.org/10.54691/bcpbm.v24i.1512>
- Elton, E.J. and Gruber, M.J. (1997) ‘Modern portfolio theory, 1950 to date’, *Journal of Banking & Finance*, 21(11–12), 1743–1759. [https://doi.org/10.1016/S0378-4266\(97\)00048-4](https://doi.org/10.1016/S0378-4266(97)00048-4)
- Estrada, J. (2010) ‘The Sharpe ratio revisited’, *The Journal of Portfolio Management*, 36(4), 71–84. <https://doi.org/10.3905/jpm.2010.36.4.071>
- Federal Reserve Bank of St. Louis (n.d.) 10-year Treasury constant maturity rate [Data set]. FRED, Federal Reserve Bank of St. Louis. Available at: <https://fred.stlouisfed.org/series/DGS10> (Accessed: 4 July 2025).
- İş Yatırım (n.d.) Tarihsel fiyat bilgileri [Historical price information]. Available at: <https://www.isyativim.com.tr/tr-tr/analiz/hisse/Sayfalar/Tarihsel-Fiyat-Bilgileri.aspx> (Accessed: 4 July 2025).
- Lintner, J. (1965) ‘The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets’, *The Review of Economics and Statistics*, 47(1), 13–37. <https://doi.org/10.2307/1924119>
- Markowitz, H. (1952) ‘Portfolio selection’, *The Journal of Finance*, 7(1), 77–91. <https://doi.org/10.2307/2975974>
- Mossin, J. (1966) ‘Equilibrium in a capital asset market’, *Econometrica*, 34(4), 768–783. <https://doi.org/10.2307/1910098>
- Sharpe, W.F. (1964) ‘Capital asset prices: A theory of market equilibrium under conditions of risk’, *The Journal of Finance*, 19(3), 425–442. <https://doi.org/10.2307/2977928>
- Vanini, P. and Vignola, L. (2001) *Markowitz model – Optimal portfolio selection (Chapter 1)*. Unpublished manuscript. Available at: <https://www.bm.com.tn/ckeditor/files/markowitz.pdf> (Accessed: 22 April 2001).
- Vuković, D., Vyklyuk, Y., Matsiuk, N. and Maiti, M. (2020) ‘Neural network forecasting in prediction Sharpe ratio: Evidence from EU debt market’, *Physica A: Statistical Mechanics and Its Applications*, 542, 123331. <https://doi.org/10.1016/j.physa.2019.123331>
- Zhang, P., Chen, Z., Jin, S. and Li, S. (2016) ‘Markowitz's mean-variance optimization with investment and constrained reinsurance’, *Journal of Industrial and Management Optimization*, 12(1), 22–22. <https://doi.org/10.3934/jimo.2016.12.22>