Application of Advanced Panel Threshold Regression Model to Estimate Optimal Capital Structure

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Abstract
Modern capital structure theory started in 1958, when Modigliani and Miller (1958) first brought out “Capital Structure Irrelevance Theory”, advocated that the firm value and weighted average cost of capital (WACC) is unaffected by the financial structure of the firm. The goal in this paper is to analyze whether leverage affects firm value for a panel of Persian listed companies during the period 2009 to 2015. ROA, ROE, EPS and Tobing’s q are adopted as proxy variables for firm value. The result shows that there exists a single threshold effect between debt ratio and firm value only when Tobing’s q is selected as the proxy variable for the firm value. The estimated threshold value (γ) is found to be 37.84% and two coefficients (α₁ and α₂) are all positive with the evidence that the α₁ in the low debt level is significant, while the α₂ in the high debt level is not. Advanced panel threshold regression model is performed to test if there exists an “optimal” debt ratio, which may result in threshold effects and asymmetrical relationships between debt ratio and firm value.

Keywords: panel threshold regression model, Tobin’s q, Firm Value, Capital Structure

Introduction
The extension of M&M and Miller’s model is the trade off theory between the tax advantage of debt and various leverage-related costs (such as debt-issuing costs, bankruptcy costs, agency costs, and loss of non-debt tax shields). Direct bankruptcy costs include the costs that are associated with bankruptcy, such as legal and administrative costs. In addition, though borrowing saves a firm’s money on its corporate taxes, but the more a firm borrows, the firm increases its risk causing the firm’s bond rating to decrease, and its costs of debt to increase. The more likely it is that the firm becomes bankrupt and finally even has to pay the “bankruptcy tax”. Indirect bankruptcy costs include the difficulties of running a business that is experiencing financial distress. Moreover, Jensen and Meckling (1976) specified the existence of “agency costs” which arise due to the conflicts either between managers and shareholders (agency costs of equity) or between shareholders and debtholders (agency costs of debt).

Modern capital structure theory started in 1958, when Modigliani and Miller (1958) first brought out “Capital Structure Irrelevance Theory”, advocated that the firm value and weighted average cost of capital (WACC) is unaffected by the financial structure of the firm. However, M&M’s perfect market assumptions: such as no transaction costs, no taxes, symmetric information and identical borrowing rates, and risk free debt, are contradictory to the operations in the real world. Modigliani and Miller (1963) later modified their original M&M’s model and considered the tax deductibility of interest (tax shields effect). According to modified M&M theory with taxes, value of levered firm equals the value of un-levered plus the value of the tax shields. In this case, the more the debt in the capital structure, the higher will be the value of a levered firm. One can always increase firm value by increasing leverage, implying a capital structure of 100% debt is optimal to maximize the firm’s value. Miller (1977) further added personal taxes to the analysis and demonstrated that tax deductibility of interest at the firm level is offset by personal income taxes at the investor level.

Review of Literature
In the Static Tradeoff Theory (Myers, 1977) there is a static or balance amount of debt and equity for the manager to decide, by analyzing the trade-off between the benefits of more debt versus the cost of additional
probabilistic approach to the problem of optimization of corporate capital structure. The approach enables bankruptcy in the future as a function of the time interval remaining until the bankruptcy. The probability is then benefits, which may also increases the degree of the firm’s bankruptcy and agency costs. Therefore, they argued positive personal taxes on equity and on bond income, 2. expected costs of financial distress, and 3. positive non-

profits of shareholders, they possibly will refuse issuing new shares of stock (equity) and debts, and prefer patterns of external financing.

The relationship between capital structure and firm value has been the subject of considerable debate throughout the literature. There are two issues to discuss: 1. Whether there is an optimal capital structure for an individual firm; 2. Whether the proportion of debt usage is irrelevant to the individual firm’s value. Castanias (1983) emphasized the possibility of bankruptcy has a negative effect on the value of the firm. As the proportion of debt in the firm’s capital structure is increased, the probability of bankruptcy also increases. Consequently, the rate of return required by bondholders increases with leverage. The optimal ratio of debt to equity is determined by taking an increasing amount of debt until the marginal gain from leverage is equal to the marginal expected loss from the bankruptcy costs. Altman (1984) compared the present value of expected bankruptcy costs with the present value of expected tax benefits from interest payments on leverage, and concluded that the potential impact of bankruptcy costs on firm valuation and capital structure issues is very important. Jensen (1986) emphasized the agency conflicts between top managers and shareholders. These conflicts are especially severe in firms with “large” free cash flows—more cash than profitable investment opportunities. Top managers may waste cash on organization inefficiencies or invest it at the projects that the net present value (NPV) of them is small than zero. In this case, increasing of debt levels lower free cash flows, consequently increase the value of firms. Leland and Toft (1996) pointed out the use of long-term debt financing, though generates more tax benefits, which may also increases the degree of the firm’s bankruptcy and agency costs. Therefore, they argued that using short-term debt reduces agency conflicts, thus reducing the associated degree of risk.

With respect to finding the optimal capital structure, Philosopnov and Philosopnov (1999) developed a probabilistic approach to the problem of optimization of corporate capital structure. The approach enables quantitative assessment of optimal Debt/Equity ratio, and includes calculation of probability of corporate bankruptcy in the future as a function of the time interval remaining until the bankruptcy. The probability is then used in a modified formula of discount share valuation to calculate the share or value of a corporation. In addition, modern “dynamic” capital structure model (Goldstein, Ju and Leland, 1998), extending “static” tradeoff models, simulated an optimal capital structure by Monte Carlo approach. Most traditional capital structure models assume that the decision of how much debt to issue is a static choice. In practice, however, firms adjust outstanding debt levels in response to changes in firm value. The study demonstrated the optimal strategy of a firm when it has the option to increase debt levels in the future. Due to the target debt ratio changes over time, and implying reversion to previous debt levels. In particular, companies investigated the contingent cash flows for arbitrary capital structure strategies, and managers choose the one that maximizing current shareholders’ wealth.

Contradictory to Tradeoff Models, the Pecking Order Model (Myers and Majluf, 1984) emphasized asymmetric information between managers and outside investors, and predicts external debt financing driven by the internal financial deficit but not interest tax shield benefit. Since the managers have the information that the outside investors do not have, and they make decisions usually based upon the objective of maximizing the profits of shareholders, they possibly will refuse issuing new shares of stock (equity) and debts, and prefer internal financing. Shyam-Sunder and Myers (1999) further demonstrated the moving of the capital structure or the changes in debt ratios are driven by the need for external funds, not by any attempt to reach an optimal capital structure. It is the result of the financial hierarchy, which descends from internal funds (retained earnings), to debt (safe debt, risky debt), to external equity. In particular, a firm that realizes a reduction in value because of very poor profits may become more highly levered because of a reluctance to issue new equity. However, Chirinko and Singha (2000) specifically made a critical comment to Shyam-Sunder and Myers (1999), and considered their simple test and conclusions generated misleading inferences when evaluating “plausible” patterns of external financing.

There are a few papers studied the determinants of the choice of capital structure. Bradley, et al. (1984) developed a model that synthesizes the modern balancing theory of optimal capital structure and incorporates: 1. positive personal taxes on equity and on bond income, 2. expected costs of financial distress, and 3. positive non-debt tax shields. Using simulation analysis, the results indicated that firm leverage ratios will be negatively
related to the volatility of firm earnings if the costs of financial distress are nontrivial. The analysis also showed strong industry influences across firm leverage ratios. Concern is raised whether focusing on leverage ratios is the best way to uncover the determinants of capital structure. Castanias (1983) finds that ex ante default costs are large enough to induce the typical firm to hold an optimum mix of debt and equity. Titman and Wessels (1988) conducted analysis of measures of short-term, long-term, and convertible debt instead of an aggregate measure of total debt. It was found that debt levels are related negatively to the "uniqueness" of a firm's line of business. The results further indicated short-term debt ratios were demonstrated to be related negatively to firm size.

Morellec (2001) investigated the impact of asset liquidity on the valuation of corporate securities and the firm’s financing decisions. The empirical studies showed that asset liquidity increases debt capacity only when bond covenants restrict the disposition of assets. However, with unsecured debt, greater liquidity increases credit spreads on corporate debt and reduces optimal leverage. The model also determined the extent to which pledging assets increases firm value. Lie (2002) investigated whether companies use self-tender offers to optimize their capital structure. The debt ratios of the firms around the offers, are compared with predicted debt ratios by static trade-off model. The results showed that self-tender offers undertaken to defend against takeovers reach a debt ratio that reduces the probability that the firm will be acquired; while non-defensive self-tender offers reach an optimal debt ratio. However, to effectively deter takeovers, the debt ratio may have to be higher than optimal as predicted by the static trade-off model, in which tax benefits are traded off against financial distress costs. Bergman and Callen (1991) found out when a company’s ratio of intangible assets to total assets increases, the debt ratio appears to become relatively lower. Debt ratio is related negatively to growth of intangible assets. Burgman (1996) examined unique factors that may help explain the capital structure choice of multinational corporations (MNCs). The results suggested that specific international factors such as political risk and exchange rate risk are relevant to the capital structure decision, that multinationals have higher agency costs than purely domestic firms, and that international diversification does not lower earnings volatility for MNCs.

Taiwan, a typical island-style export-led country, is a main supplier of electronics and Information-Technology (IT) related products to the U.S. and the rest of the world. Taiwanese economy is now relies more on capital-intensive goods than ever. Among different industries, Whiting (1991) pointed out that the weighted average debt as a percentage of total capital within the electronic industry is higher than within other type of industries. Therefore it is worth exploring the effect of the use of financial leverage on firm value of electronics companies in Iran.

Aiming at investigating whether application of financial leverage affects corporate performance or firm value of electronic listed firms in Taiwan, we apply threshold regression model to the observed “balanced panel data” to test if there exists an optimal Debt/Total Assets ratio (D/TA ratio hereafter) which may result in threshold effect and asymmetrical responses of the corporate performance to the D/TA ratio. If this “threshold” value of $\gamma$ is verified, the financial managers should take steps to increase debt levels in the low debt regime of D/TA ratio lower than the $\gamma$. Conversely, they should take steps to reduce debt levels in the high debt regime of D/TA ratio higher than $\gamma$.

This paper contributes to previous literature in four aspects. First, we apply advanced panel threshold regression model developed by Hansen (1999) to test if there exists a “threshold” of optimal debt usage. In contrast with traditional linear model, this nonlinear threshold model can describes the “trade-off” between the benefits of tax shields of more debts and the disadvantages of costs from additional debts that may damage the corporate performance or value. Second, we consider panel data of listed companies to fully examine the financial characteristics of the electronic industry and to solve the short period sample problem. Third, we use both accounting measurements of ROA, ROE and EPS and Tobin’s q to serve as proxies for firm value. Finally, four related control variables are considered to make our nonlinear function form more persuadable.

The remainder of this paper is organized as follows: Section II describes the selected variables and data. Methodologies are introduced Section III. Section IV presents and analyzes the empirical results. Section V concludes this paper.

**Data Description**

This paper explores if there exists an optimal D/TA ratio, which may result in threshold effect and asymmetrical responses of the firm value to the D/TA ratio through employing threshold regression model. The investigation has been performed using “balanced panel data” for a sample of 20 selected companies listed on the Tehran Stock Exchange. A total of 200 observations are adopted for each variable considered. For the firm value, we choose accounting financial ratios: Return on Assets (ROA), Return on Equity (ROE), Earnings Per Share (EPS) as the indicators or proxy variables to evaluate the corporate performance or firm value. Besides, in order to consider the effect of market valuation of a firm, Tobin’s q, which defined as the ratio of the market value of a firm to the replacement cost of its assets, is also selected as the proxy variable for the firm performance or value.
The calculations of the approximated \( q \), following the suggestions by Chung and Pruitt (1994), is defined as follows:

\[
\text{Approximated } q = \frac{(\text{MVE} + \text{PS} + \text{DEBT})}{\text{TA}},
\]

where MVE is the product of a firm's share price and the number of common stock shares outstanding, PS is the liquidating value of the firm's outstanding preferred stock, DEBT is the value of the firm's short-term liabilities net of its short-term assets, plus the book value of the firm's long-term debt, and TA is the book value of the total assets of the firm. There are two categories of explanatory variables in our panel data examination. The first is the threshold variable, which is the key variable to be investigated whether there exists an asymmetric threshold effect of the financial leverage on firm value. The debt to total assets ratio (D/TA Ratio) is selected as the indicator for the debt usage of the firms since it is widely used in the literature. Second category of explanatory variable is the control variables, which we adopt to make our function form more persuadable. In this paper, four control variables, including dividend payout ratio, management ownership ratio, growth rate of total assets, and switch-out investment ratio, which are presumed to have influences upon the firm value, are applied in our examination.

**Methodologies**

1. **Panel Unit Root Models**

Hansen’s (1999) panel threshold regression model is in fact an extension of the traditional least squared estimation method. It requires that variables considered in the model need to be stationary in order to avoid the so-called spurious regression. We thus process the unit root test in our first step. Since the data are all panel in our investigation, both well known LLC (Levin, Lin and Chu, 2001) and IPS (Im, Pesaran and Shin, 1997) techniques are employed for the panel unit root tests.

The result of the stationary test for each panel (explained variables, threshold variable, and control variables) shows that all the variables are most likely to be presumed to carry stationary characteristics since the null of unit root are mostly rejected, especially in the findings from LLC test. These stationary findings enable us to go further estimations of the panel threshold regression.

2. **Threshold Autoregressive Model**

Modern dynamic capital structure model proposed an idea of finding the “target” optimal debt ratio and firms will adjust outstanding debt levels in response to changes in firm value. This paper applies a newly-developed, alternative method: panel threshold regression model to solve this problem, to strike a “balance” between the tax benefit and the potential costs that comes along with this benefit. Since Tong (1978) proposed Threshold Autoregressive model, thereafter, this non-linear time series model has become very popular for economic and financial research.

When the Threshold Autoregressive Model is estimated, first we should test if there exists threshold effects. If we can not reject the null hypothesis, the threshold effect doesn’t exist. Again, the existence of nuisance will make the testing statistic follow non-standard distribution, which was called “Davies’ Problem. Hansen (1999) suggested a “bootstrap” method to compute the asymptotic distribution of testing statistics in order to test the significance of threshold effect. Furthermore, when the null hypothesis doesn’t hold, which means, the threshold effect does exist, Chan (1993) proved that OLS estimation of threshold is super consistent, the asymptotic distribution is derived. However, nuisance influences this distribution and makes it non-standard. Hansen (1999) used simulation likelihood ratio test to derive the asymptotic distribution of testing statistic for a threshold.

Hansen (1999) proposed to use two-stage OLS method to estimate the panel threshold model. On the first stage, for any given threshold \( \gamma \), compute the sum of square errors (SSR) separately. On the second stage, try to find the estimation of \( \gamma \) by minimization of the sum of squares. At last, use the estimation of threshold to estimate the coefficient for every “regime” and do analysis.

2.1 **Threshold Model Construction**

According to the “Tradeoff Theory” of Capital Structure, when debt ratio increases, the interest tax shield increases; however, on the other side, leverage related costs increase to offset the positive effect of debt ratio to the firm value. Thus, this paper aims at examining whether threshold effect exists between the financial leverage and firm’s performance or value. We assume that there exists an optimal D/TA ratio, and try to use threshold model to estimate this ratio, which can capture the relationship between financial leverage and firm performance as well as help financial managers make decisions.

Thus we set up single threshold model as follows:
\[ v_{it} = \begin{cases} \mu_i + \theta' h_{it} + \alpha_1 d_{it} + e_{it} & \text{if } d_{it} \leq \gamma \\ \mu_i + \theta' h_{it} + \alpha_2 d_{it} + e_{it} & \text{if } d_{it} > \gamma \end{cases} \] (1)

\[ \theta = (\theta_1, \theta_2, \theta_3, \theta_4)' \], \( h_{it} = (s_{it}, m_{it}, g_{it}, c_{it})' \)

Where \( v_{it} \) represents proxy variables of the firm value, which are \( a_{it} \): ROA, \( e_{it} \): ROE, \( p_{it} \): EPS, \( q_{it} \): Tobin’s q, \( d_{it} \): D/TA ratio, which is also the threshold variable, \( \gamma \), the specific estimated threshold value.

There are four “control variables”(\( h_{it} \)) that they may have influences upon the firm value, which are \( s_{it} \): stock dividend per share, \( m_{it} \): management ownership ratio, \( g_{it} \): growth rate of total assets, \( c_{it} \): long-term investment ratio. Besides, \( \mu_i \), the fixed effect, represents the heterogeneity of companies under different operating conditions; The errors \( e_{it} \) is assumed to be independent and identically distributed with mean zero and finite variance \( \sigma^2 \) \( (e_{it} \sim iid(0, \sigma^2)) \); \( i \) represents different companies; \( t \) represents different periods.

Another threshold regression model of (1) is to set:
\[ v_{it} = \mu_i + \theta' h_{it} + \alpha_1 d_{it} I(d_{it} \leq \gamma) + \alpha_2 d_{it} I(d_{it} > \gamma) + e_{it} \] (2)

where \( I(.) \) represents indicator function,
\[ v_{it} = \mu_i + \left[ \theta', \alpha' \right]' \begin{bmatrix} h_{it} \\ d_{it}(\gamma) \end{bmatrix} + e_{it} \]
\[ v_{it} = \mu_i + \left[ \beta', \gamma \right]' x_{it} (\gamma) + e_{it} \]
\[ d_{it}(\gamma) = \begin{bmatrix} d_{it} I(d_{it} \leq \gamma) \\ d_{it} I(d_{it} > \gamma) \end{bmatrix} \]

where \( \alpha = (\alpha_1, \alpha_2)' \), \( \beta = (\theta', \gamma)' \), \( x_{it} = (h_{it}', d_{it}(\gamma))' \).

The observations are divided into two “regimes” depending on whether the threshold variable \( d_{it} \) is smaller or larger than the threshold value( \( \gamma \) ). The regimes are distinguished by differing regression slopes, \( \alpha_1 \) and \( \alpha_2 \). We will use known \( v_{it} \) and \( d_{it} \) to estimate the parameters \(( \gamma, \alpha, \theta, \text{and } \sigma^2)\).

2.2 Estimation
Note that taking averages of (3) over the time index \( t \) to derive:
\[ \overline{v}_{it} = \frac{1}{T} \sum_{t=1}^{T} v_{it} = \frac{1}{T} \sum_{t=1}^{T} \overline{e}_{it} \] (4)

where \( \overline{v}_{it} = \frac{1}{T} \sum_{t=1}^{T} v_{it} \), \( \overline{e}_{it} = \frac{1}{T} \sum_{t=1}^{T} e_{it} \), and
\[ \overline{d}_{it}(\gamma) = \frac{1}{T} \sum_{t=1}^{T} d_{it}(\gamma) = \begin{bmatrix} \frac{1}{T} \sum_{t=1}^{T} d_{it} I(d_{it} \leq \gamma) \\ \frac{1}{T} \sum_{t=1}^{T} d_{it} I(d_{it} > \gamma) \end{bmatrix} \]

Taking the difference between (3) and (4) yields:
\[ v_{it}^* = \alpha' d_{it}(\gamma) + \overline{e}_{it} \] (5)

where \( v_{it}^* = v_{it} - \overline{v}_{it} \), \( d_{it}(\gamma) = d_{it}(\gamma) - \overline{d}_{it}(\gamma) \), and \( \overline{e}_{it} = e_{it} - \overline{e}_{it} \)

Let
\[
V^*_i = \begin{bmatrix}
  v^*_1 \\
  \vdots \\
  v^*_T
\end{bmatrix}, \quad D^*_i(\gamma) = \begin{bmatrix}
  d^*_1(\gamma) \\
  \vdots \\
  d^*_T(\gamma)
\end{bmatrix}, \quad \varepsilon^*_i = \begin{bmatrix}
  \varepsilon^*_1 \\
  \vdots \\
  \varepsilon^*_T
\end{bmatrix}
\]

Denote the stacked data and errors for an individual with one time period deleted. Then let \( V^* \), \( D^*(\gamma) \) and \( e^* \) denote the data stacked over all individuals.

\[
V^* = \begin{bmatrix}
  v^*_1 \\
  \vdots \\
  v^*_n
\end{bmatrix}, \quad D^*(\gamma) = \begin{bmatrix}
  d^*_1(\gamma) \\
  \vdots \\
  d^*_n(\gamma)
\end{bmatrix}, \quad e^* = \begin{bmatrix}
  \varepsilon^*_1 \\
  \vdots \\
  \varepsilon^*_n
\end{bmatrix}
\]

Use this notation, (5) is equivalent to

\[
V^* = D^*(\gamma)\alpha + e^*_n
\]

The equation (6) represents the major estimation model of threshold effect. For any given \( \gamma \), the slope coefficient \( \alpha \) can be estimated by ordinary least squares (OLS). That is,

\[
\hat{\alpha}(\gamma) = \left( D^*(\gamma)'D^*(\gamma) \right)^{-1} D^*(\gamma)V^*
\]

The vector of regression residuals is

\[
\hat{e}^*(\gamma) = V^* - D^*(\gamma)\hat{\alpha}(\gamma)
\]

and the sum of squared errors, SSE is

\[
SSE_i(\gamma) = \hat{e}^*(\gamma)'\hat{e}^*(\gamma) = V^* ( I - D^*(\gamma)'D^*(\gamma) )^{-1} D^*(\gamma)'V^*
\]

Chan(1993) and Hansen (1999) recommend estimation of \( \gamma \) by least squares. This is easier to achieve by minimization of the concentrated sum of squared errors (9). Hence the least squares estimator of \( \gamma \) is

\[
\hat{\gamma} = \arg \min_{\gamma} SSE_i(\gamma)
\]

Once \( \hat{\gamma} \) is obtained, the slope coefficient estimate is \( \hat{\alpha} = \hat{\alpha}(\hat{\gamma}) \). The residual vector is \( \hat{e}^* = \hat{e}^*(\hat{\gamma}) \), and the estimator of residual variance is

\[
\hat{\sigma}^2 = \hat{\sigma}^2(\hat{\gamma}) = \frac{1}{n(T-1)} \hat{e}^*(\hat{\gamma})'\hat{e}^*(\hat{\gamma}) = \frac{1}{n(T-1)} SSE_i(\hat{\gamma})
\]

where \( n \) indexes the number of sample, \( T \) indexed the periods of sample.

2.3 Testing for a threshold

This paper hypothesizes that there exists threshold effect between the D/TA ratio and firm value. It is important to determine whether the threshold effect is statistically significant. The null hypothesis and alternative hypothesis can be represented as follows:

\[
\begin{cases}
H_0 : \alpha_1 = \alpha_2 \\
H_1 : \alpha_1 \neq \alpha_2
\end{cases}
\]

When the null hypothesis holds, the coefficient \( \alpha_1 = \alpha_2 \) the threshold effect doesn’t exist. When the alternative hypothesis holds, the coefficient \( \alpha_1 \neq \alpha_2 \) the threshold effect exists between the D/TA ratio and firm value.

Under the null hypothesis of no threshold, the model is

\[
v_{it} = u_i + \theta^* h_{it} + \alpha' d_{it}(\gamma) + \varepsilon_{it}
\]

After the fixed-effect transformation is made, we have

\[
V^*_{it} = \alpha'H^*_{it} + e^*_{it}
\]
The regression parameter is estimated by OLS, yielding estimate $\hat{\alpha}$, residuals $\hat{e}^*$ and sum of squared errors $SSE_0 = \hat{e}^*/\hat{e}^*$. Hansen (1999) suggests that we use the F Test Approach to test the existence of threshold effect, and use the sup-Wald statistic to test the null hypothesis.

$$F = \sup_F(\gamma)$$

$$F(\gamma) = \frac{(SSE_0 - SSE_1(\hat{\gamma}))/1}{SSE_1(\hat{\gamma})/n(T - 1)}$$

Under the null hypothesis, some coefficients (e.g. the pre-specified threshold $\gamma$) do not exist, therefore, the nuisance exists. According to “Davies’ problem” (1977,1987), the F statistic becomes non-standard distribution. Hansen (1996) showed that a bootstrap procedure attains the first-order asymptotic distribution, so p-values constructed from the bootstrap are asymptotically valid. Treat the regressors $x_i$ and threshold variable $d_i$ as given, holding their values fixed in repeated bootstrap samples. Take the regression residuals $\hat{e}^*_i$, and group them by individual: $\hat{e}^*_i = (\hat{e}^*_1, \hat{e}^*_2, \ldots, \hat{e}^*_T)$. Treat the sample $\{\hat{e}^*_1, \hat{e}^*_2, \ldots, \hat{e}^*_n\}$ as the empirical distribution to be used for bootstrapping. Draw a sample of size n from the empirical distribution and use these errors to create a bootstrap sample under $H_0$. Using the bootstrap sample, estimate the model under the null (13) and alternative (5) and calculate the bootstrap value of the likelihood ratio statistic $F(\gamma)$ (15). Repeat this procedure a large number of times and calculate the percentage of draws for which the simulated statistic exceeds the actual. This is the bootstrap estimate of the asymptotic p-value for $F(\gamma)$ under $H_0$. The null of no threshold effect is rejected if the p-value is smaller than the desired critical value.

$$P = P(\tilde{F}(\gamma) > F(\gamma) | \zeta)$$

where $\zeta$ is the conditional mean of $\tilde{F}(\gamma) > F(\gamma)$.

### 2.4 Asymptotic distribution of threshold estimate

Chan (1993) and Hansen (1999) showed that when there is a threshold effect $\alpha_0 \neq \alpha_1$, $\hat{\gamma}$ is consistent for $\gamma_0$, and that the asymptotic distribution is highly non-standard. Hansen (1999) argued that the best way to form confidence intervals for $\gamma$ is to form the ‘no-rejection region’ using the likelihood ratio statistic for tests on $\gamma$. To test the hypothesis

$$H_0: \gamma = \gamma_0$$

$$H_1: \gamma \neq \gamma_0$$

We construct the testing model:

$$LR_1(\gamma) = \frac{SSE_1(\gamma) - SSE_1(\hat{\gamma})}{\hat{\sigma}^2}$$

Hansen (1999) pointed out that when $LR_1(\gamma_0)$ is too large and the p-value exceeds the confidence interval, the null hypothesis is rejected. Besides, Hansen (1999) indicated that under some specific assumptions and $H_0: \gamma = \gamma_0$, $LR_1(\gamma) = d\zeta$ as $n \to \infty$, where $\zeta$ is a random variable with distribution function

$$P(\zeta \leq x) = (1 - \exp(-x/2))^2$$

The asymptotic p-value can be estimated under the likelihood ratio. According to the proof of Hansen(1999), the distribution function (18) has the inverse

1. Note that the statistic (17) is testing a different hypothesis from the statistic (15) introduced in the previous section. 

\( c(\alpha) = -2\log(1 - \sqrt{1 - \alpha}) \) \hspace{1cm} (20)

from which it is easy to calculate critical values. For a given asymptotic level \( \alpha \), the null hypothesis \( \gamma = \gamma_0 \) rejects if \( LR(\gamma) \) exceeds \( c(\alpha) \).

### 2.5 Multiple thresholds Model

If there exist double thresholds, the model is modified as:

\[
v_{it} = \begin{cases} 
\mu_i + \theta h_{it} + \alpha_1 d_{it} + e_{it} & \text{if } d_{it} \leq \gamma_1 \\
\mu_i + \theta h_{it} + \alpha_2 d_{it} + e_{it} & \text{if } \gamma_1 < d_{it} \leq \gamma_2 \\
\mu_i + \theta h_{it} + \alpha_3 d_{it} + e_{it} & \text{if } \gamma_2 \leq d_{it}
\end{cases}
\] \hspace{1cm} (21)

where threshold value \( \gamma_1 < \gamma_2 \). This can be extended to multiple thresholds model \( (\gamma_1, \gamma_2, \gamma_3, \cdots, \gamma_n) \).

### Empirical Results

This paper applies the threshold theory proposed by Hansen (1999) and assumes that debt ratio and corporate performance have asymmetric nonlinear relationship. First we test if there exists threshold effect. We test double threshold and single threshold effect, respectively, and the formulas for both models are as follows:

\[
v_{it} = \begin{cases} 
\mu_i + \theta h_{it} + \alpha_1 d_{it} + e_{it} & \text{if } d_{it} \leq \gamma_1 \\
\mu_i + \theta h_{it} + \alpha_2 d_{it} + e_{it} & \text{if } \gamma_1 < d_{it} \leq \gamma_2 \\
\mu_i + \theta h_{it} + \alpha_3 d_{it} + e_{it} & \text{if } \gamma_2 \leq d_{it} \leq \gamma_3 \\
\mu_i + \theta h_{it} + \alpha_4 d_{it} + e_{it} & \text{if } \gamma_3 \leq d_{it} < \gamma_4 \\
\mu_i + \theta h_{it} + \alpha_5 d_{it} + e_{it} & \text{if } d_{it} \geq \gamma_4
\end{cases}
\]

The dependent variable \( v_{it} \) represents corporate performance or firm value, which uses ROA, ROE, EPS, and Tobin’s q as proxies, respectively. The independent variable \( d_{it} \) represents debt ratio (D/TA ratio), which is indeed the threshold variable. \( h_{it} \) is a control variable vector that contains four variables of dividend payout ratio, management ownership ratio, growth rate of total assets, and switch-out investment ratio. Besides, \( \mu_i \), the fixed effect, represents the heterogeneity of companies under different operating conditions. The errors \( e_{it} \) is assumed to be independent and identically distributed with mean zero and finite variance \( \sigma^2 (e_{it} \sim iid(0, \sigma^2)) \). i and t are symbols for firms and time periods.

This paper follows the bootstrap method to get the approximation of F statistic and then calculate the p-value. Table 1 presents the empirical results of test for both single threshold and double threshold effects. After repeating bootstrap procedure 200 times for each of the two panel threshold tests, we find that the tests for double threshold are all statistically insignificant for any of the dependent variables-ROA, ROE, EPS, or Tobin’s q served as the proxy variable of the firm value. However, the significant finding at the 10% level with a bootstrap p-value of 0.06 occurs only when Tobin’s q is selected as the proxy for firm value in the testing of single threshold. We thus conclude that there exists a single threshold effect of the debt ratio on firm value when Tobin’s q is selected. For the remainder of the analysis we work with this single threshold model.

**Table 1** Tests for threshold effects between the debt ratio and proxy variables of the firm value

<table>
<thead>
<tr>
<th>Firm Value Variables</th>
<th>Single threshold effect test</th>
<th>Double threshold effect test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Threshold - value</td>
<td>F</td>
</tr>
<tr>
<td>ROA</td>
<td>41.12</td>
<td>2.01</td>
</tr>
<tr>
<td>ROE</td>
<td>41.12</td>
<td>2.70</td>
</tr>
<tr>
<td>EPS</td>
<td>41.12</td>
<td>3.16</td>
</tr>
<tr>
<td>Tobin’s q</td>
<td>37.84</td>
<td>12.40</td>
</tr>
</tbody>
</table>

Notes:
1. F Statistic and P-value result from repeating bootstrap procedure 200 times for each of the two bootstrap tests.
2. The symbol ***, **, and *, represent the significant at 1%, 5%, and 10% levels, respectively.
When there exists a single threshold effect of the debt ratio on firm value, all observations are split into two regimes, a low debt level and a high debt level, depending on whether the threshold variable \( d_e \) is smaller or larger than the threshold value \( \gamma \). The regimes are distinguished by differing regression slopes, \( \alpha_1 \) and \( \alpha_2 \). Table 2 represents the regression slope estimates together with the conventional OLS standard errors and White-corrected standard errors for two regimes.

<table>
<thead>
<tr>
<th>Table 2 Estimated Coefficients for Each Proxy Variable of the Firm Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>------------------</td>
</tr>
<tr>
<td>ROA</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>ROE</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>EPS</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Tobin’s q</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Notes: 1. \( \hat{\alpha}_1 \) and \( \hat{\alpha}_2 \) represent coefficient estimate that smaller and larger than threshold value \( \gamma \).

2. The symbol ***, **, and *, represent the significant at 1%, 5%, and 10% levels, respectively.

The estimated model from above empirical findings can be expressed as follows:

\[
v_i = u_i + 0.1848d_{it}I(d_{it} \leq \hat{\gamma}) + 0.0319d_{it}(d_{it} > \hat{\gamma}) + \epsilon_i
\]

The estimated threshold value \( \hat{\gamma} \) is 37.84%, and thus all of the observations can be divided into two regimes depending on whether the D/TA ratio is smaller or larger than the threshold value. Two coefficients \( \alpha_1 = 0.1848 \) and \( \alpha_2 = 0.0319 \) are all positive with the evidence that the \( \alpha_1 \) in the low debt level is significant, while the \( \alpha_2 \) in the high debt level is not.

Under the situation without considering threshold effect, from Table 2 we still can find the asymmetric nonlinear relationships between debt ratio and corporate performance, when ROA and ROE are selected as the proxy variables (ROA: \( \alpha_1 = 0.0290, \alpha_2 = -0.0126 \); ROE: \( \alpha_1 = 0.0557, \alpha_2 = -0.0548 \)). In both cases, the coefficient estimates of \( \alpha_1 \) (at the first regime) are positive; and \( \alpha_2 \) (at the second regime) are negative. These results are consistent with the trade-off theory, for which we may search a “balance” that the interest tax shield is equal to the incremental costs through debt financing.

This paper further investigates the influences of four control variables upon the firm value. The empirical results are observed in Table 3, which shows that only switch-out investment ratio when ROA is selected as the proxy for firm value has significant negative impact on firm value. Among the findings when ROE and EPS are selected as proxy, no apparent relationships between all of the four control variables and firm value are observed. Finally, dividend payout ratio is shown to have significant negative relationship with the firm value when Tobin’s q is selected as the proxy for firm value.
Table 3 Estimation of Coefficients of Control Variables

<table>
<thead>
<tr>
<th>Firm Value</th>
<th>Coefficients Estimated Value</th>
<th>OLS se</th>
<th>White se</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{\theta}_1$</td>
<td>-0.7720</td>
<td>0.3642</td>
</tr>
<tr>
<td></td>
<td>$\hat{\theta}_2$</td>
<td>0.8246</td>
<td>0.8093</td>
</tr>
<tr>
<td>ROA</td>
<td>$\hat{\theta}_3$</td>
<td>-5.7917*</td>
<td>3.3336</td>
</tr>
<tr>
<td></td>
<td>$\hat{\theta}_4$</td>
<td>-0.0003</td>
<td>0.0142</td>
</tr>
<tr>
<td></td>
<td>$\hat{\theta}_5$</td>
<td>-1.2211</td>
<td>0.8359</td>
</tr>
<tr>
<td>ROE</td>
<td>$\hat{\theta}_1$</td>
<td>1.0069</td>
<td>1.8576</td>
</tr>
<tr>
<td></td>
<td>$\hat{\theta}_2$</td>
<td>-12.0659</td>
<td>7.6515</td>
</tr>
<tr>
<td></td>
<td>$\hat{\theta}_3$</td>
<td>0.0166</td>
<td>0.0326</td>
</tr>
<tr>
<td>EPS</td>
<td>$\hat{\theta}_1$</td>
<td>-0.2890</td>
<td>0.1297</td>
</tr>
<tr>
<td></td>
<td>$\hat{\theta}_2$</td>
<td>0.1882</td>
<td>0.2882</td>
</tr>
<tr>
<td></td>
<td>$\hat{\theta}_3$</td>
<td>-1.9156</td>
<td>1.1869</td>
</tr>
<tr>
<td></td>
<td>$\hat{\theta}_4$</td>
<td>0.0026</td>
<td>0.0051</td>
</tr>
<tr>
<td>Tobin’s q</td>
<td>$\hat{\theta}_1$</td>
<td>-0.8117*</td>
<td>0.5158</td>
</tr>
<tr>
<td></td>
<td>$\hat{\theta}_2$</td>
<td>0.4806</td>
<td>1.1470</td>
</tr>
<tr>
<td></td>
<td>$\hat{\theta}_3$</td>
<td>3.0801</td>
<td>4.7062</td>
</tr>
<tr>
<td></td>
<td>$\hat{\theta}_4$</td>
<td>-0.0080</td>
<td>0.0201</td>
</tr>
</tbody>
</table>

Notes: 1. $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4$ represent the estimated coefficients: stock dividend per share, management ownership ratio, long-term investment ratio, and growth rate of total assets.

2. The symbol ***, **, and *, represent the significant at 1%, 5%, and 10% levels, respectively.

Conclusion
There are two major different capital structure theories: the Trade-off and Pecking Order theory. The Trade-off theory suggested that value-maximizing financial managers should employ capital structures for which the interest tax shield is equal to the incremental costs through debt financing. In other words, finds the “optimal debt ratios”. The pecking Order theory, on the other hand, argued that high-profitable firms prefer internal financing from external. Therefore, the purpose of this paper intends to test if there exists an optimal debt ratio; and explores whether application of financial leverage affects corporate performance or firm value of listed firms in Tehran stock exchange. The optimal debt ratio is found through using newly developed threshold regression model proposed by Hansen (1999). We found out that there exists a single “threshold” of optimal debt usage, which is equal to the trade-off between the benefits of more debts to increase the firm value versus the costs of additional debts that may deteriorate the corporate performance or value. The results of this paper are more consistent with the theoretical background of M&M’s model (1963), Myers (1977), and Ross (1977) as presented in the first section of this paper. In contrast with traditional linear model, nonlinear relationship between variables is investigated in this study. We found out there exists single threshold effect between debt ratio and firm value only when Tobing’s q is selected as the proxy variable for the firm value. The estimated threshold value ($\gamma$) is 37.84%, while all of the observations can be divided into two regimes, a low debt level and a high debt level, depending on whether the D/TA ratio is smaller or larger than the specific threshold value. Two coefficients ($\alpha_1$ and $\alpha_2$) are all positive with the evidence that the $\alpha_1$ in the low debt level is significant, while the $\alpha_2$ in the high debt level is not. However, the positive effect decreases when debt ratio increases. This may be explained that following with the interest tax shield increases; on the other side, leverage-related costs increase to counteract the positive effect of debt ratio to the firm value. This suggests that the financial managers should take steps to increase debt levels when the current debt percentage of total assets is below the threshold value of 37.84%; conversely, they should take steps to lower debt levels when the current debt usage is higher than the threshold value of 37.84%, for there is no further apparent net benefit due to the incremental leverage-
related costs in the future. Under the situation without considering threshold effect, we still can find the asymmetric nonlinear relationships between debt ratio and corporate performance, when ROA and ROE are selected as the proxy variables. In both cases, the coefficient estimates of $\alpha_1$ are positive and $\alpha_2$ are negative. These results are consistent with the trade-off theory, for which we may search a “balance” that the interest tax shield is offset by the incremental costs through debt financing. The empirical results of testing for the influences of four control variables upon the firm value indicate that only switch-out investment ratio when ROA is selected as the proxy for firm value has significant negative impact on firm value. Among the findings when ROE and EPS are selected as proxy, no apparent relationships between all of the four control variables and firm value are observed. Finally, dividend payout ratio is shown to have significant negative relationship with the firm value when Tobin’s q is selected as the proxy for firm value.

References
Davies, R. B. (1977), “Hypothesis testing when a nuisance parameter is present only under the alternative,” *Biometrika*, 64, 247-254.
______ (1987), “Hypothesis testing when a nuisance parameter is present only under the alternative,” *Biometrika*, 74, 33-43.
Hansen, B. E. (1996), “Inference when a nuisance parameter is not identified under the null hypothesis,” *Econometrica*, 64, 413-430.
ROE-Single Threshold

EPS-Single Threshold

Tobin’s q-Single Threshold