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Some Fixed point Theorems in Generalization Metric space

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Abstract
In this paper we establish some fixed point results for mapping satisfying sufficient contractive conditions on a complete G-metric space.
Key words and phrases: Metric space, generalized metric space,

1. Introduction

In 1992, Bapure Dhage in his Ph.D. thesis introduced the concept of a new class of generalized metric space called D-metric spaces[2-3]. In 2005 Mustafa and Sims[6] shows that most of the results concerning Dhage's D-metric spaces are invalid. Therefore, they introduced a improved version of the generalized metric space structure, which are called G-metric spaces as generalization of metric space (X, d), to develop and to introduce a new fixed point theory for a variety of mappings in this new setting, also to extend known metric space theorems to a more general setting.
For more details on G-metric spaces, one can refer to the papers [6]-[9].
Now, we give preliminaries and basic definitions which are used throughout the paper.
In 2004, Mustafa and Sims [7] introduced the concept of G-metric spaces as follows:

Definition 1.1[7] Let X be a nonempty set, and let $G : X \times X \times X \to R^+$, be a function satisfying the following properties:
(G1) $G(x, y, z) = 0$ if $x = y = z$;
(G2) $0 < G(x, x, y)$; for all $x, y \in X$, with $x \neq y$;
(G3) $G(x, x, y) \leq G(x, y, z)$, for all $x, y, z \in X$, with $z \neq y$;
(G4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \ldots$, (symmetry in all three variables);
(G5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$, for all $x, y, z, a \in X$, (rectangl inequality).
Then the function G is called a generalized metric, or, more specifically a G-metric on X, and the pair (X, G) is called a G-metric space.
Definition 1.2. [7] Let (X, G) be a G-metric space, and let (x_n) be a sequence of points of X. A point $x \in X$ is said to be the limit of the sequence (x_n) if
$\lim_{n,m\to\infty} G(x, x_n, x_m) = 0$, and one say that the sequence (x_n) is G-convergent to x. Thus, that if $x_n \to 0$ in a G-metric space (X, G), then for any $\epsilon > 0$, there exists $N \in \mathbb{N}$ such that $G(x, x_n, x_m) < \epsilon$, for all $n, m \geq N$, (we mean by $\mathbb{N}$ the Natural numbers).
Proposition 1.3.[7] Let (X, G) be G-metric space. Then the following are equivalent.
(1) (x_n) is G-convergent to x.
(2) $G(x_n, x_n, x) \to 0$, as $n \to \infty$.
(3) $G(x_n, x, x) \to 0$, as $n \to \infty$.
(4) $G(x_m, x_n, x) \to 0$, as $m, n \to \infty$.
Definition 1.4.[7] Let (X, G) be a G-metric space, a sequence (x_n) is called G-Cauchy if given $\epsilon > 0$, there is $N \in \mathbb{N}$ such that $G(x_n, x_m, x_l) < \epsilon$, for all $n, m, l \geq N$. That is $G(x_n, x_m, x_l) \to 0$ as $n, m, l \to \infty$.
Proposition 1.5. .[7] In a G-metric space, (X, G), the following are equivalent
1. The sequence (x_n) is G-Cauchy.
2. For every $\epsilon > 0$, there exists $N \in \mathbb{N}$ such that $G(x_n, x_m, x_m) < \epsilon$, for all $n, m \geq N$.

Definition 1.6. [7] A G-metric space (X, G) is said to be G-complete (or complete G metric) if every G-Cauchy sequence in (X, G) is G-convergent in (X, G).
Proposition 1.7. .[7] A G-metric space (X, G) is G-complete if and only if (X, d^G) is a complete metric space.
Theorem 1.8 .[7] Let (X, d) be a complete metric space, and R be a function mapping X into it self, satisfy the following condition,
$d(R(x), R(y)) \leq ad(x, R(x)) + bd(y, R(y)) + cd(x, y), \forall x, y \in X$.
where a, b, c are nonnegative numbers satisfying $a + b + c < 1$. Then, R has a unique fixed point (i.e., there exists $u \in X$; $Ru = u$).

3. *Main Results*

In this section, we will present several fixed point results on a complete G-metric space.

Theorem 2.1. Let (X,G) be a complete G-metric space, and let $R : X \rightarrow X$ be a mapping satisfies the following condition

$$a\,G(Rx,Ry,Rz) + b\,[G(x,Rx,Rx) + G(y,Ry,Ry) + G(z,Rz,Rz)] \leq c\,G(x,y,z) \qquad (2.1(i))$$

for all $x,y,z \in X$, where the constants a,b,c satisfy $a,b,c > 0$;
$0 < c < a + b;\ a \neq 0$.

Proof. Take an arbitrary and define a sequence $x_{n+1} = x_n$, $n = 0,1,2 \dots\dots$

Substituting $x = x_n.\ y = x_{n+1},\ z = x_{n+2}$, then we have

$$a\,G(Rx_n,Rx_{n+1},Rx_{n+2}) + b\,[G(x_n,Rx_n,Rx_n) + G(x_{n+1},Rx_{n+1},Rx_{n+1}) + G(x_{n+2},Rx_{n+2},Rx_{n+2})] \leq c\,G(x_n,x_{n+1},x_{n+2})$$

$$a\,G(x_{n+1},x_{n+2},x_{n+3}) + b\,[G(x_n,x_{n+1},x_{n+1}) + G(x_{n+1},x_{n+2},x_{n+2}) + G(x_{n+2},x_{n+3},x_{n+3})] \leq c\,G(x_n,x_{n+1},x_{n+2})$$

$$a\,G(x_{n+1},x_{n+2},x_{n+3}) + b\,G(x_n,x_{n+3},x_{n+3}) \leq c\,G(x_n,x_{n+1},x_{n+2})$$

$$G(x_{n+1},x_{n+2},x_{n+3}) \leq \frac{c-b}{a} G(x_n,x_{n+1},x_{n+2})$$

Since $0 < c < a + b \Rightarrow 0 < c - b < a \Longrightarrow 0 < \frac{c-b}{a} < 1$

We assume that $\frac{c-b}{a} = k$ then

$$G(x_{n+1},x_{n+2},x_{n+3}) \leq k\,G(x_n,x_{n+1},x_{n+2})$$

Similarly we can show that

$$G(x_n,x_{n+1},x_{n+2}) \leq kG(x_{n-1},x_n,x_{n+1}).$$

Processing n times

$$G(x_{n+1},x_{n+2},x_{n+3}) \leq k^{n+1}G(x_0,x_1,x_2).$$

Next we show that $\{x_n\}$ is Cauchy sequence. Without loss of generality assume that $n > m$,
Then

$$\begin{aligned}
G(x_n,x_m,x_{m+1}) &\leq G(x_n,x_{n+1},x_{n+1}) + G(x_{n+1},x_m,x_{m+1}) \\
&\leq G(x_n,x_{n+1},x_{n+1}) + G(x_{n+1},x_{n+2},x_{n+2}) + G(x_{n+2},x_m,x_{m+1}) \\
&\leq G(x_n,x_{n+1},x_{n+1}) + G(x_{n+1},x_{n+3},x_{n+3}) + \\
&\qquad \dots\dots.. + G(x_{m-1},x_m,x_{m+1}) \\
&\leq k^n G(x_0,x_1,x_2) + k^{n-1}G(x_0,x_1,x_2). + \dots\dots. + k^m G(x_0,x_1,x_2). \\
&\leq k^m(1 + k + k^2 + \dots\dots. + k^{n-m})\,G(x_0,x_1,x_2). \\
&\leq \frac{k^m}{1-k^{n-m}}\,G(x_0,x_1,x_2).
\end{aligned}$$

Hence, limit $m,n \rightarrow \infty$

$$\lim_{m,n\rightarrow\infty} G(x_n,x_m,x_{m+1}) = 0$$

i.e. $\{x_n\}$ is Cauchy sequence.

Since (X,G) is complete, so there exists $w \in X$ such that $x_n \rightarrow w$, which implies,

$$\lim_{n\rightarrow\infty} G(x_n,x_n,w) = 0.$$

Next we will show that w is fixed point of R. we take $x = x_n$ and $y = z = w$ *in* (2.1(i)) then

$$a\,G(Rx_n,Rw,Rw) + b\,[G(x_n,Rx_n,Rx_n) + G(w,Rw,Rw) + G(w,Rw,Rw)] \leq c\,G(x_n,w,w)$$

$$a\,G(x_{n+1},Rw,Rw) + b\,[G(x_n,x_{n+1},x_{n+1}) + 2\,G(w,Rw,Rw)] \leq c\,G(x_n,w,w)$$

As $n \rightarrow \infty$ we have

$$(a + 2b)G(w,Rw,Rw) \leq 0$$

Which is contradiction, so $Rw = w$ i.e. w is fixed point of R.

Uniqueness:

Let p and q are two more fixed points of R, different from w, i.e. $w \neq p \neq q$.

We take $x = w, y = p, z = q$ in (2.1(i)) then

$$a\,G(Rw,Rp,Rq) + b\,[G(w,Rw,Rw) + G(p,Rp,Rp) + G(q,Rq,Rq)] \leq c\,G(w,p,q)$$

$$\Rightarrow G(w,p,q) \leq \frac{c}{a} G(w,p,q)$$

Which is contradiction, so $w = p = q$, i.e. w is unique fixed point of R.

This complete the proof of theorem .

Theorem 2.2. Let (X,G) be a complete G-metric space, and let $R : X \rightarrow X$ be a mapping satisfies the following condition

$min\{G(Rx,Ry,Rz), G(x,Rx,Rx), G(y,Ry,Ry), G(z,Rz,Rz)\}$
$\leq \alpha G(x,y,z)$ ……… … ……………………….. (2.2(i))

for all $x,y,z \in X$, where $0 \leq \alpha < 1$.

Proof. Take an arbitrary and define a sequence $x_{n+1} = x_n$, $n = 0,1,2 \ldots\ldots$

Substituting $x = x_n$, $y = x_{n+1}$, $z = x_{n+1}$, in (2.2(i)) then we have

$$min\left\{\begin{matrix} G(Rx_n,Rx_{n+1},Rx_{n+1}), G(x_n,Rx_n,Rx_n), G(x_{n+1},Rx_{n+1},Rx_{n+1}), \\ G(x_{n+1},Rx_{n+1},Rx_{n+1}) \end{matrix}\right\} \leq \alpha G(x_n,x_{n+1},x_{n+1})$$

$$min\left\{\begin{matrix} G(x_{n+1},x_{n+2},x_{n+2}), G(x_n,x_{n+1},x_{n+1}), G(x_{n+1},x_{n+2},x_{n+2}), \\ G(x_{n+1},x_{n+2},x_{n+2}) \end{matrix}\right\} \leq \alpha G(x_n,x_{n+1},x_{n+1})$$

$$min\{G(x_{n+1},x_{n+2},x_{n+2}), G(x_n,x_{n+1},x_{n+1})\} \leq \alpha G(x_n,x_{n+1},x_{n+1}) \ldots\ldots (2.2(ii))$$

If we take

$$min\{G(x_{n+1},x_{n+2},x_{n+2}), G(x_n,x_{n+1},x_{n+1})\} = G(x_{n+1},x_{n+2},x_{n+2})$$

Then from (2.2(ii))

$$G(x_{n+1},x_{n+2},x_{n+2}) \leq \alpha G(x_n,x_{n+1},x_{n+1})$$

And if we take

$$min\{G(x_{n+1},x_{n+2},x_{n+2}), G(x_n,x_{n+1},x_{n+1})\} = G(x_n,x_{n+1},x_{n+1})$$

Then

$$G(x_n,x_{n+1},x_{n+1}) \leq \alpha G(x_n,x_{n+1},x_{n+1})$$

Which is contradiction, so that

$$G(x_{n+1},x_{n+2},x_{n+2}) \leq \alpha G(x_n,x_{n+1},x_{n+1})$$

Similarly we can show that

$$G(x_n,x_{n+1},x_{n+1})) \leq \alpha G(x_{n-1},x_n,x_n) .$$

Next we show that $\{x_n\}$ is Cauchy sequence. Without loss of generality assume that $n > m$,

Then

$$\begin{aligned} G(x_n,x_m,x_m) &\leq G(x_n,x_{n-1},x_{n-1}) + G(x_{n-1},x_m,x_m) \\ &\leq G(x_n,x_{n-1},x_{n-1}) + \cdots \ldots\ldots\ldots + G(x_{m-1},x_m,x_m) \\ &\leq k^n G(x_0,x_1,x_2) + k^{n-1} G(x_0,x_1,x_2) + \ldots\ldots + k^{m-1} G(x_0,x_1,x_2). \\ &\leq k^n(1 + k + k^2 + \ldots\ldots + k^{n-m})\, G(x_0,x_1,x_2). \\ &\leq \frac{k^n}{1-k^{n-m}}\, G(x_0,x_1,x_2). \end{aligned}$$

Hence, limit $m,n \to \infty$

$$\lim_{m,n\to\infty} G(x_n,x_m,x_m) = 0$$

i.e. $\{x_n\}$ is Cauchy sequence.

Since (X,G) is complete G – metric space which gives $w \in X$ such that $\{x_n\} \to w$, as $n \to \infty$

Next we will show that w is fixed point of R. for this we take $x = x_n$ and $y = z = w$ in $(2.2(i))$ then

$$min\{G(Rx_n,Rw,Rw), G(x_n,Rx_n,Rx_n), G(w,Rw,Rw), G(w,Rw,Rw)\} \leq \alpha G(x_n,w,w)$$

$$min\{G(Rx_n,Rw,Rw), G(x_n,Rx_n,Rx_n), G(w,Rw,Rw)\} \leq \alpha G(x_n,w,w)$$

As $n \to \infty$ we have

$$min\{G(Rw,Rw,Rw), G(w,Rw,Rw), G(w,Rw,Rw)\} \leq \alpha G(w,w,w)$$

Which is contradiction, so $Rw = w$ i.e. w is fixed point of R.

Uniqueness:

Let p and q are two more fixed points of R, different from w, i.e. $w \neq p \neq q$.

We take $x = w, y = p, z = q$ in $(2.2(i))$ then

$$min\{G(Rw,Rp,Rq), G(w,Rw,Rw), G(p,Rp,Rp), G(q,Rq,Rq)\} \leq \alpha G(w,p,q)$$

$$\Rightarrow G(w,p,q) \leq \alpha G(w,p,q)$$

Which is contradiction, so $w = p = q$, i.e. w is unique fixed point of R.

This complete the proof of theorem .

Theorem 2.3. *Let (X,G) be a complete G-metric space, and let $R : X \to X$ be a mapping satisfies the following condition*

$$\frac{min\{G(Rx,Ry,Rz), G(x,Rx,Rx),\ G(y,Ry,Ry), G(z,Rz,Rz)\}}{min\{G(x,Rx,Rx), G(y,Ry,Ry), G(z,Rz,Rz)\}} \leq \alpha G(x,y,z) \quad \ldots\ldots\ \ldots(2.3(i))$$

for all $x,y,z \in X$, where $0 \leq \alpha < 1$.

***Proof**. Take an arbitrary and define a sequence $x_{n+1} = x_n$, $n = 0,1,2 \ldots\ldots$*

Substituting $x = x_n$, $y = x_{n+1}$, $z = x_{n+1}$, in(2.3(i))then we have

$$\frac{min\{G(Rx_n,Rx_{n+1},Rx_{n+1}), G(x_n,Rx_n,Rx_n),\ G^2(x_{n+1},Rx_{n+1},Rx_{n+1})]}{min\{G(x_n,Rx_n,Rx_n), G(x_{n+1},Rx_{n+1},Rx_{n+1}), G(x_{n+1},Rx_{n+1},Rx_{n+1})\}}$$

$$\leq a\,G(x_n,x_{n+1},x_{n+1})$$

$$\frac{\min\{G(x_{n+1},x_{n+2},x_{n+2}), G(x_n,x_{n+1},x_{n+1}),\ G^2(x_{n+1},x_{n+2},x_{n+2})\}}{\min\{G(x_n,x_{n+1},x_{n+1}), G(x_{n+1},x_{n+2},x_{n+2}), G(x_{n+1},x_{n+2},x_{n+2})\}}$$

$$\leq a\,G(x_n,x_{n+1},x_{n+1})$$

$$\frac{\min\{G(x_{n+1},x_{n+2},x_{n+2}), G(x_n,x_{n+1},x_{n+1}), G^2(x_{n+1},x_{n+2},x_{n+2})\}}{\min\{G(x_{n+1},x_{n+2},x_{n+2}), G(x_n,x_{n+1},x_{n+1})\}} \leq a\,G(x_n,x_{n+1},x_{n+1}) \quad \ldots\ldots(2.3(ii))$$

Case I: If we take

$$\frac{\min\{G(x_{n+1},x_{n+2},x_{n+2}), G(x_n,x_{n+1},x_{n+1}),\ G^2(x_{n+1},x_{n+2},x_{n+2})\}}{\min\{G(x_{n+1},x_{n+2},x_{n+2}), G(x_n,x_{n+1},x_{n+1})\}} = \frac{G(x_{n+1},x_{n+2},x_{n+2}) . G(x_n,x_{n+1},x_{n+1})}{G(x_n,x_{n+1},x_{n+1})}$$

Then from (2.3(ii))

$$\frac{G(x_{n+1},x_{n+2},x_{n+2}) . G(x_n,x_{n+1},x_{n+1})}{G(x_n,x_{n+1},x_{n+1})} \leq a\,G(x_n,x_{n+1},x_{n+1})$$

$$G(x_{n+1},x_{n+2},x_{n+2}) \leq a\,G(x_n,x_{n+1},x_{n+1})$$

Case II:

if we take

$$\frac{\min\{G(x_{n+1},x_{n+2},x_{n+2}), G(x_n,x_{n+1},x_{n+1}),\ G^2(x_{n+1},x_{n+2},x_{n+2})\}}{\min\{G(x_{n+1},x_{n+2},x_{n+2}), G(x_n,x_{n+1},x_{n+1})\}}$$

$$= \frac{G(x_{n+1},x_{n+2},x_{n+2}) . G(x_n,x_{n+1},x_{n+1})}{G(x_{n+1},x_{n+2},x_{n+2})}$$

Then from (2.3(ii))

$$\frac{G(x_{n+1},x_{n+2},x_{n+2}) . G(x_n,x_{n+1},x_{n+1})}{G(x_{n+1},x_{n+2},x_{n+2})} \leq a\,G(x_n,x_{n+1},x_{n+1})$$

Then $G(x_n,x_{n+1},x_{n+1}) \leq a\,G(x_n,x_{n+1},x_{n+1})$

Which is contradiction, so that

Case III:

if we take

$$\frac{\min\{G(x_{n+1},x_{n+2},x_{n+2}), G(x_n,x_{n+1},x_{n+1}),\ G^2(x_{n+1},x_{n+2},x_{n+2})\}}{\min\{G(x_{n+1},x_{n+2},x_{n+2}), G(x_n,x_{n+1},x_{n+1})\}} = \frac{G^2(x_{n+1},x_{n+2},x_{n+2})}{G(x_n,x_{n+1},x_{n+1})}$$

Then from (2.3(ii))

$$\frac{G^2(x_{n+1},x_{n+2},x_{n+2})}{G(x_n,x_{n+1},x_{n+1})} \leq a\,G(x_n,x_{n+1},x_{n+1})$$

$$G(x_{n+1},x_{n+2},x_{n+2}) \leq b\,G(x_n,x_{n+1},x_{n+1}) \quad \text{where } b = \sqrt{a}$$

Case IV:

if we take

$$\frac{\min\{G(x_{n+1},x_{n+2},x_{n+2}), G(x_n,x_{n+1},x_{n+1}),\ G^2(x_{n+1},x_{n+2},x_{n+2})\}}{\min\{G(x_{n+1},x_{n+2},x_{n+2}), G(x_n,x_{n+1},x_{n+1})\}} = \frac{G^2(x_{n+1},x_{n+2},x_{n+2})}{G(x_{n+1},x_{n+2},x_{n+2})}$$

Then from (2.3(ii))

$$\frac{G^2(x_{n+1},x_{n+2},x_{n+2})}{G(x_{n+1},x_{n+2},x_{n+2})} \leq a\,G(x_n,x_{n+1},x_{n+1})$$

$$G(x_{n+1},x_{n+2},x_{n+2}) \leq a\,G(x_n,x_{n+1},x_{n+1})$$

From case I, II, III and IV , we have

$$G(x_{n+1},x_{n+2},x_{n+2}) \leq a\,G(x_n,x_{n+1},x_{n+1})$$

By induction we have

$$G(x_{n+1},x_{n+2},x_{n+2}) \leq a^{n+1}\,G(x_0,x_1,x_1)$$

Similarly we can show that

$$G(x_n,x_{n+1},x_{n+1})\,) \leq a\,G(x_{n-1},x_n,x_{n+2})\ .$$

Next we show that $\{x_n\}$ *is Cauchy sequence. Without loss of generality assume that* $n > m$*,*

Then

$$G(x_n,x_m,x_m) \leq G(x_n,x_{n-1},x_{n-1}) + G(x_{n-1},x_m,x_m)$$

$$\leq G(x_n,x_{n-1},x_{n-1}) + \cdots \ldots\ldots\ldots + G(x_{m-1},x_m,x_m)$$

$$\leq k^n G(x_0,x_1,x_2) + k^{n-1} G(x_0,x_1,x_2) + \ldots\ldots + k^{m-1} G(x_0,x_1,x_2).$$

$$\leq k^n (1 + k + k^2 + \ldots\ldots + k^{n-m})\ G(x_0,x_1,x_2).$$

$$\leq \frac{k^n}{1-k^{n-m}}\ G(x_0,x_1,x_2).$$

Hence, limit $m, n \to \infty$
$\lim_{m,n\to\infty} G(x_n, x_m, x_m) = 0$
i.e. $\{x_n\}$ is Cauchy sequence.
Since (X, G) is complete *G* – metric space which gives $w \in X$ such that $\{x_n\} \to w$, as $n \to \infty$.
Next we will show that w is fixed point of R. for this we take $x = x_n$ and $y = z = w$ *in* $(2.3(i))$ then
$min\{G(Rx_n, Rw, Rw), G(x_n, Rx_n, Rx_n), G(w, Rw, Rw), G(w, Rw, Rw)\} \leq a\, G(x_n, w, w)$
$min\{G(Rx_n, Rw, Rw), G(x_n, Rx_n, Rx_n), G(w, Rw, Rw)\} \leq a\, G(x_n, w, w)$
As $n \to \infty$ *we have*
$min\{G(Rw, Rw, Rw), G(w, Rw, Rw), G(w, Rw, Rw)\} \leq a\, G(w, w, w)$
Which is contradiction, so $Rw = w$ *i.e. w is fixed point of* R.

Uniqueness:

Let p and q are two more fixed points of R, different from w, *i.e.* $w \neq p \neq q$.
We take $x = w, y = p, z = q$ in $(2.3(i))$ then
$min\{G(Rw, Rp, Rq), G(w, Rw, Rw), G(p, Rp, Rp), G(q, Rq, Rq)\} \leq a\, G(w, p, q)$
$\Rightarrow G(w, p, q) \leq aG(w, p, q)$
Which is contradiction, so $w = p = q$, i.e. w is unique fixed point of R.
This complete the proof of theorem.

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