Modeling with ARIMA-ARCH/GARCH Techniques to Estimate Weekly Exchange Rate of Liberia

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Abstract
The current research employs the estimations of univariate linear time series, ARIMA and two traditional volatility time series models, ARCH and GARCH to analyze the behavior of exchange rate volatility in the Liberian economy using weekly time series observation spanning from January 07, 2013 to December 25, 2017. This study estimated the parameters of the selected models and detected the irregular pattern the financial series portrays in the Liberian economy. Evidently, the paper finds huge volatility and fat tail distribution in the exchange rate series of Liberia and as such the series behavior is worrisome which needs to be expediently modeled well. Additionally, the ARCH and GARCH models were estimated separately to capture the volatility pattern in the series. The results show that there is persistence volatility in the financial series as the estimated ARCH parameter was equal to unity and the sum of the ARCH and GARCH terms were close to unity in the GARCH(1,1) specification. On the other hand, after assuming generalized error distribution for the exchange rate series due to its fat tail, the parameters on the volatility models reduced significantly but the means of these equations were practically zero. In addition, the two volatility models were re-estimated to the residuals of the ARIMA to model the noise in the univariate time series model. The results reveal that the models performed remarkably well when fitted to the residuals of the ARIMA(1,1,2) model. The recommendation from this empirical research is that with the high persistence in the series and the risk as well as the very low returns it comes with, modelers and policymakers should estimate the parameters of the exchange rate effectively and with care before any point forecast can come into play because knowledge about the distribution and the calculated returns will all aid in better prediction.
Keywords: Exchange rate volatility, ARIMA model, ARCH model, GARCH model, Volatility clustering, Liberia

Introduction

The issue of tackling exchange rates irregular nature in the economic arena has been extensively studied and there are more new developments still evolving in the time series literature. The need of exchange rate forecasting in order to prevent its disruptive movements has engrossed many policy makers and economist for many years (Fahimifard, et al., 2009). Furthermore, (Fahimifard, et al., 2009) state that the determinant of exchange rate have grown manifold making its behavior complex, nonlinear and volatile so that nonlinear models have better performance for its forecasting. A recent report in Liberia (International Monetary Fund, 2017)shows that the deterioration in the supply of foreign exchange in the Liberian economy poses risk to both internal and external balance. In December 2016, International Monetary Fund (IMF) provided help to secure additional foreign currency for intervention purposes in the midst of the foreign exchange shortage and depreciation pressures, the Central Bank of Liberia (CBL) introduced a 25 percent surrender requirement on inward remittances through money transfer companies. Up to mid-2017, the CBL has used a major portion of the U.S. dollars acquired from the surrender requirement to intervene in the foreign exchange market and the rest to augment its reserves. The net foreign exchange position of the CBL, computed at program exchange rates, declined from US$178 million in June 2016 to US$146 million in June 2017 (Lagarde, 2017).

(Stockman, 1978) explains in his thesis that exchange rates and their rates of change in the course of time have been more volatile than relative prices and rates of inflation, and therefore, they are often as reported in the literature to be inconsistent with equilibrium. “Attempts to manage exchange rate volatility and its overshooting tendencies started after the failure of the Breton Woods System in 1971” (Stockman, 1978). (Panda & Narasimhan, 2003) recent work describes that in order to keep inflation stable at appropriate level and economic activity at higher level, the monetary authority must have confidence, which will come through the better understanding of the movements of exchange rate, in conducting the monetary policy. Recently, there are growing numbers of research on exchange rate volatility in the literature with uncountable methods and approaches. The quest to model exchange rate volatility is essential for policymakers, researchers and academicians to continue monitoring the exchange rate dynamics in order to keep safe in making comprehensive monetary policy because it is a vehicle which plays a pivotal role in every economy. Concisely, (Pelinescu, 2014) states that the applications of some models are not robust in emergent countries than developed ones. The author narrates that the generalized autoregressive conditional heteroscedastic (GARCH) model and its variant of family were used to study exchange rate volatility in developed countries but conclusively generalized it to emerging economies which resulted to models not fitting better to developing countries data as compared to the developed nations using these same models. Policy makers need accurate forecasts about future values of exchange rates (Epaphra, 2017). This is due to the fact that exchange rate volatility is a useful measure of uncertainty about the economic environment of a country (Epaphra, 2017). Also, in his work he reveals that previous day's volatility in exchange rate can affect current volatility of exchange rate.

In the exchange rate market, there are continuous studies that have focused on capturing the asymmetry and leverage effect of exchange rate series. However, the asymmetrical nature of exchange rate continues to pare down growth in African economies most especially, the Liberian economy which is the sample under investigation in this research. "Economies around the globe are prone to various shocks that lead to higher levels of volatility and uncertainty. This can render the traditional models inefficient in gauging the volatility, because the relationships among economic variables are expected to alter with changes in economic conditions" (Rofael & Hosni, 2015). Financial time series such as exchange rate often exhibits the phenomenon of volatility clustering, that is, periods in which its prices show wide swings for an extended time period followed by periods in which there is calm (Epaphra, 2017). In addition, researchers most often return to the traditional famous autoregressive conditional heteroscedastic (ARCH) model proposed by (Engle, 1982) and the generalized autoregressive conditional heteroscedastic (GARCH) model initiated by (Bollerslev, 1986) to effectively model the volatility nature of financial series. Conditional heteroskedascity is a key character in most of economic and financial real data and basically, it narrates that the conditional variance of the data is changing over time (Zhu, et al., 2015). (Hill, et al., 2011) state that in any particular problem, one challenge is to determine a functional form that is compatible with economic theory and data. Further, (Wei, 2006) states that the "two central properties of many economic time series are nonstationarity and time-volatility and that these two properties have led to many applications in both economics and statistics."

1.1 Conceptual issues

Forecasting techniques help to assimilate important information from current data to past data for better decision making in an environment. According to (Box & Jenkins, 1976), they state that the use of current observation at time t to forecast some future values provide a) economic and business planning, b) production panning, c) inventory and production control, and d) control and optimization of industrial processes. As tension erupts in developing and even developed economies, decision makers are most often under pressure to decide which time series models is right to make predictions of economic variables as it is the primary application of many econometric models in the literature. It has been written extensively in the time series literature that some time series models effectively forecast the behavior of exchange rate. This paper does try to investigate the volatility nature of such a series and presents clear empirical applications of the estimates for the weekly exchange rate series of Liberia using three econometric and time series models. Furthermore, the focus is to estimate, measure and compare the results performance of these models in the Liberian economy during the economic meltdown adopting similar approaches of (Montgomery, et al., 1998). This study will be modeled on the postulates of some univariate and nonlinear time series methods of weekly exchange rate with has strong unique wandering pattern, or move countercyclically, either during economic slowdowns and contractions and speedups during expansions. Volatility clustering is one of the most common stylized facts in financial time series; this phenomenon has intrigued many researchers and oriented in a major way the development of stochastic models in finance (Niyitegeka & Tewari , 2013). (Grier & Perry, 1998), without much surprise, provides empirical evidence that inflation raises inflation uncertainty, as measured by the conditional variance of the inflation rate, for all G7 countries in the period from 1948 to 1993. In examining foreign exchange markets, time-varying volatility models have been widely adopted to study various issues ranging from time-varying risk premia, volatility spillovers between the spot and forward exchange market, hedging strategies, to the effect of monetary policy, (Wang, 2005).

The approach of this paper proves vital to the Liberian economy as it moves further away from the innumerable vector autoregressive (VAR) and the non-technical component in investigating the dynamic nature of exchange rate volatility on growth in Liberia. In view of this, the contribution of this paper is to specify linear univariate autoregressive integrated moving average (ARIMA) models and nonlinear time series models such as ARCH and GARCH to the residuals of the univariate model. The specifications are proper for this key macroeconomic variable modeling in the case of Liberia and other settings. This strategy of estimating these models abilities using these techniques prove sensible, as evidently high frequency data analysis in the context of robust time series models have evolved as a standard instrument in econometrics.

Several useful studies have proposed the used of the ARCH, GARCH, State Space, time-varying parameters models among the many robust time series models to estimate and forecast volatility of exchange rate series. As it is mentioned earlier, there are no robust methods but new methods can aid to capture the volatility nature of the exchange rate series, therefore, this is while, the approaches used in this research prove reliable and effective to serve the wandering pattern in the sample under consideration. Over time, there have been extensive decry from public opinions about the lackadaisical approach of the Central Bank of Liberia (CBL) and other reputable institutions to help alleviate the problems of the dual regime systems, that is, the United States Dollars (USD) and the Liberian Dollar (LRD), where the latter has served as subordinate to the former for over several years now. Most citizens are infuriated about the way in which the government handles such a vital economic variable which can significantly reduce the purchasing power of households as well as, drives away investors that will be more chaotic to the growth and development of the economy. Owing to these factors and many others, this current study has been motivated to handle this irregular behavior of this financial series in the below highlighted ways: firstly, the Liberian economy as a dollarized economy needs to be investigated appropriately for its exchange rate floating capacity and which has rendered the series an active policy series or monetary transmission mechanism channel. Secondly, this study shows the conflicting issues in the regime system of this key financial variable from the context of the approaches used to analyze this series.

The Liberian exchange rate series has been depreciating quite a lot against the USD which it has been used as its national currency till now. It is against these defaults that this work is focusing on estimating the exchange rate volatility in Liberia using these class of models which are flexible to solve real life problems, namely; the ARIMA-ARCH/GARCH models may help determine the nature that generate the exchange rate series. With these useful time series methods, I believe this paper has significantly contributed to academics and researchers who intend to model less developed nations time series variables as with the case of Liberia and thereby, the recommendation highlighted here will be key for governments of various countries and analysts who are in dare need to adopt such methods that will minimize the disturbances of such a time series variable. These models are all known in the literature to be quality models that can accurately handle time series variables with their powerful modeling abilities. The paper in is structured in the following way. Section 2 gives the literature review, Section 3 gives some review on the ARIMA, ARCH and GARCH models, correspondingly; while Section 4 presents the stylized facts and applications, as well as the data description and source; Section 5 presents the techniques and results of the estimated models. Finally, Section 6 and 7 discusses the research and concludes with policies implications, as well as limitations of the study and recommendation for future work.

2 Literature Review

(Montgomery, et al., 1998) compare the forecast performances using variety of linear and nonlinear time series for the U.S. unemployment rate with quarterly and monthly data. Their results indicated that significant improvements in forecasting accuracy can be obtained over existing methods. (Rofael & Hosni, 2015) estimate and forecast the volatility of exchange rate in Egypt adopting the ARCH type models and the State Space (SS) models and its class of family using daily time series between January 2003 to June 2013. The results showed that exchange rate returns for the sample investigated suffer from the volatility clustering phenomena with the existence of time-varying variance. They further stated in their work that stock market was insignificant to predict the volatility of the exchange rate series. (Zhu, et al., 2015) empirically examine daily exchange rates between January 1, 2006, to December 31, 2011 of United States Dollars per Russian Ruble (USD/RUB), USD per Taiwan Dollar (USD/TWD), USD per Bulgarian Lev (USD/BGN), and USD per Polish Zloty (USD/PLN) respectively. (Zhu, et al., 2015)proposed the buffered autoregressive model with conditional generalized autoregressive heteroscedasticity (BAR-GARCH) which can help capture the buffering nature of time series in both the conditional mean and variance.

(Epaphra, 2017) applied univariate nonlinear time series analysis to the daily Tanzanian currency per USD (TZS/USD) exchange rate data spanning from January 4, 2009 to July 27, 2015 to examine the behavior of exchange rate in Tanzania in order to capture the symmetry effect in exchange rate data, and used the ARCH, GARCH and the exponential GARCH (EGARCH) models to effectively model the asymmetry in volatility clustering and the leverage effect in exchange rate. (Epaphra, 2017) found that exchange rate series exhibits the empirical regularities such as clustering volatility, nonstationarity, non-normality and serial correlation that justify the application of the ARCH methodology. Further stated, the author also suggests that exchange rate behavior is generally influenced by previous information about exchange rate. (Fahimifard, et al., 2009) apply both linear and nonlinear time series models with daily exchange rates data ranging from March 20, 2002 to November 21, 2008 a total of 2436 realizations to investigate the Iran Rial per USD and Rial per Euro. Their analysis showed that nonlinear models outperformed the linear model, GARCH outperformed ARIMA model and ANFIS outperformed ANN model. Accordingly, they also outlined other useful functions of nonlinear models and finally concluded that the accuracy of the ANFIS model must not be denied.

(Onanuga & Onanuga, 2016) empirically examine the volatility of exchange rates in the West African Monetary Zone (WAMZ) using monthly data spanning from 1960M01 to 2011M12. (Onanuga & Onanuga, 2016) findings after applying the GARCH models to comparatively explore the behavior of exchange rate volatility of currencies revealed that the Ghanaian cedi was the most volatile currency in the zone. In addition, (Onanuga & Onanuga, 2016) found that leverage effect existed for Gambian dalasi, but did not exist for the Nigerian naira; but also realized that the impact of central banks intervention on exchange rate volatility was found to be inconclusive for Ghana, Guinea, and Liberia. (Pelinescu, 2014) employed the variant or different class of ARCH-GARCH procedures to circumvent the volatility of the leu/euro exchange rate including other economic variables. The author used daily dates from January 5, 2000 to August 31, 2013, and the results revealed that the volatility of the leu/euro rate follows an ARCH process, thereby making the series to have a high symmetry posture. (Zorzi, et al., 2016) applied the DSGE model and forecasted the real exchange rate, but further argued that forecasts should not replicate high volatility of exchange rates observed in sample and that models should be used to exploit the mean reversion of the real exchange rate over long time horizons. (Zorzi, et al., 2016) further proved that DSGE model performs well in real exchange rate forecasting but failed to forecast nominal exchange rates better than the random walk.

Furthermore, (Bergman & Hansson 2005; as cited in; Brooks, 2008) applied a Markov switching model with an AR(1) structure for the real exchange rate, which allows for multiple switches between two regimes using quarterly obervations observations from 1973Q2 to 1997Q4 (99 data points) are used on the real exchange rate (in units of foreign currency per US dollar) for the UK, France, Germany, Switzerland, Canada and Japan, but their model estimated using the first 72 observations (1973Q2-1990Q4) with the remainder retained for out-of-sample forecast evaluation. The authors use 100 times the log of the real exchange rate, and this is normalised to take a value of one for 1973Q2 for all countries. They found that estimated models did allow parameters and variance to vary across the states, but the restriction that the parameters are the same across the two states cannot be rejected and hence the values presented in the study assume that they are constant.

A sophisticated study of handling exchange rates with MSA was done by (Ismail & Isa, 2006). (Ismail & Isa, 2006) considered the two regime switching models, SETAR model and the MS-AR model to estimate monthly exchange rates of three ASEAN countries (Malaysia, Singapore and Thailand) between from February 1990 to June 2005 for a total of 185 observations. They tested for structural breaks in the time series variables and proved that there was structural breaks, thereby applied the regime switching model which they realized performed better than simple autoregressive model in-sample fitting. Recently, (Gbatu, et al., 2017) exposited that there was no significant relationship between exchange rate volatility (ERV) and real gross domestic product (RGDP) in Liberia. They used annual observations spanning from 1980 to 2015 a total of 36 observations. They adopted the unrestricted vector autoregressive (VAR) model to analyze the dynamic associations between ERV and RGDP and observed that innovations to the country RGDP led to fluctuations in ERV based on the results of the variance decomposition.

(Gbatu, et al., 2017) exploit panel data of Economic Community of West African States (ECOWAS) countries from 1980 to 2015 to model the influence of exchange rate volatility (erv) and real oil price shocks on economic output. (Gbatu, et al., 2017) data revealed that that linear and asymmetric effects of oil price shocks on real gross domestic product (rgdp) showed that erv negatively and significantly influence rgdp.

2.1 Brief Development of Liberia's Exchange Rate Policy

The intervention of International Monetary Fund (IMF) to adapt a Memorandum of Economic and Financial Policies (MEFP) to review Liberia's economic developments and performance identified that monetary policy will continue to focus on smoothing the inflation path by containing excess volatility in the foreign exchange market. Over the medium term, the Central Bank of Liberia (CBL) aims to increase reserve cover above the three months of imports to buttress macroeconomic stability. The government also intends to further develop financial markets and strengthen market-based policies aimed at improving confidence and encouraging the wider use of the Liberian dollar (Lagarde, 2017).

The Liberian economy experienced shortage of US dollars, particularly since end-2016 which led to depreciation of the exchange rate and a spike in inflationary pressures. The fiscal year FY2015/16 and FY2016/17 led to a 28 percent decline in total foreign exchange inflows due to UNMIL withdrawal and declines in both net remittance inflows and aid disbursements. The resulting shortage of foreign exchange has been felt clearly in the depreciation of the Liberian dollar, which is now significantly weaker, having depreciated by about 20 percent since the last review in December 2016. As a consequence, price pressures intensified, and inflation is expected to close 2017 at about 12½ percent (Lagarde, 2017). “High levels of dollarization limit the scope of monetary policy, which has focused on weekly foreign exchange auctions to reduce liquidity and maintain broad exchange rate stability. Financial sector indicates the country has low access to credit, particularly for the rural poor” (AFDB, 2013).

Moving further, from 2008 the Liberian economy witnessed a sharp increase in average rate of inflation which stood at 15.2 percent during the quarter, and from 9.9 percent in the preceding quarter. Moreover, local currency that circulated the Liberian economy amounted to L$3,383.4 million during the reporting quarter, but there was a fall of 5.9 percent when compared to the fourth quarter of 2007. The decline in currency in circulation was mainly influenced by an 11.0 percent fall in currency outside banks. Money supply increased by 12.6 percent to L$9,972.0 million during the quarter, from L$8,859.2 million in the previous quarter. The rise in money supply was largely a result of the ongoing economic recovery process and increased economic activities in the country (CBL, 2017). In 2017, the Liberian dollar was more volatile as it depreciated on average by 3.2 percent to L$104.02/ US$1.00 at end-March, 2017, from L$100.80/US1.00 at end-December, 2016. (CBL, 2017) report shows that on a year-on-year basis, the average exchange rate depreciated by 14.9 percent largely on account of high demand for foreign exchange (FX) to facilitate import payments and the global decline in the prices of the country's major commodity exports" (CBL, 2017).

Furthermore, the exchange rate system of Liberia has been difficult to manage as the increasing rate continues to fluctuate on a day-to-day basis. In addition, one USD is equivalent to 127.879 LRD currently in the economy making economic activities very much difficult. Despite the continuous fight to mitigate the volatility of exchange rates in the economy, the de jure floatation of USD dollars in the economy led to several interventions by CBL and other prominent financial institutions to deal with the adverse effects on the exchange rate regime.

In 2003, the inflation rate was reduced by 4.2 percentage points, from 14.6 percentage point during that period as there was hope in CBL but there was massive unemployment in the economy as both public and private sector investments shrunk (CBL, 2003). In 2013, through the tight policy of Central Bank under the mentorship of Bank Governor, Mills Jones, the economy further experienced a sharp depreciation of the Liberian dollar, but the economy grew at an estimated 8.1 percent, and inflation remained in single digit throughout the year, averaging 7.6 percent see (CBL, 2013). This motive was meant to contain the pressure on the Liberian dollar during the CBL intervention tool in the foreign exchange which led to a sale of US$72.3 million through its auction program. Also, the CBL introduced the sale of CBL bills, which was the first of its kind, serving as an additional policy instrument in managing Liberian-dollar liquidity (CBL, 2013). Little hope was restored in the policies implemented by the CBL which further led to expansion in local businesses as loans were given to local marketers with low end of year interest payment, and this resulted to the employment of some local skilled and semi-skilled citizens through this implementation.

Following the outbreak of the Ebola epidemic and political strives, the Liberian economy has continued to be unstable even though, the interventions of CBL to stabilize the exchange rate but it continues to depreciate further against the USD. Recently, the October elections has further mounted pressure on the Liberian economy as most businesses and investments were halted due to the growing speculations of war and other deleterious economic factors. Aside from the elections issues and political factors, the CBL is faced with several other factors that has reduced the hope of the public.

2.2 Modeling Exchange Rate Volatility in Liberia

This study objectives are: 1) to estimate the weekly exchange rate series with all the specified models, 2) secondly, to model the ARCH and GARCH to the residuals of the ARIMA model of the exchange rate series and compare the results. Following the footpaths of (L-Stern, 2013) and (Epaphra, 2017), this paper uses weekly exchange rate data of Liberia, that is LRD/USD. In financial series modeling, for number of statistical reasons, it is admissible not to work directly with the raw series, so that the raw series are usually converted into series of returns (Brooks, 2008). Owing to this fact, this paper represents the changes in the weekly natural logarithms the exchange rates of Liberia in calculated returns form. If it is deduce that x_t is the exchange rate series under consideration and that it is given to have time invariant percent pattern over the time period, then one can infer that

$$\log(x_t) - \log(x) = \log\left(\frac{x_t}{x_{t-1}}\right) \tag{1}$$

where the x_t denotes the log price relative to the previous series value $X_{t-1}$to its current value X_t. The current research contributes to the vast number of growing empirical work in the literature, and as such, this paper synthesizes the weekly exchange rate series adopting robust time series methods to estimate the variable with these models and thereafter compare their estimating ability of each model. The overly used VAR approach to model the exchange volatility in the context of Liberia, prompted this study to adapt the approach highlighted here which is a prefer way to analyze such a financial series using weekly data.

2.3 Related Methodology

The wide ranger of literature on exchange rates volatility of using robust time series methods and models that can handle the effects of exchanges rates fluctuations in a particular, the analysis in here is most closely linked to four different papers. (Epaphra, 2017) applies the ARCH, GARCH and the exponential GARCH (EGARCH) models to time series analysis to the daily Tanzanian currency per USD (TZS/USD) exchange rate data spanning from January 4, 2009 to July 27, 2015. On the other hand, (Montgomery, et al., 1998) compared the forecast performances using variety of linear and nonlinear time series for the U.S. unemployment rate with quarterly and monthly data. The difference between this paper and previous work is that I use weekly exchange rates LRD/USD while the former uses monthly, daily and quarterly data for analyzes. (Rofael & Hosni, 2015) estimate and forecast the volatility of exchange rate in Egypt adopting the ARCH type models and the State

Space (SS) models and its class of family using daily time series between January 2003 to June 2013. (L-Stern, 2013) study considers fitting the ARCH/GARCH to the residuals of the univariate time series model (ARIMA) to the log stock price of Apple using monthly data of January 1, 2007 to July 7, 24 2012 using data from the United States of America.

3 Methods and Models for Estimation

This section summarizes statistical methods and models outlined for this study to estimate the weekly exchange rates data of Liberia. In this section, the orderly arrangements of the models specifications are as follows; the univariate (ARIMA) linear time series model is firstly. Next is the specifications of two parametric volatility models that is the ARCH and GARCH models.

3.1 The ARIMA models

An easy to follow specification of univariate linear time series models is (Vogelvang, 2005) integrated autoregressive moving average model. Suppose it is deduced that $\Delta^d X_t$ is a stationary process which can be presented by an the autoregressive moving average ARMA(p,q) model. With this in hand X_t is a time series that follows the autoregressive moving average (ARIMA) model, then one can be certain that X_t is an ARIMA(p,d,q) process. This random process X is the variable to be modeled or forecast, therefore the Box-Jenkins procedure of stationarity can be applied by differencing the variable to make it stationary where both the first and second moments of the variable will all be time invariant. In practical works, the stationary part of the variable leads to differencing the series d-times but most for applied works, the process is d=1 or at most d=2. First starting with an ARMA(p,q) procedure can be written as

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \cdots + \alpha_p X_{t-p} + e_t - \beta_1 e_{t-1} - \beta_2 e_{t-2} - \cdots - \beta_q e_{t-q} \tag{2}$$

$$\emptyset(L) X_t = \theta(L) e_t \tag{2.1}$$

or using the lag operator L, this representation can be rewritten as

$$\emptyset(L) = 1 - \alpha_1 L - \alpha_2 L^2 - \cdots - \alpha_p L^p \tag{2.1.1}$$

$$\theta(L) = 1 - \beta_1 L - \beta_2 L^2 - \cdots - \beta_p L^q \tag{2.1.2}$$

where the $\emptyset(L)$ and $\theta(L)$ are the polynomials of orders p and q, which can be written in the below form $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are unknown parameters and the e are the independent and identically distributed normal errors with zero mean and σ^2. The p represents the number of lagged values in the autoregressive (AR) part and the d is the part that shows the number of time the series should be differenced to make it stationary, while the q denoted the lagged values of the white noise process which represents the order of the moving average (MA) dimension of the model. It is worth noting that the representation (2.1) does not include independent variable, but only the past values of the series are included. Since most macroeconomic series are plagued by unit root, the exchange rates series X_t might result to be nonstationary in $\emptyset(L)$, therefore, it is best to rewrite the polynomial $\emptyset(L)$ to obtain an ARIMA(p-1, q) model for ΔX_t. The pure ARIMA(p,1,q) specification with no structural components for exchange rates series X_t can be written as

$$\emptyset^*(L)\Delta X_t = \theta(L) e_t \tag{2.2}$$

where $\emptyset(L)$ is known to be the autoregressive operator and assumed to be stationary if the roots of $\emptyset(L) = 0$ remain outside the unit circle. The $\emptyset(L) = \emptyset(L)\Delta^d$ is the generalized autoregressive operator; and this representation shows that it is a unit root operator with the d denoting the roots of $\emptyset(L) = 0$ which must be unity; whereas the $\theta(L)$ is the moving average operator, and it is deduced to be invertible and that its roots of $\theta(L) = 0$ lie outside the unit circle. Note that when d=0, the representation (2.2) is a stationary process. It we infer that the process is integrated of order one I(1), due to a unit root process in the $\emptyset(L)$. Processes involving $d(d \leq 2)$ unit roots in autoregressive polynomial are known as ARIMA(p,d,q) processes

$$\emptyset^*(L)\Delta^d X_t = \theta(L) e_t \tag{2.2.1}$$

with $\emptyset^*(L)$ polynomial of p-d. There are three basic steps to the development of an ARIMA model: 1) Identification/model selection: the values of p, d, and q must be determined. The principle of parsimony is adopted; mostly stationary time series can be modeled using very low values of p and q. 2) Estimation: the $\emptyset$ and θ parameters must be estimated, usually by employing a least squares approximation to the maximum likelihood estimator. 3) Diagnostic checking: once the considered models have been estimated, the residuals of the models must be checked for its adequacy and revised if necessary, or that this process may have to be repeated until a satisfactory model is found see (Kennedy, 2008) for all the details outlined.

3.2 The ARCH model Specification

Specifying the ARCH model proposed by (Engle, 1982) is to model the volatility of the exchange rate series which is not constant through time due to periods of relatively low volatility and periods of relative high volatility which tend to be grouped together. The traditional ARCH model attempts to estimate the time-dependent volatility which is a function of observed prior volatility (Anon., 2013). Displaying the ARCH model to capture the variance of the exchange rate equation with its error term serving as lagged values of the squared equation error term. The simplest ARCH(1) model is written as to model the return of the $\log(x_t)$

$$\log(x_t) = \sigma_t \epsilon_t \tag{3}$$

$$\sigma_{\epsilon t}^2 = \gamma_0 + \gamma_1 \epsilon_{t-1}^2 \tag{3.1}$$

where ϵ_t is the standard Gaussian white noise; which can also be written as $\epsilon_t \sim \text{iid } N(0,1)$. The ϵ^2 is the squared innovations and the γ_1 is the lagged one parameter of the ARCH. Eq. (3) is the conditional mean, while (3.1) is the conditional variance function. There is no explanatory variable included in Eq. (3), for the exchange rates returns. By assuming that the error variance at time t is equal to the constant term plus a term times the squared of the error term in past period one. If the $\gamma_1 = 0$, then the classical theory framework of homoscedasticity of the error variance is realized. It is worth noting that the coefficient must be nonnegative, that is $0 \leq \gamma_1 < 1$. In other specification work as adopted from (Hill, et al., 2011), the equation could be written with a constant term included as

$$x_t = c + \varepsilon_t, \qquad \varepsilon_t | I_{t-1} \sim N(0, h_t). \tag{4}$$

$$h_t = \gamma_0 + \gamma_1 \varepsilon_{t-1}^2 \tag{4.1}$$

$\hat{\gamma}_0 > 0;\ 0 \leq \hat{\gamma}_1 < 1$ for stationarity and positive variance. If the error or the residual return is very large, then the forecast values of next period volatility will also be very large. Also, the tails of the return series distribution can be fat with the normal violation of 3 for the kurtosis and thereby can lead to fatter tails nonnormal conditional distributional assumptions. In this paper, I estimate the return series assuming normal distribution and generalized error distribution (GED) due to the fat tail component of the series. The estimate of the $\hat{\sigma}_t^2$ is considered because we cannot observe the σ_t^2 , (see, Gujarati & Porter, 2009) and (Hill, et al., 2011) for more discussion on this; however, the specification from Eq.(3.1) or (4.1), that is, ARCH(1) function is a special representation of the ARCH(p) and can be carried over to the ARCH(p) specification. The ARCH(1) specification can include many p-lags as written below as ARCH(p) model with the p lagged squared error terms.

$$\sigma_t^2 = \gamma_0 + \gamma_1 \epsilon_{t-1}^2 + \gamma_2 \epsilon_{t-2}^2 + \cdots + \gamma_p \epsilon_{t-p}^2 \tag{4.2}$$

when this condition $E(\epsilon_t | \epsilon_{t-p}) = 0$ is satisfied for the conditional mean and the conditional variance $Var(\epsilon_t | \epsilon_{t-p}) = \gamma_0 + \gamma_1 \epsilon_{t-1}^2 + \gamma_2 \epsilon_{t-2}^2 + \cdots + \gamma_p \epsilon_{t-p}^2$ with hetereoscedastic error term ϵ_t conditional on its previous p-lagged error ϵ_{t-p}. The variance will seem to be meaningful only when $\gamma_0 > 0$ and $|\gamma_p| < 1$ thereby abiding by the classical homoscedasticity assumption (see, Vogelvang, 2005) for more details. The ARCH(1) representation can be tested with a t-test statistic to check for arch effects, whereas the F-test test the parameters of the ARCH(p) jointly for any arch effects. The null hypotheses and alternative hypotheses for both the ARCH(1) and ARCH(p) specifications are

$$H_0: \gamma_1 = 0 \tag{5}$$

$$H_1: \gamma_1 \neq 0 \tag{5.1}$$

This the hypothesis for ARCH(1) model, while the below hypothesis if the ARCH(p) models.

$$H_0: \gamma_1 = \gamma_2 = \cdots = \gamma_p = 0 \tag{5.2}$$

$$H_1: \text{at least one } \gamma\text{s coefficient is statistically different from zero.} \tag{5.3}$$

3.3 The GARCH model Specification

The ARCH(p) model does not eradicate all the problems of as it poses problems of estimation to volatility series. In the literature, the ARCH(p) order model consume several degrees of freedom, as well as the difficulty in interpreting all the estimated coefficients due to the changing signs in the values, and the estimation issues when ordinary least squares (OLS) is used to estimate the mean and variance of the equation (Gujarati & Porter, 2009). The lagged effects of the $E(\epsilon_t | \epsilon_{t-1})$ of the observed series can be considered as assumed that the innovations of the error terms ϵ_t all have the same variance σ_t^2 which maybe unrealistic (see, Heij, Boer, Franses, Kloek, & Dijk, 2004) for further reading. The GARCH model has been found to be more useful because of its lower parameter specification as compared to the ARCH that uses more parameters in estimation.The generalized ARCH model which was proposed by edowed econometrician (Bollerslev, 1986) can be written in a simple GARCH(1,1) form in mean as

$$x_t = c + \epsilon_t, \qquad \epsilon_t \sim N(0, \sigma_t^2). \tag{6}$$

$$\sigma_t^2 = \gamma_0 + \gamma_1 \epsilon_{t-1}^2 + \alpha_1 \sigma_{t-1}^2 \tag{6.1}$$

with $\gamma_0 > 0$, $\gamma_1 > 0$, and $\alpha_1 > 0$ as nonnegative parameters, and the sum of $\gamma_1 + \alpha_1 < 1$. The model (6.1) shows that the conditional variance at time t depends not only on the lagged squared error in the past time period but also on the lagged variance in past time period. When one is using the GARCH model, it is possible to interpret the current fitted variance as a weighted function of a long-term average value, information about volatility during the previous period (Brooks, 2008). With this backdrop, the GARCH(1,1) can also be expanded to a GARCH(p,q) model where the current conditional variance is parameterized to depend upon q lags of the squared error and p lags of the conditional variance.

$$\sigma_{\epsilon t}^2 = \gamma_0 + \gamma_1 \epsilon_{t-1}^2 + \cdots + \gamma_p \epsilon_{t-p}^2 + \alpha_1 \sigma_{t-1}^2 + \alpha_2 \sigma_{t-2}^2 + \cdots + \alpha_q \sigma_{t-q}^2 \tag{6.2}$$

where the γ_i are defined as the ARCH parameters; the α_i are the GARCH parameters. Where empirically, it is assumed that the error term ϵ_t followed a Gaussian (normal) distribution $\epsilon \sim N(o, \sigma^2)$. Also, the restrictions$\gamma_0, \gamma_1, \gamma_2, \ldots, \gamma_p \geq 0$, and $\alpha_0, \alpha_1, \alpha_2, \ldots, \alpha_q \geq 0$ are restrictions imposed in order for the variance σ^2 to be a nonnegative integer. The GARCH(1,1) model is necessary to detect the volatility clustering in the data (Brooks, 2008).

3.4 Estimation of the Parameters of ARCH and GARCH models

Assuming that the ARCH model follow the ARCH(1) process and to estimate the parameters of the ARCH(1) model is typically accomplished by conditional maximum likelihood MLE. The conditional likelihood of the data $x_2, \ldots, x_n$ given x_1, is defined by the specification of (Brooks, 2008) as follows: AR(1)-GARCH(1,1) model is defined as

$$x_t = c + \beta x_{t-1} + \epsilon_t, \epsilon_t \sim N(0, \sigma_t^2) \tag{7}$$

$$\sigma_t^2 = \gamma_0 + \gamma_1 \epsilon_{t-1}^2 + \alpha \sigma_{t-1}^2 \tag{8}$$

where the variance of the errors, σ_t^2, time-varying (see, Bollerslev, T. and Woodridge, J.M. (1992); Weiss, A.A. (1986); as cited in; Epaphra, 2017). Based on the appraoch of (Brooks, 2008), the log-likelihood function (LLF) that maximizes under a normality assumption for the disturbances is presented as

$$L = -\frac{1}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\log(\sigma_t^2) - \frac{1}{2}\sum_{t=1}^{T}(x_t - c - \beta x_{t-1})^2 / \sigma_t^2 \tag{9}$$

where $-\frac{1}{2}\log(2\pi)$ is a constant with respect to the parameters, T is the number of observations and x_t is the exchange rate return in natural logarithms. The maximization of the LLF necessitates minimization of $\sum_{t=1}^{T}\log(\sigma_t^2)$, $\sum_{t=1}^{T}(x_t - c - \beta x_{t-1})^2 / \sigma_t^2$ and the error variance (Brooks, 2008; as cited in; Epaphra, 2017). Similarly, (Tsay, 2010) presents more detailed specifications for the estimation of the ARCH process. Furthermore, the author presents in his work that there are several likelihood functions that are used in modeling ARCH depending on the distributional assumption of the error term, ϵ_t. It can also be assumed that ϵ_t follows a generalized error distribution (GED) with probability density function see (Tsay, 2010) for more discussion on the various distributions of the error of the ARCH estimation. Model diagnostic checking can be done by applying the Ljung-Box test, the QQ line plots, the skewness and the kurtosis can also provide useful approach for the estimated model.

3.5 Analyzing the Nonnormal Conditional Return Series

In modeling financial series volatility, the distributional assumptions come into play as one may infer different distribution to capture the excess kurtosis observe in modeling a time series volatility. (Reider, 2009) describes many useful distribution of the conditional returns. For example, the author states that an analyst can assume returns follow a student's t-distribution or a Generalized Error Distribution (GED), both of which can have fat tails, see (Reider, 2009). Now following the specification of (Reider, 2009), the density function for the GED is written as

$$f(x) = \frac{v \exp\left(-\frac{1}{2}\left|\frac{x}{\lambda\sigma}\right|^v\right)\frac{1}{\sigma}}{\lambda 2^{(1+1/v)}\Gamma(1/v)} \tag{10.1a}$$

where $v > 0$ and

$$\lambda = [2^{-(2/v)}\Gamma(1/v)/\Gamma(3/v)]^{1/2} \tag{10.1b}$$

where $\Gamma(.)$ Gamma function which is defined as

$$\Gamma(x) = \int_0^\infty y^{x-1} e^{-y} dy \tag{10.1c}$$

so that $\Gamma(x) = (x - 1)!$ if x is an integer. The parameter, ν, is a measure of fatness of the tails. The Gaussian distribution is a special case of the GED when $\nu = 2$, and when $\nu < 2$, the distribution has fatter tails than a Gaussian distribution and the log-likelihood representation for the GED can also be found in the work of (Reider, 2009). Instead of using a three parameter distribution for conditional returns that captures kurtosis, one could use a four parameter distribution for conditional returns that captures both skewness and kurtosis (Reider, 2009).

4 Application to Liberia's Weekly Exchange Rates

The author begins with varieties of pre-tests including the time series plots, descriptive statistics of the natural logarithmic exchange rates series, the correlogram plot of the series, the unit root tests, that is, the Augmented Dickey Fuller (ADF) and Phillips and Perron (PP) unit root tests versions. Furthermore, the univariate ARIMA will be the first model to be estimated; next the ARCH test equation will be modeled to enable the author identify any ARCH effect that will lead to both estimations of the volatilities models-that is, ARCH and GARCH. Henceforth, the paper has proposed to estimate these models for their useful modeling abilities to be compared to the benchmark model which is the ARIMA model. Noting that these models need to be properly justified based on statistical methods, so the selection process includes the (AIC, HQC, SBC and likelihood ratio test). Finally, the diagnostics tests of these models will be performed in order to detect any unusual process in their residuals.

4.1 Data

The data used in this study is weekly exchange rates (local currency per US$) for the Republic of Liberia over the period from January 7, 2013 to December 25, 2017 for a total of 260 observations. All the exchange rates data are sourced from **https://www.oanda.com.** The variable under investigation is exchange rate returns in percentage $x_t = 100^*[\ln x_t] - [\ln x_{t-1}]$ where x_t the weekly exchange rates is. This paper uses weekly data to estimate the parameters of the three considered time series models to study the exchange rate return volatility of Liberia. The various time plots of the series are displayed below with years or time on the horizontal axis and the values of the series on the vertical axis.

Visualizations of time series variables are very useful in detecting the nature of the series. To detect higher-order serial dependence structure in data, one can also examine the autocorrelation structure of the absolute returns which might help to identify if the returns are independently and identically distributed (Cryer & Chan, 2008). Clinching onto this, several different plots of the log exchange rates returns (Δlnx) are displayed in 'Appendix A1'. Hence, these plots will assist in properly determining the order of an ARCH estimation. The time plot in 'Figure 1a' of the series seasonal pattern in the number of Liberian Dollar (LBR) per United States Dollar (USD) rates seems to be time variant over time and probably does depend on the level of time series under investigation. Also, the plot in 'Figure 1a' is the log of LRD/USD over the period.

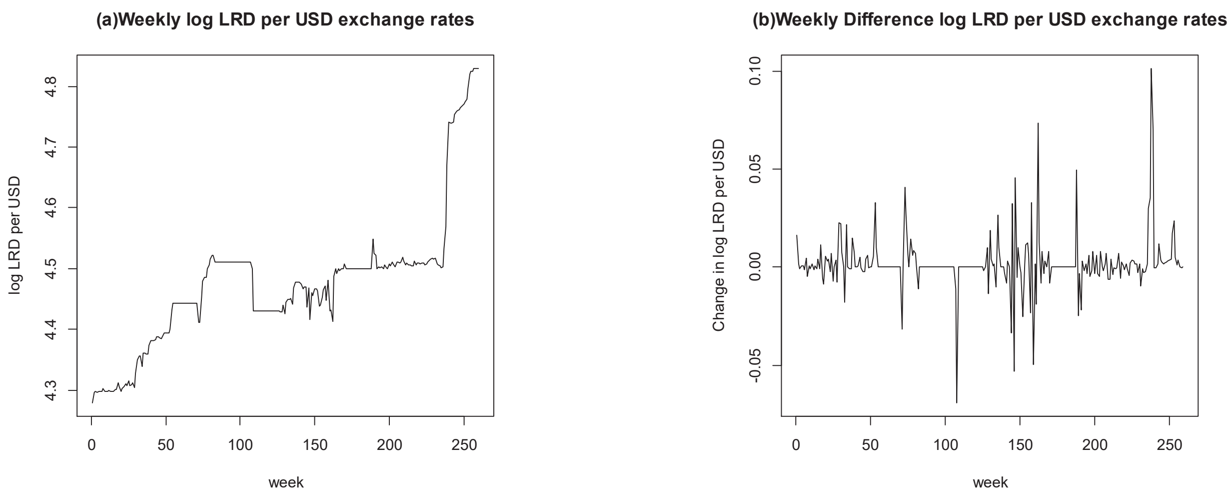


Figure 1: LBR/USD weekly exchange rate from January 7, 2013, to December 25, 2017. Weekly series of Liberia exchange rates in natural logarithms {lnx in (a)), and the weekly growth rates {LRD-USD = Dlnx) in (b).

This figure shows the relative percentage of the USD against the LBR in the early periods but it hugely appreciated against the LBR and quite interestingly with some fixed exchange rates periods on average, but there are ups and downs in the series over the entire sample period. The volatility clustering observed in the exchange rates series of 'Figure 1b' which is the first differences of the logarithms of the series in this study, clearly depicts that the time series shows clusters of volatility which indicates that it is a white noise process. In other words, the series seems not to be independently and identically distributed as the variance is not constant over time. Hence, the amplitude seen in the diagram looks small at the beginning and in the middle of the sample period. Furthermore, 'Figure 1b' shows that the LBR/USD returns were more volatile over some time periods and became less volatile almost toward the end of the study period.

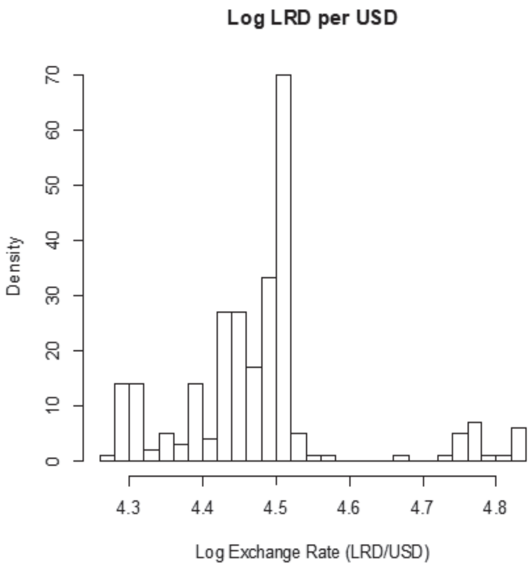
The average returns on the LRD/USD is equal to 0.002122 with a standard error of 0.000905. Thus the mean of the return process is not statistically significantly different from zero. This remarkably shows volatility in the exchange rate series. The appearance of the plot shows that the series was tranquil between weeks II to II of 2014 but in 2015 weeks I, II to V there was some longer time period with high volatilities in those months with some positive and negative returns. In 2/15/2016 the returns was high with a figure of 0.073 or 7.3%, but in 3/07/2016 that week of the same year, the returns was seen to be 0.008 or 0.8% and dropped thereafter. During the end of 2016 beginning from the first week to the fifth week, the returns were low and constant but rose highly again. These rising and falling features show level of volatility in the series.

The non-rejection of normality for the log of LRD/USD based on the extensively used normality test means that the series is far from normality with the peakedness of the series high and its flat skewed tails. Descriptive statistics are good way to start analyzing a data and as such 'Table 1' shows the output of summary statistics calculated for the exchange rate return series. The summary statistics in Table (1) shows that the normality tests including the Shapiro-Wilk and JB tests statistics reject the null hypothesis of normality at the 5% levels. The sample skewness of the returns of the LRD-per-USD is defined as 1.5936. The series shows a very heavy tail due to its excess kurtosis calculated to be 18.03945 which is far more beyond the value 3.0. In order words, the exchange rates returns has a heavy-tailed distribution. The coefficient of variation which is the ratio of standard deviation to mean measures the relative spread of the variable as its value is computed to be 6.682724. Also, this indicates that the series is more volatile. The mean and the standard deviation (std.dev) of the returns series are both low with the mean calculated to be 0.002122, while the std.dev which shows relative lower variability is computed to be 0.015. In summary, the 'LRD per USD' return observations are remarkably identified to contain volatility clustering.

Table 1: Descriptive statistics of exchange rate return series.

Statistic	**Value**	**Statistic**	**Value**	**Statistic**	**Value**	**Jarque-Bera p-value [ ]**
min	-0.068993	median	0.0000	Skewness	1.5936	2551.8 [2.2 x 10^{-16}]
max	0.1013414	std.dev	0.014565	Kurtosis	18.03945	
mean	0.002122	coef.var	6.682724	Shapiro-Wilk p-value	0.6623 [0.000]	

Source: Author's construction



JB-108.67; p-value=0.000000; Skewness=1.14;Kurtosis=2.15

Figure 2: Histogram & Normality Test of returns to LRD/USD

Source: Author's computation using data from **https://www.oanda.com.**

Aside from the descriptive statistics and plots of the series, it more appropriate and practical to test for unit root in time series variables. Owing to the fact that most economic and financial time series variables are not stationary in which they may show signs of nonstationarity, therefore, the Augmented Dickey Fuller (ADF) and Phillips-Perron (PP) unit root tests approach. It is worth mentioning that the specification of appropriate lags length for the ADF test are based on the statistical methods of Akaike Information Criterion (AIC) and the Schwarz Information Criterion (SIC). 'Table 2' show results of the level data and its difference.

The results in 'Table 2' shows that the logarithmic exchange rate series of Liberia is nonstationary from the ADF test statistic in Panel A and also from the PP-test statistic in Panel B of the same table respectively. On the other hand, Panel B and Panel D show the results of the first difference unit root test of both ADF and PP test computed to the log exchange rate returns of weekly exchange rate of Liberia. The series is declare to be difference stationary, I(1). With this in handy, we can further proceed to model the linear univariate time series models and the volatilities models in this study.

Table 2: The Unit Root Test results for log-Levels and First Difference Return Exchange Rates Series

Panel A: ADF unit root test results of log levels exchange rates data (lnX)			
ADF test statistic	-0.7964	1% critical value	-3.98
		5% critical value	-3.42
		10% critical value	-3.13
Panel B: ADF unit root test results of first differenced exchange rates data (ΔlnX)			
ADF test statistic	-8.9714***	1% critical value	-2.58
		5% critical value	-1.95
		10% critical value	-1.62
Panel C: PP unit root test results of log levels exchange rates data (lnX)			
PP test statistic	-0.8894	1% critical value	-3.996758
		5% critical value	-3.42846
		10% critical value	-3.137349
Panel D: PP unit root test results of levels exchange rates data (ΔlnX)			
PP test statistic	-15.0405***	1% critical value	-3.996889
		5% critical value	-3.428523
		10% critical value	-3.137386

Note: 'lnX' denotes the exchange rate series as was described in the equations. '***' indicates that the series is stationary at all the critical significance levels.

5 Estimation Techniques and Results

5.1 ARIMA Model & Identification Procedures

This section and the remaining sections display the outputs of the univariate ARIMA model, the two volatilities models (ARCH & GARCH) in order to capture any clustering components of the series. The first thing here is the correlogram plots of the exchange rates series (lnx) and the diff(log(X)) that will enable us to properly specify the ARIMA p and q models as well as, the ARCH family models which are based on the correlation between the series current values of residuals and its previous values. It is of necessity to make the series stationary as well as, critically analyze the autocorrelations and the partial autocorrelations to determine the maximum lags for further modeling. The horizontal dotted lines or the confidence interval of the sample ACF represent the zero axis in 'Figure 3'. Similarly, the below figures depict the sample ACF and PACF of weekly log exchange rates data for Liberia. After specifying up to 50 lags, it can be clearly seen that the observations are all above the dotted line which shows a positive values of the series. In addition, the ACF plot of ln(x) does not tail off fast but rather shows very strong positive autocorrelations over time and their autocorrelation (ac) coefficients are statistically different from zero for the specified lags and even up to lags of 40 and beyond. The ac coefficients for lags 1, 7 and 36 are 0.968, 0.760 and 0.163 respectively.

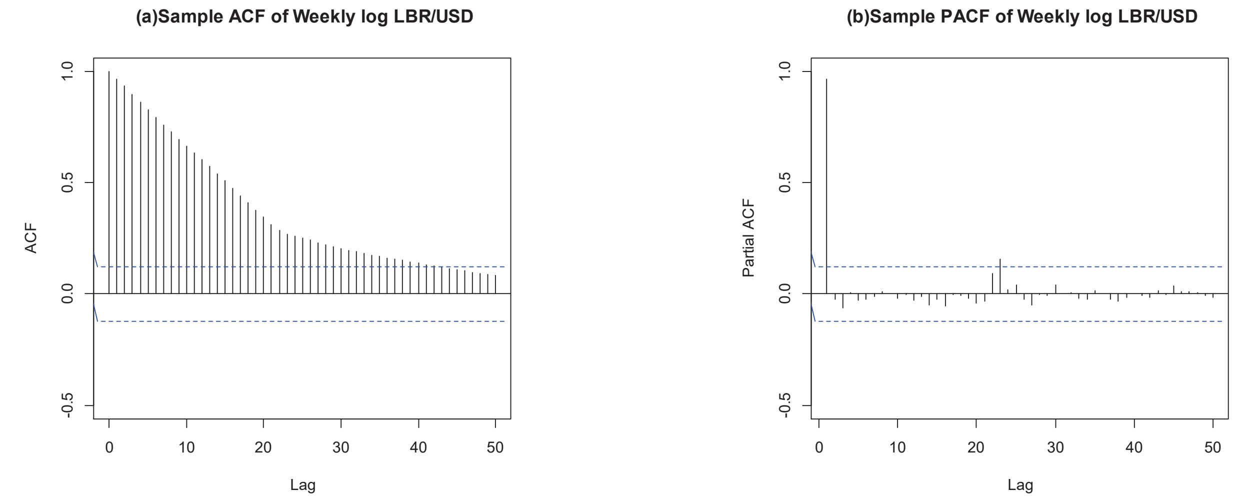


Figure 3: Sample ACF and PACF of log exchange rate (lnx) of Weekly data for Liberia

Moreover, the plots indicates that the series has been volatile over time and shows a nonstationary pattern. In the words of (Hanke & Wichern, 2005), they discussed that ACF can help to identify if a data is random, trend or nonstationary, stationary and if there are seasonal components in the data. Interestingly, PACF of the series declines very fast but show some spikes at lag 23 and this both exceeds the significance bounds with their PACF coefficients being positive. The necessary parsimonious model to choose here is based on the methodological approach of (Box & Jenkins, 1976). The decision applied here to choose the necessary model is based on the first-differenced plots of the sample ACF and PACF as shown below in 'Figure 4'. After specifying from lags 1-50 of the first differenced time series of the weekly logarithmic LRD/USD exchange rates of Liberia, the autocorrelation at lags 1,2 and 15 exceed the significance bounds but the rest with the same lags specification from 1-50 do not cross the dotted interval line. Note, that the lag of 29 exceeding the interval could be due to chance. The correlogram plot of the partial autocorrelation illustrates that lags 2 and 3 exceed the significance bounds with both positive correlations. Notice, that the lags 2 and 3 value of the sample PACF are positive and negative values of 0.172 and -0.177 which cuts off immediately with all other lags from 1-50 lying within the significance bounds.

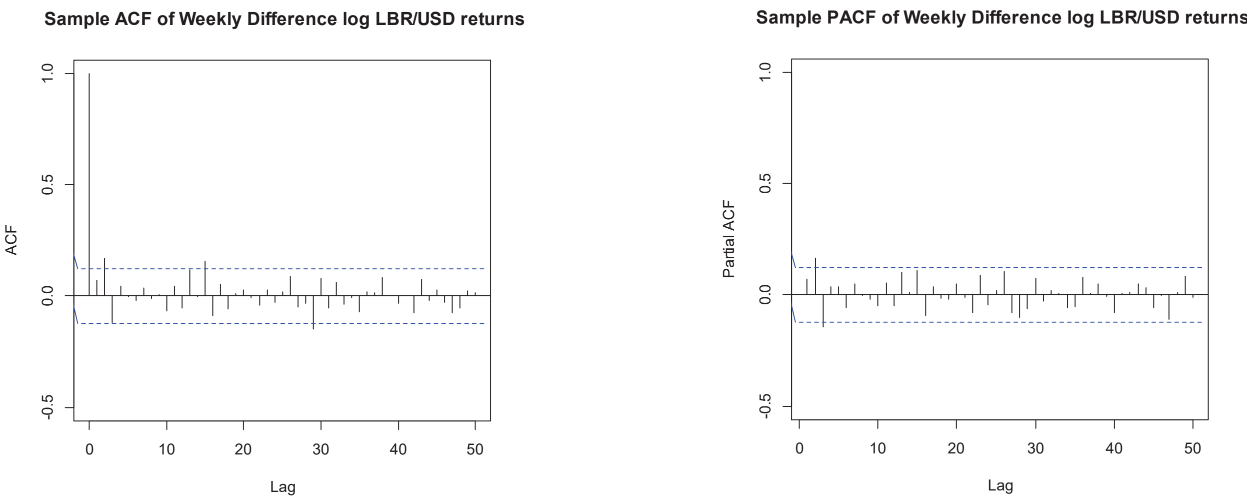


Figure 4: 1st differenced PACF plot of log returns and 1st differenced ACF plot of log returns

Since the sample ACF plot cuts off after lag 2, and the partial autocorrelations tails off, this means that the following ARIMA (autoregressive integrated moving average) model: ARIMA(p,d,q)=MA(q=2) is identified, see (Coghlan, 2017). The three basic steps to developing an ARIMA model are useful decision of properly selecting the right model to be chosen (Kennedy, 2008). An MA(2) seems plausible here since an autocorrelogram or ACF cuts off at lag 2 and the PACF dies down. This could be a manifestation that the returns series follows an MA(2) which could be written as the log(x) in the following manner as

ARIMA(p=0,d=1,q=2)=(0,1,2) specification. On the other hand, the ACF tails off while the PACF cuts off at lags 2 and 3 are both positive and negative, but the partial autocorrelation tail off to zero after lag 3. In this light, an ARMA(3,0) model could be appropriate where p=3 since the PACF is zero after lag 3 and ACF tails to zero; likewise, an ARMA(2,0) model could also be necessary with p=2 specification, since autocorrelation is zero after lag 2 and PACF cuts off for the ln(x) which is also equivalent to ARIMA(p=2,d=1,q=0)=(2,1,0).

According to (Coghlan, 2017), in practice we use the principle of parsimony to decide which model is best: that is, we assume that the model with the fewest parameters is best. The ARMA(3,0) model has 3 parameters, the ARMA(2,0) model has 2 parameters, and the ARMA(p,q) model has at least 2 parameters. With this in hand, models to be estimated are ARIMA(0,1,2)=MA(2),ARIMA(3,1,0)=AR(3) and ARIMA(2,1,0)=AR(2). Thereafter, ARIMA(3,1,2); ARIMA(2,1,2). The best competing model will be chosen by the results of the AIC, BIC and AICc - which defined as the small-sample-size corrected version of AIC of the estimated models. The information criteria statistics, and other useful diagnostic tests provided to aid in choosing the appropriate model. The below table illustrates the estimated models with their selection criteria based on the AICc and other useful statistics (i.e., sigma squared, Box-Ljung test statistics and p-values.

5.2 ARIMA model selection

The five models show white noise residuals and their Ljung-Box statistics values with the degree of freedom is not significant as there are large p-values for the estimated models.

Table 3: Model Selection Based on AICc & Other Statistics

Fitted Model	AICc	$\hat{\sigma}_\varepsilon^2$	Box-Ljung test	p-value
(0, 1, 2)	-7.464670	0.0002059	Q(24)=23.26	0.2762
(3, 1, 0)	-7.468878	0.0002034	Q(24)=17.063	0.6489
(2, 1, 0)	-7.459743	0.0002069	Q(24)=23.022	0.2877
(3, 1, 2)	-7.461418	0.0002017	Q(24)=16.75	0.6692
(2, 1, 2)	-7.469051	0.0002018	Q(24)=16.621	0.6774

Source: Author's construction

To choose the final model, the author compares the AICc, of all the five models. The AICc prefers the ARIMA(2,1,2). Working with mixed autocorrelation model is useful as it is easy to work with in this work. The result from the estimated ARMA(2,1,2) model is given below in ‘Table 4’.

Table 4: The ARIMA estimates of LRD/USD returns with no intercept term

	Estimate	SE	t-value	p-value
ar1	-0.6109	0.2106	-2.9006	0.0040
ar2	-0.1969	0.2411	-0.8167	0.4148
ma1	0.7164	0.1913	3.7447	0.0002
ma2	0.4527	0.2215	2.0434	0.0420
sigma^2 estimated as 0.0002018:				
$AIC [1] -7.477652 ; $AICc[1] -7.469051; $BIC[1] -8.422872				

Source: Author’s construction

$$\Delta \ln X_t = \emptyset_1 \Delta \ln X_{t-1} + \emptyset_1 \Delta \ln X_{t-2} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} \quad (11)$$

$$(X_t - X_{t-1}) = -0.6109_{[-2.9006]} (X_t - X_{t-2}) - 0.1969_{[-0.8167]} (X_t - X_{t-3}) + 0.7164_{[3.7447]} \epsilon_{t-1} + 0.4527_{[2.0434]} \epsilon_{t-2} + \epsilon_t \quad (12.1)$$

The t-statistics of the parameter estimate are enclosed in parentheses beside the estimated coefficients. The regression coefficient on the AR(2) is not statistically different from zero. The ACF plot of the residuals from the ARIMA(2,1,2) fitted model shows that all the correlations are within the 95% confidence interval limits of the residuals which shows a whit noise pattern. The Ljung-Box statistic for the estimated model is Q(24) =16.621 with chi-squared distribution with 20 degrees of freedom (df) and a computed large p-value which also suggests that the residuals of the fitted equation is Guassian white noise. Notice that the estimated model (12.1) does not contain a constant term because the series under investigation was tested to contain unit root and was not a stationary process which could have been hard to justify for the Liberian economy exchange rates due to the way in which the rates fluctuated within the specified time frame. Moreover, according to Box-Jenkins, when d > 0, constant should not be included except for series showing significant trend; see (L-Stern, 2013).

Checking the estimated model critically by looking at its diagnostic plots presented in 'Appendix A2' in 'Figure 7 ', the time plot of the standardized residuals seems to be very stable and it displays no irregular patterns. Contrast to the standard unique pattern, there are outliers that are detected. The correlogram plot of the ACF of the standardized residuals displays no wandering format from the fitted model. Also, the Q-Q plot in the same appendix of the estimated model (12.1) shows a fixed horizontal line and it moves away with an elongated S-shaped form which could possibly be due to some nonlinear pattern and the outliers detected. Since time plot displayed in 'Figure 1a' shows upward trend and also (b) is the growth or the percentage changes of the financial series in this study with a high percentage value of 0.101 or 10.1% during the third weeks of 2017, it is prudent enough to include a constant term in the Eq. (12.1). Although, there were fixed periods with zero percent but there are trend.

Owing to this fact, the model (12.1) is re-estimated with a constant term included. One key thing to remember in modeling economic variables using most especially financial series with ARIMA procedures is that, ARIMA is a method to linearly model the data and the forecast width remains constant because the model does not reflect recent changes or incorporate new information. In other words, it provides best linear forecast for the series, and thus plays little role in forecasting model nonlinearly. In order to model volatility, ARCH/GARCH method comes into play; see (L-Stern, 2013).

Table 5: Estimated ARIMA(2,1,2) of lnx with constant term

	Estimate	SE	t-value	p-value
ar1	-0.6568	0.2040	-3.2194	0.0015
ar2	-0.2458	0.2290	-1.0733	0.2841
ma1	0.7482	0.1826	4.0973	0.0001
ma2	0.4867	0.2086	2.3332	0.0204
Constant	0.0021	0.0010	2.0747	0.0390

sigma^2 estimated as 0.0001985:
$AIC[1] -7.486129; $AICc [1] -7.47716; $BIC [1] -8.417654

Source: Author's construction

In 'Table 5', there is constant term in the ARIMA (2,1,2) estimated model. The estimated $\alpha, \emptyset$, and θ are shown in the above table. All estimated values of the AR(1), the constant term and the MA(1) and MA(2) terms are significant except the AR(2) term. The parameter estimates are not very close to unity. To properly choose the model deterministic term, I estimated two separate models without and with constant term and extracted the information criteria (IC) from the separate model presented in the above two Tables. But, interestingly, the AR(2) term is not significant in the models with and without constant term. With this backdrop, I present the output of two separate results after dropping the AR(2) term and the diagnostic tests based on the Ljung-Box test statistic. Also, the sigma squared results are computed and indicated in the below Table 6. Carefully, looking at the results in the above 'Table 6', the model with constant term seems to be a good one and therefore, it is maintained in this study. Fitting another model excluding the AR(2) term, the result is given in the below 'Table 7'. Interestingly, the estimates of the moving averages are lower as compared to the ones in Tables 4 and 5 respectively. Another useful thing to note is that the IC all choose the ARIMA(1,1,2) model instead of the former. This model is the final maintained for the analysis of the returns LRD/USD in this study.

Table 6: ARIMA deterministic model selection

ARIMA(1,1,2) without constant term				
$AIC	$AICc	$BIC	sigma^2 estimated	Ljung-Box test
[1] -7.482936	[1] -7.47464	[1] -8.441851	0.0002022:	X-squared = 29.927, df = 32, p-value = 0.5718
ARIMA(1,1,2) with constant term				
$AIC	$AICc	$BIC	sigma^2 estimated	Ljung-Box test
[1] -7.48992	[1] -7.48132	[1] -8.435141	0.0001993:	X-squared = 29.389, df = 32, p-value = 0.5993

Source: Author's construction

Now, we realized that all the estimated values are all statistically significant at the 5% levels as shown in 'Table 7'. The Ljung-Box statistic calculated p-value is larger than the 5% critical value, therefore we fail to reject the null hypothesis of autocorrelation in the residuals of the series. The plots of the residuals of the ARIMA(1,1,2) is given in Appendix A2 of the text. The ACF shows no spikes but the QQ plot shows nonlinear relationship.

Table 7: ARIMA Estimation with nonsignificant estimates exclusion

	Estimate	SE	t-value	p-value
ar1	-0.5128	0.1616	-3.1738	0.0017
ma1	0.6112	0.1578	3.8722	0.0001
ma2	0.2560	0.0642	3.9889	0.0001
Constant	0.0021	0.0011	1.9709	0.0498

sigma^2 estimated as 0.0001993:
$AIC [1] -7.48992; $AICc [1] -7.48132; $BIC [1] -8.435141

Source: Author's Construction

5.3 The ARCH-GARCH Estimations

5.3.1 Testing for ARCH Effects

In Figure 1 a to b the time plots of log LRD/USD in (a) and its returns plot in (b) have been shown already. The idea behind plotting the log of the exchange rate instead of the exchange rate is that the change in the log of a variable represents a relative change (or rate of return), whereas a change in the variable itself represents an absolute change; see (Gujarati & Porter, 2009). To capture the volatility component of the log exchange rates denoted log(x) and following the simple methodological approach of (Gujarati & Porter, 2009), where the series can be represented as $x_t = c + \epsilon_t$; x_t as the percentage change and ϵ_t as the random disturbance term. Given the LRD against USD with weekly sample ranging from January 7, 2013 to December 25, 2015, the least squares (LS) output is given below is to measure the volatility of the series

$$\hat{x}_t = 0.002122$$
$$se = (0.000905) \qquad (12.1)$$

Durbin-Watson stat (DW) = 1.852 p-value=0.224

The mean of monthly returns from the fitted simple ARCH model is 0.002122 or about 0.2122% and its variance is about 0.0001402. This is practically zero which means that in the market, that the market has low returns for the sample period. Only the constant parameter is included in the estimated model with no exogenous variables. The intercept term as may also be called is the average percent rate of return for the exchange rate series (lnx). This estimated model (13.1) is not really a model of interest but the squared residuals which measures the deviation of monthly returns from the mean value is important. There are swings in this process which continues over the sample period, and this is a possible sign of volatility in the series and seems to be autocorrelated. The unusual very low and tranquil pattern from the above image is alarming and therefore, the estimated ARCH effects model below will show the investigation of this volatility. The plot of the squared residuals is displayed below in Figure 5.

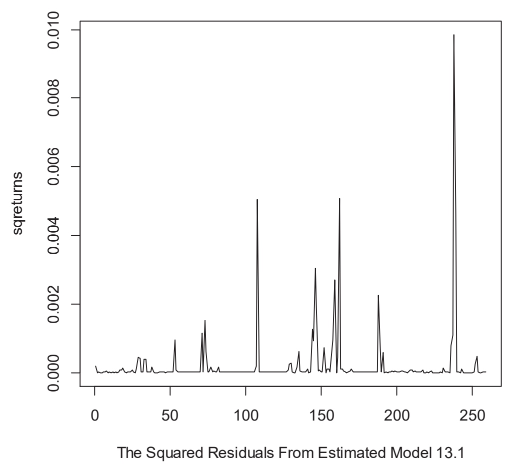


Figure 5: Squared residuals from regression 13.1

The estimated ARCH(1) test model from the residuals in regression model (13.1) is presented below.

$$\hat{\epsilon}_t^2 = 0.0001402_{[2.648]} + 0.3352_{[5.693]}\hat{\epsilon}_{t-1}^2 \qquad (13.2)$$
$$R^2 = 0.1124 \;\; DW = 1.97; p - value = 0.516$$

The $\hat{\epsilon}_t$ is the residual from (13.1) estimated regression. The t-values are enclosed in brackets beside their corresponding estimated parameters. The lagged one valued on the squared disturbance term in model (13.2) is statistically different from zero, and its calculated Durbin-Watson statistic is higher suggesting that there is no information in the lagged of the error term to its current value. In order words, the estimated ARCH(1) effects model is statistically significant since its lagged one residuals is significantly different from zero. Furthermore, to determine whether the returns series in this study is predictable, the author plots the ACF of the ($Dlnex^2$) labeled in 'Appendix A1'. Evidently, the squared residuals is predictable. However, in the context of the ARCH(1) process, knowing the squared error in the previous period $\hat{\epsilon}_{t-1}^2$ improves our knowledge about the likely magnitude of the variance in period t, which is useful for situations when it is important to understand risk, as measured by the volatility of the variable; see (Hill, et al., 2011). The LM test is applied on (T-q)*Rsq; where the q is the degrees of freedom and the T is the sample size, and the Rsq is the usual statistic in regression model. Since the Lagrange Multiplier (LM) test shows a test statistic of $F \approx 32.40641$ a *p*-value is practically zero, and very well below 0.05, the null hypothesis of no conditional hetereoskedascity, ARCH effects is rejected suggesting that there is ARCH effects in LRD-USD returns. Moreover, this proves that we can further estimate the ARCH*(*p*)* parameter and estimate jointly the mean function and the variance function of either higher order ARCH(p) model since there is ARCH effects in the residuals of the mean equation of this univariate time series.

Also, Figure 5 illustrates the volatility nature in the observed financial asset in this study. Furthermore, volatility in the exchange rate is the due to the risk of the financial series as its variance changes overtime, thus the ARCH effects can be seen as the measurement of the risk the financial series poses. Note that Eq. (13.2) is defined as the ARCH(1) effects model because there is only a lag attached to the squared residuals but the Eq. (13.2) can be expanded to include many lagged values as defined in Eq.(3.2). In the literature, the expected sign(s) of the parameters to be estimated are to be nonnegative since variance cannot be negative, see (Gujarati & Porter, 2009) for further discussion on this.

Another key issue that is considered here is that we have presented the squared returns of the series to detect autocorrelation because the Figure (5) shows clustered volatility which will affect the next time period(week). The Ljung-Box computed statistic for the squared returns is Q(12)=33.65, p-value = 0.00077 which is less than 0.01, 0.05 significance levels which shows strong ARCH effects. The different kinds of plots for the returns and squared returns series are illustrated in 'Appendix A1' where the ACF of the squared log returns series show spikes, that is significant autocorrelation at lags 1 and 2. It is the predictability nature of the series that showed those significant correlation. In the words of (Tsay, 2010), if the ARCH effects is determined to be significant, one can use the PACF of the σ^2 to determine the order of the ARCH model. Adding additional number of lags to the specification as in Eq. (3.2) can also be done on the basis of the Akaike information criterion (AIC) and Schwarz information criterion (SIC). In 'Appendix A1' the plots of the returns (LRD/USD) series show that the variable is not independent but rather the percentage changes of the series is conditional hetereoskedastic. With this in handy, the author fits and ARCH(1) based on the sample PACF in the Appendix A1 of the squared returns using the maximum likelihood method (MLM) assuming that the distribution is not normal but rather a generalized error distribution. The results of the estimated mean and the estimated variance of the ARCH(1) model is given below. The PACF of the ϵ_t^2 is a useful tool to determine the order p of the ARCH model (Tsay, 2010).

5.4 Normal innovation results of the ARCH(1) estimated model

The estimated values are presented in the below equations from the ARCH(1) estimates for the log returns with the normal distribution assumption.

$$\hat{x}_t = \hat{c} = 0.001353 \quad (13.3a)$$

Where the standard error is computed to be 5.199E-04; and the t-value is given as 2.603 respectively. The estimate of the mean equation is highly significant at the 5% level. Further shown in equation (13.3b) is the variance equation of the estimated model with calculated standard errors for the estimates given as 7.084E-06 for the intercept term and 9.027 as the t-value on the intercept term . Also, the standard error for the ARCH term, Resid(-1)^2 is 1.976E-01 and the t-value is 5.062 which is statistically different from zero.

$$\hat{h}_t = \gamma_0 + \hat{\gamma}_1 \hat{\varepsilon}_{t-1}^2 = 0.00006395 + 1.000\ \hat{\varepsilon}_{t-1}^2 \quad (13.3b)$$

The results from the estimated ARCH(1) is not satisfactory as the estimated variances estimate is equal one which implies that there is persistent volatility in the returns series. One point to note is that, this estimation is based on the normal distribution assumption. The estimates are all significant at the 1% levels. Moreover, the Ljung-Box statistics of the standardized residuals of Q(10)= 5.49 with p-value of 0.86. Similarly, the squared standardized residuals of Q(10)= 4.07 with p-value of 0.94. The Lagrange Multiplier (LM) statistic is TR^2 = 5.11 and the computed p-value is 0.95 show that the model is adequate; likewise the LM ARCH test is statistically not different from zero supporting the Ljung-Box statistics. Note, that all the estimated parameters of the ARCH(1) specification are statistically significant at the 5% level.

The estimated ARCH(1) expected weekly returns of the logarithmic exchange rates is 0.001345% or approximately 0.1353% which is not different from zero. This low return is not surprising as the LRD/USD has been very much volatile which has led to fluctuations in economic activities in the Liberian economy for a very long time. If we were to take a look at the unconditional standard deviation of the returns series $x_t = \sqrt{0.00006395/(1-1.000)} \approx$ undefined. In the literature, when $\alpha_1 > 0$, it means that the conditional variance is high due to a high lagged squared error which could lead to high expected errors. Thus, this might be due to outliers in the data or the distribution of normality is too strong to hold. Also, the estimated ARCH(1) function based on the normality condition can not help to predict the weekly volatility of the LRD/USD returns. The Jarque-Bera test rejects the null hypothesis that the conditional distribution of the return is normal distribution.

5.5 GED innovation of the ARCH(1)

Assuming the nonnormal distributional condition of the ARCH(1) with AR(1) term seems to be necessary for this study since the kurtosis is also very high and significant. Also with the nonstationary term of the arch term could also mean that that the normality condition is too harsh for the return LRD/USD series for the sample under investigation. Now re-estimating the ARCH(1) model assuming the GED, the result is given as

Table 8: Estimates of ARCH(1) function with GED

Variable	Coefficient	Std. Error	z-Statistic	Prob.
mu	5.71E-25	4.17E-06	1.37E-19	1.0000
	Variance Equation			
omega	5.97E-05	1.90E-05	3.132902	0.0017***
alpha1	0.360950	0.173204	2.083956	0.0372**
GED PARAMETER	0.073753	0.000815	90.53643	0.0000***

The computed Ljung-Box statistics for the standardized and squared standardized residuals are Q(10) = 7.2289 with p-value of 0.704; and 2.9587 with computed p-value of 0.982 respectively. The LM statistic TR^2 is 0.097 with p-value of 0.7551. As for the skewness and kurtosis of the estimated ARCH(1) with four parameters, there is high kurtosis value of 13.66 and low skewness value of 0.303. Another interesting realized from the ARCH(1) estimates assuming GED is that the mean equation value is practically zero and not even significant, but the volatility equation and the term on the ARCH term has reduced as compared to the former estimated value.

5.6 ARCH(1) estimates from ARIMA(1,1,2) with normal distribution

To circumvent this process and to compare the usual of the ARCH(1) model estimated with the normality distribution assumption, this study presents the estimated model using the residuals from the ARIMA(1,1,2) model specified in 'Table 9' not to the original series or the log or differenced log series because we only want to model the noise of ARIMA model; see (L-Stern, 2013) for similar method and discussion. Since, the primary objective research is to estimate the series with the models already discussed previously, and to compare their estimating abilities to the univariate ARIMA, this approach seems necessary for this volatility modeling of nonlinear ARCH/GARCH models. To aid in this process, I plotted several diagrams of the residuals from the fitted ARIMA(1,1,2) model with the constant term and the diagrams are all displayed in 'Appendix A3a' of this work. Thereafter, I estimated the ARIMA(1,1,2) models with and without constant. The results are presented below:

The results in the equations below show the ARCH fitted to ARIMA(1,1,2) previously fitted including a constant term. The p-value of the constant term (conditional mean) is greater than 0.05 which cannot be rejected, while the p-values of the conditional variance equation are all less than 0.05. Non-rejection of the null hypothesis implies statistical insignificance; whereas, rejecting the null at the 5% level means statistical significance of the estimates. Furthermore, several diagnostics test results are displayed below the coefficient estimates. The Ljung-Box at different chi-squared df are all statistically insignificant, which means the model is adequate and free of autocorrelation. The Q(10) = 8.88 with calculated p-value of 0.54; while the Q(10) = 5.31, p-value of 0.87 for the squared standardized residuals of the series. Also, normality test rejects the null. In equations form, the fitted ARCH(1) model in 'Table 9' can be written as

$$\hat{x}_t = c = -0.0000441 \tag{13.1a}$$

$$\hat{h}_t = \hat{\gamma}_0 + \hat{\gamma}_1 \hat{\epsilon}_{t-1} = 0.0000814 + 0.978\widehat{\epsilon^2}_{t-1} \tag{14.1b}$$

The unconditional standard deviation is approximately $\sqrt{0.0000814/(1-0.978)} \approx 0.0037$. Note that the equations (14.1a) and (14.1b) are the conditional mean and the conditional variance functions respectively. With the statistically nonsignificant estimates, the model is re-estimated after dropping the constant term and the result is given as follows:

$$\hat{h}_t = \hat{\gamma}_0 + \hat{\gamma}_1 \hat{\epsilon}_{t-1} = 0.0000815 + 0.976\widehat{\epsilon^2}_{t-1} \tag{14.1c}$$

Where the estimated parameters values standard errors are 9.775E-06, and 2.590E-01 respectively; whereas, the t-values are [8.336]*** and [3.767]*** which are statistically significant at the 1%, 5% and 10% critical levels. The re-estimated model is not normally distributed as the JB and Shapiro-Wilk p-values are practically zeros. Similarly, the Ljung-Box test statstics at the Q(10) is 8.84 with p-value given as 0.55; while the squared residuals statistic of Q(10) is also computed to be 5.39, p-value is 0.86. These results show that there is no serial correlation is the residuals of the estimated model. The ARCH test for heteroscedasticity is calculated to be 5.91 with p-value computed to be 0.92 which is nonsignificant. As for the model criteria statistics, the AIC, BIC, and the HQIC are computed as -6.074, -6.046, and -6.063 which show improvement over the model estimated with drift term. Further stated, the ARCH(1)-ARIMA(1,1,2) model is adequate for the heteroskedastic feature of the data at the 5% critical value. The expected weekly log LRD/USD returns as given in Table 6 is practically zero which means that which is not surprising as the dual currencies in the economy has been very much volatile and with this sample that includes the Ebola outbreak year(s) and the economic melt down periods of 2015 and so forth. The same can be said about the equation with a 0.00815% log returns to the LRD/USD. The equation estimated conditional heteroskedastic variance aggregated for only one past period is, $\bar{\epsilon}^2$= 0.976^2 is greater than $\frac{1}{3}$ which does violate the conditional fourth moment of the series weekly returns. Computing the unconditional standard deviation of the relative value for the series, x_t which can be computed as $\sqrt{0.0000815/(1-0.976)} \approx 0.0583$.

5.7 Estimating the Generalized (ARCH)

The GARCH model has been extensively discussed in the time series literature as well as an overview has also been discussed in this study. This model is identified to be very popular because it fits many economic and financial series very well. The GARCH model can also be extended to higher order but the GARCH(1,1) has been widely considered as a benchmark to capture the volatility component of most time series variables. There are uncountable number of studies that have applied the GARCH(1,1). The critical assumption of a time invariant and zero mean of a series can at time be violated most often. Since the variance measures the uncertainty or risk on future values of most economic and financial time series variables, it is of essence to carefully model the variance which poses threat due to its changing nature over time in the business setting as well as the economic environment; see (Heij, et al., 2004). For instance, more recent studies of (Epaphra, 2017), (Hill, et al., 2011) and (Tsay, 2010) all demonstrated the GARCH(1,1) specification in modeling different time series variables.

5.7.1 Estimated GARCH(1,1) with both Normal (Gaussian) and GED Assumptions

Firstly, I present the result of the estimated GARCH(1,1) from the residuals of the previously considered ARIMA(1,1,2) model are displayed below.

$$\hat{x}_t = 1.299 \times 10^{-03} + \epsilon_t \qquad t = (2.624) \tag{14.1}$$

$$\sigma_t^2 = 5.013 \times 10^{-05}\ 1.000 \times 10^{+00}\epsilon_{t-1}^2 + 7.43 \times 10^{-02}\sigma_{t-1}^2 \qquad t = (7.53)\quad (5.18)\quad (1.908) \tag{15.2}$$

The standard error of Eq(15.1) is 4.95×10^{-04}; while the standard errors of the estimated parameters of the variance equation, Eq(15.2) are 6.66×10^{-06}, 1.93×10^{-01} and 3.89×10^{-02} respectively. The estimated GARCH(1,1) follows a normal distribution. The estimated parameter of the GARCH term is not statistically different from zero and this is reduced to ARCH(1) model. Some useful statistics computed from the estimated model such as the Ljung-Box statistics of the standardized residuals, Q(10)= 4.201739 with p-value computed as 0.9377876. Similarly, the squared standardized residual $\{\bar{\epsilon}^2\}$ computed value of Q(10) =1.914191 with a calculated p-value of 0.9969565. The Lagrange Multiplier statistic of TR^2 = 2.903172 with p-value computed to be 0.9961899. The Jarque-Bera and Shapiro-Wilk normality test statistics are 1555.1638 with computed p-value zero, 0.739 with computed p-value of zero. Secondly, the below Table 9 illustrates the estimated GARCH(1,1) function for the log returns series assuming both normal distribution of the error and the generalized error distribution due to the heavy tail process of the series. Surprisingly, the all parameters for the normal distribution assumption are all statistically significant, but the ARCH term is greater than 1. On the other side as can be seen in Panel B and BB, the GED assumption fitted to the GARCH(1,1) function is remarkably different from its counterpart. Only the GARCH(-1) term and the GED parameter are statistically significant. Also, the mean equation in Panel B is zero when the GED assumption is imposed. But this is not different from the normal distribution assumption mean in Panel A.

Table 9: The results from the Estimated GARCH(1,1) with Normal distribution and GED

	Panel A: mean equation				**Panel B: mean equation: GED**			
	coefficient	std.error	z-statistic	p-value	coefficient	std.error	z-statistic	p-value
C	0.001267	0.000461	2.75	0.0060	1.73e-25	4.00e-06	4.33e-20	1.0000
	Panel AA: variance equation				Panel BB: variance equation			
C	4.92e-05	2.66e-06	18.50	0.0000	8.46e-05	6.16e-05	1.37	0.1697
Resid(-1)^2	1.288	0.146	8.828	0.0000	0.205	0.157	1.307	0.1913
Garch(-1)	0.059	0.018	3.167	0.0015	0.672	0.195	3.442	0.0006
					GED parameter			
					0.062	0.000	102.180	0.0000

These findings are interesting as the series is very much volatile with very low returns and that with different error distributions, there is variance in the estimates.

5.8 Generalized ARCH (1, 1) estimates from ARIMA (1, 1, 2) with normal distribution

$$\hat{x}_t = -4.408 \times 10^{-05} + \epsilon_t \qquad (15.3)$$
$$t = (-0.082)$$

$$\sigma_t^2 = 4.010 \times 10^{-05}\ 8.521 \times 10^{-01} \epsilon_{t-1}^2 + 2.620 \times 10^{-01} \sigma_{t-1}^2 \qquad (15.4)$$
$$t = \quad (6.359) \qquad (4.288) \qquad\qquad (4.916)$$

After fitting the residuals of the GARCH(1,1)-ARIMA(1,1,2), the parameter of the mean equation is not statistically significant, but the estimated terms of the variance equation are all statistically significant at the 5% level. Dropping the nonsignificant term of the mean equation, the model is re-estimated and the output is given as

$$\sigma_t^2 = 4.015 \times 10^{-05} + 8.490 \times 10^{-01} \epsilon_{t-1}^2 + 2.621 \times 10^{-01} \sigma_{t-1}^2 \qquad (15.5)$$
$$t = (6.367) \qquad (4.366) \qquad\qquad (4.904)$$

Interestingly, the re-fitted GARCH(1,1)-ARIMA(1,1,2) model estimated parameters of the variance equation are all statistically different from zero and their estimates are not too different from the estimates of Eq(15.4). The Ljung-Box statistics of the residuals of the re-fitted model are Q(10) =3.067907 with p-value of 0.9797805; whereas, the $\{\bar{\epsilon}^2\}$ is calculated to be Q(10) = 2.175326 with p-value of 0.9948112 respectively. The model seems to be an adequate representation of the log exchange rate returns for the Liberian economy without a constant term. The LM statistic is computed to be TR^2 = 3.295477 with given p-value of 0.993074. From Eq (15.5) which is the volatility function of the estimated GARCH(1,1)-ARIMA(1,1,2) to the residuals of the log returns series, the unconditional variance of the error term ϵ_t is a stationary process that is defined as

$$Var(\epsilon_t) = \frac{\gamma_0}{1 - \gamma_1 - \alpha_1} \qquad (15.6a)$$

since the $\epsilon_t = \sigma_t c_t$, therefore the unconditional variance of the returns series, $E(\sigma_t^2) = E(\epsilon_t^2)$ which is written as $\gamma_0/(1 - \gamma_1 - \alpha_1)$.

$$\frac{4.015 \times 10^{-05}}{1 - 8.490 \times 10^{-01} - 2.621 \times 10^{-01}} = -0.000361 \qquad (15.6b)$$

6 Conclusion and Policy Implications

In this current work, the core thesis was to estimate the exchange rate volatility of Liberia (i.e. LRD/USD) using the weekly observations. The paper modeled univariate ARIMA model and two volatilities time series model which were separately modeled in their original form and also to the residuals of the considered ARIMA(1,1,2) model which was maintained after some previous scrutiny were justified in this study. Further stated, the two volatilities models, ARCH and GARCH model applied in this study were estimated to the log returns exchange rate series of Liberia with two assumed distributions, that is, the normal distribution and the GED respectively. The GED was assumed and analyzed with the ARCH(1) and GARCH(1,1) models because preliminary tests mentioned in the previous sections find it useful and accurate for this distribution to be analyzed to the series. The approach of this paper is expedient to investigate the persistent volatility pattern of the Liberian economy because most empirical applications that have investigated the volatility pooling of the financial series have not justifiably shown that normal distribution, outliers and other forms of nonstationarity can hinder the accurate modeling process of the volatile series in the Liberian setting. Importantly in this study, two traditional unit root test show that the series is nonstationary and the correlogram plots displayed in previous sections also gauged the volatility proofs of the series.

The estimated value of the ARCH term and the GARCH term are both statistically significant at the 5% levels. The ARIMA(2,1,2) model without constant term seems to be more adequate than the one including the constant term. The conditional mean equation of the fitted GARCH(1,1,) in Eq.(15.1a) is approximately zero. The sum of theARCH and GARCH terms estimates is 1.1131 which further indicates how volatile the LRD/USD returns series has been. In order words, the returns series is nonstationary or has a unit root-which is an integrated process. Another thing to notice is that the fitted ARIMA (1,1,2) residuals fitted to the GARCH(1,1) model including a constant, the standardized residuals are normally distributed. The two popular normality tests that is Jarque-Bera and Shapiro-Wilk both reject the null of normality. In the words of (Brooks, 2008), he states that nonnormality is not really a great deal in a well-specified GARCH estimation model but that the computed usual standard error estimates will be inappropriate, and a different variance--covariance matrix estimator that is robust to nonnormality, due to Bollerslev and Wooldridge (1992), should be used. This procedure (i.e. maximum likelihood with Bollerslev--Wooldridge standard errors) is known as quasi-maximum likelihood, or QML; see (Brooks, 2008).

Moving further on, the fat-tailed distribution as was clearly computed in the summary statistics for the relative and the changes in the log exchange rate return series prove clearly that the series has fat tail distribution with high value calculated that was displayed in the previous Table 1. Similarly, the JB test with its very high statistical value and highly significant p-value further proves that the LRD-USD series modeled with this high frequency data is very much volatile. Another useful findings from this empirical investigation is the estimation of the simple linear model to the log returns to test for ARCH effects in the series. The fitted model with only a constant term with no other regressors and the squared residuals tested to the lag one for ARCH effects show that there is huge heteorskedastic pattern in the series labelling the series to me more nonstationary over time. On the other end, the current study orderly displayed the pattern in which the study objectives were to be achieved both investigating useful empirical studies and employing the methods to accurately model the return series of Liberia.

The estimated ARCH(1) model chosen was based on the PACF of the squared returns in this study proves vital; even though, the persistence of the series was immediately seen after the estimation of the ARCH model. The study reintroduced another method following the approach of (L-Stern, 2013) and re-estimated the residuals obtained from the ARIMA(1,1,2) model and fitted and ARIMA(1,1,2)-ARCH model as well as GARCH. The methodological approach proves sensitive as the ARCH parameter and the GARCH parameter that were greater or equal to unity reduced drastically. This approach is sensitive to the model specification but the Ljung-Box and the LM statistics all indicated that the model was free of autocorrelation and heteroscedasticity.

Insightfully, the unconditional standard deviation calculated for the ARCH(1) original model was undefined due to the high persistence of the estimated parameter which clearly shows that the ARCH model is limited to capture the full volatility sequence in the series. The time series plots and the diagnostic plots presented in the appendices further show that the series is much more volatile and that accurate or proper investigation as shown in this study will clearly display the behavior of the financial series which has been very much unstable in the Liberian economy.

In conclusion, the volatility pattern of the exchange rate series of Liberia is clearly defined and that accurate and robust methods can shown the true behavior of the series, and in which, when identified can be accurately and reliably modeled and predicted for future periods ahead. Our approach in here was not to forecast the time series under investigation, but to show the actual and proper estimating patterns that will unveil the structure of the time series behavior which could help ease the stress of adopting the wrong methods to investigate this volatile series. Accurate measurement and forecasting of s-step ahead exchange rate volatility in Liberia needs to be strongly considered because the country is highly dependent on imports and foreign direct investment and the need to control the volatility component should be considered greatly.

7 The Research Limitations and Future Work

Despite the several useful literature and the modeling of these robust time series models and methods applied to estimating the weekly exchange rate of Liberia, the current study does not model the leverage and the nonlinear nature of the series which could have been better estimated with some more time series methods other than just the three methods employed here. However, this study has just added to the several financial applications in the literature; and the future task is to estimate and forecast the integrated components and the some macroeconomic impact of the financial series controlling for more financial variables to investigate in more detail the effects of volatility in the Liberian economy. The policy recommendation highlighted in here is that the exchange rate behavior in the Liberian economy is worrisome and that policy makers and modelers should consider leverage effects and nonlinearity when modeling the financial series in order to capture the true pattern of the underlying series.

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Appendix A1

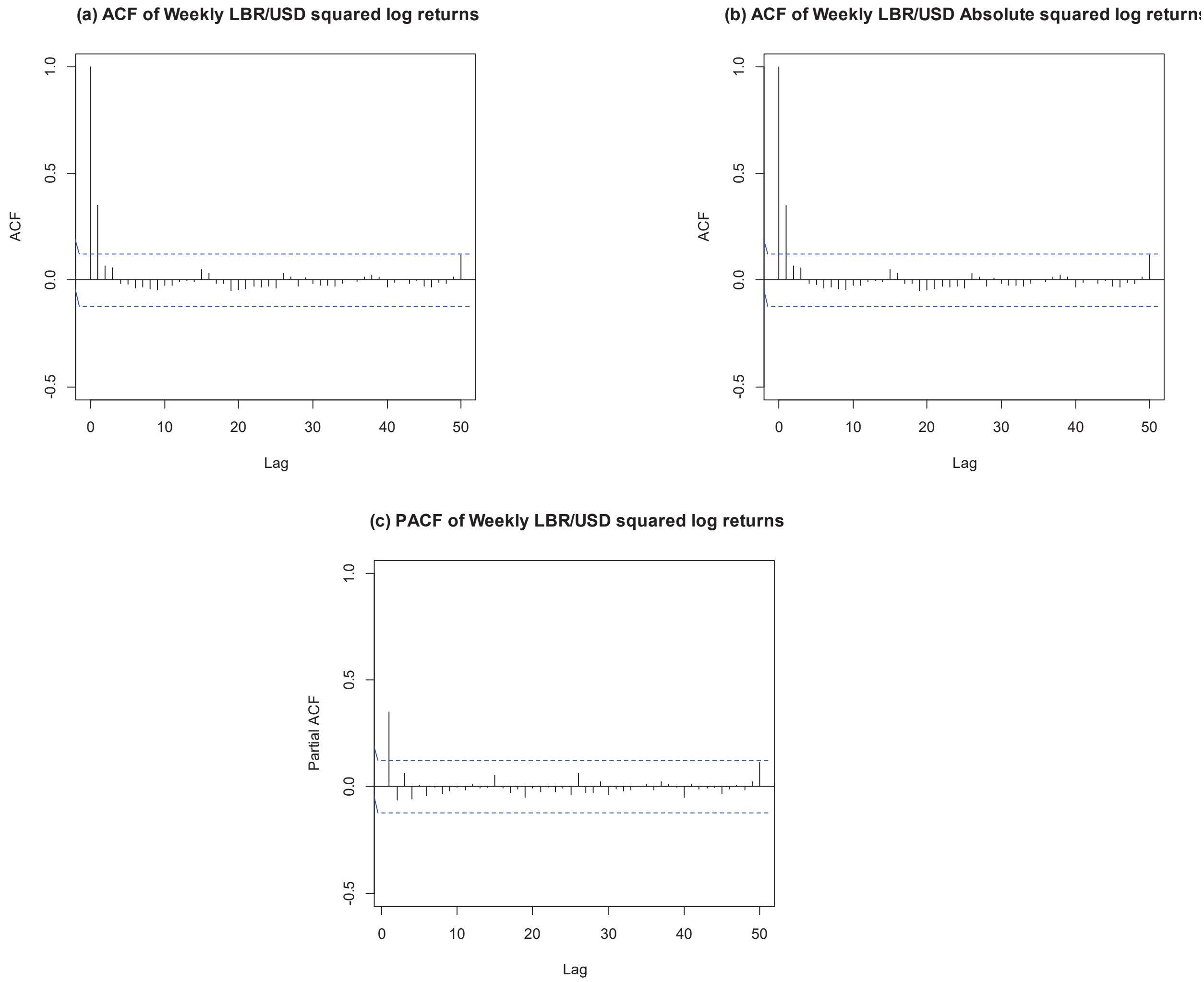
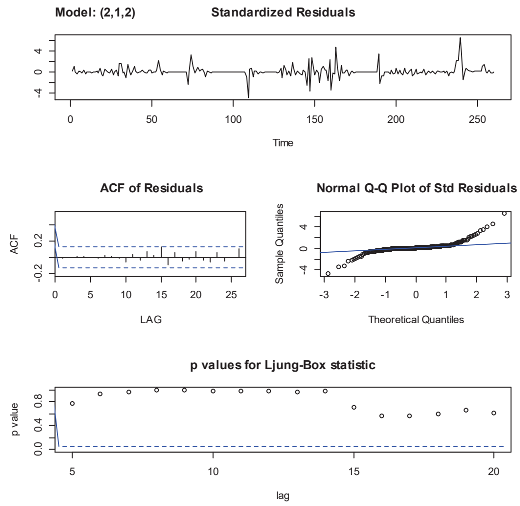
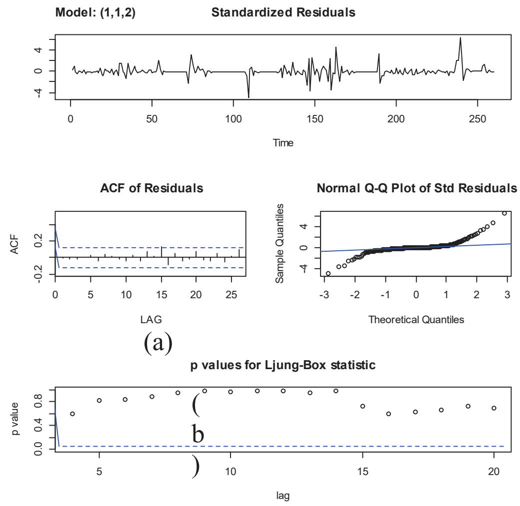


Figure 6 Different Sample ACFs and PACF of weekly log exchange rates returns of Liberia from January 1, 2013 to December 25, 2017: (a) ACF of the squared Δlnx; (b) ACF of the absolute squared returns Δlnx and (c) PACF of the squared Δlnx.

Appendix A2



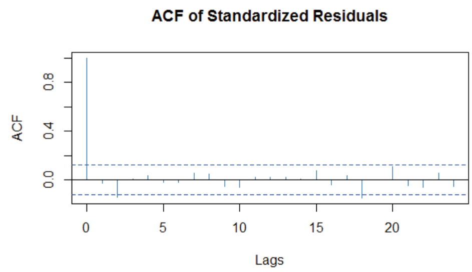


(a)

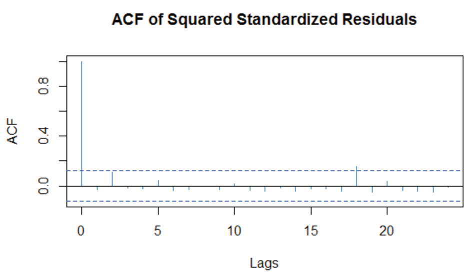
(b)

Figure.7.
Diagnostics plots of the residuals of the ARIMqa EDCER E HDFA(2,1,2) in (a) and ARIMA(1,1,2) in (b)log LRD/USD growth rate series

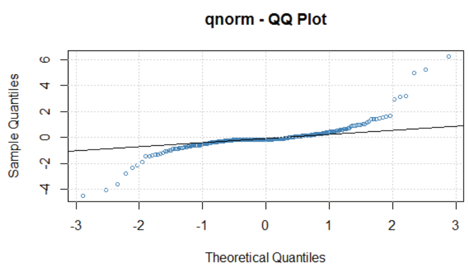
Appendix A3(a)



(a)
(b)



(c)



(d)

Figure. 8. Model diagnostics of the residuals from ARIMA(1,1,2)-ARCH(1) on log LRD/USD growth rate from Jan. 07, 2013 to Dec. 25, 2017: Parts (a), (b) (c) and (d) are the standardized residuals, ACF of standardized residuals, ACF of squared standardized residuals, and the Q-Q plots of the fitted model.

Appendix A3(b)

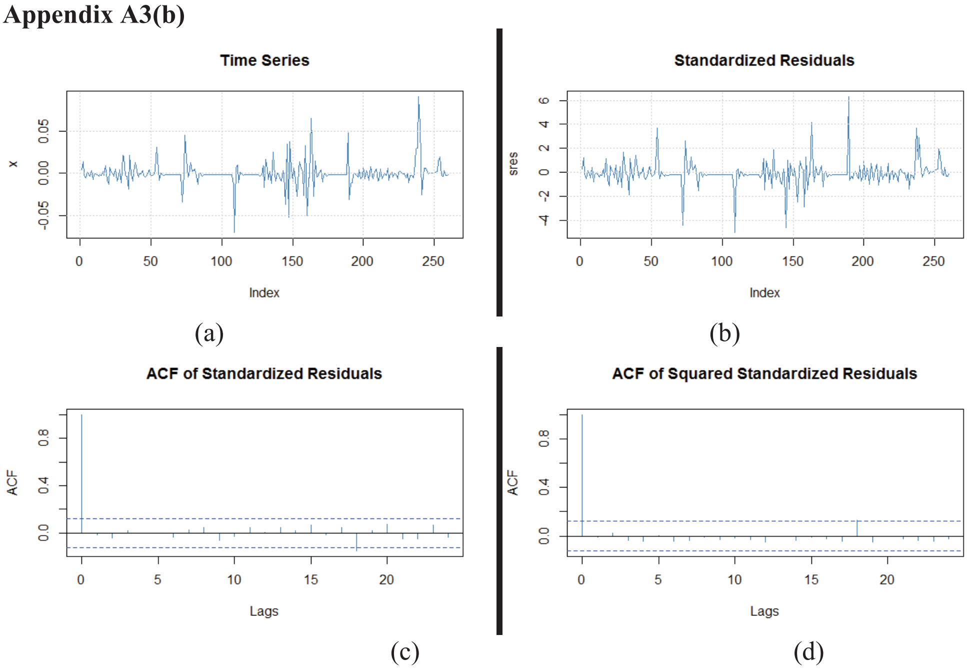


Figure. 9. Model diagnostics of the residuals from ARIMA(1,1,2)-GARCH(1,1) on log LRD/USD growth rate from Jan. 07, 2013 to Dec. 25, 2017: Parts (a), (b) (c) and (d) are the time plot of the volatility component estimated for the series returns, standardized residuals that represents the shocks of the weekly series returns of Liberia, ACF of standardized residuals and ACF of squared standardized residuals.