Credit default swaps, regulatory arbitrage and banking regulation

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Abstract
This paper analyzes theoretically the impact of credit default swaps (CDS) on the regulatory capital required in a banking system. We develop a simple capital requirement model with and without CDS, which shows that CDS reduces banking system capital ratio. The model also highlights the regulator’s ability to use prudential ratios at their disposal to limit regulatory arbitrage. An enhancing of this model via a reduced-form default model, inspired by Duffie and Singleton (1999), shows the possibility for banks facing the regulator’s desire to reduce regulatory arbitrage, to affect the level of regulatory capital savings. This becomes possible if the CDS market gives them the opportunity to influence some parameters related to the CDS market value: the default intensity, the partial recovery rate, the spread, the risk-free rate. Therefore, in addition to its intervention on prudential ratios, the regulator should limit regulatory arbitrage effectively by intervening also in the CDS market to counter the strategic use of CDS by banks.

Key words: bank, prudential regulation, regulatory arbitrage, credit default swaps.

JEL Classification: G19, G28, G33

1. Introduction
Credit default swaps (CDS), a form of credit risk insurance, were engineered in 1994 by the US bank J. P. Morgan Inc. to transfer credit risk exposure from its balance sheet to protection sellers. Specifically, they are swap or over-the-counter (OTC) contracts that transfer, through a contract between two counterparties, all or part of the credit risk and the yield to a third party called a reference entity, without actually giving up the ownership of the underlying asset (International monetary fund IMF, 2002). A development of the credit derivatives market in the financial system could make it possible to take advantage of the growth potential of the debt market, which represents a large volume of trade: bank loans to individuals, corporate loans, so-called sovereign loans or sovereign debts (Greenspan, 2004). The establishment of such a market requires a critical size in the financial market to provide a certain depth of market, liquidity, dynamism and potential for geographic diversification. However, mispriced or poorly managed credit derivatives can lead to increased financial instability or systemic risk (Bruyère, 1998; Duffie, 1999; Duffee and Zhou, 2001; Rey, 2009). In fact, if banks run a liquidity risk that they cover on the interbank market, the transfer of credit risk may lead to a contagion effect between the banking sector and the insurance sector and, as a result, create a systemic risk (Allen and Carletti, 2006). This is a paradox, especially since credit derivatives, conceived theoretically to spread risk, end up leading to an increased concentration of risks (Laurent, 2000). By distorting the incentives for monitoring and risk management, CDS can lead to an increase in operational risk and greater difficulties in setting up an operational plan for monitoring and controlling risks. Credit derivatives are also considered to be one of those responsible for the 2007-2008 financial crisis (Tett, 2009). Warren Buffett (2003) denounced derivatives as weapons of mass destruction.

These disadvantages, however, should not obscure the benefits of credit derivatives. Like any financial instrument that allows the level and type of risk to be adapted to those desired, credit derivatives are able to facilitate the diversification of risks borne by banks (Prato, 2002; Bomfim, 2002; Vinod, 2003). When used well, they increase the liquidity and overall efficiency of markets by improving the ability of operators to optimize their exposure to credit risk (Kiff and Morrow, 2000; Finger, 2002). It is possible that the banking system, provided it is able to control the risks of financial instability induced by the use of credit derivatives, can take advantage of the original virtues of credit derivatives (O’Kane and Mc Adie, 2001). Prior to the invention of credit derivatives, banks managed the insolvency risk of the debtor both through various collateral and the risk-based capital requirements of loan contracts. In case of default, the required capital cashes the loss. The credit derivatives mechanism allows a bank to lend without assuming the full risk of the transaction and without increasing its capital requirements (Flesaker, Schreiber and al, 1994). In doing so, it reduces its exposure to credit risk when it considers capital adequacy standards disproportionate to the actual risk incurred (Kessler and Levenstein, 2001). The adoption of credit derivatives by banks corresponds to a paradigm shift in the activity of bank credit that contrasts with the standard model of bank credit (Morrisson, 2002).

Theoretical and empirical research have been conducted to advance our knowledge of credit derivative products, and to improve our understanding of the economic role of CDS contracts. These studies were initially concerned with models for the pricing of CDS using the fundamental principles of replicating strategies (Das, 1995; Duffie, 1999; Duffie and Singleton, 2003). Then, research on CDS has quickly expanded into a broad research field.
ranging from corporate finance to market finance going through financial intermediation and regulation (Jarrow, 2011). Studies have examined the effect of CDS on the capital structure. CDS trading reduces borrowing costs in equilibrium (Goderis and Wagner, 2011; Salomao, 2014) and can affect default risk and bankruptcy costs (Che and Sethi, 2014). Another strand of research has examined the impact of CDS on the efficiency, quality, and liquidity of the bond market (Das, Kalimipalli and Nayak, 2014) and the interactions between the CDS market and the stock market in terms of information flow (Hilscher, Pollet and Wilson, 2015). Some studies focus on the strategic use of CDS, a real revolution in banking and risk management. A model proposed by Hakenes and Schnabel (2010) shows that the insurance provided by CDS increases the credit supplied to risky borrowers in the presence of risk sharing. Such practices may also lead to excessive risk taking, with destabilizing effects on aggregate risk (Biais, Heider and Hoerova, 2016). CDS can be used for regulatory capital relief, in order to improve regulatory capital adequacy for individual banks. They thus make it possible for banks to save capital and to improve their profitability (Froot, 2001). The strategic use of CDS may lead to other unintended externalities with ambiguous welfare implications in the context of financial intermediation, such as efficient risk sharing (Thompson, 2010) and improved risk management (Norden, Buston and Wagner, 2014), but also reduced monitoring incentives for banks (Arping, 2014), adverse selection and moral hazard in the bank–debt relationship (Chakraborty, Chava and Ganduri, 2015) and counterparty risk (Stephens and Thompson, 2014).

The strategic use of CDSs to improve regulatory capital adequacy for individual banks, or regulatory arbitrage, enables banks to legally circumvent the prudential constraints imposed on their balance sheet assets. However, it can be done at the expense of making them more vulnerable to systemic shocks (Yorulmazer, 2013) and of creating contagion because of credit risk transfer (Allen and Carletti, 2006). For regulators, arbitrage regulatory is simply exploiting the legal vacuum linked to the complexity of transactions involving CDS (Ringe, 2015). For this reason, they warn banks against the temptation to conduct regulatory arbitrage transactions that would allow them to save on capital (Basel Committee, 2016). In this context, regulators seek to increase the stability and transparency of the CDS market, by requiring regulated financial institutions to limit their use of CDSs, and by imposing on them capital requirements and strict oversight (Shadab, 2009; Carlson and Margaret, 2014). But Regulators and other decision-makers can implement new financial regulations that often worsen the problem, if they have not before accumulated sufficient theoretical analysis on CDS (Augustin and al., 2016). In this paper, we develop a theoretical model that sheds light on the difficulty regulators may have in putting in place measures that limit regulatory arbitrage. Indeed, this model shows the regulator's ability to use prudential rules to limit regulatory arbitrage in the banking system. But despite these prudential constraints, banks can still do regulatory arbitrage if they have the opportunity to influence the CDS market in their favor. The rest of the article is structured as follows: Section 2 models the capital savings due to CDS and how the regulator can limit such regulatory arbitrage. In Section 3, we highlight the possibility for banks facing the regulator’s desire to reduce regulatory arbitrage, to affect the level of regulatory capital savings. Section 5 makes recommendations and concludes.

2. Regulatory arbitrage and banking regulation: a theoretical model

2.1. The assumptions

Consider a banking system composed of \( n \) commercial banks. Without loss of generality, we can restrict ourselves to the case of two banks A and B. Bank A whose capital value is noted \( N \) has a nominal value bond fixed, for the sake of simplicity, at 1. This obligation is issued by a company C with a well-defined rating. Bank A purchases protection at Bank B in the form of a CDS whose reference asset is this bond. It is assumed that the regulator supports the development of the credit derivatives market depending on whether the position is "short" or used to cover another. The capital requirement for a short position is the same as for an equivalent cash position on the reference asset. The principles that imply the regulatory ratios set by the regulator in this model are in line with those of Basel Committee. Here, the ratio values are taken in their general expression. The regulator imposes capital requirements on the three actors A, B and C as follows:

- The weighting applicable to issuer C of the bond is set to \( c (c = 100\% = 1 \text{ in Basel 2}) \);
- Issuer C is subject to the maintenance of a minimum capital ratio of \( a \) (in %);
- Bank A wishing to hedge against C's obligation must then constitute a reserve of \( a \);
- The weighting applicable to bank B is \( b \) (in %);

After the conclusion of the CDS, the weighting applicable to bank B, that is, \( b \), replaces that of the issuer of the bond. This CDS seller bank has a higher credit rating than the issuer of the reference asset. So, regulated bank B is subject to a lower ratio of \( a \times b \) (with \( ab < b \)). In this case, the capital requirement to which bank A will be submitted will be not the ratio \( a \) but the ratio \( ab \). Let us make the additional assumption of a zero correlation between the failures of entities B and C so that they are not simultaneously deficient as to their commitments. In addition, the regulator sets a regulatory surcharge (or add-on) as a counterparty risk requirement. This
coefficient denoted \( d \) is that applicable to interest rate contracts with a remaining term of more than 5 years. The counterparty weighting is capped at \( e \), as in all off-balance sheet derivative transactions.

It is assumed that the market value of the CDS is always lower than the face value of the underlying bond. This is because the CDS is a form of insurance purchased to protect against the default of the underlying bond. A simple common sense rule is that one buys insurance at a lower price than what worth the product insured. Formally, we have:

\[
MtM < 1 \quad (1)
\]

In addition, the value of the equity of bank A is considered to be greater than or equal to the nominal value of the bond. In fact, the bond that bank A has acquired from the issuing company C is generally accounted for as part of the bank’s own funds. Thus, we have:

\[
N \geq 1 \quad (2)
\]

### 2.2. Determination of the capital savings ratio

Taking into account the assumptions mentioned above, it follows that the capital requirement for bank A, before the conclusion of the CDS, is:

\[
K^\text{without CDS}_A = aN
\]

That of bank B is zero: \( K^\text{without CDS}_B = 0 \). Thus, in the absence of a CDS contract, the capital requirement at the level of the banking system \( K^\text{without CDS}_B \) is:

\[
K^\text{without CDS}_B = K^\text{without CDS}_A + K^\text{without CDS}_B = aN
\]

How are these results affected when introducing CDS into the banking system? With the zero correlation between the failures of Entities B and C, the buying bank A of CDS does not suffer a default loss, and its capital requirement is:

\[
K^\text{with CDS}_A = abN \quad (4)
\]

For Bank B, which classifies CDS in its trading portfolio and goes back to an insurance company, the capital requirement corresponds to the counterparty risk assumed by Bank B on the insurance or reinsurance undertaking. Bank B does not record a specific risk given the strict identity of its buying position (vis-à-vis bank A) and its short position (vis-à-vis the insurance company or reinsurance). This counterparty risk reduction, subject to certain capital requirements, is done without altering the existing commercial relationship with this counterparty.

Noting \( MtM \) the market value of the CDS, the capital requirement for bank B amounts to \( K^\text{with CDS}_B = [MtM + (N \times d)] \times e \times a \). We obtain:

\[
K^\text{with CDS}_B = aeMtM + dN \quad (5)
\]

Let us make the additional assumption that the only risks that the use of CDS affects are credit risks. Other risks (operational risk, market risk, systemic risk, etc.) are not expected to be affected by the use of CDS. This implicitly implies that the regulator has managed to contain the risks of instability related to CDS. This can be done by prohibiting or restricting the speculative dimension of CDS.

In the case of a CDS contract, the capital requirements at the level of the banking system are obtained by adding those of banks A and B:

\[
K^\text{with CDS}_B = K^\text{with CDS}_A + K^\text{with CDS}_B \Rightarrow K^\text{with CDS}_B = abN + aeMtM
\]

Having reached this stage of the analysis, it is possible to compare the regulatory capital of the non-CDS banking system with the regulatory capital of the banking system with CDS. We obtain the following proposition:

**Proposition 1:** CDSs can be used for regulatory arbitrage purposes, thus allowing banks to legally circumvent prudential constraints on their balance sheet assets. Indeed, the use of CDS as an instrument for hedging bank credit risk leads to a decrease in the regulatory capital requirement in the banking system. We have:

\[
K^\text{with CDS}_B < K^\text{without CDS}_B \quad (7)
\]

**Proof of proposition 1:** just compare the equations (3) and (6): \( acN \) and \( (ab + aed)N + aeMtM \).

To compare \( acN \) and \( (ab + aed)N + aeMtM \), we proceed in several stages. From relation (1), we can write that:

\[
(ab + aed)N + aeMtM < (ab + aed)N + ae \quad (8)
\]

Let’s compare \( (ab + aed)N + ae \) and \( acN \). Consider the difference \( \mathcal{D} = acN - (ab + aed)N - ae \). This is simplified to \( \mathcal{D} = (ac - ab - aed)N - ae \). \( \mathcal{D} = 0 \) for \( N = \frac{e}{c-b-ed} \); \( \mathcal{D} > 0 \) for \( N > \frac{e}{c-b-ed} \); \( \mathcal{D} < 0 \) for \( N < \frac{e}{c-b-ed} \). It follows that:

\[
\begin{cases}
(ab + aed)N + ae & \leq acN \quad \text{for } N \geq \frac{e}{c-b-ed} \\
(ab + aed)N + ae & \geq acN \quad \text{for } N \leq \frac{e}{c-b-ed}
\end{cases}
\]
By virtue of relation (2), we have \( N \geq 1 \). This implies that one can never have \( N \leq \frac{e}{c-b-ed} \). It follows that
\[
N \geq \frac{e}{c-b-ed},
\]
which implies \((ab + aed)N + aeN \leq acN\) (9). By combining the relations (8) et (9), it comes that:
\[
(ab + aed)N + aeMtM < acN
\]
And finally:
\[
K^\text{with CDS}_\text{SB} < K^\text{without CDS}_\text{SB} \quad \text{QED}
\]
Define \( \Pi \) as the regulatory capital saving ratio. It is equal to the capital regulatory capital ratio with CDS on regulatory capital capital without CDS. This ratio is bounded lower by 0 and higher by 1: \( 0 < \Pi < 1 \). The closer it gets to 1, the less capital there is. The more \( \Pi \) tends to 0, the greater the savings in own funds. Formally, this ratio is written:
\[
\Pi = \frac{K^\text{with CDS}_\text{SB}}{K^\text{without CDS}_\text{SB}} = \frac{(ab + aed)N + aeMtM}{acN} = \frac{(b + ed)N + eMtM}{cN} < 1
\]
This expression makes it possible to highlight the impact of prudential rules on the capital savings that can be realized through CDS. This impact is summarized in the following proposition (proof in Appendix A1):

**Proposition 2**: The regulatory capital savings that the banking system can achieve through the use of CDS are all the greater as the levels of the following regulatory ratios decrease: the weighting \( b \) applicable to the selling bank of the CDS, the coefficient \( d \) and the weighting \( e \) applicable to the counterparty. The regulatory capital savings increase when the level of weighting \( c \) applicable to issuer C of the bond, increases. On the other hand, in the model, the minimum capital ratio \( a \) has no effect on capital savings.

This result shows that unlike the other regulatory ratios used in the model, the level of the minimum capital ratio \( a \) does not affect the level of capital savings. Indeed, if all other ratios and other parameters of the model are held constant, the equity levels payable with or without CDS are allocated in the same proportions by the minimum capital ratio. This prudential ratio is therefore in no way related to the use or not of the CDS. Proposition 2 also shows that regulators can use the regulatory ratios available to them to limit regulatory arbitrage. In fact, regulators, increasingly thinking that regulatory arbitrage is simply exploiting the legal vacuum linked to the complexity of transactions involving CDS, are increasingly trying to warn banks against the temptation to achieve regulatory arbitrage (Ringe, 2015, Basel Committee, 2016).

Can banks counterbalance the ability of regulators to limit regulatory arbitrage through prudential ratios? We show in the following of this article that, despite prudential ratios deliberately set by the regulator, banks can still influence the level of regulatory capital savings, if they are able to influence the market value of the CDS, \( MtM \), via certain variables to be determined. To do this, we determine a more complete expression of the regulatory capital ratio \( \Pi \) by expressing the market value of the CDS, \( MtM \), according to certain characteristics of the CDS market (default intensity, partial recovery rate, spread, risk free rate).

### 3. A possibility of regulatory arbitrage despite banking regulation

#### 3.1. Determination of the market value of the CDS

The market value of the CDS, ie, \( MtM \), is the payment made by the selling bank B to the acquiring bank A in the event of default by the issuer C of the underlying bond. If the default occurs before the maturity of the swap, Bank B makes a payment to Bank A, equivalent to the difference between the nominal of the debt covered by the swap and the recovery rate observed at the moment of default. To determine the market value of the CDS, a reduced form model is used. The reduced-form models date back to Pye (1974) and have been popularized by many works, particularly those of Duffie and Singleton (1999). In these models, the default is considered an unpredictable event whose law is governed by a stochastic process called arrival intensity or hazard rate. The simplest example of a reduced-form model is that where the moment of default \( \tau \) is defined as the first moment of arrival of a Poisson process of intensity default \( \lambda \). In other words, the moment of default \( \tau \) follows an exponential law of parameter \( \lambda > 0 \):

\[
P[\tau > t] = e^{-\lambda t} \quad ; \quad E[\tau] = \frac{1}{\lambda} \quad ; \quad P[\tau \in (t, t + dt)|\tau > t] = \lambda dt
\]

The market value of the CDS is the difference between the purchase price of the CDS (the fixed leg) and the future revenue stream generated by the CDS (the variable leg). The fixed leg of the CDS is the spread pay leg on regular dates \( t \), until maturity \( T \) of the CDS, except in the event of default by the reference entity. Assuming that the premium is paid up to default, and without taking into account the accrued coupon denoted \( CC \) (with \( CC = 0 \)), and considering the parameters \( s \) (CDS spread) and \( I_{[\tau > t]} \) (indicator variable equal to 1 when \( \tau > t \) and equal to 0 otherwise), the fixed leg of the CDS is a net present value (NPV) of the spread whose expression is (proof in appendix A2):

\[
JF(s) = E \left[ e^{-rt} \sum t \in T(t - t_{i-1})I_{[\tau > t]} \right] + CC = \sum t \left( t - t_{i-1} \right) e^{-(\rho + \lambda) t_i}
\]

This expression of the fixed leg of the CDS is a Riemann sum which converges, assuming that the payment of
the spread is done continuously, to an integral of the form:

\[
JF(s) \approx \int_0^T \frac{e^{-(r+\lambda)t}}{r+\lambda} dt = \int_0^T \frac{1}{r+\lambda} dt = s(t,T)DV(0,T)
\]

where \( DV(0,T) = \frac{1 - e^{-(r+\lambda)t}}{r+\lambda} \) (14)

The expression \( DV(0,T) \) is a generalization of the notion of duration in case of credit risk. This risky duration, called Dollar Value (DV), is equal to the duration of the bond that remunerates the risk-free asset continuously if the default intensity is zero. It takes into account the absence of spread payment for cases where default occurs before maturity. Thus, the expression of the fixed leg is equal to the spread multiplied by the dollar value.

The variable leg, defined as the future income stream generated by the CDS, is equal to the discounted loss in case of default of issuer C of the bond, multiplied by the nominal of the bond assumed to be 1. Here we assume that this future stream is paid at the instant \( \tau \) of occurrence of the default. Assuming a fractional recovery of market value in case of default with \( \delta \) the recovery rate, the Loss Given Default is equal to:

\[
\text{LGD} = 1 - \delta \]

The expression of the variable leg of the CDS is then:

\[
PV = \frac{1 - e^{-(r+\lambda)t}}{r+\lambda} \]

At this point, we can calculate the market value of the CDS defined as the difference between the fixed leg of the CDS and its variable leg. Formally, it gives:

\[
MtM = JF(s) - PV = s(t,T) \frac{1 - e^{-(r+\lambda)t}}{r+\lambda} - (1 - \delta)(1 - e^{-\lambda t}) \]

3.2. A complete expression of the capital savings ratio

By integrating equation (16) into equation (11), we determine a more complete expression of the regulatory capital saving ratio \( \Pi \) as follows:

\[
\Pi = \frac{b + ed}{c} + e \frac{1}{cN} \left[ s(t,T) \frac{1 - \exp(- (r + \lambda)T)}{r + \lambda} - (1 - \delta)(1 - \exp (-\lambda t)) \right] \]

It is considered that, to counter the regulatory arbitrage made possible by the use of CDS, the regulator sets the ratios \( (b, c, d, e) \) to \( (b, c, d, e) \). We get the relation:

\[
\Pi(\delta, \lambda, \tau) \]

Equations (17) and (18) highlight the possibility of lowering the ratio of regulatory capital savings, by influencing the values of the parameters that are the spread \( s(t,T) \), the intensity of default \( \lambda \), the partial recovery rate \( \delta \) and the risk-free rate \( r \). More specifically, this ratio \( \Pi \) decreases when these 4 parameters decrease (proof in Appendix A3). But these parameters can relatively be affected by the banks. The risk-free rate in the money markets can be influenced by the behavior of banks. Banks can also influence the partial recovery rate and the default intensity of the bond issuer. Proposition 3 summarizes these results:

**Proposition 3:** For fixed values of prudential ratios, it is still possible for the banking system to make regulatory arbitrage by downwardly influencing the risk-free rate \( r \), the default intensity \( \lambda \), the spread \( s \) and the partial recovery rate \( \delta \). These effects on these parameters lead to a decline in the market value of the CDS and an increase in regulatory capital savings, ceteris paribus. Therefore, the intervention of the regulator on prudential ratios is not enough to limit regulatory arbitrage effectively. The regulator should intervene also in the CDS market in order to counter the strategic use of CDS by banks.

4. Concluding Remarks

In this paper, we have developed a model to assess the capital savings that the banking system can achieve by using CDS. The use of CDS allows the banking system to do regulatory arbitrage by saving capital. The model also shows that regulators can use regulatory ratios at their disposal to limit regulatory arbitrage. However, it is possible that, in the face of this possibility for regulators to limit regulatory arbitrage, banks may relatively affect the level of regulatory capital savings. This becomes possible if the CDS market gives them the opportunity to act on certain parameters related to the CDS market value: the default intensity, the partial recovery rate, the spread, the risk-free rate. Therefore, in addition to its intervention on prudential ratios, the regulators should limit regulatory arbitrage effectively if they intervene also in the CDS market to counter the strategic influence of banks in this market.
The model developed analyzes the issue of regulatory arbitrage in the presence of CDS by statically considering the relationship between the regulator and the banks: the regulator sets the regulatory ratios so as to minimize regulatory arbitrage due to CDS. From these fixed ratios, banks can always react by saving equity if the market offers them the opportunity to act on the market value of CDS. With a view to deepening this model, it is possible to analyze this issue of banking regulation in the presence of regulatory arbitrage by resorting to the game theory to take into account the strategic interactions between the regulator and the banks. The equilibrium values of the prudential ratios and those of the parameters related to the CDS market will then be determined from the reaction functions of the regulator and the representative bank, each of these actors optimizing its objective function by anticipating the behavior of the other.

In addition, another extension of the model is to model the behavior of the regulator who wishes to minimize regulatory arbitrage by taking into account the effects of CDS on the credit supply, the cost of credit and the transmission mechanisms of monetary policy to credit market. Indeed, CDSs can help expand credit supply capacity and reduce the risk of credit rationing (Duffee and Zhou, 2001, Aglietta, 2008). Such an increase in lending capacity may be tempered somewhat by the fact that loan prices include the cost of hedging credit risk via credit derivatives. This is likely to increase the cost of intermediation with the risk of exacerbating foreclosure effects. In addition, the ability of credit derivatives to increase banks' lending potential through better risk diversification/dispersion can reduce the effectiveness of monetary policy in influencing credit supply (Estrella and Jeffrey, 2002). Further study is expected in the future.

References


Shadab, H. S., 2009, “Credit Default Swaps and Regulatory Reform”, Mercatus on Policy, n°56, August.


Appendix A1:
The expression of the capital savings ratio is written as:

\[ \Pi = \frac{(b + e d)N + e M t M}{c N} \]  \hspace{1cm} (A1.1)

From this expression, we highlight the 4 variables that affect this ratio:

\[ \Pi \equiv \Pi(b, c, d, e) \]  \hspace{1cm} (A1.2)

We calculate the partial derivatives with respect to all the explanatory variables of this ratio.

\[ \frac{\partial \Pi}{\partial e} = \frac{d}{c} + \frac{e M t M}{c N} > 0 \]  \hspace{1cm} (A1.3)

\[ \frac{\partial \Pi}{\partial b} = \frac{1}{c} > 0 \]  \hspace{1cm} (A1.4)

\[ \frac{\partial \Pi}{\partial d} = \frac{e}{c} > 0 \]  \hspace{1cm} (A1.5)

\[ \frac{\partial \Pi}{\partial c} = -\frac{b + e d}{c^2} - \frac{1}{c^2} \frac{e M t M}{N} < 0 \]  \hspace{1cm} (A1.6)

\[ \frac{\partial \Pi}{\partial a} = 0 \]  \hspace{1cm} (A1.7)

Appendix A2:
The fixed leg of CDS is equal to:

\[ F(s) = E\left[e^{-r t_i} \sum_i s(t_i - t_{i-1}) \mathbb{1}_{[t > t_i]} \right] + CC \]  \hspace{1cm} (A2.1)

If we do not take into account the accrued coupon, knowing that the premium is paid until the default, we have

\[ CC = 0. \]  \hspace{1cm} (A2.2)

Knowing that the moment of defect \( \tau \) follows an exponential law of parameter \( \lambda > 0 \):

\[ P[\tau > t] = e^{-\lambda t} \]  \hspace{1cm} (A2.4)

We get:

\[ F(s) = e^{-r t_i} \sum_i s(t_i - t_{i-1}) \times P[\tau > t] \]  \hspace{1cm} (A2.3)

Which one gets:

\[ F(s) = e^{-r t_i} \sum_i s(t_i - t_{i-1}) \times e^{-\lambda t} \times e^{-r t_i} \]  \hspace{1cm} (A2.5)

\[ F(s) = \sum_i s(t_i - t_{i-1}) e^{-(r + \lambda) t_i} \]  \hspace{1cm} QED  \hspace{1cm} (A2.6)

Appendix A3:
The expression of the capital savings ratio is written as:

\[ \Pi = \frac{b + e d}{c N} + e \left[ \frac{1}{T} \ln \left(1 - \exp(-(-r + \lambda)T)ight)\right] \]  \hspace{1cm} (A3.1)

Fixing the ratios \((b, c, d, e)\) to \((\tilde{b}, \tilde{c}, \tilde{d}, \tilde{e})\), we get the functional relation that highlights the 4 variables that affect this capital savings ratio:

\[ \Pi \equiv \Pi(s, \lambda, \delta, r) \]  \hspace{1cm} (A3.2)

From this expression, we calculate the partial derivatives with respect to all the explanatory variables of this ratio:

\[ \frac{\partial \Pi}{\partial r} = \frac{e}{c N} \left[ \frac{T(r + \lambda) \exp(-(r + \lambda)T) + \exp(-(r + \lambda)T) - 1}{(r + \lambda)^2} \right] > 0 \]  \hspace{1cm} since \( T > 1 \)  \hspace{1cm} (A3.3)

\[ \frac{\partial \Pi}{\partial \lambda} = \frac{e}{c N} \left[ \frac{T(r + \lambda) \exp(-(r + \lambda)T) + \exp(-(r + \lambda)T) - 1}{(r + \lambda)^2} \right] > 0 \]  \hspace{1cm} since \( T > 1 \) and \( T \geq t \)  \hspace{1cm} (A3.4)

\[ \frac{\partial \Pi}{\partial \delta} = \frac{e}{c N} \left(1 - \exp(-\lambda t)\right) > 0 \]  \hspace{1cm} (A3.5)

\[ \frac{\partial \Pi}{\partial s} = \frac{e}{c N} \left[1 - \exp(-(r + \lambda)T)\right] > 0 \]  \hspace{1cm} (A3.6)

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