Rebalancing Return and Hedge Effectiveness of Dynamic Portfolio Insurance Strategies: A simulation Based Study

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Abstract
Dynamic portfolio insurance strategies can be used to protect a stock market exposure against large losses. The implementation of these strategies entails a regular rebalancing which serves to adjust the current asset allocation to the one desired according to the strategy. An alternative to regular rebalancing could be a one-time portfolio allocation at the beginning of the investment period with no further adjustment up to the end of the period (“buy-and-hold”, B&H). An essential criterion for deciding on a portfolio insurance strategy is its performance in comparison to the B&H strategy. The aim of this paper is to study the performance of different dynamic portfolio insurance methodologies through a simulation process based on normally distributed stock returns. The analysis focuses on the Constant Proportion Portfolio Insurance (CPPI), Time Invariant Portfolio Protection (TIPP), and a modified version of the Time Invariant Portfolio Protection methodology (TIPP-M). The performance is measured using a rebalancing return and a hedge effectiveness measure. The results of the simulation analysis suggest that CPPI is the best strategy according to the rebalancing return while the TIPP strategy leads to the best hedge effectiveness results. TIPP-M is similar to TIPP but seems to be slightly worse.

Keywords: Portfolio insurance, Constant Proportion Portfolio Insurance, CPPI, Time Invariant Portfolio Protection, TIPP, Buy-and-Hold, Rebalancing Return, Hedge Effectiveness, Monte Carlo Simulation

JEL Classification: G11

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1. Introduction
Not option-based dynamic portfolio insurance (PI) strategies are designed to protect portfolios against large falls by a contractually guaranteed predetermined floor through a dynamic allocation. On the one hand they can be defined to guarantee a minimum level of wealth and on the other hand they can allow to participate in rising stock markets (Hoque and Meyer-Bullerdiek, 2016, p. 80). Hence, PI strategies can reduce portfolio risk while the investor can participate to rises in the market.

The most popular approach of not option-based dynamic PI strategies in the asset management industry seems to be the Constant Proportion Portfolio Insurance (CPPI) (Dichtl and Drobetz, 2010, p. 41). This methodology was introduced by Perold (1986) on fixed income assets and extended by Black and Jones (1987) for equity based underlying assets. The Time Invariant Portfolio Protection (TIPP) methodology was introduced by Estep and Kritzman (1988) and modified (TIPP-M) by Meyer-Bullerdiek and Schulz (2003). The difference between the CPPI and the TIPP methodology is the assumption in respect to the initial floor which can vary. The TIPP-M methodology is supposed to enhance the use of the market trend for a portfolio.

There has been a lot of research work on PI strategies. For example, Brennan and Schwartz (1988, p. 283) pointed out that a PI strategy may be of considerable significance to portfolio managers whose investment performance is monitored periodically and to investors who need to meet liabilities in the future.

In several studies the efficiency of not option-based PI strategies was examined by using a Monte Carlo simulation approach. Among them, Zhu and Kavee (1988) showed that CPPI has the ability to reduce downward risk and to retain a certain part of upward gains. In a simulation comparison of popular dynamic strategies of asset allocation Cesari and Cremonini (2003) use alternative measures for risk, return and risk-adjusted return. Their simulations show – independently of the performance measure adopted – a dominant role of constant proportion strategies in bear and no-trend markets.

Annaert, Van Osselaer, and Verstraete (2009) provide a simulation based performance evaluation of different PI strategies including the stochastic dominance framework. Their results indicate that portfolio insurance strategies outperform a buy-and-hold (B&H) strategy with regard to downside protection and risk/return trade-off.

Running Monte Carlo simulations and historical simulations Dichtl and Drobetz (2011) find that the traditional portfolio insurance strategies Stop Loss, Synthetic Put, and CPPI are the preferred investment strategies for a prospect theory investor.

Hoque and Meyer-Bullerdiek (2016) analyse the performance of different dynamic portfolio insurance methodologies for securities in the German market in different time periods by comparing them to the B&H
strategy, the stock only strategy and the bonds only strategy. Based on the Sortino ratio, the TIPP strategy turned out to deliver the best results.

In a more recent paper, Hoque, Kämmer and Meyer-Bullerdiek (2018) run Monte Carlo simulations for the B&H and several PI strategies using different levels of low interest rates. According to the Sortino Ratio or LPM performance measures the strength of the CPPI strategy becomes obvious when interest rates are reduced, especially when it comes to negative rates.

The above mentioned authors analyse the performance of PI methodologies under different perspectives. This paper provides a performance evaluation of the CPPI-, TIPP- and TIPP-M-techniques based on a Monte Carlo simulation. Apart from more traditional performance measures, this study considers the rebalancing return of the different approaches as well as the hedge effectiveness according to Johnson (1960) whereas the value at risk is used because this downside risk measure is more appropriate in a portfolio insurance context.

This paper is structured as follows: Section 2 provides a brief overview of the portfolio insurance strategies considered in the simulation analysis. Section 3 shows the Monte Carlo simulation design and presents the performance measures used in this study. The results from the simulations are presented and discussed in section 4. Section 5 summarizes the main results of the study.

2. Overview of PI strategies considered in this study

In portfolio management practice, CPPI strategies are quite popular. To keep the risk exposure constant, the CPPI portfolio is invested in various proportions in a risky asset (e.g. a stock portfolio) and in a non-risky one (e.g. a risk-free bond). The exposure $E$ which should be invested in the risky asset can be defined by the following equation (Hoque and Meyer-Bullerdiek, 2016, 80):

$$E = m \times C = m \times \max \left( V - F; 0 \right),$$ (1)

where $m$ is the multiplier which represents the risk aversion of the investor ($m \geq 1$, the greater $m$, the less risk averse is the investor), $V$ is the total portfolio value, $F$ is the floor, and $C$ is the cushion which can be defined as follows:

$$C = \max \left( V - F; 0 \right)$$ (2)

In equations (1) and (2) it is also possible to use the present value of the floor instead of the floor value (Hoque, Kämmer and Meyer-Bullerdiek (2018, p. 14-15).

At the beginning of the period, the multiplier and the floor are defined. They will remain constant during the time period (Lee, Hsu and Chiang, 2010, p. 221). After each change of the value of the total assets, the new exposure (amount of the risky asset) can be calculated using the abovementioned equation. Thereafter, the volume of the risk-free asset will be allocated by the difference between the total assets and the risky asset (Meyer-Bullerdiek and Schulz, 2004, p. 55). In this paper, the risky asset is represented by a notional stock portfolio, and the non-risky asset is taken to be a risk-free bond that earns no interest.

Very similar to the CPPI is the TIPP methodology. According to Estep and Kritzman (1988) TIPP has the following characteristics:

1. The portfolio can never decline below a preset floor
2. The floor is adjusted continuously to be a specified percentage of the highest value the portfolio reaches
3. Protection is continuous and has no ending date

The floor is set as a fixed percentage of the total value of the portfolio and the strategy follows this process:

1. Calculation of the total value of the portfolio
2. Multiplication of the total value with the preset floor percentage rate
3. Setting of a new floor if the result of (2) is greater than the previous floor
4. Calculation of the cushion: $C = \max \left( V - F; 0 \right)$
5. Calculation of the exposure: $E = m \times C$
6. Purchase / Sell of the risky asset according to (5)

Consequently, TIPP and CPPI only differ in the assumption concerning the initial floor which is not constant for the TIPP methodology. The preset floor will increase in case of an increase of the total value of the portfolio. If the portfolio value decreases, this value should not be less than the floor. Thus, TIPP can be regarded as a more passive strategy than CPPI (Tiefeng and Rwegasira, 2006, p. 98).

Meyer-Bullerdiek and Schulz (2003) presented the so-called TIPP-M strategy which uses a variable multiple factor and where “M” in the title stands for “modified”. This approach is characterized in that not only the cushion changes from one rebalancing period to the other but also the multiplier (“m”) depending on the market trend. This multiplier of a certain period $t$ (“$M_{t}$”) can be calculated e.g. for a certain stock portfolio according to the following equation (Hoque and Meyer-Bullerdiek, 2016, p. 81):

$$M_{t} = \frac{PSP}{\text{Mean}^{\text{PSP}}_{t-n+5} \times M_{t-1}}$$ (3)
where $M_{t-1}$ is the multiplier of the previous period, i.e. before adjustment, PSP is the current price of the stock portfolio, and $\text{Mean}_{t-5 \text{ to } n}^{\text{PSP}}$ is the mean of the stock portfolio prices of the previous 5 periods and the current period.

Meyer-Bullerdiek and Schulz (2003) recommend the following interval within which the calculated multiplier can fluctuate: $1 \leq M_t \leq 2 \times m$, where $m$ is the initial multiplier. The algorithm to calculate the multiplier can be expressed as follows:

```
Figure 1. TIPP-M strategy: Algorithm for multiplier calculation
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Following this algorithm, the multiplier will be increased until it reaches the defined maximum in case of bullish markets so that the investor can benefit from increasing stock prices. If markets are bearish, the M-factor can drop to a minimum to reduce the stock exposure.

3. Monte Carlo simulation design and performance measures

In the empirical analysis a Monte Carlo simulation is used to generate weekly logarithmic stock returns which are normally distributed. It is based upon a weekly logarithmic stock market return of 0.13% and a weekly standard deviation of stock market returns of 2.8%. This is similar to the market return and volatility of the German stock index DAX from January 1971 to February 2018. Dimson, Marsh and Staunton (2006) found in their study of equity premiums in 17 countries similar long-run standard deviations, while real equity returns were more diverse in the different countries.

The weekly logarithmic stock returns are generated with MS Excel. For each PI strategy these logarithmic returns of the risky asset are transformed into weekly absolute returns.

In this study, a period of 500 weeks is used. Transaction costs are neglected. The initial investment is assumed to be € 1,000,000. For the considered strategies a weekly rebalancing is assumed.

The PI strategies are implemented with an initial protection level of 90% of total wealth. The risk-free asset is assumed to earn 0% interest. According to the different risk appetite of investors the study uses five different initial multipliers ($m = 1, 2, 3, 4, \text{ and } 5$). Hence, the following initial stock investments (Exposure, E) can be calculated:

$m = 1$ => $E = m \times (V - F) = 1 \times (€1,000,000 - €900,000) = €100,000$

$m = 2$ => $E = m \times (V - F) = 2 \times (€1,000,000 - €900,000) = €200,000$

$m = 3$ => $E = m \times (V - F) = 3 \times (€1,000,000 - €900,000) = €300,000$

$m = 4$ => $E = m \times (V - F) = 4 \times (€1,000,000 - €900,000) = €400,000$

$m = 5$ => $E = m \times (V - F) = 5 \times (€1,000,000 - €900,000) = €500,000$

With regard to the TIPP-M strategy, the double value of the initial multiplier is defined as maximum so that the interval – as shown above – is for the new factor: $1 \leq M_t \leq 2 \times m$. Not only the mean of the stock portfolio prices of the previous 5 periods and the current period is calculated ($\text{Mean}_{t-5 \text{ to } n}^{\text{PSP}}$) but also of the previous 11 periods and the current period (i.e. $\text{Mean}_{t-11 \text{ to } n}^{\text{PSP}}$, “TIPP-M12”). This seeks to offset large fluctuations in the stock portfolio prices in a volatile market environment and to prevent excessive market fluctuations between individual periods from having a direct impact on the portfolio.

In total, 10,000 simulations are performed for each PI strategy including the B&H portfolio. For performance measurement purposes logarithmic returns of each PI strategy are calculated for each week, the mean logarithmic return and the standard deviation of weekly returns over the 500 weekly periods for each simulation, and the average of all 10,000 mean logarithmic returns. Furthermore, the maximum and the minimum portfolio value of
the 10,000 simulations is determined for each PI strategy. Finally, the value at risk is calculated at the 95% confidence level for each simulation according to the following formula (Bruns/Meyer-Bullerdiek, 2013, p. 33):

\[
\text{VaR} = Q \times \left( e^{(\mu + z\sigma)} - 1 \right)
\]

(4)

where \( Q \) is the initial value of the total portfolio (here: 1,000,000), \( \mu \) is the mean logarithmic return and \( \sigma \) is the standard deviation of weekly returns over the 500 weekly periods, \( z \) is the z-value from the normal distribution.

The outcomes of the PI strategies are measured with different approaches. On the one hand the performance can be measured by comparing it to a Buy and Hold (B&H) strategy. While the PI strategy is rebalanced on a regularly basis, the B&H portfolio is based on a onetime portfolio allocation at the beginning of the investment period with no further adjustment up to the end of the total period.

Therefore, the rebalancing return according to Hallerbach (2014, pp. 6-10) can provide a suitable measure to calculate the performance of a PI strategy. The rebalancing return can be described as the full difference between the geometric mean returns of a rebalanced and a B&H portfolio. Hallerbach postulates that the rebalancing return is composed of the volatility return and a dispersion discount which is shown in the following equation:

\[
\text{RR}_H = \bar{r}_g - \bar{r}_{B&H} = \left( \frac{1}{n} \sum_{i=1}^{n} W_{i0} \times \bar{r}_g \right) - \left( \frac{1}{n} \sum_{i=1}^{n} W_{i0} \times \bar{r}_{B&H} \right)
\]

(5)

where \( \bar{r}_g \) is the geometric mean return of the portfolio (which is rebalanced), \( \bar{r}_{B&H} \) is the geometric mean return of the B&H portfolio, and \( W_{i0} \) are the initial fixed weights of the assets. Hence, the volatility return contributes positively and the dispersion discount contributes negatively to the rebalancing return.

To determine if the rebalancing return is significantly positive (or negative), a t-test with the following statistic is used (Bleymüller and Weißbach, 2015, p. 135-136, Bruns and Meyer-Bullerdiek, 2013, p. 772):

\[
t = \frac{\text{RR}_H - \mu}{\sigma_{\text{RR}_H}} \sqrt{n}
\]

(6)

where \( \text{RR}_H \) is the average rebalancing return, \( \sigma_{\text{RR}_H} \) is the standard deviation of the average rebalancing return, \( \mu \) is the specified value (here it is taken to be 0), and \( n \) is the sample size (which is 10,000 in this study). The null hypothesis and the alternative hypothesis are defined as follows:

Null hypothesis: \( \mu = 0 \)

Alternative hypothesis: \( \mu > 0 \)

Accordingly, it is tested whether the rebalancing return is significantly positive, based on a significance level (error rate) of \( \alpha = 5\% \) which is often used in the economic and social sciences. Correspondingly, the relevant critical value for \( t \) can be taken from the t-distribution table. At values below this critical value, the null hypothesis is maintained; because then it cannot (significantly) be rejected. If the values are above the critical value, it can be assumed that the rebalancing return is significantly positive (Poddig, Dichtl and Petersmeier, 2003, pp. 338-339, 344, and 767).

On the other hand, the performance is measured using an approach which is based on the hedge effectiveness measure according to Johnson (1960, p. 144):

\[
\text{HE}_{PI} = \frac{\text{VaR}_{B&H} - \text{VaR}_{PI}}{\text{VaR}_{B&H}} = 1 - \frac{\text{VaR}_{PI}}{\text{VaR}_{B&H}}
\]

(7)

Thus, the PI strategy is considered as a mechanism to reduce portfolio risk which is measured by the value at risk (VaR). In this study the VaR is based on weekly standard deviations. The hedge effectiveness is measured by the extent to which the value at risk of a B&H strategy (“unhedged position”) is reduced by a PI strategy (“hedged position”). A complete portfolio hedge would be available if the value at risk of the hedged position is zero, resulting in a hedge effectiveness of 100%.

4. Monte Carlo simulation results

In this section the simulation results for the B&H portfolio and for each PI strategy are presented. Table 1 shows the results of the B&H portfolio.
Table 1: B&H results

<table>
<thead>
<tr>
<th></th>
<th>m=1</th>
<th>m=2</th>
<th>m=3</th>
<th>m=4</th>
<th>m=5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\bar{r}_{m}^{\max})</td>
<td>0.0232%</td>
<td>0.0419%</td>
<td>0.0577%</td>
<td>0.0715%</td>
<td>0.0838%</td>
</tr>
<tr>
<td>VaR</td>
<td>-6,574</td>
<td>-12,077</td>
<td>-16,942</td>
<td>-21,377</td>
<td>-25,504</td>
</tr>
<tr>
<td>(V_{\max})</td>
<td>3,088,073</td>
<td>5,176,147</td>
<td>7,264,220</td>
<td>9,352,294</td>
<td>11,440,367</td>
</tr>
<tr>
<td>(V_{\min})</td>
<td>919,617</td>
<td>839,234</td>
<td>758,851</td>
<td>678,469</td>
<td>598,086</td>
</tr>
</tbody>
</table>

In the table \(\bar{r}_{m}^{\max}\) is the arithmetic average of 10,000 mean logarithmic returns (of each 500 weeks period) which is almost identical with the arithmetic average of the 10,000 mean geometric returns. \(\text{VaR}\) is the arithmetic average of 10,000 value at risks, \(V_{\max}\) is the maximum portfolio value after 500 periods over 10,000 simulations, and \(V_{\min}\) is the corresponding minimum portfolio value.

Please note that there are the following relationships between the different \(V_{\max}\) and the different \(V_{\min}\) respectively:

\[
V_{\max}^{m=2} - V_{\max}^{m=1} = 5,176,147 - 3,088,073 = 2,088,073
\]
\[
V_{\max}^{m=3} - V_{\max}^{m=2} = 7,264,220 - 5,176,147 = 2,088,073
\]
\[
V_{\max}^{m=4} - V_{\max}^{m=3} = 9,352,294 - 7,264,220 = 2,088,073
\]
\[
V_{\max}^{m=5} - V_{\max}^{m=4} = 11,440,367 - 9,352,294 = 2,088,073
\]

\[
V_{\min}^{m=2} - V_{\min}^{m=1} = 839,234 - 919,617 = -80,383
\]
\[
V_{\min}^{m=3} - V_{\min}^{m=2} = 919,617 - 839,234 = 80,383
\]
\[
V_{\min}^{m=4} - V_{\min}^{m=3} = 839,234 - 919,617 = -80,383
\]
\[
V_{\min}^{m=5} - V_{\min}^{m=4} = 919,617 - 839,234 = 80,383
\]

The equality of the differences results from the fact that each of the five B&H strategies (i.e. B&H with \(m = 1, 2, 3, 4, 5\)) in each of the 10,000 simulation runs is based on the same 500 normally distributed, random returns, with the returns, of course, differing from run to run. At the same time, due to the assumption of 0% interest, the amount of the risk-free bond remains constant over all 500 periods. Since this consistency is not the case for the PI strategies (due to the changed exposure in equities and risk-free bond after each rebalancing), the above-mentioned relationships cannot occur in the PI strategies.

It can be seen that with increasing risk appetite (i.e. higher value for \(m\)) the average return grows, but the average value at risk also increases. Table 2 presents the results of the CPPI strategy.

Table 2: CPPI results

<table>
<thead>
<tr>
<th></th>
<th>m=1</th>
<th>m=2</th>
<th>m=3</th>
<th>m=4</th>
<th>m=5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\bar{r}_{m}^{\max})</td>
<td>0.0232%</td>
<td>0.0524%</td>
<td>0.0685%</td>
<td>0.0733%</td>
<td>0.0731%</td>
</tr>
<tr>
<td>VaR</td>
<td>-6,574.02</td>
<td>-16,829.37</td>
<td>-23,177.90</td>
<td>-25,605.24</td>
<td>-26,336.97</td>
</tr>
<tr>
<td>(V_{\max})</td>
<td>3,088,073</td>
<td>11,496,296</td>
<td>16,364,275</td>
<td>17,693,899</td>
<td>18,319,735</td>
</tr>
<tr>
<td>(V_{\min})</td>
<td>919,617</td>
<td>902,581</td>
<td>900,224</td>
<td>900,013</td>
<td>900,000</td>
</tr>
<tr>
<td>(\text{RR}_t)</td>
<td>0.0000%</td>
<td>0.0105%</td>
<td>0.0108%</td>
<td>0.0018%</td>
<td>-0.0106%</td>
</tr>
<tr>
<td>(t_{\text{RR}})</td>
<td>---</td>
<td>35.37</td>
<td>25.22</td>
<td>3.72</td>
<td>-20.55</td>
</tr>
<tr>
<td>(HE_{\text{RR}})</td>
<td>0.0000%</td>
<td>-30.6838%</td>
<td>-28.9854%</td>
<td>-13.8197%</td>
<td>1.1139%</td>
</tr>
</tbody>
</table>

In the table \(\bar{r}_{m}^{\max}\) is the arithmetic average of 10,000 rebalancing returns (of each 500 weeks period), \(t_{\text{RR}}\) is the t-statistic to determine if the rebalancing return is significantly positive (or negative). The error rate is \(\alpha = 5\%\). \(HE_{\text{RR}}\) is the arithmetic average of 10,000 hedge effectiveness measures.

First of all, it is noticeable that the portfolio value after 500 periods in none of the 10,000 simulations falls below the initial floor value of € 900,000. This makes the CPPI strategy at least successful in terms of hedging against a portfolio loss of more than € 100,000.

It is striking that the CPPI strategy for the cases \(m = 2\) to \(m = 4\) outperforms the B&H portfolio in terms of the average geometric returns of all simulations. Thus, the average of all 10,000 rebalancing returns in these cases is statistically significantly positive. In the case of \(m = 1\), CPPI and B&H are the same. The frequency distribution of rebalancing returns over all 10,000 simulations is shown in Figure 2 using the example of \(m = 3\).
Figure 2. CPPI with m=3: Distribution of the rebalancing returns of 500 rebalancing periods. 10,000 simulations are considered.

On the basis of the frequency distribution, it becomes obvious that the rebalancing return is in many cases negative, but then has only very low absolute values. On the other hand, numerous fairly high positive rebalancing returns occur, so that for m = 3 a significantly positive average rebalancing return results.

However, the CPPI strategy involves a higher risk than the B&H portfolio (measured as the average value at risk). This is also suggested by the average hedge effectiveness. For m = 2, 3 and 4, the average hedge effectiveness is negative which means that the risk is increased in comparison to B&H. Only in the case of m = 5 the average hedge effectiveness is slightly positive, which corresponds to a slight risk reduction (compared to the B&H portfolio) as an average over all simulations although the (absolute) average value at risk of the CPPI strategy is still higher than the one of the B&H portfolio.

To what extent the TIPP and TIPP-M strategies lead to better results are shown in the following sections. Table 3 presents the results of the TIPP strategy.

Table 3. TIPP results

<table>
<thead>
<tr>
<th>m</th>
<th>( \bar{\text{r}} )</th>
<th>( \text{VaR} )</th>
<th>( \text{V}_{\text{max}} )</th>
<th>( \text{V}_{\text{min}} )</th>
<th>( \text{RR} )</th>
<th>( \text{IE} )</th>
<th>( \text{HE} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0141%</td>
<td>-3.819,03</td>
<td>1.368.878</td>
<td>919.994</td>
<td>-0.0091%</td>
<td>-62,09</td>
<td>36,8934%</td>
</tr>
<tr>
<td>2</td>
<td>0.0232%</td>
<td>-6.545,42</td>
<td>1.810.155</td>
<td>902.940</td>
<td>-0.0187%</td>
<td>-81.39</td>
<td>43,9610%</td>
</tr>
<tr>
<td>3</td>
<td>0.0281%</td>
<td>-8.402,17</td>
<td>2.309.592</td>
<td>900.281</td>
<td>-0.0297%</td>
<td>-97.68</td>
<td>50,2039%</td>
</tr>
<tr>
<td>4</td>
<td>0.0295%</td>
<td>-9.579,76</td>
<td>2.837.025</td>
<td>900.018</td>
<td>-0.0420%</td>
<td>-106,9</td>
<td>55,7023%</td>
</tr>
<tr>
<td>5</td>
<td>0.0286%</td>
<td>-10.240,19</td>
<td>3.345.743</td>
<td>900.000</td>
<td>-0.0552%</td>
<td>-109,88</td>
<td>60,5534%</td>
</tr>
</tbody>
</table>

As with the CPPI strategy, the final portfolio value after 500 periods in none of the 10,000 simulations falls below the initial floor value of € 900,000. Thus, the TIPP strategy also appears to be successful, at least in terms of downward hedging. Compared to the B&H strategy, however, it is worse as far as the average geometric returns of all simulations are concerned. Thus, the average of all 10,000 rebalancing returns is negative. The extent to which this negative deviation from zero is statistically significant can be checked again with the t-test. The corresponding critical t-value in this case is an absolute value of 1.645. Therefore, in all cases, the rebalancing return is significantly negative, i.e. the average geometric return after 500 periods is significantly lower for the TIPP strategy than for the B&H strategy. The frequency distribution of rebalancing returns over all 10,000 simulations is shown in Figure 3 using the example of m = 3.
Compared to the B&H strategy, the TIPP strategy leads to a significant reduction in risk, which is indicated by the values for the hedge effectiveness. The risk reduction effect increases with increasing multiplier values.

The following section shows the results of the TIPP-M strategy. Firstly, the strategy is considered, in which the mean value is calculated from the stock portfolio prices of the past five periods and the current price (i.e. from the last six stock portfolio prices available) ("TIPP-M6"). The values shown in Table 4 can be determined for this strategy.

<table>
<thead>
<tr>
<th>m</th>
<th>m=1</th>
<th>m=2</th>
<th>m=3</th>
<th>m=4</th>
<th>m=5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_{\text{mean}} )</td>
<td>0.0194%</td>
<td>0.0258%</td>
<td>0.0270%</td>
<td>0.0242%</td>
<td>0.0199%</td>
</tr>
<tr>
<td>( \text{VaR} )</td>
<td>-5.58556</td>
<td>-8.37206</td>
<td>-9.99615</td>
<td>-10.62660</td>
<td>-10.63512</td>
</tr>
<tr>
<td>( V_{\text{max}} )</td>
<td>1.727724</td>
<td>2.679896</td>
<td>3.742391</td>
<td>4.816412</td>
<td>5.551146</td>
</tr>
<tr>
<td>( V_{\text{min}} )</td>
<td>914.901</td>
<td>909.899</td>
<td>904.874</td>
<td>900.552</td>
<td>899.700</td>
</tr>
<tr>
<td>( \text{RR}_{\text{m}} )</td>
<td>-0.0038%</td>
<td>-0.0161%</td>
<td>-0.0307%</td>
<td>-0.0474%</td>
<td>-0.0640%</td>
</tr>
<tr>
<td>( t_{\text{RR}} )</td>
<td>-39.80</td>
<td>-86.49</td>
<td>-98.05</td>
<td>-100.10</td>
<td>-100.33</td>
</tr>
<tr>
<td>( \text{HE}_{\text{m}} )</td>
<td>9.8543%</td>
<td>30.160%</td>
<td>41.9076%</td>
<td>51.3115%</td>
<td>59.0878%</td>
</tr>
</tbody>
</table>

It is also noticeable with the TIPP-M6 strategy that the final portfolio value after 500 periods only falls below the initial floor value of 900,000 € in the case of m=5 (and there only in one of 10,000 simulations). Thus, the TIPP-M6 strategy also appears to be successful with regard to a downward hedge.

However, even with the TIPP-M6 strategy, the average of all 10,000 rebalancing returns is negative. With very high absolute t-values, the rebalancing returns for all values of m are statistically significantly negative, i.e. the average geometric return after 500 periods is significantly lower with the TIPP-M6 strategy than with the B&H strategy.

The frequency distribution of the rebalancing return over all 10,000 runs – again in the case that m = 3 – can graphically indicate this significance (Figure 4).
The risk is significantly lower with the TIPP-M6 strategy than with the B&H strategy. It is striking that the TIPP strategy leads to higher hedge effectiveness values for all multipliers than the TIPP-M6 strategy. Thus, the risk reduction effect is higher for the TIPP strategy. This is also true for the TIPP-M12 strategy, in which the mean value is calculated from the stock portfolio prices of the past 11 periods and the current price (i.e. from the last 12 stock portfolio prices available). This strategy comes to similar conclusions as the TIPP-M6 strategy (Table 5).

<table>
<thead>
<tr>
<th></th>
<th>m=1</th>
<th>m=2</th>
<th>m=3</th>
<th>m=4</th>
<th>m=5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{r}_{\text{log}}^m$</td>
<td>0.0194%</td>
<td>0.0251%</td>
<td>0.0256%</td>
<td>0.0226%</td>
<td>0.0182%</td>
</tr>
<tr>
<td>VaR</td>
<td>-5.638,79</td>
<td>-8.411,75</td>
<td>-9.949,71</td>
<td>-10.514,75</td>
<td>-10.492,14</td>
</tr>
<tr>
<td>$V_{\text{max}}$</td>
<td>1.730,153</td>
<td>2.626,569</td>
<td>3.655,536</td>
<td>4.585,519</td>
<td>5.163,407</td>
</tr>
<tr>
<td>$V_{\text{min}}$</td>
<td>913,760</td>
<td>905,635</td>
<td>900,416</td>
<td>900,058</td>
<td>899,693</td>
</tr>
<tr>
<td>$RR_m$</td>
<td>-0.0038%</td>
<td>-0.0168%</td>
<td>-0.0321%</td>
<td>-0.0490%</td>
<td>-0.0656%</td>
</tr>
<tr>
<td>$\text{IQR}$</td>
<td>-38,21</td>
<td>-81,56</td>
<td>-94,79</td>
<td>-98,61</td>
<td>-99,86</td>
</tr>
<tr>
<td>$HE_{\text{PI}}^m$</td>
<td>8,8680%</td>
<td>29,6165%</td>
<td>41,9145%</td>
<td>51,6804%</td>
<td>59,5528%</td>
</tr>
</tbody>
</table>

The frequency distribution of the rebalancing return over all 10,000 runs – again for the case that $m = 3$ – is shown in Figure 5. It is similar to that of the TIPP-M6 strategy.

Figure 4. TIPP-M6 with $m=3$: Distribution of the rebalancing returns of 500 rebalancing periods. 10,000 simulations are considered.

Figure 5. TIPP-M12 with $m=3$: Distribution of the rebalancing returns of 500 rebalancing periods. 10,000 simulations are considered.
The above mentioned results are summarized in Table 6. For each case, the table displays which strategy produced the best result, i.e. the highest values for average logarithmic return, maximum and minimum final portfolio value, average rebalancing return and average hedge effectiveness. With regard to the value at risk, the strategies that lead to the lowest (absolute) values are displayed.

Table 6: Overall results of B&H, CPPI, TIPP, TIPP-M6 and TIPP-M12

<table>
<thead>
<tr>
<th>m=1</th>
<th>m=2</th>
<th>m=3</th>
<th>m=4</th>
<th>m=5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_{\text{log}} )</td>
<td>CPPI/B&amp;H</td>
<td>CPPI</td>
<td>CPPI</td>
<td>CPPI</td>
</tr>
<tr>
<td>VaR</td>
<td>TIPP</td>
<td>TIPP</td>
<td>TIPP</td>
<td>TIPP</td>
</tr>
<tr>
<td>( V_{\text{max}} )</td>
<td>CPPI/B&amp;H</td>
<td>CPPI</td>
<td>CPPI</td>
<td>CPPI</td>
</tr>
<tr>
<td>( V_{\text{min}} )</td>
<td>TIPP</td>
<td>TIPP-M6</td>
<td>TIPP-M6</td>
<td>TIPP-M6</td>
</tr>
<tr>
<td>( RR_{m} )</td>
<td>CPPI/B&amp;H</td>
<td>CPPI</td>
<td>CPPI</td>
<td>CPPI</td>
</tr>
<tr>
<td>( HE_{m} )</td>
<td>TIPP</td>
<td>TIPP</td>
<td>TIPP</td>
<td>TIPP</td>
</tr>
</tbody>
</table>

5. Conclusion

In this paper the Monte Carlo simulation is used to analyse portfolio insurance strategies. These strategies are designed to protect portfolios against large losses by a contractually guaranteed predetermined floor through a dynamic allocation. The goal of these strategies is to reduce downside risk and to participate in rising markets. The analysis shows that the CPPI strategy leads to the highest average returns of the portfolio and thus to the highest rebalancing returns. At the same time, the risk – measured as value at risk with a confidence level of 95% – is lowest for the TIPP strategy. Thus, the hedge effectiveness measure used in this study is the highest for the TIPP strategy. Compared to the B&H strategy, the value at risk can be reduced by up to 60% (in case of a multiplier of 5). The CPPI strategy, on the other hand, leads to the highest maximum final portfolio values at the end of the 500 (weekly) periods considered. The TIPP and TIPP-M6 strategies lead to the highest results for the minimum portfolio end values, although it should be noted that the end values differ only to a small extent from each other, as they are all close to € 900,000.

In this study, transaction costs are neglected. In addition, an interest rate of 0 % is assumed for the risk-free position. Future studies on the rebalancing return and hedge effectiveness of portfolio insurance strategies should include both transaction costs and different risk-free interest rates.

References


