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Common Fixed Point Theorems of Integral Type in Menger Pm Spaces

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Abstract: In this paper, we propose integral type common fixed point theorems in Menger spaces satisfying common property (E.A). Our results generalize several previously known results in Menger as well as metric spaces.
Keywords: Menger space; Common property (E.A); weakly compatible pair of mappings
and t-norm.

1. Introduction:

in 1942 Menger [21] initiated the study of probabilistic metric space (often abbreviated as PM space) and by now the theory of probabilistic metric spaces has already made a considerable progress in several directions (see [32]). The idea of Menger was to use distribution functions (instead of nonnegative real numbers) as values of a probabilistic metric. This PM space can cover even those situations where in one can not exactly ascertain a distance between two points, but can only know the possibility of a possible value for the distance (between a pair of points).
In 1986, Jungck [13] introduced the notion of compatible mappings and utilized the same to improve commutativity conditions in common fixed point theorems. This concept has been frequently employed to prove existence theorems on common fixed points. However, the study of common fixed points of non-compatible mappings was initiated by Pant [32]. Recently, Aamri and Moutawakil [1] and Liu et al. [39] respectively defined the property (E.A) and the common property (E.A) and proved interesting common fixed point theorems in metric spaces. Most recently, Kubiaczyk and Sharma [15] adopted the property (E.A) in PM spaces and used it to prove results on common fixed points. Recently, Imdad et al. [26] adopted the common property (E.A) in PM spaces and proved some coincidence and common fixed point results in Menger spaces.
The theory of fixed points in PM spaces is a part of probabilistic analysis and continues to be an active area of mathematical research. Thus far, several authors studied fixed point and common fixed point theorems in PM spaces which include [5, 7, 8, 10, 16, 17, 18, 24, 26, 28, 30, 34, 31, 36, and 37] besides many more. In 2002, Branciari [3] obtained a fixed point result for a mapping satisfying an integral analogue of Banach contraction principle. The authors of the papers [2, 4, 6, 26, 11, and 29] proved a host of fixed point theorems involving relatively more general integral type contractive conditions. In an interesting note, Suzuki [35] showed that Meir-Keeler contractions of integral type are still Meir-Keeler contractions. The aim of this paper is to prove integral type fixed point theorems in Menger PM spaces satisfying common property (E.A).

2 Preliminaries:

Definition 2.1 [7] A mapping $F:\Re\rightarrow\Re^+$ is called distribution function if it is non-decreasing, left continuous with $\inf\{F(t) : t\in\Re\} = 0$ and $\sup\{F(t) : t \in\Re\} = 1$.
Let L be the set of all distribution functions whereas H be the set of specific distribution functions (also known as Heaviside function) defined by

$$H(x) = \begin{cases} 0, & \text{if } x \leq 0 \\ 1, & \text{if } x > 0 \end{cases}$$

Definition 2.2 [21] Let X be a nonempty set. An ordered pair (X, F) is called a PM space if F is a mapping from X × X into L satisfying the following conditions:
(i) $F_{p,q}(x) = H(x)$ if and only if $p = q$,
(ii) $F_{p,q}(x) = F_{q,p}(x)$,
(iii) $F_{p,q}(x) = 1$ and $F_{q,r}(y) = 1$, then $F_{p,r}(x + y) = 1$, for all p, q, r ∈ X and x, y ≥ 0.
Every metric space (X, d) can always be realized as a PM space by considering $F : X \times X \rightarrow L$ defined by $F_{p,q}(x) = H(x - d(p, q))$ for all p, q ∈ X. So PM spaces offer a wider framework (than that of the metric spaces) and are general enough to cover even wider statistical situations.
Definition 2.3. [7] A mapping $\Delta : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is called a t-norm if
(i) $\Delta (a, 1) = a, \Delta (0, 0) = 0$,
(ii) $\Delta (a, b) = \Delta (b, a)$,
(iii) $\Delta (c, d) \geq \Delta (a, b)$ for $c \geq a, d \geq b$,

(iv) $\Delta(\Delta(a, b), c) = \Delta(a, \Delta(b, c))$ for all a, b, c $\in [0, 1]$.

Example 2.4. The following are the four basic t-norms:

(i) The minimum t-norm: $T_M(a, b) = \min\{a, b\}$.

(ii) The product t-norm: $T_P(a, b) = a.b$.

(iii) The Lukasiewicz t-norm: $T_L(a, b) = \max\{a + b - 1, 0\}$.

(iv) The weakest t-norm, the drastic product:

$$T_D(a, b) = \begin{cases} \min\{a, b\}, & \text{if} \max\{a, b\} = 1 \\ 0, & \text{otherwise} \end{cases}$$

In respect of above mentioned t-norms, we have the following ordering:

$T_D < T_L < T_P < T_M$.

Definition 2.5. [21] A Menger PM space (X, F,Δ) is a triplet where (X, F) is a PM space and Δ is a t-norm satisfying the following condition:

$$F_{p,r}(x + y) \geq \Delta(F_{p,q}(x), F_{q,r}(y)).$$

Definition 2.6. [28] A sequence $\{p_n\}$ in a Menger PM space (X, F,Δ) is said to be convergent to a point p in X if for every $\in > 0$ and $\lambda > 0$, there is an integer $M(\in,\lambda)$ such that $F_{p_n,p} > 1-\lambda$ for all $n \geq M(\in,\lambda)$.

Lemma 2.7. [7, 9] Let (X, F,Δ) be a Menger space with a continuous t-norm Δ with $\{x_n\}$, $\{y_n\} \subset X$ such that $\{x_n\}$ converges to x and $\{y_n\}$ converges to y. If $F_{x,y}(.)$ is continuous at the point t_0, then $\lim_{n\to\infty} F_{x_n,y_n}(t_0) = F_{x,y}(t_0)$.

Definition 2.8. Let (A, S) be a pair of maps from a Menger PM space (X,F, Δ) into itself. Then the pair of maps (A, S) is said to be weakly commuting if $F_{ASx,SAx}(t) \geq F_{Ax,Sx}(t)$, for each $x \in X$ and $t > 0$.

Definition 2.9. [34] A pair (A, S) of self mappings of a Menger PM space (X, F, Δ) is said to be compatible if $F_{ASp_n,SAp_n}(x) \to 1$ for all $x > 0$, whenever $\{p_n\}$ is a sequence in X such that $Ap_n, Sp_n \to t$, for some t in X as $n \to 1$.

Clearly, a weakly commuting pair is compatible but every compatible pair need not be weakly commuting.

Definition 2.10. [19] A pair (A, S) of self mappings of a Menger PM space (X, F,Δ) is said to be non-compatible if and only if there exists at least one sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t \in X \text{ for some } t \in X$$

Implies that $\lim_{n\to\infty} F_{ASx_n,SAx_n}(t_0)$ (for some $t_0 > 0$) is either less than 1 or non-existent.

Definition 2.11. [15] A pair (A, S) of self mappings of a Menger PM space (X, F,Δ) is said to satisfy the property (E.A) if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t \in X$$

Clearly, a pair of compatible mappings as well as noncompatible Mappings satisfies the property (E.A).

Inspired by Liu et al. [39], Imdad et al. [26] defined the following:

Definition 2.12. Two pairs (A, S) and (B, T) of self mappings of a Menger PM space (X,F, Δ) are said to satisfy the common property (E.A) if there exist two sequences $\{x_n\}$, $\{y_n\}$ in X and some t in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ty_n = \lim_{n\to\infty} By_n = t$$

Definition 2.13. [12] A pair (A, S) of self mappings of a nonempty set X is said to be weakly compatible if the pair commutes on the set of their coincidence points i.e. Ap = Sp (for some $p \in X$) implies ASp = SAp.

Definition2.14. [24] Two finite families of self mappings $\{A_i\}$ and $\{B_j\}$ are said to be pairwise commuting if:

(i) $A_iA_j = A_jA_i$, $i, j \in \{1, 2...m\}$,

(ii) $B_iB_j = B_jB_i$, $i, j \in \{1, 2...n\}$,

(iii) $A_iB_j = B_jA_i$, $i \in \{1, 2...m\}$, $j \in \{1, 2...n\}$.

3. Main Result

The following lemma is useful for the proof of succeeding theorems.

Lemma 3.1. Let (X,F, Δ) be a Menger space. If there exists some $k \in (0, 1)$ such that for all p, q $\in$ X and all x > 0,

$$\int_0^{F_{p,q}(kx)} \phi(t)dt \geq \int_0^{F_{p,q}(x)} \phi(t)dt \qquad (3.1.1)$$

where $\phi: [0,\infty) \to [0,\infty)$ is a summable non-negative Lebesgue integrable function such that

$\int_c^1 \phi(t)dt > 0$ for each $t \in [0, 1)$, then p = q.

Remark 3.2. By setting $\phi(t) = 1$ (for each $t \geq 0$) in (3.1.1) of Lemma 3.1, we have

$\int_0^{F_{p,q}(kx)} \phi(t)dt = F_{p,q}(kx) \geq F_{p,q}(x) = \int_0^{F_{p,q}(x)} \phi(t)dt,$

which shows that Lemma 3.1 is a generalization of the Lemma 2 (contained in [8])

Lemma 3.3. Let A, B, S and T be four self mappings of a Menger space (X,F, Δ) which satisfy the following conditions:
(i) the pair (A, S) (or (B, T)) satisfies the property (E.A),
(ii) $B(y_n)$ converges for every sequence $\{y_n\}$ in X whenever $T(y_n)$ converges,
(iii) for any p, q ∈ X and for all x > 0,

$$\int_0^{F_{Ap,Bq}(kx)} \phi(t)dt \geq \int_0^{\min\{F_{Sp,Tq}(x),F_{Sp,Ap}(x),F_{Tq,Bq}(x),F_{Sp,Bq}(x),F_{Tq,Ap}(x),\frac{F_{Sp,Tq}(x)F_{Tq,Bq}(x)}{F_{Sp,Bq}(x)},\frac{F_{Sp,Tq}(x)F_{Sp,Ap}(x)}{F_{Tq,Ap}(x)}} \phi(t)dt \quad (3.3.1)$$

where $\phi : [0,\infty)\to[0,\infty)$ is a non-negative summable Lebesgue integrable function such that $\int_c^1 \phi(t)dt > 0$ for each t ∈ [0, 1), where 0 < k < 1 and
(iv) A(X) ⊂ T(X) (or B(X) ⊂ S(X)).
Then the pairs (A, S) and (B, T) share the common property (E.A).

Proof. Suppose that the pair (A, S) enjoys the property (E.A), then there exists a sequence $\{x_n\}$ in X such that

$$lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = t, \text{ for some } t \in X.$$

Since A(X) ⊂ T(X), for each x_n there exists $y_n \in$ X such that $Ax_n = Ty_n$, and hence

$$lim_{n\to\infty} Ty_n = lim_{n\to\infty} Ax_n = t$$

Thus in all, we have $Ax_n \to t$, $Sx_n \to t$ and $Ty_n \to t$. Now we assert that $By_n \to t$.
To accomplish this, using (3.3.1), with $p = x_n$, $q = y_n$, one gets

$$\int_0^{F_{Ax_n,By_n}(kx)} \phi(t)dt \geq \int_0^{\min\{F_{Sx_n,Ty_n}(x),F_{Sx_n,Ax_n}(x),F_{Ty_n,By_n}(x),F_{Sx_n,By_n}(x),F_{Ty_n,Ax_n}(x),\frac{F_{Sx_n,Ty_n}(x)F_{Ty_n,By_n}(x)}{F_{Sx_n,By_n}(x)},\frac{F_{Sx_n,Ty_n}(x)F_{Sx_n,Ax_n}(x)}{F_{Ty_n,Ax_n}(x)}\}} \phi(t)dt$$

Let $lim_{n\to\infty} B(y_n)$. Also, let x > 0 be such that $F_{t,l}(\cdot)$ is continuous in x and kx. Then, on making n→∞ in the above inequality, we obtain

$$\int_0^{F_{t,l}(kx)} \phi(t)dt \geq \int_0^{\min\{F_{t,t}(x),F_{t,t}(x),F_{t,l}(x),F_{t,l}(x),F_{t,t}(x),\frac{F_{t,t}(x)F_{t,l}(x)}{F_{t,l}(x)},\frac{F_{t,t}(x)F_{t,t}(x)}{F_{t,t}(x)}\}} \phi(t)dt$$

or

$$\int_0^{F_{t,l}(kx)} \phi(t)dt \geq \int_0^{F_{t,l}(x)} \phi(t)dt$$

This implies that $l = t$ (in view of Lemma 3.1) which shows that the pairs (A, S) and (B, T) share the common property (E.A).

Theorem 3.4. Let A, B, S and T be self mappings of a Menger space (X, F, Δ) which satisfy the inequality (3.3.1) together with
(i) the pairs (A, S) and (B, T) share the common property (E.A),
(ii) S(X) and T(X) are closed subsets of X.
Then the pairs (A, S) and (B, T) have a point of coincidence each. Moreover, A, B, S and T have a unique common fixed point provided both the pairs (A, S) and (B, T) are weakly compatible.

Proof. Since the pairs (A, S) and (B, T) share the common property (E.A), there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = lim_{n\to\infty} Ty_n = lim_{n\to\infty} By_n = t, \text{ for some } t \in X.$$

Since S(X) is a closed subset of X, hence $lim_{n\to\infty} Sx_n = t \in S(X)$. Therefore, there exists a point $u \in$ X such that $Su = t$. Now, we assert that $Au = Su$. To prove this, on using (3.3.1) with $p = u$, $q = y_n$, one gets

$$\int_0^{F_{Au,By_n}(kx)} \phi(t)dt$$

$$\geq \int_0^{\min\{F_{Su,Ty_n}(x),F_{Su,Au}(x),F_{Ty_n,By_n}(x),F_{Su,By_n}(x),F_{Ty_n,Au}(x),\frac{F_{Su,Ty_n}(x)F_{Ty_n,By_n}(x)}{F_{Su,By_n}(x)},\frac{F_{Su,Ty_n}(x)F_{Su,Au}(x)}{F_{Ty_n,Au}(x)}\}} \phi(t)dt$$

which on making n→∞, reduces to

$$\int_0^{F_{Au,t}(kx)} \phi(t)dt \geq \int_0^{\min\{F_{t,t}(x),F_{t,Au}(x),F_{t,t}(x),F_{t,t}(x),F_{t,Au}(x),\frac{F_{t,t}(x)F_{t,t}(x)}{F_{t,t}(x)},\frac{F_{t,t}(x)F_{t,Au}(x)}{F_{t,Au}(x)}\}} \phi(t)dt$$

or $\int_0^{F_{Au,t}(kx)} \phi(t)dt \geq \int_0^{F_{Au,t}(x)} \phi(t)dt$

Now appealing to Lemma 3.1, we have $Au = t$ and hence $Au = Su$. Therefore, u is a coincidence point of the pair (A, S).
Since T(X) is a closed subset of X, therefore $lim_{n\to\infty} Ty_n = t \in T(X)$ and hence one can find a point $w \in$ X such that $Tw = t$. Now we show that $Bw = Tw$. To accomplish this, on using (3.3.1) with $p = x_n$, $q = w$, we have

$$\int_0^{F_{Ax_n,Bw}(kx)} \phi(t)dt$$

$$\geq \int_0^{\min\{F_{Sx_n,Tw}(x),F_{Sx_n,Ax_n}(x),F_{Tw,Bw}(x),F_{Sx_n,Bw}(x),F_{Tw,Ax_n}(x),\frac{F_{Sx_n,Tw}(x)F_{Tw,Bw}(x)}{F_{Sx_n,Bw}(x)},\frac{F_{Sx_n,Tw}(x)F_{Sx_n,Ax_n}(x)}{F_{Tw,Ax_n}(x)}\}} \phi(t)dt$$

which on making n→∞, reduces to

$$\int_0^{F_{t,Bw}(kx)} \phi(t)dt \geq \int_0^{\min\{F_{t,t}(x),F_{t,t}(x),F_{t,Bw}(x),F_{t,Bw}(x),F_{t,t}(x),\frac{F_{t,t}(x)F_{t,Bw}(x)}{F_{t,Bw}(x)},\frac{F_{t,t}(x)F_{t,t}(x)}{F_{t,t}(x)}\}} \phi(t)dt$$

or

$$\int_0^{F_{t,Bw}(kx)} \phi(t)dt \geq \int_0^{F_{t,Bw}(x)} \phi(t)dt$$

On employing Lemma 3.1, we have $Bw = t$ and hence $Tw = Bw$. Therefore, w is a coincidence point of the pair (B, T).
Since the pair (A, S) is weakly compatible and $Au = Su$, therefore $At = ASu = SAu = St$.
Again, on using (3.3.1) with $p = t$, $q = w$, we have

$$\int_0^{F_{At,Bw}(kx)} \phi(t)dt$$

$$\geq \int_0^{\min\{F_{St,Tw}(x),F_{St,At}(x),F_{Tw,Bw}(x),F_{St,Bw}(x),F_{Tw,At}(x),\frac{F_{St,Tw}(x)F_{Tw,Bw}(x)}{F_{St,Bw}(x)},\frac{F_{St,Tw}(x)F_{St,At}(x)}{F_{Tw,At}(x)}} \phi(t)dt$$

or

$$\int_0^{F_{At,t}(kx)} \phi(t)dt \geq \int_0^{\min\{F_{At,t}(x),F_{At,At}(x),F_{t,t}(x),F_{At,t}(x),F_{t,At}(x),\frac{F_{At,t}(x)F_{t,t}(x)}{F_{At,t}(x)},\frac{F_{At,t}(x)F_{At,At}(x)}{F_{t,At}(x)}} \phi(t)dt$$

or

$$\int_0^{F_{At,t}(kx)} \phi(t)dt \geq \int_0^{F_{At,t}(x)} \phi(t)dt$$

Appealing to Lemma 3.1, we have $At = St = t$ which shows that t is a common fixed point of the pair (A, S).
Also the pair (B, T) is weakly compatible and $Bw = Tw$, hence

$$Bt = BTw = TBw = Tt.$$

Next, we show that t is a common fixed point of the pair (B, T). In order to accomplish this, using (3.3.1) with $p = u$, $q = t$, we have

$$\int_0^{F_{Au,Bt}(kx)} \phi(t)dt$$
$$\geq \int_0^{\min\{F_{Su,Tt}(x), F_{Su,Au}(x), F_{Tt,Bt}(x), F_{Su,Bt}(x), F_{Tt,Au}(x), \frac{F_{Su,Tt}(x)F_{Tt,Bt}(x)}{F_{Su,Bt}(x)}, \frac{F_{Su,Tt}(x)F_{Su,Au}(x)}{F_{Tt,Au}(x)}} \phi(t)dt$$

or

$$\int_0^{F_{t,Bt}(kx)} \phi(t)dt \geq \int_0^{\min\{F_{t,Bt}(x), F_{t,t}(x), F_{Bt,Bt}(x), F_{t,Bt}(x), F_{Bt,t}(x), \frac{F_{t,Bt}(x)F_{Bt,Bt}(x)}{F_{t,Bt}(x)}, \frac{F_{t,Bt}(x)F_{t,t}(x)}{F_{Bt,t}(x)}} \phi(t)dt$$

or

$$\int_0^{F_{t,Bt}(kx)} \phi(t)dt \geq \int_0^{F_{t,Bt}(x)} \phi(t)dt$$

Using Lemma 3.1, we have $Bt = t$ which shows that t is a common fixed point of the pair (B, T). Hence t is a common fixed point of both the pairs (A, S) and (B, T). Uniqueness of common fixed point is an easy consequence of the inequality (3.3.1). This completes the proof.

Theorem 3.5. Let A,B, S and T be self mappings of a Menger space (X,F, Δ) satisfying the inequality (3.3.1). Suppose that
(i) the pair (A, S) (or (B, T)) has property (E.A),
(ii) $B(y_n)$ converges for every sequence $\{y_n\}$ in X whenever $T(y_n)$ converges,
(iii) $A(X) \subset T(X)$ (or $B(X) \subset S(X)$),
(iii) S(X) (or T(X)) is a closed subset of X.
Then the pairs (A, S) and (B, T) have a point of coincidence each. Moreover, A, B, S and T have a unique common fixed point provided both the pairs (A, S) and (B, T) are weakly compatible.
Proof. In view of Lemma 3.3, the pairs (A, S) and (B, T) share the common property (E.A), i.e. there exists two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = lim_{n\to\infty} Ty_n = lim_{n\to\infty} By_n = t \text{ for some } t \in X.$$

If S(X) is a closed subset of X, then proceeding on the lines of Theorem 3.5, one can show that the pair (A, S) has a coincidence point, say u, i.e. $Au = Su = t$.
Since $A(X) \subset T(X)$ and $Au \in A(X)$, there exists $w \in X$ such that Au = Tw. Now, we assert that Bw = Tw.
On using (3.3.1) with $p = x_n$, $q = w$, one gets

$$\int_0^{F_{Ax_n,Bw}(kx)} \phi(t)dt$$
$$\geq \int_0^{\min\{F_{Sx_n,Tw}(x), F_{Sx_n,Ax_n}(x), F_{Tw,Bw}(x), F_{Sx_n,Bw}(x), F_{Tw,Ax_n}(x), \frac{F_{Sx_n,Tw}(x)F_{Tw,Bw}(x)}{F_{Sx_n,Bw}(x)}, \frac{F_{Sx_n,Tw}(x)F_{Sx_n,Ax_n}(x)}{F_{Tw,Ax_n}(x)}\}} \phi(t)dt$$

which on making $n\to\infty$, reduces to

$$\int_0^{F_{t,Bw}(kx)} \phi(t)dt \geq \int_0^{\min\{F_{t,t}(x),F_{t,t}(x),F_{t,Bw}(x),F_{t,Bw}(x),F_{t,t}(x),\frac{F_{t,t}(x)F_{t,Bw}(x)}{F_{t,Bw}(x)},\frac{F_{t,t}(x)F_{t,t}(x)}{F_{t,t}(x)}\}} \phi(t)dt$$

or

$$\int_0^{F_{t,Bw}(kx)} \phi(t)dt \geq \int_0^{F_{t,Bw}(x)} \phi(t)dt$$

Owing to Lemma 3.1, we have $t = Bw$ and hence $Tw = Bw$ which shows that w is a coincidence point of the pair (B, T). Rest of the proof can be completed on the lines of the proof of Theorem 3.4. This completes the proof.

Corollary 3.6. Let A and S be self mappings on a Menger space (X,F, Δ). Suppose that
(i) the pair (A, S) enjoys the property (E.A),
(ii) for all p, q ∈ X and for all x > 0,

$$\int_0^{F_{Ap,Aq}(kx)} \phi(t)dt \geq \int_0^{\min\{F_{Sp,Sq}(x),F_{Sp,Ap}(x),F_{Sq,Aq}(x),F_{Sp,Aq}(x),F_{Sq,Ap}(x),\frac{F_{Sp,Sq}(x)F_{Sq,Aq}(x)}{F_{Sp,Aq}(x)},\frac{F_{Sp,Sq}(x)F_{Sp,Ap}(x)}{F_{Sq,Ap}(x)}\}} \phi(t)dt, \quad 0<k<1$$

(iii) S(X) is a closed subset of X.
Then A and S have a coincidence point. Moreover, if the pair (A, S) is weakly compatible, then A and S have a unique common fixed point.

References

1. A. Aamri and D. El Moutawakil, Some new common fixed point theorems under strict contractive conditions, J. Math. Anal. Appl. 270(2002), 181-188.
2. A. Aliouche, A common fixed point theorem for weakly compatible mappings in symmetric spaces satisfying a contractive condition of integral type, J. Math. Anal. Appl., 322(2)(2006), 796-802.
3. A. Branciari, A fixed point theorem for mappings satisfying a general contractive condition of integral type, Int. J. Math. Math. Sci., 29(9)(2002), 531-536.
4. A. Djoudi and A. Aliouche, Common fixed point theorems of Gregus type for weakly compatible mappings satisfying contractive conditions of integral type, J. Math. Anal. Appl., 329 (1) (2007), 31-45.
5. A. Razani and M. Shirdaryazdi, A common fixed point theorem of compatible maps in Menger space, Chaos, Solitions and Fractals, 32 (2007), 26-34.
6. B. E. Rhoades, Two fixed point theorems for mappings satisfying a general contractive condition of integral type, Internat. J. Math. Math. Sci., 63(2003), 4007-4013.
7. B. Schweizer and A. Sklar, Probabilistic metric spaces, Elsevier, North Holand, New York, 1983.
8. B. Singh and S. Jain, A fixed point theorem in Menger spaces through weak compatibility, J. Math. Anal. Appl., 301 (2005), 439-448.
9. D. Mihet, A generalization of a contraction principle in probabilistic metric spaces (II), Int. J. Math. Math. Sci 5 (2005), 729-736.
10. D. Mihet, Fixed point theorems in fuzzy metric spaces using property E.A., Nonlinear Anal., 73 (2010), 2184-2188.
11. D. Turkoglu and I. Altun, A common fixed point theorem for weakly compatible mappings in symmetric spaces satisfying an implicit relation, Matematica Mexicana. Boletin.Tercera Serie., 13 (1) (2007), 195-205.
12. G. Jungck, Common fixed points for noncontinuous nonself maps on nonmetric spaces. Far East J. Math. Sci., 4 (2) (1996), 199-215.
13. G. Jungck, Compatible mappings and common fixed points, Int. J. Math. Math. Sci., 9 (4) (1986), 771-779.
14. I. Altun, M. Tanveer and M.Imdad, Common fixed point theorems of integral type in Menger PM spaces, Journal of Nonlinear Analysis and Optimization, vol.(3) No.1(2012),55-56.

15. I. Kubiaczyk and S. Sharma, Some common fixed point theorems in Menger space under strict contractive conditions, Southeast Asian Bull. Math., 32 (2008), 117-124.
16. J. Ali, M. Imdad and D. Bahuguna, Common fixed point theorems in Menger spaces with common property (E.A), Comp. Math. Appl., 60(2010), 3152-3159.
17. J. Ali, M. Imdad, D. Mihe¸t, and M. Tanveer, Common fixed points of strict contractions in Menger spaces, Acta Math. Hung., 132(4)(2011), 367-386.
18. J.K. Kohli and S. Vashistha, Common fixed point theorems in probabilistic metric spaces, Acta Math Hung., 115 (2007), 37-47.
19. J.X. Fang and Y. Gao, Common fixed point theorems under strict contractive conditions in Menger spaces, Nonlinear Analysis, 70 (2009). 184-193.
20. K. Menger, Probabilistic geometry, Proc. Nat. Acad. Sci. USA, 37 (1951), 226-229.
21. K. Menger, Statistical metrics, Proc. Nat. Acad. Sci. USA, 28 (1942), 535-537.
22. Lj. B. Ciri´c, A generalization of Banach's contraction principle, Proc. Amer. Math. Soc., 45 (1974), 267-273.
23. M. Imdad and J. Ali, Jungck's common fixed point theorem and E.A property, Acta Math. Sinica, 24 (2008), 87-94.
24. M. Imdad, Javid Ali and M. Tanveer, Coincidence and common fixed point theorems for nonlinear contractions in Menger PM spaces, Chaos, Solitons & Fractals, 42 (2009), 3121-3129.
25. M. Imdad, M. Tanveer and M. Hasan, Erratum to "Some common fixed point theorems in Menger PM Spaces", Fixed Point Theory Appl., 2011, 2011:28.
26. M. Imdad, M. Tanveer and M. Hasan, Some commn fixed point theorems in Menger PM spaces, Fixed Point Theory Appl., Vol. 2010, 14 pages.
27. MS El Naschie, A review of applications and results of Einfinity theory, Int. J. Nonlinear Sci. Numer. Simul. 8(2007), 11-20.
28. O. Hadzic and E. Pap, Fixed point theory in probabilistic metric spaces, Kluwer Academic Publishers, Dordrecht, 2001.
29. P. V_ayaraju, B.E. Rhoades and R. Mohanraj, A fixed point theorem for a pair of maps satisfying a general contractive condition of integral type, Internat. J. Math. Math. Sci., 15(2005), 2359-2364.
30. R. Chugh and S. Rathi, Weakly compatible maps in probabilistic metric spaces, J. Indian Math. Soc., 72 (2005), 131-140.
31. R.A. Rashwan and A. Hedar, On common fixed point theorems of compatible maps in Menger spaces, Demonst. Math., 31 (1998), 537-546.
32. R.P. Pant, Common fixed points of noncommuting mappings, J. Math. Anal. Appl., 188 (1994), 436-440.
33. S. Kutukcu and S. Sharma, Compatible maps and common fixed points in Menger probabilistic metric spaces, Commun. Korean Math. Soc., 24 (2009), 17-27.
34. S.N. Mishra, Common fixed points of compatible mappings in PMspaces, Math. Japon., 36 (1991), 283-289.
35. T. Suzuki, MeirKeeler contractions of integral type are still MeirKeeler contractions, Internat. J. Math. Math. Sci., (2007), Article ID 39281, 6 pages.
36. T.L. Hicks, Fixed point theory in probabilistic metric spaces, Univ. u Novom Sadu Zb. Rad. Prirod.Mat. Fak. Ser. Mat., 13 (1983), 63-72.
37. V.M. Sehgal and A.T. Bharucha-Reid, Fixed point of contraction mappings on probabilistic metric spaces, Math. Systems Theory, 6 (1972), 97-102.
38. V.W. Bryant, A remark on fixed point theorem for iterated mappings, Amer. Math. Monthly, 75 (1968), 399-400.
39. Y. Liu, Jun Wu and Z. Li, Common fixed points of single-valued and multivalued maps, Int. J. Math. Math. Sci., 19 (2005), 3045-3055.