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Application of Linear Programming Techniques to Practical Decision Making

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Abstract:

In spite of the overwhelming number of real life applications, linear programming techniques are yet to receive the desired level of recognition and acceptance. The general public does not seem to understand nor appreciate the contributions of operations research and its component studies to work place decision making process. It has become necessary to review some of these applications and to reconcile the seeming apathy towards linear programming techniques and the benefits derived therein. That is the main thrust of this study! As a way out of the woods, there is need to evolve a national policy that will make the teaching and learning of operations research and its applications a national priority. This should in the long run, inculcate a culture of bias towards quantitative thinking and its applications. The phobia associated with the study of mathematics and other related subjects is still far from being over. It is high time our tertiary institutions shy away from soft sell degree programs for more practically oriented courses. Presently, not many of our Nigerian universities run a degree program in operations research and decision sciences. The general public and more especially the private sector should invest in the study of operations research and its' component studies in programming techniques. This will evolve a culture of record keeping and statistics, thereby reducing the drudgeries associated with '' guess works '' and going for a more informed decision making process. Readers are enjoined not only to appreciate and study the concept of linear programming techniques but also to apply this all important and time tasted problem solving approach that could enhance decision making of managers, especially where there are restrictions or constraints in the decision making process.

Keywords: Objective function, Decision Variable, Parameters, Minimization, Maximization, Structured Constraints, Optimal Solutions.

1.0 Introduction

Linear programming is a mathematical technique concerned with the allocation of scarce resources. It is a procedure adopted to optimize the value of some objectives subject to some constraints. The objectives may be to maximize profit or to minimize costs. The program is designed to help the production and operations manager in planning and decision making relative to resource allocation. The manager who is restricted in terms of solutions to be adopted is however expected by management to produce the best of optimal solutions. A linear programming model enables the manager to make optimal use of limited available resources. These scarce resources can be in form of manpower, money, materials, skills, time and facilities etc. The decision maker then aims at finding the best decision or outcomes with the scarce resources. The decision or outcome may be with respect to costs, profits, sales, return or investments, general welfare of the society etc. The technique involves formulating a given problem as a linear programming model with the variables clearly identified and using standard techniques to solve the problem. As is often formulated, linear programming seeks to find non – negative values of the variables X_1, X_2, X_3, X_4 X_5, X_6………, X_n that satisfy the constraints: $a_{11}x_1 + a_{12}x_2 + a_{i3}x_3 + \ldots\ldots\ldots\ldots a_{in}x_n = b$,where I = 1,2,3…..m, that maximizes or minimizes the linear form $C_1X_1 + C_2X_2 + \ldots\ldots\ldots C_nX_n$

Against the above backdrop, the objective of this study is to expose and arouse the interest of readers to practical applications and usage of linear programming techniques in real life situations .The essence is to reconcile the seeming apathy towards linear programming techniques with the benefits derived therein. The technique assumes a linear or straight line relationship between variables. These variables are manipulated to produce solution which is improved upon until the optimum solution is reached. To capture all the issues involved, our discussion will focus on the following key areas: The historical origin of linear programming techniques, basic requirements and assumptions made in a linear programming model. The study will also take more than a cursory look at formulation of linear programming models, different methods of solving a linear programming model and its application to practical decision making process. Thereafter, policy implications will be drawn and necessary suggestions and recommendations proffered.

2.1 Historical Origin of linear programming Techniques

George B. Dantzig(1947) is commonly credited with being the "father" of linear programming techniques. Dantzig was involved in military strategic challenges in the US Air force during the world war 11 and the areas he emphasized at the initial stage were on transportation, assignment and deployment decisions. He developed and formulated the Simplex method as a basic solution of the Linear programming model in 1947. The use of linear programming technique was extended to business organizations after the Second World War and has since found applications in various fields of human endeavor.

Koopmans (1949) further developed the linear programming theory while working with the planning of optimal transportation of ships back and forth the Atlantic Ocean during the world war 11.

Kuhn and Tucker (1950) developed further the technique of linear programming independently in the United States of America. They were the first to use linear programming on a large scale, in the scheduling of iron and steel production, paper and oil industries and in the minimization of freight or transport costs.

Kantorovich (1953) showed how linear programming can be used to improve economic planning in Russia; he analyzed efficiency conditions for an economy as a whole, demonstrating the connection between the allocation of resources and the price system.
Assar (1957) extended the work of Kantorovich to deal with more than one commodity and can also provide a measure of the opportunity costs of different combinations of inputs and outputs which must be linear. **Cooper (1957**) made an application of linear programming in the area of petroleum –scheduling refineries, while **Nueman** in **(1958)** worked on problems involving linear inequalities and the game theory.

In 1984, Karmarker, the Bell Laboratory Mathematician devised a radically new procedure that may speed up the routine handling of problems by business and government agencies and also make it possible to tackle problems that can be applied to include a variety of commercial and government applications ranging from allocating time on a communication satellite to routing millions of telephone calls over long distances, or whenever time must be allocated most efficiently among competing users.
The above trend has continued with varying degree of studies going on in the area of and application of linear programming techniques to daily life situations.

2.2. Basic Requirements for the use of a linear programming Technique.

To solve some problems using the linear programming approach, some basic conditions must be met. These requirements can be grouped into two main categories, namely the **components and assumptions** of linear programming model. The components can be grouped into 4 major categories. They are the objective function, the decision variables, constraints and the parameters. These are briefly discussed below:

Objective Function: The linear programming problem must have a quantitatively specified linear objective function or criterion to be optimized. For example, one may wish to optimize returns on investments, productivity, profit, costs etc. This is done by either maximizing or minimizing the objective function. The objective of the problem must first be identified and then translated into mathematical function in order to assess the degree of effectiveness of the system. The aim is to find the optimal value of the objective function.

Decision Variable: These are the unknown variables to be computed in the models. Decision variables, represent choices that are available to the decision maker and are measured in terms of inputs or outputs. Decision variables can be represented by unit power of alphabets like $X_{I,}$ X_2 or X, Y, Z etc .If $X_{I,}$ X_2…….. X_n are decision variables and P is the profit to be maximized, then the objective function can be expressed as $P = a_1 X_I + a_2 X_2 + \ldots\ldots a_n X_n$

Structural constraints: Decision problems involving the use of linear programming technique usually require the optimization of limited resources. These limited resources are expressed as constraints in a linear programming problem. The constraints limit the value of decision variables in the objective function. The constraints are represented as linear equation or linear inequalities.

Parameters: A linear programming model has both the mathematical statement of the objectives as well as that of the constraints. These mathematical statements contain symbols representing the decision variables and also numerical values in the form of constants called parameters.

2.3. Assumptions made in linear programming models

Linearity: It is assumed that decision variables are of the first power. There is no provision for higher powers like squares and cubes in any of the equations and inequalities**.**

Divisibility: Values of the decision variables are allowed to be fractions and need not be integers alone.

Deterministic Parameters: It is assumed that the values of the parameters are known and are constant. This means that the model assumes a static state. It is pertinent to point out that in real life situation, there are probabilistic model parameters.

Non negativity: All decision variables must take on non negative values**.**

Proportionality: It is assumed that the output obtained from a resource is proportional to the input of the resource.

Additivity: The linearity concept also implies that the total measures of the objective function and the total usage are additive in nature.

Independence of variables: It is assumed that the various components of the model work independently.

Existence of an optimal solution: There exists an optimal solution to the objective function, taking into consideration the identified constraints (Adedayo et al:2006).

2.4. Formulation of a linear programming model:

Many linear programming problems are not stated in mathematical forms. They'll need to be formulated as a linear programming problem using the following steps: First, list and define the decision variables, second, State the objective function to be optimized and identify the constraints on one or more variable. Third, write the mathematical expression relating the terms and the constraints using the appropriate relational signs. Fourthly, express the non negativity constraints mathematically. This enhances the feasibility of the solution. Thus, the general form of a linear programming model with n decision variables and 'n' constants are given as follows:

Maximize $P = a_1 X_I + a_2 X_2 + \ldots\ldots\ldots\ldots\ldots\ a_n X_n$

OR

Minimize $C = a_1 X_I + a_2 X_2 + \ldots\ldots\ldots\ldots\ldots\ a_n X_n$

Subject to $b_{11}X_I + b_{21}X_2 + \ldots\ldots\ldots\ldots. + b_{ai}X_n \leq d_1$

$$b_{12}X_I + b_{22}X_2 + \ldots\ldots\ldots. + b_{n2}X_n \leq d_2$$

$$\ldots. \quad \ldots. \qquad \ldots. \quad \ldots.$$

$$b_{1m}X_I + b_{2m}X_2 + \ldots\ldots\ldots. + b_{mn}X_n \leq d_m$$

With $X_{I,}\ X_{2}\text{-}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \quad X_n \geq 0$

2.5. Methods of solving a linear programming model:

There are two basic methods of solving a linear programming model. This includes the graphical and simplex methods. These are briefly reviewed below:

Graphical Method

This method involves plotting the constraints on a graph and identifying the region that satisfies all of the constraints. This region is called the feasible solution space. After plotting the constraints and identifying the feasible space the objective function is then plotted and used to identify the optimal point in the feasible solution space. This point may be read directly from the graph or determined by substituting all the coordinates of the joints into the objective functions as these joints form the feasible solution space of the linear programming problem. If it is a maximization problem, the highest of these values is the optimum solution. However, if it is a minimization problem, the smallest of the values give the optimum solution of the linear programming problems.

The Simplex Method

The Simplex method is a general purpose linear programming Algorithm widely used to solve large scale problems. It is an iterative procedure that progressively approaches and ultimately reaches an optimal to linear programming problems. In other words, in a maximization problem the last solution yields a total profit than yielded by the previous solution. The steps involved in a Simplex method include:

- Formulation of the problem into an objective function and the constraints
- Setting up of an initial Simplex tableau.
- Selection of the key/ Pivot column
- Development of main row and balance for a new table.
- Iterative process of the above steps until all index numbers (Not including the constraint column are positive) – An optimum solution then results.

Having undertaken a bird's eye view of what linear programming technique entails; we will now dwell more extensively on some basic applications of a Linear programming model in real life situations.

3.0 Some basic Applications of Linear Programming Techniques

3.1 Financial Application

Financial applications involving the use of linear programming effort will be discussed under three sections – Portfolio Selection problem, financial mix and investment strategies.

3.1.1 Portfolio Selection problem: Portfolio selection problems are financial management situations in which a manager must select specific investments from a variety of investment alternatives for example: stocks, bonds etc. Situations like this are usually encountered by managers of mutual funds, credit unions, insurance companies and banks. The objective functions for these problems are usually aimed at maximization of expected return or minimization of risk. The constraints usually take the form of restrictions on the type of permissible investments, state laws, company policy and, maximum permissible risks and so on

Sample of a practical application problem:

Chief Okoro wants to invest a certain sum of money that would generate an annual yield of at least N100, 000.Two stock groups are available: Blue chips limited and Umanna high tech, with an average annual yield of 10% and 25% respectively. Though Umanna high –Tech stock provides higher yield; they are more risky; thus, Chief Okoro wants to limit the amount invested in this stock to no more than 45% of the total investment. What is the minimum amount that, Chief Okoro should invest in each stock group to accomplish the investment goal.

3.1.2 Financial Mix Strategy

Financial mix strategies involve the selection of means for financing company projects, inventories, production operations and various other activities. This is a particular application of a financial decision model which must be made with regards to how much production is to be supported by internally generated funds and how much is to be supported by external funds.

Sample of a Practical Application problem:

Uchenna is an aspiring entrepreneur. He has the sum of N100, 000 in his savings account, but has realized that, that is not enough to finance his block molding business and has decided to borrow the sum of N200, 000 from a Micro finance Bank. Uchenna is expected to defray his indebtedness to the bank within a period of 12 months on a straight-line simple interest charge of 10% per month. The maximum number of blocks he could mould in a day is 400pieces.The maximum daily demand is 360 blocks per day. The profit margin is N15 per block

In your capacity as his account officer, you are required to determine the optimal rate at which he repays his outstanding loan indebtedness/ interest and at the same time retain enough cash flow to service his operational tools and to take care of his family.

3. I .3 investments Application

Today's investors are presented with multitudes of investment opportunities. Linear programming can be used to select the optimal mix of opportunities that will maximize return while meeting the investment conditions set by the investor. (Taya: 2007)

Sample of a practical application problem- (Loan Policy Model):

First bank of Nigeria Plc is in the process of devising a sustainable loan policy and has devoted the sum of N15billion to that effect .The following table provides the pertinent data on available loan types.

Type of loan	**Interest rate**	**Bad debt ratio**
Personal	15%	0.01
Car	16%	0.07
Home	14%	0.03
Agricultural	12%	0.05
Commercial	19%	0.10

A recent Central Bank of Nigeria's regulation requires that commercial banks allocate at least 40% of their funds to Agriculture and the commercial sectors. To assist the housing industry in the country, home loans must equal at least 50% of the personal, car and home loans. First Bank of Nigeria Plc has a policy of not allowing the overall ratio of bad debts on all loans to exceed 4%. One is required to determine the amount of loans in each category that will maximize its net returns.

3.2 Marketing Application:

Marketing Application involving linear programming model will be discussed under two sections – Media selection and marketing strategy.

3.2.1 Media Selection:

This is aimed at helping marketing managers allocate a fixed advertising budget across various media. Potential advertising media include newspapers, magazines, radio commercials, television commercials, direct mailing and others. The essence is aimed at maximization of audience exposure

Sample of a Practical Application problem:

ABC Confectionaries Limited is a new and upcoming fast food Industry. The company is desirous of making its brand a household name within the nearest possible time and has hired the services of an advertising agency – Afromedia Nigeria Limited to design her promotional campaign. /strategies. Potential advertising media include Daily Newspapers, Monthly magazines, Radio commercials, Television Adverts and Direct SMS Messages via Telecommunication companies. The essence is aimed at maximization of audience exposure.

Afromedia Nigeria Limited has collected data on potential purchase families to be reached via each advertising medium, the cost per advertisement, the maximum number of times per month each medium will be made available for use and the expected exposure unit per medium .These data are supplied in the table below.

Advertising Media	Number of potential families reached	Cost per Advertisement	Maximum times available per month	Expected Exposure units
Daily Newspapers	1500	N45000	40	40
Monthly Magazines	7500	N65000	1	20
Radio jingles/ Advertisement	4500	N15000	300	70
Television Adverts/ Commercials	3000	N25000	5	25
Direct SMS Messages.	10000	N3	1000	80

ABC Confectionaries Limited has provided the Advertising Agency with an Advertising budget of N10million for the first month's campaign and has imposed the following restrictions on how the advertising agency may allocate the funds:

- At least 25 adverts must be aired on the National Television in course of the month.
- An advert must be placed in at least 1 daily newspaper per week
- The radio jingles should cut across wide audience participation and must be aired in English language, Pidgin English and Native mother tongues.
- The Direct SMS messages must be shared in the ratio of 40% to MTN, 25% to AIRTEL, 25% to GLO Mobile and 10% to ETISALAT network.
- In addition, not more than N1.8Million should be spent on television advertisement.

What advertising media selection plan should Afromedia Nigeria limited adopt to suit the aspirations of his client?

3.2.2 Marketing strategy

This involves an optimal allocation of sales force and advertising efforts .This is aimed at increased sales and profit.

Sample of a practical application problem:

ShopRite Superstores operates 7 days a week. The sales manager estimates that the minimum number of salespersons required to provide prompt and efficient service is 25 for Monday, 30 for Tuesday, 35 for Wednesday, 38 for Thursday, 40 for Friday and 45 for each of Saturday and Sunday. Each salesperson works 5 days a week, with two consecutive off days staggered throughout the week. For example if 15 sales persons start on Monday, Five can take their off duty on Tuesday and Wednesday, Eight on Wednesday and Thursday and Seven on Saturday and Sunday. How many salespersons should be contracted and how should their off days be allocated to maximize an efficient service delivery at the departmental store?

3.3 Management Applications:

Under this category, we shall consider the production scheduling and manpower planning applications**.**

3.3.1 Production Scheduling

A product scheduling problem can be viewed as a product mix problem for each of several periods in the near future. The manager must determine the production levels that will allow the company to meet product demand requirements, given limitations on productive capacity and storage space. At the same time, it is desired to minimize the total cost of carrying out the task. In most cases a production scheduling task is recurring in nature. A general linear programming model for a production scheduling procedure may be frequently applied once the model has been formulated.

Sample of a practical application problem:

Alpha quarries Nigeria Plc operates two plants, each of which produces three different sizes of granites named size A, B and C. The hourly output of the two plants is as follows:

	Size "A"	**Size "B"**	**Size "C"**
Plant 1	5 tonnes	3tonnes	2tonnes
Plant 1	10tonnes	1tonnes	15tonnes

The two plants are run intermittently as required. On a particular day, the company has orders in hand for 400 tonnes of size "A" product, 100 tonnes of size "B" product and 270 tonnes of size "C" product. The two plants will run simultaneously until these quantities are made available. If it costs N1, 000 an hour to run plant 1 and N1, 250 an hour to run plant 11, for how long should each plant be run in order to provide sufficient output to meet the correct orders**?**

3.3.2 Manpower Panning:

A manpower planning or scheduling problem occurs when a manager needs to make decisions involving departmental staffing requirements for a given period of time. This is particularly true when manpower assignments have some flexibility and at least some manpower can be assigned to more than department or work center. This is often the case when employees have been cross-trained on two or more jobs.

Sample of a practical application problem:

Lever brothers Nigeria plc is fine-tuning her production process and intends to produce only two types of detergents in the upcoming months. The detergents are processed in four separate departments. Excess equipment capacity is available and will not be a constraining factor. However, the company's manpower resources are limited and will probably constrain the production volume for the two products. The man-hour requirements per carton of detergent are shown in the table below. The company makes a profit of N500.00 per carton of detergent1 and N400 per carton of detergent 2, if the number of man –hour available in each department is fixed. We are required to (i) Find the optimal product mix for the planning period (ii) Locate manpower to the departments in such a fashion that profits will be maximized.

Man- Hours per carton of detergent type.

Department	Detergent 1	Detergent 2	Possible labor assignment	Man hours Available
1	0.070	0.100	Department 1 only	430
2	0.050	0.084	Department 2 only	400
3	0.100	0.067	Department 3 only	500
4	0.010	0.025	Department 4 only	135
			Department 1 or 2	570
			Department 3 or 4	300
				Total = 2335

3.4 Transportation Model

Transportation models are possibly the most widely used linear programming models. Problems and solutions presented in this model are concerned with the transportation or physical distribution of goods and services from several supply locations to several customers' locations.

The classic example of a transportation problem involves the shipment of some "homogenous commodity" from m sources of supply or origins to n points of demand "destinations". The objective of a transport model is to minimize the total transportation or delivery costs while ensuring that the number of units shipped from an origin does not exceed the available number of units at the point of origin and that demand at each destination is satisfied.

There are two major assumptions in the transportation model. They are (i) The model assumes a homogenous commodity.ie, there are no significant differences in the characteristics of the commodity at each point of origin and (ii) The model assumes that total supply and total demand are equal.

Solution methods of transportation model: There are several approaches to solving a transportation problem. Many of these problems can be put into two major groups. The first group which is normally referred to as initial solution methods; do not guarantee optimal solution, although they are feasible. The second group which usually takes off from the first group can tell when one has gotten at an optimal solution. The initial solution methods include the following: Northwest-Corner Rule, Least- Cost Method (or low cost first) and the Vogel Approximate Method (VAM).

The list for the optimal solution group comprises of the stepping stone method, Modified distribution method (MODI) and the Simplex method.

Sample of a practical application problem:

A company manufactures a product in three factories namely, F_I, F_2 and F_3. The three factories ship units of this product to four warehouses, W_I, W_2, W_3 and W_4. The demand at each warehouse, the capacity of each factory and the cost per unit from each factory to each Warehouse are as follows:

Factories	Warehouses W_1	W_2	W_3	W_4	Totals
F_1	N4	N8	N7	N5	2000
F_2	3	8	8	4	1700
F_3	4	9	7	4	1400
Total	1000	800	2100	1200	5100

One is required to solve this problem and to ascertain how best the company can minimize cost of shipment of products from the factories to the various warehouses.

3.5 An Assignment Model

An assignment model is a special case of a transportation model. It is most appropriate in problems that involve the assigning of resources (from a point of origin) to n tasks (destination). A typical example of an assignment problem include the assignment of sales persons to sales territories, airline crews to flights, Ambulance units to calls for service, Referee and officiating officers to a sporting event and lawyers within a law firm to cases or clients. Usually the objective of an assignment problem is that of minimization or maximization. For example an organization may wish to minimize time and costs and to maximize profit. There are three assumptions significant in the formulation of assignment problems. They are:

i) Each resource is assigned exclusively to one task, (ii) Each task is assigned exactly to one resource and iii) For the purpose of ascertaining solutions to a problem, the number of resources available for assignment must equal the number of tasks to be performed.

There are various methods of solution to an assignment model. These includes: A total enumeration of all solutions, a programming method and the simplex methods. Others are the stepping stone variant of transportation model and special purpose algorithm. In most cases, an assignment problem is completely defined by the following table:

	Machine or Work Centre					
Job	1	2	3	.	.	n
1	C_{11}	C_{12}	C_{13}	.	.	C_{1n}
2	C_{21}	C_{22}	C_{23}	.	.	
3	C_{31}	C_{32}	C_{33}	.	.	
.	.	.	.	.	.	
n	Cn_1	Cn_2	Cn_3	.	.	

Where C_{ij} are the costs of assigning job "I" to machine "j" or the time taken by machine "j" to work on job" I".

Sample of a practical application problem:

The Human Resource Manager of XYZ Bank Nigeria Plc has hired the services of three new Business Managers to man their three different Regional offices located in Lagos, Abuja and Port Harcourt. The three new Business Managers are equally qualified to serve as Regional Managers. The decision to assign them to any of the Regional offices will be based on the cost of relocating their individual families. Hence the cost data in Naira are given below:

Manager	**Lagos**	**Abuja**	**Port Harcourt**
Michael	80,000	110,000	120,000
Anthony	50,000	160,000	130,000
Collins	50,000	100,000	230,000

You are required to determine the posting options that will minimize costs to the bank

3.6 .Urban planning

Urban planning contends with three major areas (1) Building of new housing estates to ease off accommodation problems (2) Upgrading of city plans, deteriorating housing and recreational areas and (3) the planning of public facilities such as public parks, schools airports etc. The constraints associated with these projects are both economic (Land, construction, financing) and social (schools, parks, income level). The objectives in urban planning could be profit oriented .It could equally have some social, political, economic and cultural considerations.

Sample of a practical application problem:

Owerri Municipal Council is in the process of approving the construction of a new multipurpose hall. Two sites have been proposed within the city centre and both require the acquisition of adjoining properties. The following table provides data about proposed (contiguous) properties in both sites together with the acquisition costs**.**

	Site 1		**Site 2**	
Property	**Area(1000ft^2)**	**Cost(N'M)**	**Area(1000ft^2)**	**Cost(N'M)**
1	20	1000	80	2800
2	50	2100	60	1900
3	50	2350	50	2800
4	30	1850	70	2500
5	60	2950		

Two key properties are precedent to the selection of a particular site**.** At least 75% of property 4 must be acquired if site I is selected and at least 50% of property 3 must be acquired if site 2 is selected. Although site 1 property is more expensive (on a per ft^2 basis), the construction cost is less than at site 2, because the infrastructures (Access road, Street lights and drainage facility) at site 1 is in a much better shape. Construction cost is N25million at site 1 and N27 Million at site 2.One is required to ascertain which of the two sites should be selected and what properties should be acquired.

3.7. Diet Problem

One of the earliest problems solved by linear programming technique was that of a diet problem. It got its name from the fact that it was first applied to determining an economical diet. The diet model involves determining the ingredients and all that need be included in a meal so as to minimize the cost of the meal while at the same time, satisfying certain nutritional requirements. In its industrial form, a diet model consists of determining the most economical mixture of raw materials that will result in a product with a desired chemical formula. Next, we will take a look at an example that will vividly explain a practical diet problem

Sample of a practical application problem:

Mrs. Dike's six year old son has just been discharged from the hospital. To enable him convalescence faster, he was placed on a special diet. Mrs. Dike is trying to select the cheapest combination of two foods, "A" and "B" that could meet the daily vitamin needs of her son. This need calls for at least 40 units of vitamin type 1, 50 units of vitamin type2, and 49 units of vitamin type 3.Each gram of food "A" provides 4 units of type 1 vitamin, 10 units of type 2 vitamin and 7 units of type 3 vitamins. Also each gram of food "B" provides 10 units of type 1 vitamin, 5 units of type2 vitamin and 7 units of type 3 vitamins. Food "A" costs N5 per gram and food "B", N8per gram. One is required to determine the least expensive way to satisfy the vitamin needs of Mrs. Dike's son.

3.8 Currency Arbitrage

In today's global economy, Bureau de change, commercial banks and multinational companies must deal with currencies of countries in which they operate in and that of other nations. Currency arbitrage, or simultaneous purchase and sale of currencies in different markets offer opportunities for advantageous movement of money from one currency to another.

Sample of a practical application problem:

Mallam Sidi's Bureau De Change was presented with an initial sum of N250 million by one of it's customers to be converted optimally to a combination of Pounds, Dollars and Euros. The final mix should not include less than 240,000 pounds, 480,000 Dollars and 450,000 Euros. As the Head Teller, you are required to determine an optimal buying mix of the three currencies using the current buying rates.

3.9. Blending Model:

Linear programming has found wide application in an area referred to as "blending model" Blending models are formulated to determine an optimal combination of component ingredients to be blended into a final product. Among the areas that blending model could be used includes Petroleum products, Feed mill (Mixing of feed for agricultural products) and Fertilizers. Others are in the area of blending of spirits, Tea and Coffee.

The objective of a blending model is usually to minimize the cost of the blend. A typical constraint in a blending model includes lot size requirements for each blend, technological or recipe requirements and limited availability of component ingredients. A typical blending problem that will require the use of a blending model is cited below:

Sample of a practical application problem:

Eleme Petroleum Oil Refinery Port Harcourt has a capacity of 1,500,000 barrels of crude oil per day. The final products from the refinery include three types of unleaded gasoline with different Octane numbers (ON): Regular with ON =87, Premium with ON = 89 and super with ON = 92.The refining process encompasses three stages (1) A distillation tower that produces feedstock (ON=82) at the rate of 0.2barrel per barrel of crude oil (2) A cracker unit that produces gasoline stock(ON=98) by using a portion of the feedstock produced from the distillation tower at the rate of 0.5 barrel per barrel of feedstock and (3) a blender unit that blends the gasoline stock from the cracker unit and the feedstock from the distillation tower. The company estimates that the net profit per barrel of the three types of gasoline to be N6.70, N7.20 and N8.10 respectively. The input capacity of the cracker unit is 200,000 barrels of feedstock a day .The demand limits for regular, premium and super gasoline are 50,000, 30,000 and 40,000 barrels per day. We are required to develop a model for determining the optimum production schedule for the refinery.

3.10. Environmental Protection: Application of linear programming technique is not limited to the business and industrial settings only. It equally has need in the heath sector as well as in environmental protection

Sample of a practical application problem:

Over time Exxon Mobil Unlimited company has been surcharged by the Nigerian National Petroleum Corporation (Department of Petroleum Resources) for gas flaring. The company is contemplating on the next line of action to take with regards to this breach of regulatory directives. A cost benefit analyses of their actions and in actions are briefly summarized below:

What does the company stand to lose with the continued flaring of gas? First, they will be seen as not being environmentally friendly. Second, this is a continued breach of regulatory directives .Thirdly; the regulatory authorities will continue to surcharge them for gas flaring and environmental degradation.

What does the company stand to gain by not flaring of gases? : First, they will be seen as helping to save the eco system from environmental pollution and degradation. Second, the regulatory bodies will stop billing them for gas flaring. Thirdly, the gas reinjection scheme will assist them recover some oil that could have been lost due to depleted or reduced reservoir pressure.

It is pertinent to mention here, that a gas reinjection scheme has some cost involvements i.e. costs associated with the building and installation of compressors and gas lift equipments. Other necessary information is as contained and summarized in the table below:

Costs Associated with the flaring of Gas	Quantum of gas being flared on a daily basis	Costs associated with the building of a Gas Reinjection Plant	Barrels of Oil recovered per day due to gas reinjection exercise	Price of oil
0.01 k per cubic meter of Gas	1million cubic meters of gas	N10 Million	10 Barrels	USD120

As the company's safety and environmental manager, you are required to advise Exxon Mobil Unlimited Company accordingly on what line of action to take.

The above examples are by no means exhaustive of the practical applications of linear programming techniques.

4.0 Limitations of Linear programming Techniques

In spite of the overwhelming number of real life applications, linear programming techniques do have some limitations. These are highlighted below:

Linear programming is based on a set of rather restrictive assumptions. Although these assumptions will be valid for many practical problems, there are numerous other situations in which one or more of the suppositions may not be factual. Some of the limitations associated with linear programming technique include the following (i) indivisibility (ii) multiple objectives (iii) Uncertainty (iv) Non Linear relationships and (v) sequential problems. Some of these limitations are briefly discussed below

Indivisibility: The linear programming model (assumes that each decision variable is divisible into fractional value. Such assumption generally will be valid when these variables a physical measure (Weight, capacity, length, area or volume), time, monetary values or percentages. Linear programming may not provide such whole –unit or integer solutions. For example, a solution may result in 3.56 chairs which is not practical since discrete solution is desirable. The proper tool to use for such an analysis is the integer programming model.

Multiple Objectives: The linear programming model also assumes that there is only a single objective in the problem. Furthermore, it is presumed that the objective can be expressed in terms of a numerical criterion such as quantity, time or revenue. However, an objective could be expressed in terms of different criterion and may not be possible to quantify. For instance, it is hard to measure social benefits, customer satisfaction, quality and similar criteria in numerical terms. These objectives/ goals may be equal in dimension, complimentary or even conflicting. The appropriate tool to use in such a situation is the goal programming model

Uncertainty: The assumption of certainty has its limitation in that some situations occur where factors like costs and other constrain requirements may not be known before hand. In such a situation, probabilistic linear programming technique is more applicable.

The Non Linear Relationships: As a result of expansion and contraction, decision variable constraint utilization rate may fluctuate, rather than remain constant as the level of activity changes. In addition, each decision does not always make a constant and independent contribution to the objective. Furthermore, decision variables frequently have interrelated, rather than independent effects on the criterion value. In such situations, the appropriate tool to use is the non linear programming techniques.

Sequential Problems: The linear programming model assumes that a problem involves single, one-time decision making. In real life situation, it is not always so as most problems involve a series of interrelated decisions.

5.0. Conclusion and Recommendations

From the foregoing analysis, it could be seen that our subject matter has an overwhelming number of real life applications. It has become necessary to reconcile the seeming apathy towards linear programming techniques and the benefits derived therein. The general public does not seem to understand nor appreciate the contributions of operations research and its component studies on programming techniques to work place decision making process. As a way out, there is need to evolve a national policy that will make the teaching and learning of operations research and its applications a national priority. This should in the long run, inculcate a culture of bias towards quantitative reasoning, thinking and its associated applications. The phobia associated with the study of mathematics and other related subjects is still far from being over. It is high time our tertiary institutions shy away from soft sell degree programs for more practically oriented courses. Presently, not many of our Nigerian universities run a degree program in operations research and decision sciences.

The public as well as the private sector should invest in the study of operations research with its component studies in programming techniques. This will evolve a culture of record keeping and statistics, thereby reducing the need to resort always to qualitative decision making in preference for a more informed and quantitatively backed decision making process. Readers are enjoined to get beyond the realms of merely appreciating the subject matter to the level of studying and applying this all important and time tasted problem solving approach that could enhance decision making of managers, especially where there are restrictions or constraints in the decision making process.

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