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The Role on Fixed Income in Pension Scheme Investment in Ghana: A Possible Adoption for the United States Economy

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Abstract
The paper investigates the role of fixed income in pension schemes investment in Ghana by specifically looking at the asset allocation and the initial investment required to make the scheme solvent in the future at a specified high probability after matching all liabilities in Ghana.Looking at the assets-only analysis of pension schemes without matching their liabilities, equity appears to be an attractive asset classes to invest in. However, considering asset-liability analysis, there is a general trend of asset allocation shifting from equities to bonds (specifically One-year bonds).
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1. Background

Pension scheme investment in Ghana meant government securities, corporate bonds/debts including (REITs, Mortgage and Assets Backed securities and debentures), money market, ordinary shares and open and closed funds (National Pension Act, 2008). However, the investment for pension schemes in this paper has been limited to fixed income investment (that is investment whose returns are known at the time of making the investment like bond and treasury bill) as well as non-fixed income investment like equity from the Ghana Stock Exchange.

Over the years until now, pension scheme providers including SSNIT which is trying to achieve the investment policies which include: a) to implement an optimal asset allocation policy; b) to maintain a long-term optimum fund ratio; c) to protect the corpus of the assets in the scheme and the value of those assets; d) to achieve a real return on the investment of the least +2.25% per annum and, e) to attract, train and retain competent investment talents; and are still faced with a challenge as to how to maximize the returns on the investments to meet the benefits and cost of running the scheme. (SSNIT Annual Report, 2012)

1.1 Investment portfolio of SSNIT

Now looking critically at one of Ghana's largest pension scheme providers, SSNIT, a large amount of surplus funds accumulated need to be invested like all funded pension schemes elsewhere. With the SSNIT Scheme under law 247, the self-sustaining investment policies of SSNIT would achieve professional attributes when: a) all restrictions on investments are removed and, b) investment responsibilities are entrusted in the SSNIT Board and devoid of Government inference (SSNIT Annual Report, 2012). Theoretically, investment returns are to be above a minimum acceptable level in aggregate over the long term. Aware of the long term nature of liabilities, investment in long-term projects are to be undertaken as long as short-term requirement are met. Generally, SSNIT's investment policy is guided by seven principles which are: a) safety of investment; b) yield or rate of return; c) liquidity; d) maintenance of the fund's monetary values; e) diversification; f) spread of investment by duration and, g) harmonization with national objectives.

With all these guided investment principles, the investment portfolio of SSNIT funds comprised investment in fixed and non-fixed income investment made up as follows: i) Short-term government instruments; ii) government bonds; iii) corporate loans; iv) student loans; v) equity and, vi) property.

Generally, on the average, SSNIT seems to adopt a 60% bond allocation (fixed income investment) and 30% equity allocation (non-fixed income investment) as an investment strategy. In fact, SSNIT is the single largest holder of shares in the Ghana Stock Exchange. SSNIT is the largest institutional investor in Ghana and is as such badly exposed in the capital market (SSNIT Annual Report, 2012)

1.2 Structure of investment income

SSNIT has the largest pool of funds that it can manage efficiently to provide an effective social protection for a greater number of Ghanaians. The investment portfolio of SSNIT is about GHC 4.07 billion but still has on-going problems with return, liquidity and asset quality particularly with equity (listed and unlisted) and fixed incomes (SSNIT Annual Report, 2012). As we noted earlier, investments in equity averaged 34.06% of SSNIT's total investment between 2005 and 2010. However, the dividend incomes to SSNIT from equity investment averaged about 11.18% between 2005 and 2012.

Similarly, investment in fixed income averaged 55.45% of SSNIT's total investment between 2005 and 2010. The investment income from term deposit and treasury bills as well as government and registered bonds which constitute fixed income investment averaged about 34.09% and 3.82% respectively between 2005 and 2012. Table 1.1 shows the investment incomes from all the assets held by SSNIT between 2005 and 2012.
It is recommended that SSNIT should restructure its assets portfolio so as to maximize return and protect the quality of its investments after matching all liabilities to ensure sustainability of the scheme as a social insurance fund.

1.3 Purpose of the study

The purpose of this paper is to investigate the role of fixed income in pension scheme investment in Ghana by specifically looking at the asset allocation and the initial investment required to make the scheme solvent in the future at a specified high probability after matching all liabilities.

2. Methodology

The data for the study are secondary data which are gathered from published and unpublished records of treasury bills (91-days), One-year, and Two-year bonds from the Bank of Ghana (BOG). All share-index from the Ghana Stock Exchange was also gathered. All data were gathered from 2007 to 2013.
I choose to use the mean-variance model to simulate asset returns for assets of pension funds. The model requires not only knowledge of the expected returns and the standard deviation of the returns on each asset, but also the correlation of returns for each and every pair of assets which helps to uncover large risk reduction opportunities through diversification.

The mean-variance model is developed consisting of four asset classes. Each individual asset class is modeled as a mean-variance time series in which the parameters are estimated using historical data, taking the future economic outlook into account. The asset classes are basically grouped into two which are equity and bond asset classes (One-year bond, Two-year bond and treasury bill).
Generally, the mean- variance model is specified as:

$$\boldsymbol{R_{it}} = \mu_i + \boldsymbol{\sigma_i Z_{it}} \qquad (1)$$

Where μ_i = mean of the return of asset *i*
$\boldsymbol{\sigma_i}$ = standard deviation of the return of asset *i*
$\boldsymbol{Z_{it}}$ = randomly generated random numbers for asset *i* over a time *t* period. $Z \sim N(0,1)$
$\boldsymbol{R_{it}}$ = returns produced on asset *i* over time period *t*.

2.1 Data set, parameters and valuation bases for asset

2.1.1 Equities.

We generate the equity returns from the GSE All share index. The returns on equity (R_e) are computed as follows:

$$\boldsymbol{R_e} = \frac{Y_t - Y_{t-1}}{Y_{t-1}} \qquad (2)$$

Where Y_t denote the current GSE is share index and Y_{t-1} denote the previous GSE share index. We model the returns as a simple random walk using the expected return computed from historical data for the whole period (2007-2013) as drift in the model projecting the equity returns forward over 40-year period and simulating 10,000 scenarios of the equity returns.
If R_{et} is the expected return on equity, then we model the returns as a walk with the drift as follows:

$$\boldsymbol{R_{et}} = \mu_e + \boldsymbol{\sigma_e Z_{et}} \qquad (3)$$

Where $Z_{et} \sim N(0,1)$, μ_e is the drift parameter (expected mean return for the whole period, 2007-2013) and σ_e denotes the volatility of the return for the whole period, 2007-2013.
It is worth noting that the future equity returns changes randomly and are independent of each other

2.1.2 Bonds

Bond returns R_t are calculated from the yields. The assumption made is that, an annual par fixed coupon bond is bought within a given time period and its held for one year, and then rolled into a new bond with a given parameter *m*.

$$\boldsymbol{R_t} = \boldsymbol{Y_t} + \frac{\boldsymbol{Y_t}(\boldsymbol{1} - (\boldsymbol{1}/(\boldsymbol{1} + \boldsymbol{Y_{t+1}})^{\boldsymbol{m-1}})}{\boldsymbol{Y_{t+1}}} + \boldsymbol{1}/(\boldsymbol{1} + \boldsymbol{Y_{t+1}})^{\boldsymbol{m-1}} - \boldsymbol{1} \qquad (4)$$

Where m denotes the duration of the bond and yield, Y_t at time t.
We model the bond returns as a simple random walk using the expected return computed from historical data for the whole period (2007-2013) as drift in the model projecting the bond returns forward over 40-years period and simulating 10,000 scenarios of the bond returns.
If R_{bt} is the expected returns on bonds, then we model the returns as a walk with the drift as follows

$$\boldsymbol{R_{bt}} = \mu_{\boldsymbol{b}} + \boldsymbol{\sigma_b Z_{bt}} \qquad \textbf{(5)}$$

Here $Z_{bt} \sim N(0,1)$ where μ_b is the drift parameter (mean of bond return for the whole period, 2007-2013) and σ_b denotes the volatility of the return for the whole period, 2007-2013.

It is worth noting that the future bond returns changes randomly and are independent of each other

When the random numbers are projected, it produces uncorrelated random numbers and this make the projected simulated returns uncorrelated. In order to make the projected simulated returns correlated, cholesky decomposition on these uncorrelated random numbers is performed.

The cholesky decomposition is carried out by multiplying the uncorrelated random numbers (error terms) by the lower or upper triangular cholesky decomposition of the correlation matrix all assets.

Now let $L = l_{ij}$ be the lower triangular cholesky decomposition of the correlation matrix A (that is $l_{ij} = 0$ for all $j > i$ and $A = LL^T$), therefore projected simulated returns for each asset (equity and bonds) will be given by:

$$\boldsymbol{R_{it}} = \mu_{\boldsymbol{i}} + \boldsymbol{\sigma_i}(\boldsymbol{Z_{it}} * \boldsymbol{L}) \qquad \textbf{(6)}$$

Where μ_i = mean of the return of asset *i*

σ_i= standard deviation of the return of asset *i*

Z_{it}= randomly generated random numbers for each asset *i* over a time period *t*. $Z \sim N(0,1)$

R_{it}= returns produced on asset *i* over time period *t*.

L= Lower triangular cholesky decomposition of the correlation matrix A

2.2. Liability determination and projection

This section explains the approach adopted to determine liabilities. The liabilities are projected forward over time across ages.

Some important assumptions made in the analysis are that, the chosen age for the members who could start contributing to the scheme to await their pensions paid to them later during their retirement age was 20 years and the age for retirement was 60. More so, all pensioners are assumed to die at age 100.

Again, one of the principal assumptions made is that, the pensioners portfolio is a closed portfolio where there are no additional contributors added to the scheme as the years go by. In view of this, the number of pensioners will run-off by 40 years time and therefore there will be no cash inflow from any other sources than the investments. However, in open pensioners' portfolio, additional contributors are added to the scheme as the years progress.

2.2.1 Data set, parameters and valuation bases for liabilities

2.2.1.1 Contributors

2.2.1.1.1 Projected Survivors for contributors.

The number of contributors for a particular age who survived in next year is given as follow:

$$\boldsymbol{l_{x+1}} = \boldsymbol{l_x} * \boldsymbol{p_x} \qquad \textbf{(7)}$$

Where l_{x+1} denotes the number of persons (contributors) at age X who will live to age $X + 1$ in the following year.

l_x denotes the number of persons (contributors) at age X.

p_x denotes the probability that a person (contributor) age X will live in one year.

2.2.1.1.2 Projected deaths for contributors

The number of death recorded as the contributors at age X move to age $X + 1$ in the following year is given as:

$$\boldsymbol{d_x} = \boldsymbol{l_x} - \boldsymbol{l_{x+1}} \qquad \textbf{(8)}$$

Where d_x denotes the number of persons (contributors) who die between age X and $X + 1$ in the following year

l_{x+1} denotes the number of persons (contributors) at age X who will live to age $X + 1$ in the following year.

l_x denotes the number of persons (contributors) at age X.

2.2.1.1.3. Projected contributors

The projected contributors that moved from age X to age $X + 1$ in the following year is given as:

$$\boldsymbol{l_{x+1}} = \boldsymbol{l_x} - \boldsymbol{d_x} \qquad \textbf{(9)}$$

Where l_{x+1} denotes the number of projected contributors at age X who move to age $X + 1$ in the following year.

l_x denotes the number of persons (contributors) at age X.

d_x denotes the number of persons (contributors) who died between age X and $X + 1$ in the following year

2.2.1.1.4 Projected total salary
The projected total salary on which contributions were paid is given as:

$$\boldsymbol{AS_{(x+1,t+1)} = AS_{(x,t)} * (1+g)} \qquad \mathbf{(10)}$$

Where $AS_{(x+1,t+1)}$ denotes the projected total salary received by contributors in the following year $t+1$ and age $X+1$

$AS_{(x,t)}$ denotes the average salary received by contributors at current time t and age X.

g denotes a fixed indexation rate of 2%

2.2.1.1.5 Projected total contributions
The projected total contributions for a particular age are given as:

$$\boldsymbol{TC_{(x+1,t+1)} = AS_{(x+1,t+1)} * m} \qquad \mathbf{(11)}$$

Where $TC_{(x+1,t+1)}$ denotes projected total contribution paid by contributors in the following year at time $t+1$ and age $X+1$

$AS_{(x+1,t+1)}$ denotes the projected average salary received by contributors in the following year $t+1$ at age $X+1$

m denotes a fixed contribution rate of 11%.

2.2.1.1.6 Projected total expenses
The total expenses made by the scheme are given as:

$$\boldsymbol{TE_{(x+1,t+1)} = TC_{(x+1,t+1)} * n} \qquad \mathbf{(12)}$$

Where $TE_{(x+1,t+1)}$ denotes the projected total expenses made by scheme for the following year $t+1$ and age $X+1$.

$TC_{(x+1,t+1)}$ denote projected total contribution paid by contributors in the following year at time $t+1$and age $X+1$.

n denotes a fixed expense rate of 24%.

2.2.1.2 Pensioners

2.2.1.2.1 Projected Survivors for pensioners.
The number of pensioners for a particular age who survived in the following year is given as follow:

$$\boldsymbol{l_{x+1} = l_x * p_x} \qquad \mathbf{(13)}$$

Where l_{x+1} denote the number of pensioners at age X who will live to age $X+1$ in the following year.

l_x denote the number of pensioners at age X.

p_x denotes the probability that a person (pensioner) age X will live in one year.

2.2.1.2.2 Projected deaths for pensioners
The number of death recorded as pensioners at age X move to age $X+1$ in the following year is given as:

$$\boldsymbol{d_x = l_x - l_{x+1}} \qquad \mathbf{(14)}$$

Where d_x denotes the number of pensioners who died between age X and $X+1$ in the following year

l_{x+1} denote the number of pensioners at age X who will live to age $X+1$ in the following year.

l_x denote the number of pensioners at age X.

2.2.1.2.3 Projected pensioners
The projected pensioners that moved from age X to age $X+1$ in the following year is given as:

$$\boldsymbol{l_{x+1} = l_x - d_x} \qquad \mathbf{(15)}$$

Where l_{x+1} denote the number of projected pensioners at age X who move to age $X+1$ in the following year.

l_x denote the number of pensioners at age X

d_x denotes the number of pensioners who died between age X and $X+1$ in the following year

2.2.1.2.4 Projected average pensions.

The projected average pensions paid to pensioners in the following year $t+1$ and age $X+1$ is given as:

$$\boldsymbol{AP_{(x+1,t+1)} = AP_{(x,t)} * (1+r)} \qquad \mathbf{(16)}$$

Where $AP_{(x+1,t+1)}$ denote the projected average pension paid to pensioners in the following year $t+1$ and age $X+1$

$AP_{(x,t)}$ denote the average pension paid to pensioners at current time t and age X.

r denote a fixed indexation rate of 2%

2.2.1.2.5 Projected total pensions.

The total pensions paid to pensioners for a particular age are derived as follows:

$$TP_{(x+1,t+1)} = AP_{(x+1,t+1)} * l_{x+1} \qquad (17)$$

Where $TP_{(x+1,t+1)}$ denote the total pensions paid to a pensioner in the following years as time $t+1$ and age $X+1$

l_{x+1} denote the number of projected pensioners at age X who move to age $X+1$ in the following year.

$AP_{(x+1,t+1)}$ denote the projected average pension paid to pensioners in the following year $t+1$ and age $X+1$

2.3 Approach to determine investment strategy (Asset-liability analysis)

This section explains the approach adopted to determine investment strategy. Based on the economic scenarios, the assets and liabilities of the pension schemes are projected forward. This step is repeated many times, each time based on a fresh simulation of a projected economic scenario. In particular, assuming a start date of 31 December 2007, 10,000 40-year scenarios projecting forward the assets and liabilities of the pension schemes are simulated from that date until run-off. Asset and liability optimization modeling is then carried out to determine the investment strategy.

2.3.1 Asset and Liability Management

Asset and liability management is a risk management technique which takes into account the assets, liabilities and interactions of policies which may be adopted by the board of trustees of a pension fund. In the early 2000s, taking pension funds into consideration, the traditional asset-only investment strategy which focused on outperforming a market index failed. This followed the perfect storm of the equity bubbles and low interest rate which led to large deficits in pension funds. The required investment strategy that ensures that the solvency of a fund is enough to pay off all liabilities is determined by the pension fund trustee. The solvency of the fund in the long run may be measured over a specified solvency probability (that is the probability that all liabilities are covered in the long run).

The sponsors of the fund adjust the contributions to compensate for the shortfalls when the fund is in deficit. Otherwise surpluses may be redistributed to sponsors or used to improve benefit levels in some circumstances. The changes in assets and liabilities of the scheme cause changes in solvency level over time. Practically, the conflicting interest of stakeholders influences investment strategy decisions. The stochastic influences from the market and economic and actuarial risks intensify the influence on investment strategy decision. In the next section, the asset and liability model incorporating these stochastic influences is described. The last two sections looked at the optimization model and determination of the investment strategy.

2.3.2 Asset and Liability Modelling

The asset and liability model considered a closed pensioners portfolio using the SSNIT 2005 male pensioner mortality. Even though, pension scheme can operate under the closed and open pensioners' portfolio, only the situation for the closed pensioners portfolio was considered in this study. Based on the pension plan design and actuarial assumption made, the liabilities are calculated. 10,000 40-year scenarios of the pension liabilities are simulated and projected forward.

The changes in the characteristics of the pension plan participants and demographics as well as risk factors such as interest risk, longevity risk and ageing affect the pension liabilities. The scope of this research does not cover these risks. The exposure of longevity risk on pensions will cause pension payment to be made for a longer period as far as the recipient (pensioner) lives longer. This may directly affect the funding status of the fund. Blake, Cairns and Dowd (2006), looked at the longevity risk into details and discussed the various ways to manage this risk exposure. Once the assets and liabilities have been calculated, the solvency of the fund is obtained at the run-off horizon. A scheme is solvent if it is able to pay all liabilities in the long run. The solvency at any point in time is measured by the difference in the market value of assets and liabilities. Once the asset and liabilities at time zero are known, the fund values are projected based on a recurring relation as follows:

$$F_{t+1} = F_t\left(1+\widetilde{R}\right) - M_t \qquad (8)$$

where $\widetilde{R}$ denotes the stochastic investment (expected) return obtained from the mean-variance model specified, assuming liabilities M_t are paid at the end of the year, where M_t denotes the sum of all liabilities in the portfolio. The procedure is projected into the future until the liabilities are paid off in 40 years time when all pensioners are assumed to be dead at age 100. The step above is repeated for 10,000 simulations of the assets and liabilities.

2.3.3 Portfolio Optimisation and Investment Strategy

Over a specified time period, an actuarial approach is used to determine the probability of solvency for a minimum investment required and then to determine the investment strategy. The investment strategy is determined after all liabilities are paid in 40 years time when all pensioners are assumed to be dead at age 100.

At time zero, the investment strategy is determined for the starting fund such that the proportion of scenarios before the assets are run off by the liabilities is, say, $(1-\beta)\%$. Then it can be said that the scheme is solvent at that level of confidence.

In this model, this translates into determining an investment strategy that will ensure that a minimum amount of assets is kept now at an agreed confidence level. In particular, the strategic asset allocations are obtained for which the amount of assets kept now is minimized to ensure that the probability of the pension fund being ruined at the run-off horizon is at most $\beta\%$, where β denotes a very small probability. Mathematically, the following optimization problem is solved:

$$\begin{gathered} \operatorname*{Min}_{w_i} (F_0) \\ \textit{subject to}: \\ P\{(F_T - M_T) \geq 0\} \geq (1-\beta), \qquad (19) \\ \sum_i^n s_i = 1, \\ s_i \geq 0 \; \forall \, i \end{gathered}$$

where s_i denotes the weights in asset i, F_0 denotes the amount of asset kept at time zero, and $P\{(F_T - M_T) \geq 0\}$ denotes the probability of solvency. In this model there is no allowance for short-selling, hence $s_i \geq 0$ for all i.This approach serves as a solvency testing tool and also provides a whole probability distribution of surplus in the long run . One of the rationales underlying asset-liability management (ALM) is the minimization of ruin probability in a DB scheme.

3. Literature Review

3.1 Pension scheme system in Ghana

Most literature reviewed in Ghana looked generally at the social security system in Ghana. Kumado and Gockel (2003) carried out a research on the social security system in Ghana where they conducted a comparative assessment of various social security systems highlighting particularly the best practices. They also investigated the law and practice of social security in Ghana in relation to the best practices elsewhere so to bring to fore the issues on ownership and control of SSNIT, membership of SSNIT board, impact of the oath of secrecy sworn by workers representatives on the SSNIT board and investment standards of SSNIT. Kumado and Gockel further determined whether there could be additional benefits under the SSNIT scheme and made recommendation for the social security in Ghana.

Other studies carried out in Ghana by Dei (2001), looked at the pension fund management in Ghana. He carried out actuarial projections and analysis to ascertain the viability and sustainability of the scheme (SSNIT) into the future. The analysis he carried out entailed the determination and projections of contributors over the next 5 years or more and the determination of the funds inflow expected from contributions and investment returns. He also carried out projections on the number of expected pensioners, invalidity and death cases to arrive at the future funds outflow. He furthered assessed the financial viability of the social security scheme by looking at the fund ratio.

Missing from the literature on pension fund investment in Ghana is specifically the role in fixed income in pension scheme investment.

3.2 Pension fund risk management

Globally several literature have looked at pension fund risk management, investment policy and decision making using asset-liability models. The literature on pension fund risk management, investment policy and decision making have usually considered market, inflation, earning and demographic (longevity) risks by embedding these risk factors in asset and liability models. In previous studies, an autoregressive, conditional, heteroskedastic (ARCH) model, used by Wilkie (1995) and further discussed by Blake, Cairns and Dowd (2001), Haberman et al.(2003) and Wright(1998) have factored in market, inflation and earning risk in the generation of returns when considering portfolio risk on pension fund decision making.

Colombo and Haberman (2005) indicated that, allowing stochastic new entrant explains the occurrence of demographic risk. They further discussed that, allowing possible scenarios for the evolution of death rates which are generated by different survival models as used by Copola et al. (2011) or allowing uncertainty pertaining to the time of death, whether or not survival probability are known with certainty as indicated by Hari et al.(2005) also explains the occurrence of longevity risk.

3.3 Role of pension scheme in fixed income investment (A case study of the US market)

However, a paper carried out by Sweeting (2004) specifically looked at the role of fixed income in pension scheme investment. He expanded on the work carried out by ABN Amro team by and extending the sample period and using the arithmetic mean instead of geometric mean as a more appropriate measure of mean-variance analysis.

He concentrated on the US market data only. There are several analyses that he carried out. The first was to compare the historical risk and return characteristics of US high yield corporate debt/bond, investment grade corporate debt/bond, treasury bonds and equities. In addition to the mean and variance of asset returns as well as correlation between asset classes, he measured the Sharpe ratio, skewness and excess kurtosis.

The paper will use the stochastic asset model (mean-variance model) to carry out risk and return analysis on assets

4. Findings

4.1. Investment returns analysis of asset classes

This section of the work concentrates on the returns of the assets only. First to look at the returns produced by the assets and the level, stability and development of the projected returns over time (projection over 40 year period).

The returns calculated from GSE indices and bond yields (2007 to 2013) are presented in Table 4.1 below. In this analysis, it was preferable to calculate annual returns for all asset classes.

4.1.1. Long-Term Results

The returns on the different asset classes (equities, treasury bills, One-year bonds and Two-year bonds) over a long term period (say 40 years) can be calculated and analyzed. First is to consider the mean and standard deviations of the returns of the various assets classes for the whole period (2007 to 2013). The calculation of returns on assets is straightforward as shown in Equations (2) and (4). The projections of simulated correlated returns on both equity and bond assets are also shown in Equation (3) and (5) respectively.

The simulated projected correlated returns for the different asset classes over 40 year period are shown in Figure 4.0, 4.1, 4.2 and 4.3

As can be seen in the Figures on the projected average returns of the different asset classes, the returns produced by the portfolio of equity is higher than that produced by bond asset classes (treasury bill, Two-year bond and One-year bond). Generally, all the asset classes seem to see a slight rise in their average returns as the years progress into the future.

Comparing all the risk and return characteristics (mean, standard deviation, Sharpe ratio, skewness and excess kurtosis) of both the historical data and projected results over the entire projection, it can be concluded that the results on the historical data are similar to that of the projected results except in the case of skewness for One-year bond which was positively skewed for the projected simulated returns and negatively skewed for historical data and secondly, equity which had poor risk-adjusted return in the historical data but good risk-adjusted returns in the projected simulated returns. On the whole, we could conclude that results on historical data be used as a good predictor to tell future outcomes without necessarily using a stochastic asset-liability model. The subsequent sections will look at which best asset to invest in, considering liabilities over time.

4.2 Liability analysis.

This section of the analysis concentrates solely on the liabilities. First to calculate the liabilities that are paid by social security scheme and the projection made into the future in order to ensure the viability and sustainability of the scheme. The analysis entails the determination and projection of contributors, total salaries and hence the projected total contributions which indicate the funds inflow expected from contributions in the future (over the next 40 years).

The projected total contributions and the investment returns makes up to the total asset of the scheme.

Based on the available data and the projection made, the number of expected pensioners, average pensions and total pensions are computed. The projected total expenses of the scheme are also computed. Combining the projected total pension and the projected total expenses results in the total liabilities incurred by the scheme.

Some important assumptions made in the analysis are that, all pensioners will die by age 100. Also, the chosen age for the members who could start contributing to the scheme to await their pension payments during their retirement age was 20 years and the age for retirement was 60.

It is worth noting that the principal assumption made in our analysis is that the pensioners portfolio was a closed portfolio where there are no additional contributors added to the scheme hence the number of pensioners will run-off by 40 years time and therefore there will be no cash inflow from any other sources than the investments.

It is also important to note that the number of projected years of 40 was chosen because per the projected pensioners analysis made, all pensioners will die by 40 years (that is pensioners at age 60 at the start of scheme will die by age 100) and this will be the only time when the scheme can determine that it has paid off all its liabilities (especially total pensions) and can then determine the sustainability of the scheme.

4.2.1 Projected total expenses.

The graph in Figure 4.4 also shows a falling trend of expenses based on the actuarial projections for the running of the scheme. This pattern seems to occur because of the closed pensioners portfolio assumption made.

4.2.2 Projected total pensions

The graph in Figure 4.5 shows a falling trend of total pension payments. The falling trend of benefit paid to pensioners (total pensions) can be expected since in a closed pensioner portfolio since the projected pensioners as well as the projected average pension decreases as the years progress.

The projected total pension is one of the two main liabilities which are incurred by the scheme with the other liability being the total expenses.

4.3. Investment strategy

This section provides an analysis of investment strategy (that is the asset allocation and the minimum initial investment that need to be kept in order to make the fund solvent at a specified probability in the future). It is worth being reminded that the closed pensioners' portfolio was considered in this case.

4.3.1. Investment strategy and solvent (ruin) probabilities.

Table 4.3 summarizes the solution reached when the basic problem in Equation (19) is solved. The minimum investment required as well as the sensitivity of the asset allocation to changing solvency probabilities for a 40-year horizon is also shown. A horizon of this length (40 years) is sufficient to examine risk and return characteristics of a selected portfolio because it is at this period that the scheme would have paid of all its liabilities and can determine the solvency of the scheme.

Tables 4.3 depict the investment strategy. The table shows the asset allocation and the minimum investment required at different solvency levels.

In general, the minimum risk tolerance portfolio, that is 97.5% solvency level consist of 100% bond allocation, specifically One-year bonds. This can be explained by the fact that One-year bonds have very good risk-adjusted returns.

On the other hand, the maximum risk tolerance portfolios vary slightly moving from 100% equity allocation at 90% solvency level to 90% equity allocation and 10% bond allocation at 92.5% solvency level. There is a general trend of asset allocation shifting from equities to bonds (specifically One-year bonds) at increasing solvency levels.

Having considered the general trend in asset allocation, the general trend in minimum investment required is considered next. From table 4.3, there is a direct relationship between solvency probability and minimum investment required. The minimum investment required increases as the risk tolerance is reduced (approaching higher solvency levels).

5. Conclusions

5.1. Asset-only investment returns analysis

The general pattern of high returns for higher risk for equity and Two-years bond and higher returns for low risk for treasury bill and One-year bond in the projected years is consistent with that of the historical data.

Now looking at other measures of risk and return characteristics of the projected average returns such as Sharpe ratio, all the asset classes appear to a good risk-adjusted return with equity having a better risk-adjusted return as compared to the bond asset classes.

Considering the skewness of the projected returns, all asset classes show positively skewed distribution indicating frequent small losses and few extreme gains hence lesser chance of negative outcomes. Equity appeared to be more normal as compared to the bond asset classes when excess kurtosis was considered.

Looking at the assets-only analysis of pension schemes without thinking about how to match their liabilities, equity appears to be an attractive asset class to invest in.

Generally, comparing all the risk and return characteristics (mean, standard deviation, Sharpe ratio and excess kurtosis) of both the historical data and projected simulated results, it can be concluded that the results on the historical data could be used as a good predictor to tell future outcomes without necessarily using a stochastic asset-liability model.

5.2. Liability analysis

Concerning total pensions and expenses which constitute the liabilities incurred by scheme under a closed pensioners' portfolio, total pensions paid to pensioners decreases as the year progress and total expenses made by the scheme also decrease as the years progress.

5.3. Investment strategy (Asset-Liability analysis)

When liabilities are taken into account, the picture changes and bonds (specifically One-year bonds) is the best-matched liabilities. The asset allocation moves from equity towards bonds (specifically One-year bonds as the solvency levels increases. The minimum investment required also increases as the solvency level increases.

6. Recommendations

The study is implemented in Ghana and also to be adopted in the United States economy to help pension fund managers to know the best asset class to invest into, in the absence of liabilities and also the investment strategy required to make the scheme solvent in the future at a specified high probability) in the presence of all liabilities

References

ABN Amro (2000). Credit bonds as an asset class. ABN Amro Global Consulting Group..

Acharya V. V., Pedersen L. H., Philippon, T. and Richardson M. (2010). *Measuring Systemic Risk*. Working Paper 10-02. Cleveland, OH: Federal Reserve Bank of Cleveland. Actuarial Society, 16 November 1999

African Development Bank Group (2012), "African Fixed Income and Derivatives Guidebook"

Bank for International Settlements (2011). *Fixed Income Strategies of Insurance Companies and Pension Funds*. Working Paper 44. Basel: Committee on the Global Financial System.

Besar, D. et al. (2010). Systemic risk in financial services. *British Actuarial Journal*, 16(2), 195-300.

Brownlees, C. T. and Engel, R. F. (2011). *Volatility Correlation and Tails for Systemic Risk Measurement*. Working Paper. New York: New York University.

Bruche, M. and Gonzalez-Aguado, C. (2010). Recovery rates, default probabilities, and the credit cycle. *Journal of Banking and Finance*, 34, 754-764.

De Jong, F. and Driessen, J. (2012). Liquidity risk premia in corporate bond markets. *Quarterly Journal of Finance*, 2(2). DOI: 10.1142/S2010139212500061.

Dimson, E. et al. (2010). *Investment Strategy and the Government Pension Fund Global*. Report by the Norwegian Strategy Council..

Dimson, E., Marsh, P. and Staunton, M. (2010). *Credit Suisse Global Investment Returns Yearbook 2010*. Zurich: Credit Suisse Research Institute.

Longstaff, F. A. et al. (2011). How sovereign is sovereign credit risk? *American Economic Journal: Macroeconomics*, 3(2), 75-103.

Moody 's Investors Service (2012). *Default and Recovery Rates of Corporate Bond Issuers, 1920-2004*. New York: Moody's Investors Service.

Moody's Investor Service (2011). *Special Comment: Sovereign Default and Recovery Rates 1983-2010*. New York: Moody's Investor Service.

Porteous, B. T., Tapadar, P. and Yang, W. (2012). Economic capital for defined benefit pension schemes: An application to the UK Universities Superannuation Scheme. *Journal of Pension Economics and Finance*, 11(4), 471-499.

Sharpe, W. F. (2010). Adaptive asset allocation policies, *Financial Analysts Journal*, 66(3), 45-59.

SSNIT (2002), Medium-Term Strategic Plan: 2002 - 2006, Social Security And National

SSNIT, Annual Report : 2006 -2010, Various Issues

Varnell, E. M. (2011). Economic scenario generators and solvency II. *British Actuarial Journal*, 16(1), 121-159.

Wilkie, A. D. et al. (2010). Yet more on a stochastic economic model: Updating and refitting, 1995-2009. *Annals of Actuarial Science*, 5(1), 777 964.

Table 1.0 Summary of SSNIT Investment portfolio allocation (Percentage of total)

Investment	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**
Equity (listed and unlisted)	30.0	29.8	31.5	42.6	46.0	30.0	30.0	32.6
Fixed Income	58.0	59.9	54.4	46.0	47.0	59.7	60.6	58.0
Real Estate	10.6	9.5	14.1	11.4	7.0	8.1	8.6	9.4
Economically Targeted Investment	1.4	0.7	0.1	0.0	0.0	1.6	0.8	0
Total	100	100	100	100	100	100	100	100

Source: Calculated from SSNIT Annual Report (2005-2012)

*Calculated from amounts that are net of provisions. Figures may not add up to 100 because of rounding

Table 1.1 Net Investment Income (in percentage)

	2005	2006	2007	2008	2009	2010	2011	2012
Government and Registered bonds	0.32	3.42	4.93	3.88	0.05	1.14	4.6	12.2
Term Deposit and Treasury Bills	57.12	28.35	49.28	33.44	38.56	33.19	22.12	10.67
Student Loan	9.41	1.64	4.11	10.29	14.31	8.06	4.46	6.22
Corporate Loan	13.24	8.29	15.09	31.08	32.80	31.06	34.95	25.76
Rent	3.45	2.50	4.92	4.55	3.19	3.86	4.42	2.27
Dividend	10.22	8.60	12.82	12.74	6.62	12.01	15.83	10.63
Profit on disposal of shares	-	-	3.77	-	-	-	-	-
Miscellaneous	6.24	47.20	5.07	4.01	4.47	10.67	13.6	32.24
Total	100.0	100.0	100.0	100.0	100.0	100.0	100	100

Source: Calculated from SSNIT Annual Report (2005-2012).
*Calculated from amounts that are net of provisions. Figures may not add up to 100 because of rounding

Table 4.0: Return on asset classes, 2007-2013

	Equity	Treasury bill	Two-year bond	One-year bond
2007	0.29	0.10	0.18	0.12
2008	0.61	0.19	0.08	0.20
2009	-0.49	0.27	0.18	0.21
2010	-0.81	0.15	0.46	0.13
2011	-0.07	0.11	0.14	0.11
2012	0.15	0.19	0.07	0.23
2013	3.34	0.12	0.30	0.22

Source: Author's calculation.

Table 4.1: Summary characteristics of simulated returns over the entire projection (40years)

	Equities	Treasury bills	Two-year bond	One-year bond
Mean	0.560083717	0.035480658	0.079664736	0.031289115
Standard deviation	0.007301187	0.000459407	0.001113587	0.000424033
Sharpe ratio	74.51761593	42.36765	42.36765	36.01710783
Skewness	0.593294918	0.651971729	0.604919068	0.456014533
Excess Kurtosis	3.664239288	-3.664239288	-3.68226	-3.88399

Source: Author's calculation

Table 4.2: Summary characteristics of returns on historical data (2007-2013)

	Equities	Treasury bills	Two-year bond	One-year bond
Mean	0.43	0.16	0.20	0.18
Standard deviation	1.266179028	0.054766519	0.125591525	0.047656625
Sharpe ratio	0.328521	2.884301355	1.447426247	3.704470557
Skewness	1.93767	0.040858	0.122994	-0.01122
Excess kurtosis	-1.94792	919.901	152.6427	2475.305

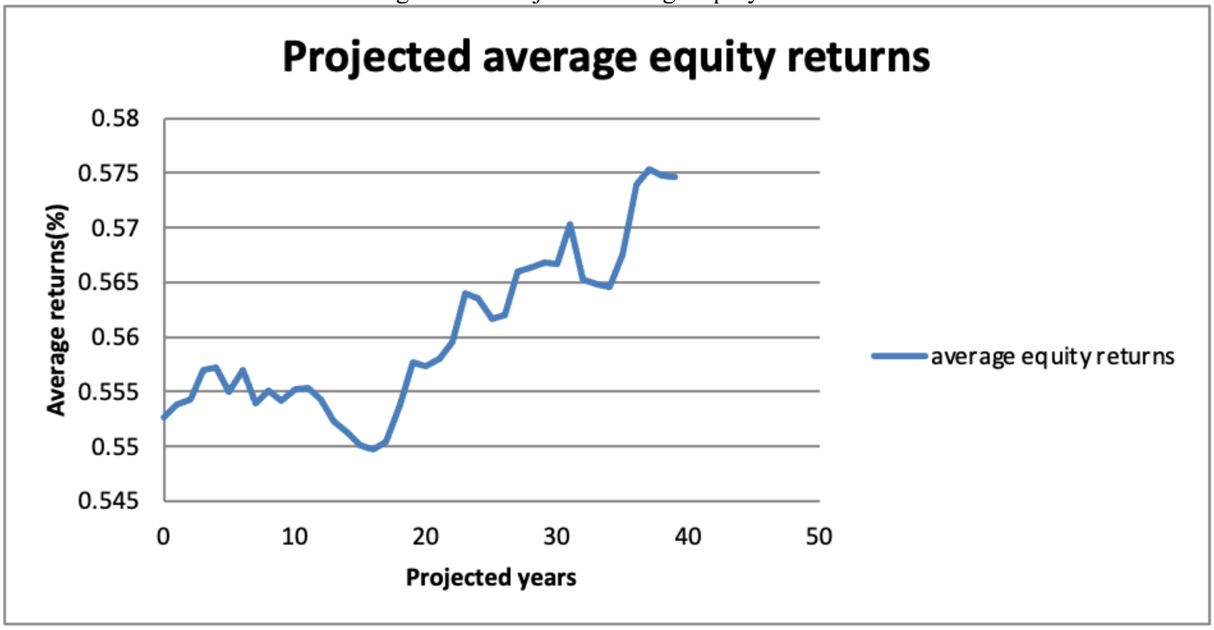
Source: Author's calculation

Table 4.3: Optimal investment strategy under varying solvent probabilities

		Asset allocations (%)			
Solvency probability	Minimum investment required	Equity	Treasury bills	Two-year bond	One-year bonds
97.5%	716,600,00	0%	0%	0%	100%
92.5%	630,000,000	90%	0%	0%	10%
90%	623,000,000	100%	0%	0%	0%

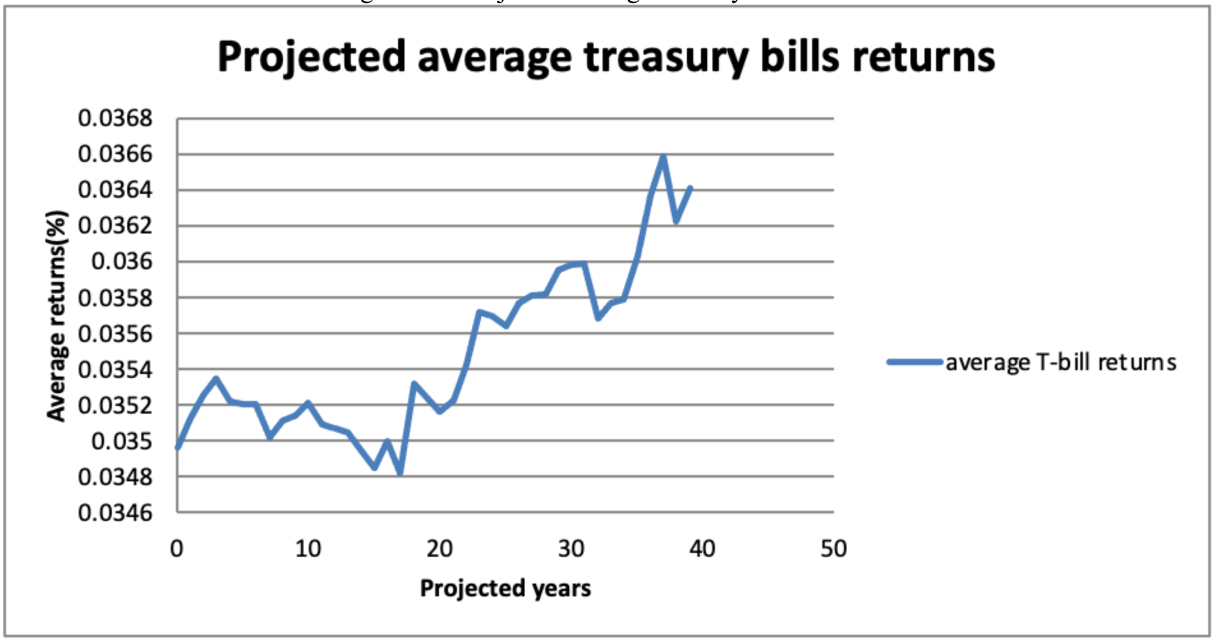
Source: Author's calculation

Figure 4.0: Projected average equity returns



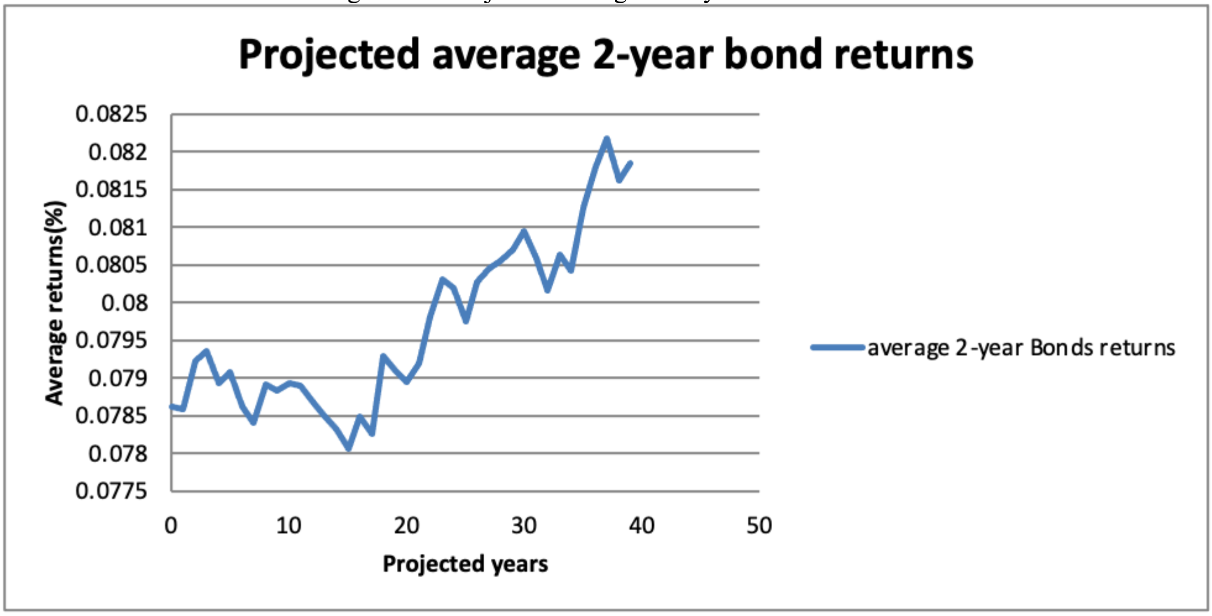
Source: Author’s construct

Figure 4.1: Projected average treasury bills returns



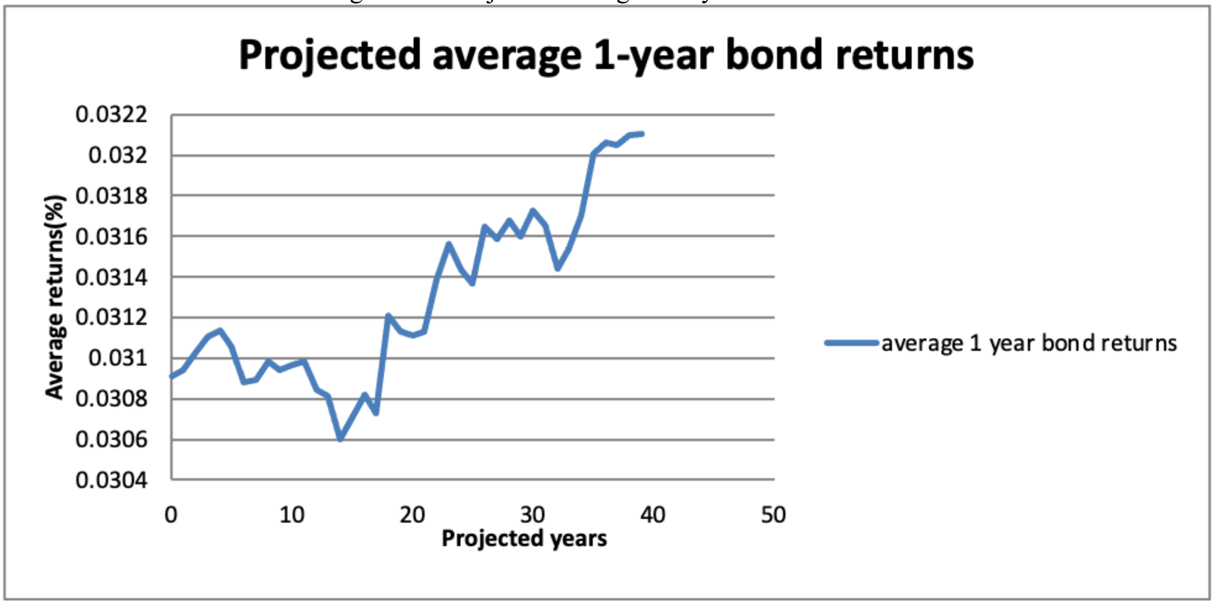
Source: Author’s construct

Figure 4.2: Projected average Two-year bond returns



Source: Author's construct

Figure 4.3: Projected average One-year bond returns



Source: Author's construct

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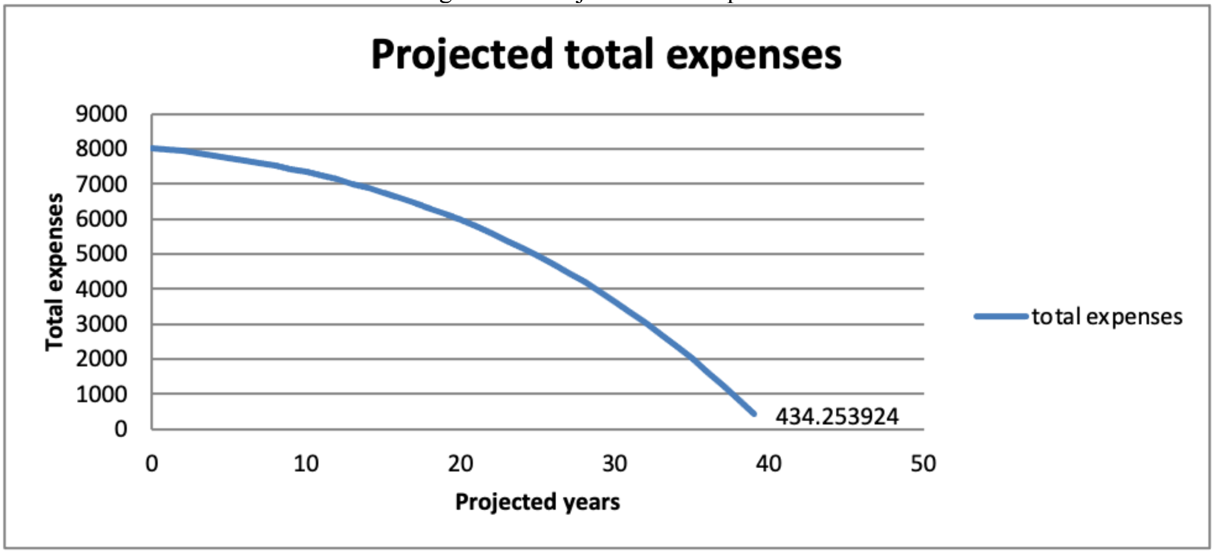
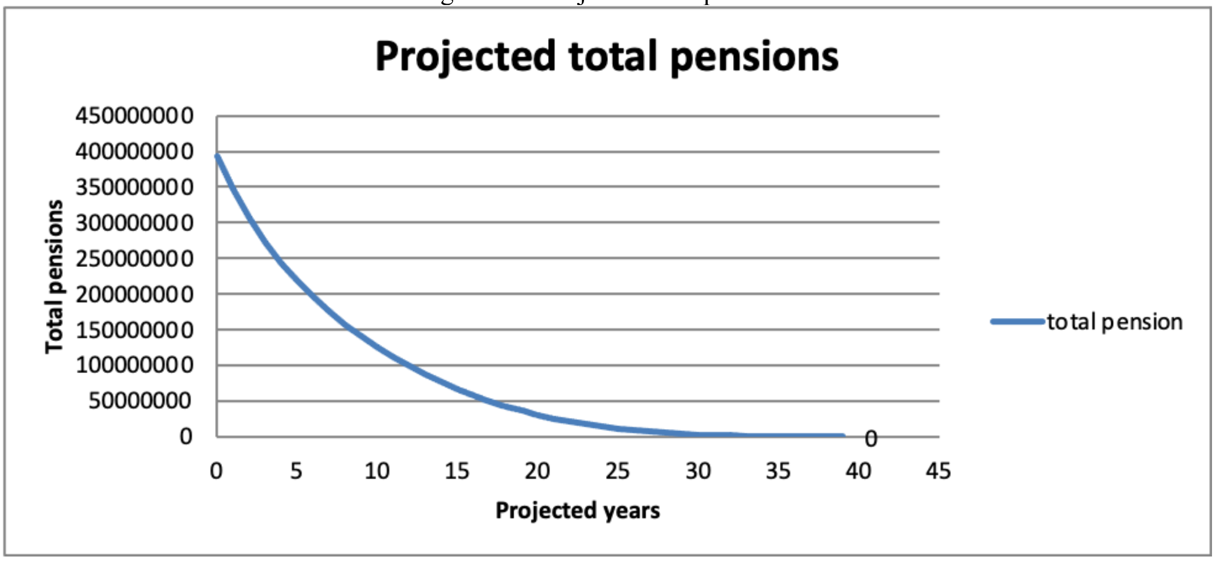
Figure 4.4: Projected total expenses



Source: Author's construct

Figure 4.5: Projected total pensions



Source: Author's construct