

# Common fixed theorems for weakly compatible mappings via an implicit relation using the common (E.A) property in intuitionistic fuzzy metric spaces

\* Kamal Wadhwa, \*\* Ramakant Bhardwaj and \*Jyoti Panthi

\*Govt. Narmada Mahavidyalaya, Hoshangabad, (M.P) India

\*\* TRUBA Institute of Engineering and Information technology, Bhopal

## Abstract:

In this chapter, we prove two common fixed theorems for weakly compatible mappings via an implicit relation using the common (E.A) property in intuitionistic fuzzy metric spaces

## 1. Introduction

The concept of Fuzzy sets was initially investigated by Zadeh [14]. Subsequently, it was developed by many authors and used in various fields. To use this concept, several researchers have defined Fuzzy metric space in various ways. In 1986, Jungck [6] introduced the notion of compatible maps for a pair of self mappings. However, the study of common fixed points of non-compatible maps is also very interesting. Aamri and El Moutawakil [15] generalized the concept of non-compatibility by defining the notion of property (E.A) and in 2005, Liu, Wu and Li [23] defined common (E.A) property in metric spaces and proved common fixed point theorems under strict contractive conditions. Jungck and Rhoades [7] initiated the study of weakly compatible maps in metric space and showed that every pair of compatible maps is weakly compatible but reverse is not true. Many results have been proved for contraction maps satisfying property (E.A) in different settings such as probabilistic metric spaces [11, 21], fuzzy metric spaces [5, 18, and 19]. Atanassov [12] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets [14] and later there has been much progress in the study of intuitionistic fuzzy sets [4, 9].

In 2004, Park [10] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to George and Veeramani [1]. Fixed point theory has important applications in diverse disciplines of mathematics, statistics, engineering, and economics in dealing with problems arising in: Approximation theory, potential theory, game theory, mathematical economics, etc. Recently, Kumar [22] established some common fixed point theorems in intuitionistic fuzzy metric space using property (E.A). In 2010, Huang et al. [24] proved some results for families of compatible mappings. In this paper, employing common (E.A) property, we prove two common fixed theorems for weakly compatible mappings via an implicit relation in intuitionistic fuzzy metric spaces.

## 2. Preliminaries

The concepts of triangular norms (t-norms) and triangular conorms (t-conorms) are known as the axiomatic skeleton that we use are characterization fuzzy intersections and union respectively. These concepts were originally introduced by Menger [14] in study of statistical metric spaces.

**Definition 2.1** [2] A binary operation  $*$  :  $[0, 1] \times [0, 1] \rightarrow [0, 1]$  is continuous t-norm if  $*$  satisfies the following conditions:

- (1)  $*$  is commutative and associative,
- (2)  $*$  is continuous,
- (3)  $a * 1 = a$  for all  $a \in [0, 1]$ ,
- (4)  $a * b \leq c * d$  whenever  $a \leq c$  and  $b \leq d$  for all  $a, b, c, d \in [0, 1]$ .

**Definition 2.2** [2] A binary operation  $\diamond$  :  $[0, 1] \times [0, 1] \rightarrow [0, 1]$  is continuous t-conorm if  $\diamond$  satisfies the following conditions:

- (1)  $\diamond$  is commutative and associative,
- (2)  $\diamond$  is continuous,
- (3)  $a \diamond 0 = a$  for all  $a \in [0, 1]$ ,
- (4)  $a \diamond b \leq c \diamond d$  whenever  $a \leq c$  and  $b \leq d$  for all  $a, b, c, d \in [0, 1]$ .

Using the concept of intuitionistic fuzzy sets Alaca et al. [3] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [8] as:

**Definition 2.3** [3] A 5-tuple  $(X, M, N, *, \diamond)$  is said to be an intuitionistic fuzzy metric space if  $X$  is an arbitrary set,  $*$  is a continuous t-norm,  $\diamond$  is a continuous t-conorm and  $M, N$  are fuzzy sets on  $X^2 \times [0, 1)$  satisfying the following conditions:

- (1)  $M(x, y, t) + N(x, y, t) \leq 1$  for all  $x, y \in X$  and  $t > 0$ ,

- (2)  $M(x, y, 0) = 0$  for all  $x, y \in X$ ,
- (3)  $M(x, y, t) = 1$  for all  $x, y \in X$  and  $t > 0$  if and only if  $x = y$ ,
- (4)  $M(x, y, t) = M(y, x, t)$  for all  $x, y \in X$  and  $t > 0$ ,
- (5)  $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$  for all  $x, y, z \in X$  and  $s, t > 0$ ,
- (6) for all  $x, y \in X$ ,  $M(x, y, \cdot) : [0, \infty) \rightarrow [0, 1]$  is left continuous,
- (7)  $\lim_{n \rightarrow \infty} M(x, y, t^n) = 1$  for all  $x, y \in X$  and  $t > 0$ ,
- (8)  $N(x, y, 0) = 1$  for all  $x, y \in X$ ,
- (9)  $N(x, y, t) = 0$  for all  $x, y \in X$  and  $t > 0$  if and only if  $x = y$ ,
- (10)  $N(x, y, t) = N(y, x, t)$  for all  $x, y \in X$  and  $t > 0$ ,
- (11)  $N(x, y, t) \diamond N(y, z, s) = N(x, z, t + s)$  for all  $x, y, z \in X$  and  $s, t > 0$ ,
- (12) for all  $x, y \in X$ ,  $N(x, y, \cdot) : [0, \infty) \rightarrow [0, 1]$  is right continuous,
- (13)  $\lim_{n \rightarrow \infty} N(x, y, t^n) = 0$  for all  $x, y \in X$ .

Then  $(M, N)$  is called an intuitionistic fuzzy metric space on  $X$ . The functions  $M(x, y, t)$  and  $N(x, y, t)$  denote the degree of nearness and the degree of non-nearness between  $x$  and  $y$  with respect to  $t$ , respectively.

**Remark 2.4** [3] Every fuzzy metric space  $(X, M, *)$  is an intuitionistic fuzzy metric space of the form  $(X, M, 1 - M, *, \diamond)$  such that  $t$ -norm  $*$  and  $t$ -conorm  $\diamond$  are associated as  $x \diamond y = 1 - ((1-x) * (1-y))$  for all  $x, y \in X$ .

**Remark 2.5** [3] In intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$ ,  $M(x, y, *)$  is non-decreasing and  $N(x, y, \diamond)$  is non-increasing for all  $x, y \in X$ .

**Definition 2.6**[3] Let  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy metric space. Then a sequence  $\{x_n\}$  in  $X$  is said to be Cauchy sequence if, for all  $t > 0$  and  $p > 0$ ,

$$(i) \quad \lim_{n \rightarrow \infty} M(x_{n+p}, x, t) = 1 \text{ and } \lim_{n \rightarrow \infty} N(x_{n+p}, x, t) = 0$$

(ii) a sequence  $\{x_n\}$  in  $X$  is said to be convergent to a point  $x \in X$  if, for all  $t > 0$ ,

$$\lim_{n \rightarrow \infty} M(x_n, x, t) = 1 \text{ and } \lim_{n \rightarrow \infty} N(x_n, x, t) = 0$$

**Definition 2.7** [3] An intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  is said to be complete if and only if every Cauchy sequence in  $X$  is convergent.

**Example 2.8** [3] Let  $X = \{\frac{1}{n} : n \in \mathbb{N}\} \cup \{0\}$  and let  $*$  be the continuous  $t$ -norm and  $\diamond$  be the continuous  $t$ -conorm defined by  $a * b = ab$  and  $a \diamond b = \min\{1, a + b\}$ , respectively, for all  $a, b \in [0, 1]$ . For each  $t \in (0, \infty)$  and  $x, y \in X$ , define  $(M, N)$  by

$$M(x, y, t) = \begin{cases} \frac{t}{t + \|x-y\|} & t > 0 \\ 0 & t = 0 \end{cases} \quad \text{and} \quad N(x, y, t) = \begin{cases} \frac{\|x-y\|}{t + \|x-y\|} & t > 0 \\ 1 & t = 0 \end{cases}$$

Clearly,  $(X, M, N, *, \diamond)$  is complete intuitionistic fuzzy metric space.

**Definition 2.9** [15] A pair of self mappings  $(T, S)$  of an intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  is said to satisfy the property (E.A) if there exist a sequence  $\{x_n\}$  in  $X$  such that

$$\lim_{n \rightarrow \infty} T x_n = \lim_{n \rightarrow \infty} S x_n = z \text{ in } X$$

**Example 2.10** Let  $X = [0, \infty)$ . Consider  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy metric space as in example 2.8. Define  $T, S: X \rightarrow X$  by  $Tx = \frac{x}{5}$  and  $Sx = \frac{2x}{5}$  for all  $x \in X$ . Clearly, for sequence  $\{x_n\} = \{1/n\}$ .  $T$  and  $S$  satisfy property (E.A).

**Definition 2.11** [1] Two pairs  $(A, S)$  and  $(B, T)$  of self mappings of an intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  are said to satisfy the common (E.A) property if there exist two sequences  $\{x_n\}$  and  $\{y_n\}$  in  $X$  such that

$$\lim_{n \rightarrow \infty} A x_n = \lim_{n \rightarrow \infty} S x_n = \lim_{n \rightarrow \infty} B y_n = \lim_{n \rightarrow \infty} T y_n = z$$

for some  $z$  in  $X$ .

**Example 2.12** Let  $X = [-1, 1]$ . Consider  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy metric space as in example 2.8. Define self mappings  $A, B, S$  and  $T$  on  $X$  as  $Ax = x/3$ ,  $Bx = -x/3$ ,  $Sx = x$ , and  $Tx = -x$  for all  $x \in X$ . Then with sequences  $\{x_n\} = \{1/n\}$  and  $\{y_n\} = \{-1/n\}$  in  $X$ , one can easily verify that

$$\lim_{n \rightarrow \infty} A x_n = \lim_{n \rightarrow \infty} S x_n = \lim_{n \rightarrow \infty} B y_n = \lim_{n \rightarrow \infty} T y_n = 0$$

Therefore, pairs  $(A, S)$  and  $(B, T)$  satisfies the common E.A. property.

**Definition 2.13** [7] A pair of self mappings  $(T, S)$  of an intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  is said to be weakly compatible if they commute at coincidence points, i.e., if  $Tu = Su$  for some  $u \in X$ , then  $TSu = STu$ .

### 3. Main Results

Implicit relations play important role in establishing of common fixed point results.

Let  $M_6$  be the set of all continuous functions  $\varphi : [0, 1]^6 \rightarrow \mathbb{R}$  and  $\psi : [0, 1]^6 \rightarrow \mathbb{R}$  satisfying the following conditions:

- (A)  $\varphi(u, 1, u, 1, u) < 0$  for all  $u \in (0, 1)$ ,
- (B)  $\varphi(u, 1, 1, u, u) < 0$  for all  $u \in (0, 1)$ ,
- (C)  $\varphi(u, u, 1, 1, u, u) < 0$  for all  $u \in (0, 1)$ ,
- (D)  $\psi(v, 0, v, 0, 0, v) > 0$  for all  $v \in (0, 1)$ ,
- (E)  $\psi(v, 0, 0, v, v, 0) > 0$  for all  $v \in (0, 1)$ ,
- (F)  $\psi(v, v, 0, 0, v, v) > 0$  for all  $v \in (0, 1)$ .

if for some constant  $q \in (0,1)$  we have

- (G)  $\varphi(u(qt), 1, 1, u(t), u(t), 1) \geq 0$  for all  $u \in (0, 1)$ ,

Or

- $\psi(v(qt), 0, 0, v(t), v(t), 0) \leq 0$  for all  $v \in (0, 1)$ ,  
 then  $u(qt) \geq u(t)$  and  $v(qt) \leq v(t)$

- (H)  $\varphi(u(qt), 1, u(t), 1, 1, u(t)) \geq 0$  for all  $u \in (0,1)$

Or

- $\psi(v(qt), 0, v(t), 0, 0, v(t)) \leq 0$  for all  $v \in (0, 1)$ ,  
 then  $u(qt) \geq u(t)$  and  $v(qt) \leq v(t)$

- (I)  $\varphi(u(qt), u(t), 1, 1, u(t), u(t)) \geq 0$  for all  $u \in (0,1)$

Or

- $\psi(v(qt), v(t), 0, 0, v(t), v(t)) \leq 0$  for all  $v \in (0, 1)$ ,  
 then  $u(qt) \geq u(t)$  and  $v(qt) \leq v(t)$

**Theorem 3.1.** Let  $J, K, P, R, S$  and  $T$  be self mappings of an intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  satisfying the following conditions

- (i)  $J(X) \subset ST(X), K(X) \subset PR(X)$  ;
- (ii)  $(J, PR)$  and  $(K, ST)$  satisfies the E. A. property
- (iii) for any  $x, y \in X, q \in (0,1), \varphi$  and  $\psi$  in  $M_6$  and for all  $t > 0$ .

$$\varphi(M(Jx, Ky, qt), M(PRx, STy, t), M(PRx, Jx, t), M(STy, Ky, t), M(PRx, Ky, t), M(STy, Jx, t)) \geq 0$$

$$\psi(N(Jx, Ky, qt), N(PRx, STy, t), N(PRx, Jx, t), N(STy, Ky, t), N(PRx, Ky, t), N(STy, Jx, t)) \leq 0$$

Then the pairs  $(J, PR)$  and  $(K, ST)$  share the common (E. A.) property.

**Proof:** Suppose that the pair  $(J, PR)$  satisfies E.A. property, then there exist a sequence  $\{x_n\}$  in  $X$  such that  $\lim_{n \rightarrow \infty} Jx_n = \lim_{n \rightarrow \infty} PRx_n = z$  for some  $z$  in  $X$ . Since  $J(X) \subset ST(X)$ , hence for each  $\{x_n\}$ , there exist  $\{y_n\}$  in  $X$  such that  $Jx_n = STy_n$ . Therefore,  $\lim_{n \rightarrow \infty} Jx_n = \lim_{n \rightarrow \infty} PRx_n = \lim_{n \rightarrow \infty} STy_n = z$ . Now, we claim that  $\lim_{n \rightarrow \infty} Ky_n = z$ . Suppose that  $\lim_{n \rightarrow \infty} Ky_n \neq z$ , then applying inequality (iii), we obtain

$$\varphi(M(Jx_n, Ky_n, qt), M(PRx_n, STy_n, t), M(PRx_n, Jx_n, t), M(STy_n, Ky_n, t), M(PRx_n, Ky_n, t), M(STy_n, Jx_n, t)) \geq 0$$

$$\psi(N(Jx_n, Ky_n, qt), N(PRx_n, STy_n, t), N(PRx_n, Jx_n, t), N(STy_n, Ky_n, t), N(PRx_n, Ky_n, t), N(STy_n, Jx_n, t)) \leq 0$$

which on making  $n \rightarrow \infty$  reduces to

$$\varphi(M(z, \lim_{n \rightarrow \infty} Ky_n, qt), M(z, z, t), M(z, z, t), M(z, \lim_{n \rightarrow \infty} Ky_n, t), M(z, \lim_{n \rightarrow \infty} Ky_n, t), M(z, z, t)) \geq 0$$

$$\psi(N(z, \lim_{n \rightarrow \infty} Ky_n, qt), N(z, z, t), N(z, z, t), N(z, \lim_{n \rightarrow \infty} Ky_n, t), N(z, \lim_{n \rightarrow \infty} Ky_n, t), N(z, z, t)) \leq 0$$

$$\varphi(M(z, \lim_{n \rightarrow \infty} Ky_n, qt), 1, 1, M(z, \lim_{n \rightarrow \infty} Ky_n, t), M(z, \lim_{n \rightarrow \infty} Ky_n, t), 1) \geq 0$$

$$\psi(N(z, \lim_{n \rightarrow \infty} Ky_n, qt), 0, 0, N(z, \lim_{n \rightarrow \infty} Ky_n, t), N(z, \lim_{n \rightarrow \infty} Ky_n, t), 0) \leq 0$$

Using (G)

$$\varphi(M(z, \lim_{n \rightarrow \infty} Ky_n, t), 1, 1, M(z, \lim_{n \rightarrow \infty} Ky_n, t), M(z, \lim_{n \rightarrow \infty} Ky_n, t), 1) \geq 0$$

$$\psi(N(z, \lim_{n \rightarrow \infty} Ky_n, t), 0, 0, N(z, \lim_{n \rightarrow \infty} Ky_n, t), N(z, \lim_{n \rightarrow \infty} Ky_n, t), 0) \leq 0$$

which is a contradiction to (B) and (E), and therefore,  $\lim_{n \rightarrow \infty} Ky_n = z$ . Hence, the pairs  $(J, PR)$  and  $(K, ST)$  share the common (E.A.) property.

**Theorem 3.2:** Let  $J, K, P, R, S$  and  $T$  be self mappings of an intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  satisfying the conditions (2) and (4) the pair  $(J, PR)$  and  $(K, ST)$  share the common (E.A.) property, (5)  $PR(X)$  and  $ST(X)$  are closed subsets of  $X$ .

Then the pairs  $(J, PR)$  and  $(K, ST)$  have a point of coincidence each. Moreover,  $J, K, P, R, S$  and  $T$  have a unique common fixed point provided both the pairs  $(J, PR)$  and  $(K, ST)$  are weakly compatible.

**Proof:** In view of (4), there exist two sequences  $\{x_n\}$  and  $\{y_n\}$  in  $X$  such that

$$\lim_{n \rightarrow \infty} Jx_n = \lim_{n \rightarrow \infty} PRx_n = \lim_{n \rightarrow \infty} Ky_n = \lim_{n \rightarrow \infty} STy_n = z \text{ for some } z \in X.$$

Since  $PR(X)$  is a closed subset of  $X$ , therefore, there exists a point  $u$  in  $X$  such that  $z = PRu$ . We claim that  $Ju = z$ . If  $Ju \neq z$ , then by (3), take  $x = u, y = y_n$ ,

$$\begin{aligned} \varphi(M(Ju, Ky_n, qt), M(PRu, Sty_n, t), M(PRu, Ju, t), M(Sty_n, Ky_n, t), M(PRu, Ky_n, t), M(Sty_n, Ju, t)) &\geq 0 \\ \psi(N(Ju, Ky_n, qt), N(PRu, Sty_n, t), N(PRu, Ju, t), N(Sty_n, Ky_n, t), N(PRu, Ky_n, t), N(Sty_n, Ju, t)) &\leq 0 \end{aligned}$$

When  $n \rightarrow \infty$

$$\begin{aligned} \varphi(M(Ju, z, qt), M(z, z, t), M(z, Ju, t), M(z, z, t), M(z, z, t), M(z, Ju, t)) &\geq 0 \\ \psi(N(Ju, z, qt), N(z, z, t), N(z, Ju, t), N(z, z, t), N(z, z, t), N(z, Ju, t)) &\leq 0 \\ \varphi(M(Ju, z, qt), 1, M(Ju, z, t), 1, 1, M(Ju, z, t)) &\geq 0 \\ \psi(N(Ju, z, qt), 0, N(Ju, z, t), 0, 0, N(Ju, z, t)) &\leq 0 \end{aligned}$$

Using (H)

$$\begin{aligned} \varphi(M(Ju, z, t), 1, M(Ju, z, t), 1, 1, M(Ju, z, t)) &\geq 0 \\ \psi(N(Ju, z, t), 0, N(Ju, z, t), 0, 0, N(Ju, z, t)) &\leq 0 \end{aligned}$$

which is a contradiction to (A) and (D). Therefore,  $Ju = z = PRu$  which shows that  $u$  is a coincidence point of the pair  $(J, PR)$ .

Since  $ST(X)$  is also a closed subset of  $X$ , therefore  $\lim_{n \rightarrow \infty} STy_n = z$  in  $ST(X)$  and hence there exists  $v \in X$  such that  $STv = z = Ju = PRu$ . Now, we show that  $Kv = z$ .

If  $Kv \neq z$ , then by using inequality (3), take  $x = u, y = v$ , we have

$$\begin{aligned} \varphi(M(Ju, Kv, qt), M(PRu, STv, t), M(PRu, Ju, t), M(STv, Kv, t), M(PRu, Kv, t), M(STv, Ju, t)) &\geq 0 \\ \varphi(M(z, Kv, qt), 1, 1, M(z, Kv, t), M(z, Kv, t), 1) &\geq 0 \end{aligned}$$

and

$$\begin{aligned} \psi(N(Ju, Kv, qt), N(PRu, STv, t), N(PRu, Ju, t), N(STv, Kv, t), N(PRu, Kv, t), N(STv, Ju, t)) &\leq 0 \\ \psi(N(z, Kv, qt), 0, 0, N(z, Kv, t), N(z, Kv, t), 0) &\leq 0 \end{aligned}$$

Using (G)

$$\begin{aligned} \varphi(M(z, Kv, t), 1, 1, M(z, Kv, t), M(z, Kv, t), 1) &\geq 0 \\ \psi(N(z, Kv, t), 0, 0, N(z, Kv, t), N(z, Kv, t), 0) &\leq 0 \end{aligned}$$

which is a contradiction to (B) and (E). Therefore,  $Kv = z = STv$  which shows that  $v$  is a coincidence point of the pair  $(K, ST)$ .

Since the pairs  $(J, PR)$  and  $(K, ST)$  are weakly compatible and  $Ju = PRu, Kv = STv$ , therefore,  $Jz = JPRu = PRJu = PRz, Kz = KSTv = STKv = STz$ .

If  $Az \neq z$ , then by using inequality (3), we have

$$\begin{aligned} \varphi(M(Jz, Kv, qt), M(PRz, STv, t), M(PRz, Jz, t), M(STv, Kv, t), M(PRz, Kv, t), M(STv, Jz, t)) &\geq 0 \\ \varphi(M(Jz, z, qt), M(Jz, z, t), M(Jz, Jz, t), M(Kv, Kv, t), M(Jz, z, t), M(z, Jz, t)) &\geq 0 \\ \varphi(M(Jz, z, qt), M(Jz, z, t), 1, 1, M(Jz, z, t), M(Jz, z, t)) &\geq 0 \end{aligned}$$

and

$$\begin{aligned} \psi(N(Jz, Kv, qt), N(PRz, STv, t), N(PRz, Jz, t), N(STv, Kv, t), N(PRz, Kv, t), N(STv, Jz, t)) &\leq 0 \\ \psi(N(Jz, z, qt), N(Jz, z, t), N(Jz, Jz, t), N(Kv, Kv, t), N(Jz, z, t), N(z, Jz, t)) &\leq 0 \\ \psi(N(Jz, z, qt), N(Jz, z, t), 0, 0, N(Jz, z, t), N(Jz, z, t)) &\leq 0 \end{aligned}$$

Using (I)

$$\begin{aligned} \varphi(M(Jz, z, t), M(Jz, z, t), 1, 1, M(Jz, z, t), M(Jz, z, t)) &\geq 0 \\ \psi(N(Jz, z, t), N(Jz, z, t), 0, 0, N(Jz, z, t), N(Jz, z, t)) &\leq 0 \end{aligned}$$

which is a contradiction to (C) and (F). Therefore,  $Jz = z = PRz$ .

Similarly, one can prove that  $Kz = STz = z$ . Hence,  $Jz = Kz = PRz = STz$ , and  $z$  is common fixed point of  $J, K, P, R, S$  and  $T$ .

**Uniqueness:** Let  $z$  and  $w$  be two common fixed points of  $J, K, P, R, S$  and  $T$ . If  $z \neq w$ , then by using inequality (3), we have

$$\begin{aligned} \varphi(M(Jz, Kw, qt), M(PRz, STw, t), M(PRz, Jz, t), M(STw, Kw, t), M(PRz, Kw, t), M(STw, Jz, t)) &\geq 0 \\ \varphi(M(z, w, qt), M(z, w, t), M(z, z, t), M(w, w, t), M(z, w, t), M(w, z, t)) &\geq 0 \\ \varphi(M(z, w, qt), M(z, w, t), 1, 1, M(z, w, t), M(z, w, t)) &\geq 0 \end{aligned}$$

and

$$\begin{aligned} \psi(N(Jz, Kw, qt), N(PRz, STw, t), N(PRz, Jz, t), N(STw, Kw, t), N(PRz, Kw, t), N(STw, Jz, t)) &\leq 0 \\ \psi(N(z, w, qt), N(z, w, t), N(z, z, t), N(w, w, t), N(z, w, t), N(w, z, t)) &\leq 0 \\ \psi(N(z, w, qt), N(z, w, t), 0, 0, N(z, w, t), N(z, w, t)) &\leq 0 \end{aligned}$$

Using (I)

$$\begin{aligned} \varphi(M(z, w, t), M(z, w, t), 1, 1, M(z, w, t), M(z, w, t)) &\geq 0 \\ \psi(N(z, w, t), N(z, w, t), 0, 0, N(z, w, t), N(z, w, t)) &\leq 0 \end{aligned}$$

which is a contradiction to (C) and (F). Therefore,  $z = w$ .

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