The Effect of the Correlation between Dimensions and Examinees Ability Level on the Methods of Detecting Local Independence in Item Response Theory

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Abstract
This study attempts to investigate the effects of the strength of the relationship between dimensions, 0.0, 0.5, 0.9 and examinees ability levels low, moderate, high on ability of four indices Q3, Fisher Z, and the mean of pair twice correlation coefficient testing locally independent items for the data generated from locally dependent items. Three group of examinee (9000) each responses are generated on basis of three dimensional tests (each dimension contains 36 uni-dimensional items); the groups are different in the strength of inter-dimensional correlation. For each group, three levels of examinees ability are chosen. The results obtained show that the percentage of local item pairs does not change on the, though the Inter-dimensional correlation strength varies. And the index means of the pair twice correlation coefficient ranks second, followed by the indices Fisher Z and Q3. This study also travels that the percentage of local item pairs does not vary when the level of examinees ability varies by using index.; the percentage decreases when Fisher Z, Q3 is used , though the inter-dimensional correlation are different

Keywords: Q3 index, index, Fisher Z index, the means of pair twice Correlation coefficient.

1.1 Introduction

The theoretical models of respond to paragraph as a contemporary psychometric models have proven their utility in overcoming many of the psychometrics and educational problems faced by the traditional model, as specialists believe that the principles of the modern theory of measurement is fundamentally different from the principles of the traditional model. In the recent development of the modern theory of measurement is considered as an important psychological measurement because it frees the measurement of the sample and the paragraphs sample, also enables specialists answering any questions about the test or the paragraphs (Embretson & Reise, 2000).

A range of models has emerged from the theory of response to paragraph known as the (Latent Trait Models), aiming to determine the relationship between the individual's performance in the test (which can be directly observed) and between the features or capabilities that underlie this performance. These models are built upon many assumptions that must be verified in the data, and linking these models as Hambleton and Swaminathan declares, between the observed performance on the paragraph’s test, and the feature or features embedded, through a specific mathematical relationship (Hambleton & Swaminathan, 1985).

Embretson & Rise summed the assumptions of the theoretical models in response to paragraph on two assumptions: the relationship between the ability (feature) and performance on paragraph, testing, and local independence (Embretson & Rise, 2000).

Hambleton and Swaminathan emphasize that there are other assumptions related to certain models of embedded features including: the presumption of one-dimensional (Unidimensionality), and performance speediness, besides the assumption of local independence as a key assumptions for all models to respond to paragraph, whether these models are single count or multi-dimension, and it is a key assumption in the traditional theory of measurement as well. This means that the assumption that the probability of the correct answer to the unexamined paragraph of the test does not affect negatively or positively on the answer to any other paragraph of the paragraphs of the test when adjusting the estimated value of the ability, and the estimated value of the difficulty of the paragraph (Hambleton & Swaminathan, 1985).

The assumption of the paragraph local independence is the key assumption for each of the traditional theory of measurement (CTT) and the theory of paragraph response (IRT) (Lee, 2004), Hambleton & Swaminathan (1985) have pointed that the assumption of local independence means that the responses of subjects on the test paragraphs to be statistically independent at a certain capacity; which means that the response for a paragraph should not affect negatively or positively on the response of another paragraph; sense that the answer paragraph does not give any hints or obstacles to answer any other paragraph.
1.2 Importance of the study

The theoretical models of respond to paragraph is one of the contemporary psychometric models that have proven their utility in overcome many of the psychological and educational measurement problems, and these models are based on a set of assumptions that must be verified in the data provided, and can be applied to even one of them. Among these assumptions, the most important is the local independence. Studies have reported on this subject that there are many ways to verify the assumption of independence in situ data (Q3, Z fisher, the center of inter- correlation coefficients), comparing two different estimates for the validity, Mental Hensl test average, the difference logarithmic likelihood test standard). The researcher noted through these studies, which focused on examining the effect of the number of dimensions of the test and the strength of the link between these dimensions to estimate parameters of the paragraphs, the lack of agreement about the effectiveness of any of the indicators that are based on the assumption of independence check in situ data. And even more the studies focused on the comparison between these four indicators of the ability to detect the assumption of local independence.

This study also aimed at comparing the indicators: (Q3, Z fisher, and the average of correlation coefficients interfaces) which are used to detect pairs paragraphs, including violation of the presumption of independence, which were not compared in previous studies under conditions of different strength of the link between dimensions (0.0, 0.9, 0.5) and different levels of ability (high capacity, medium capacity and low capacity).

1.3 Questions of the study

The study is trying to answer the following questions:

- Does the percentage of paragraphs that revealed indicators (Q3, Z fisher, and the average inter-correlation coefficients) for the link between them depending on the strength of a local correlation between the dimensions (0.0, 0.9, and 0.5)?
- Does the percentage of paragraphs that revealed indicators (Q3, Z fisher, and the average inter-correlation coefficients) for the link between them depending on the local level the ability of the respondents (high, medium, low)?

1.4 Study terminology

**Multi-dimensional test (between paragraphs):** A test that measures two features or more, and has a partial tests so that each paragraph belongs to the partial test and does not intersect with any other part paragraph.

**High ability level:** The level at which it has been the selection of high ability, where it is the top choice of 33% in terms of ability of the 9000’s population, and then 1000 were selected from this sample to represent the high level of ability.

**Medium ability level:** The level at which it has been the selection of medium ability, where it is the top choice of 33% in terms of ability of the 9000’s population, and then 1000 were selected from this sample to represent the medium level of ability.

**Low ability level:** The level at which it has been the selection of low ability, where it is the top choice of 33% in terms of ability of the 9000’s population, and then 1000 were selected from this sample to represent the low level of ability.

**Local link between a pair of paragraphs:** The correlation coefficient between the residuals of the two paragraphs at the same level of ability, and adopting the absolute value of 0.5 for the index Q3 as the maximum interval between the pairs of paragraphs, including a violation of the assumption of independence of other local paragraphs.

**Average correlation coefficients interfaces:** This method of measuring the correlation coefficients interfaces between the vertebrae is conducted after being classifying the respondents into a number of levels of ability on the basis of total marks, then calculating the number of pair paragraphs that increase the value of the link between them.

1.5 Study limitations

1. The study was limited to the use of compensatory models, and did not use the non-compensatory models.
2. The study was limited to three magnitudes of correlation between dimensions.
3. The study was limited to the compared four indicators for the detection of local independence.
4. Results reached are based on the available software of data analysis.

1.6 Methods and procedures

1.6.1 Study methodology

Data have been generated the in this study by using the RESGEN program designed by (Muraki, 2000). This program had been selected for the advantages of its potential for multiple data generation, where it requires identifying restrictions (Landmarks true vertebrae, the characteristics of the distribution parameters of the ability, the dimensionality of the test, the model used to generate the data, and the type of paragraphs) that are placed on the data for obtaining convergence and real data.

1.6.2 Data collection tool

The researcher generated responses on three-dimension test as a type of the multi-dimensional tests between the paragraphs, which contained (108) paragraphs distributed on three dimensions (each dimension with 36 paragraphs), and the procedures for generating responses are summarized as follows:

After getting real values (True Parameter) for the parameters of the difficulty and the parameters of discrimination paragraphs problem for the test, and based on the study of Aldalalah (2005), confining that the difficulty point where in the period (2 +0.2 -), and discrimination where in the period (0.18, 1.77), and the ability of subjects are distributed normally with a mean of zero and a standard deviation of one. These features were introduced to the RESGEN software to generate responses, as the assumption that the model used is a compensatory models) by entering one value for a difficulty parameter and different values of the number of dimensions to which the paragraph belongs to, so as to distinguish features of each paragraph. Because the one-dimensional paragraphs constitute to the three-dimension test between the paragraphs has been the introduction of a single value for a parameter of difficulty, and one value for the parameter of discrimination on the dimension that belongs to the paragraphs, and the 0.0 value of the rest of the dimensions that do not belong to the paragraphs. In order to control the link between dimensions it has been hypothesized that the existence of three groups of subjects each group consisted of 9,000 unexamined, distributed their normal distribution with a mean zero and standard deviation of one. It was the assumption of full independence between capacity (which represent the dimensions of the test) for the first group and the correlation coefficient between 0.0 Dimensions, and the lack of independence between the capacity of members of the second group and the correlation coefficient of 0.5, and the third member of the group and the correlation coefficient between 0.9 Dimensions. Through the output given by the software RESGEN, and for each test three sets of responses to the paragraphs were obtained, each set consists of 9000 responses to the three-dimension test consists of 108 one-dimensional paragraph. These groups vary among themselves with the strength link (0.0, 0.5, and 0.9), the researcher used the program (BILOG), a program used to estimate each of the parameters of the paragraphs (difficulty and discrimination) and the parameters of the respondents (ability), where it was limited in this study to deal only with the point trio parameter, which is a paragraph response models. After estimating parameters of the paragraphs and respondents, the parameters paragraphs estimated file was named (dad ldid) which contains parameters of difficulty and discrimination of the paragraphs, and the ability estimated file was named (doc ldid), and the reason for this naming label is to display these files on the program LDID, which does not recognize the features of respondents and paragraphs, and using this program calculate statistical indicators (Q3, Fisher Z, G2), which reveal pairs paragraphs which link locally.

After calculating the values of statistical indicators (Q3, Fisher Z, G2) by the LDID program for each indicator, and calculating the average of the correlation coefficients interfaces, has been counting the number of pairs of paragraphs, including a link local use of the all indicators above, and then calculate the ratio of the number of pairs paragraphs including local, and link with each indicator. Then splitting respondents upon their response totaling 9000 by the ability levels estimated from the data test consisting of 108 paragraphs, three groups were chosen: the abilities of high, medium and low levels of the examined1000 sample, where a random selection for examined 1000 with high ability among the 33% of the respondents of the high ability of study population totaling 9,000.

1.7 Previous studies

Specialists use in the theory of the response to paragraph several detectors to achieve the assumption of independence between pair’s paragraphs local problem for the tests and these indicators: Q3, Fisher Z, Pearsons,
and the comparison between the two estimates for different firmness, and the center of correlation coefficients for intra paragraph.

Many studies have used more than the index for the detection of local independence in their data, and a few of them compare these indicators to demonstrate the most effective, and these studies:

- Almqvs study (2008) compares between three indicators: Z, Fisher, Q3 as the researcher used the data generated in violation of the presumption of local independence, studying the impact of each dimensions (one, two, three dimensions) and the strength of correlation (0.9 , 0.5 , 0.0) on the ability of these indicators to detect localized link. The study found that the percentage of pair’s paragraphs that violate the independence using the index did not change with the dimensions. The study showed that the proportion of pair’s paragraphs of the assumptions by the two indices Fisher Z, Q3 was low when compared to the data (0.9, 0.0), while the difference was evident when comparing the data of one-dimensional with a multi-dimensional and in favor of a multi-Dimensions.

- In Chan & Wang study (2007) the index Q3 was used for the detection of the paragraphs, including violation of the presumption of independence localized through the data that have been generated according to the triple model, study results showed that what appeared violation of the presumption of localized independence between the paragraph were locally separate paragraphs, it would affect the estimated parameter of discrimination, as it is whether the violation is positive; sense that the first paragraph answers correct answers to increases the likelihood of the correct answer on the it was in violation of the assumption of independence between the paragraphs locally negative.

- Naimi (2006) examined the effect of violation of the assumption of independence localized on different estimates in response to the theory of the paragraph. Researcher conducted an achievement test in mathematics on a sample of 1210 students from the ninth grade. The researcher used two methods for the detection of local independence: the average of inter- correlation coefficients and statistical Q3. The researcher used the logistic tri model and duo correction method (when ignore the violation of the assumption of local independence). The results showed that the method of correction duo gives unbiased estimates of the highest reliability coefficients, and biased estimates of the highest coefficients of discrimination, and low coefficients of difficulty, and high transactions guessing, while estimates of the ability was not significantly affected by the violation of the assumption of local independence.

1.8 Results and Discussion

Results related to the first question: “Does the percentage of paragraphs that revealed indicators (Q3, Z fisher, and the average inter-correlation coefficients) for the link between them depending on the strength of a local correlation between the dimensions (0.0, 0.9, and 0.5)?”

Ratio of the number of pairs of paragraphs was calculated including the assumption of local independence by dividing the number of pairs of paragraphs, including violation of the presumption of local independence (local link) on the total number of pairs of paragraphs, and the following relationship describes how the proportion of the number of pairs paragraphs is calculated:

\[
\text{Proportion of the number of pairs paragraphs that violate the assumption of local pairs} = \frac{\text{number of paragraphs associated locally}}{5778}.
\]

Total number of Paragraphs pairs is equal to 5778 pairs (107x 108/2).

Table (1) shows the proportion of the number of pairs of paragraphs which included the violation of the presumption of local independence when the strength of the correlation between the dimensions (0.0, 0.5, 0.9) for each indicator (Q3, Fisher Z, G2, and the average of correlation coefficients interfaces) and all subjects generated responses totaling 10,000 unexamined.

Table (1): The proportion of the number of pairs of paragraphs which included the violation of the presumption of local independence when the strength of the correlation between the dimensions (0.0, 0.5, 0.9) for each indicator (Q3, Fisher Z, G2, and the average of correlation coefficients interfaces)

<table>
<thead>
<tr>
<th>Indicator/Strength</th>
<th>0.0</th>
<th>0.5</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fisher Z</td>
<td>0.9566</td>
<td>0.73401</td>
<td>0.788343</td>
</tr>
<tr>
<td>Q3</td>
<td>0.9823</td>
<td>0.36560</td>
<td>0.24567</td>
</tr>
<tr>
<td>G2</td>
<td>0.9900</td>
<td>0.98883</td>
<td>0.98891</td>
</tr>
<tr>
<td>average of</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>correlation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>coefficients</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>interfaces</td>
<td>0.9801</td>
<td>0.91113</td>
<td>0.31934</td>
</tr>
</tbody>
</table>
Table (1) shows that the ratio of the number of pairs of locally related paragraphs were less as possible when it was disclosed by using the indicator Q3 and the strength of the link between the dimensions of 0.9, and the highest rate detected by using the indicator G2 and the strength of correlations 0.0, 0.9. It is also noted from Table (1) that the percentage of paragraphs associated with locally varied depending on the strength of the link when using the indicators (correlation coefficients interfaces, Fisher Z, Q3) did not differ when using the indicator G2. The reason for this may be due to the indicator G2 is affected by the size of sample. results of the study are in line with the Balazs & Deboeck (2006) study, which examined the impact of correlations (0.0 , 0.5) between the dimensions on the amount of reliance localized between the paragraphs, which concluded that the local dependence between paragraphs multidimensional test least with the increasing strength of the correlation between dimensions.

Table (1) also shows that the percentage of the paragraphs locally related are decreasing with the increase of the correlation coefficient between the dimensions when the using the indicators Q3 Fisher Z and the average correlation coefficients interfaces. It is possible to attribute that these results to the test tracks became one attribute. It is noted from the table also that the proportion of the number of pairs of paragraphs in violation of the assumption of local independence did not differ greatly among the different indicators when the power of the link between the dimensions of 0.0, and otherwise when the strength of the link between the dimensions of 0.5, 0.9.

Then calculating the differences in the percentages of pairs paragraphs for each indicator depending on the strength of the link between the dimensions, these differences are shown in Table (2).

Table (2): The proportion of the number of pairs of paragraphs which included the violation of the presumption of local independence when the strength of the correlation between the dimensions (0.0, 0.5, and 0.9) for each indicator (Q3, Fisher Z, G2, and the average of correlation coefficients interfaces) between dimensions respectively

<table>
<thead>
<tr>
<th>Indicator/Strength</th>
<th>0.0 with 0.5</th>
<th>0.0 with 0.9</th>
<th>0.5 with 0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fisher Z</td>
<td>0.201112</td>
<td>0.159981</td>
<td>0.044981</td>
</tr>
<tr>
<td>Q3</td>
<td>0.609111</td>
<td>0.734881</td>
<td>0.119973</td>
</tr>
<tr>
<td>G2</td>
<td>0.009785</td>
<td>0.000222</td>
<td>0.011108</td>
</tr>
<tr>
<td>average of correlation coefficients interfaces</td>
<td>0.063681</td>
<td>0.647637</td>
<td>0.587663</td>
</tr>
</tbody>
</table>

Table (2) shows that the differences in the percentages of pair’s paragraphs index G2 did not differ greatly depending on the strength of correlation between the dimensions (0.0, 0.5, 0.9), respectively, while for indicators Fisher z, Q3 and average correlation coefficients interfaces, the differences in the percentages of pair’s paragraphs which including violation of the presumption of local independence was different, where the highest correlation between the strength of 0.0 for the index Fisher Z and the link between 0.9 and 0.0 for the link to the index and index Q3 amid correlation coefficients interfaces.

Results related to the second question: “Does the percentage of paragraphs that revealed indicators (Q3, Z fisher, and the average inter-correlation coefficients) for the link between them depending on the local level the ability of the respondents (high, medium, low)?” To answer this question the ratios of the pair’s paragraphs were calculated in the existences of the violation of the assumption of local independence between them, and table (3) shows the proportions of pair’s paragraphs that exist between them a violation of the assumption of local independence for each of the indicators depending on the ability level of the respondents upon the strength of the relationship (0.0, 0.5, 0.9) between dimensions.
Results in Table (3) show that the ratio of the number of pair’s paragraphs, including violation of the presumption of local independence with the level of ability of the respondents with the strength of correlation 0.0 between dimensions when using all indicators Q3, Fisher Z. As for when to use the indicator G2 the differences in the proportions of the pair’s paragraphs did not vary according to the ability levels.

table (3) also shows that the ratio of the number of pair’s paragraphs including violation of the presumption of local independence with the strong link connectivity 0.5 between the three dimensions, increase with the level of ability of the respondents when using the indicator Fisher Z, while the ratio of the number of pair’s paragraphs including violation of the presumption of local independence for medium ability level was the highest of the two indicators Q3 and the average correlation coefficients interfaces. The table shows that the ratio of the number of pair’s paragraphs including violation of the presumption of local independence indicators Q3 and Fisher Z and average correlation coefficients interfaces increase with the level of ability of the respondents with the strength of correlation 0.9 between the three dimensions, while there has been no significant change to this case in the ratio of the number of pair’s paragraphs in violation of the assumption of local independence change-level of ability to respondents when using the indicator G2. were then calculated the differences in the proportion of the number of pairs of paragraphs including violation of the presumption of independence localized for each indicator data depending on the level of ability, and table shows (4) these differences

Table (4): The proportions of pair’s paragraphs that exist between them a violation of the assumption of local independence for each of the indicators depending on the ability level of the respondents

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Strength</th>
<th>Medium with Low</th>
<th>Medium with Medium</th>
<th>High with Low</th>
<th>High with Medium</th>
<th>Medium with Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z Fisher</td>
<td>0.0</td>
<td>0.1109890</td>
<td>0.2653450</td>
<td>0.1100350</td>
<td>0.2653450</td>
<td>0.1100350</td>
</tr>
<tr>
<td>Q3</td>
<td>0.0</td>
<td>0.3304660</td>
<td>0.1223540</td>
<td>0.3304660</td>
<td>0.1223540</td>
<td>0.3304660</td>
</tr>
<tr>
<td>G2</td>
<td>0.0</td>
<td>0.0113000</td>
<td>0.0010000</td>
<td>0.0113000</td>
<td>0.0010000</td>
<td>0.0113000</td>
</tr>
<tr>
<td>average inter-correlation coefficients</td>
<td>0.0</td>
<td>0.0010000</td>
<td>0.0010000</td>
<td>0.0010000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Results in table (4) show that the Fisher Z indicator gained the biggest differences between the ratios between paragraphs with high ability and low- ability of the respondents with the different correlations between the
strengths of the dimensions, while the differences were less with and high and medium ability with the power of the link between the different dimensions. As for the Q3 indicator it gained the biggest differences between the proportions of pairs of high-ability and low-ability of the respondents depending on the strength of the link between dimensions.

The less difference between low ability and medium ability with the link between the different dimensions, for indicator G2 there were no clear differences between the proportions of paragraphs in violation of the assumption of local independence at different levels for different situations between dimensions. The indicator correlation coefficients interfaces gained the biggest differences between the proportions of pair’s paragraphs between high ability and medium ability for the respondents with the strength link 0.0 and 0.5 and were less differences between high ability and low ability with the strength link 0.0 and 0.5, either when the power of the link of 0.9 gained the largest differences ratios between pair’s paragraphs between high-ability and low- ability.

1.9 Conclusions and recommendations

Local independence assumption is one of the main assumptions of theory of response to a paragraph, and studies have shown the effect of non-check this assumption in the experimental data on the various estimates of the parameters of the theory of response to a paragraph, and usually measure the tests multiple attributes, those attributes are associated with each other the forces are different, and there are several indicators for the detection of local independence, and therefore this study examined that any of these indicators is less affected by the strength of the link between dimensions, and change the level of the ability of respondents.

With regard to the correlation strength between dimensions was less affected by the indicator G2 and then followed by the average correlation coefficients and then two indicators: Fisher Z and Q3. Regarding the impact of the level of ability of subjects, results showed that the proportion of pair’s paragraphs that violate the local independence did not change when using the indicator G2, and was carrying a decrease of the level of ability of subjects when using indicators: the average correlation coefficients interfaces, Q3, Fisher Z, and the strength of the link between the dimensions of 0.9, and based on the study results the researcher recommends the following:

- Recommending the use of Fisher Z indicator for the detection of paragraphs in violation of the assumption of local independence due to its sensitivity in the detection of weak violation of the presumption of local independence.
- The need to examine the strength of correlation between the dimensions of the problem of tests before choosing the indicator to detect localized independence.
- The need to examine the level of ability of the respondents by the use of the indicators: Fisher Z and the average correlation coefficients, and Q3.
- Conducting further studies dealing with different values of correlation coefficients between dimensions.

References


