Determination of Optimal Size of Casual Workers in a Production Company Using Stochastic Model

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ABSTRACT

Many big production companies engage casual workers to work with their regular employees in order to meet their daily labour needs. The researchers are quite aware that the Nigeria Labour Congress frowns at casualisation of workers which is seen as under-employment and cheating on the part of the people concerned. Since unemployment and underemployment are parts of the problems confronting Nigeria, there is the need for the issue of workers casualisation to be addressed before the nation can be transformed. This paper therefore attempts to determine the daily optimal number of casual workers that could be engaged by a production company with specific reference to Dangote Cement PLC, Ibese. This is necessary in achieving the national transformation agenda through scientific approach since policy analysis has direct impact on production. This study was done by the use of a stochastic model. In view of this, data were generated, collected and collated from Dangote Cement PLC, Ibese and used to test the validity and applicability of the model.

Keywords: Casual workers, Random numbers, Stochastic, Optimal

1.0: INTRODUCTION

Many big production companies engage casual workers to work with their regular employees in order to meet their daily labour needs whenever regular workers are absent or are in short supply. This phenomenon is fast becoming an important area of policy analysis mainly because of its direct impact on production. It is, therefore, necessary for companies to have a good knowledge of the optimal daily need of casual workers with the aim of ensuring their availability at the time of need. This will maximize man-hour utilization, avoid waste of human resources and funds, and finally assures that the daily required number of casual workers is readily available, thereby ensuring continuity in production without disruptions. These were the motivating factors for the study.

This paper attempts to formulate a stochastic model to determine the optimal number of casual workers needed by a company. The number of casual workers needed in any day is not fixed. It is a random number: Thus, the stochastic nature of the model equation. The literature mentions a large number of extensions to the stochastic problem. Early studies in this area include Erwin Kalvelagen (2003), Simon, F. (1998), Law and Kelton (1991), where computer simulations with Monte Carlo method were used.

In this paper, statistical data of previous records of the demand of casual workers and the amount payable per day were collected from Dangote Cement PLC, Ibese. This makes the study more practical and realistic than other works. Above all, the analytical solution to the model equation was obtained, thereby revealing more accurately, the variable factors required in the study.

This paper is organized as follows: section one deals with the introduction while section two is concerned with the formulation and solution of the model equation. In section three, the implementation of the equation and its interpretation is given. Finally, conclusion and recommendations were presented in section four.

2.0: FORMULATION AND SOLUTION OF THE MODEL EQUATION

Let X be the random demand for casual workers and S the possible threshold level determined by the optimal number of casual workers required by the company. In order to analyze this problem we will show that for a reasonable choice of the cost function, the optimization function, $\Phi(x)$, in the form of the cost of hiring x casual workers is given as:

$$\Phi(X) = \begin{cases} c_1 s \quad \text{for } x \le \text{s} \\ c_1 s + c_2(x - s) \text{ for } x > \text{s} \end{cases} \tag{2.1}$$

where $x \le X$ and $\Phi(X)$ is a function that describe the system and in particular the mechanism by which the system is updated, c_1 is the amount (in naira) paid to a casual worker per day and c_2 the amount paid to a regular worker as over-time for doing the work of a casual worker. The amounts c_1 and c_2 measure the level of need or importance of casual workers by the company. This is in line with the product valuation outlined by Anthony, M and Biggs, N (2000).

If the demand of casual worker of day k is a random variable x_k with probability density function of demand $f(\text{x})$ then

$$\text{P}(X \le x_k) = \int_0^{x_k} f(x)dx \tag{2.2}$$

where f is a continuous nonnegative function. However, because of the presence of x, a random parameter, $\Phi(X)$ is generally a random variable and cannot be meaningfully optimized. According to Gerchak, Y and Massman, D (1992), we therefore formulate the problem as an optimization of the expected cost.

Let the expected cost per day be G(s). Then

$$\text{G(s)} = \text{E}[\Phi(x)] \tag{2.3}$$

where $$E[X] = \int_0^{\infty} x\, f(x)dx \tag{2.4}$$

Applying (2.3) on (2.1) gives

$$G(s) = \int_0^s c_1 s f(x)dx + \int_s^{\infty} [c_1 + c_2(x - s)] f(x)dx$$

$$G(s) = \int_0^{\infty} c_1 s f(x) dx + \int_s^{\infty} [c_2 (x - s)] f(x) dx \tag{2.5}$$

Since c_1, c_2 and s are constants and $f(x)$ is a nonnegative density function we have, for $X \in [0, \infty]$,

$$G(s) = c_1 s + c_2 \int_s^{\infty} [(x - s)] f(x) dx \tag{2.6}$$

Next, we wish to find the cumulative distribution function $F(s)$ of X in order to enable us use the empirical data. From the fundamental theorem of calculus, we differentiate (2.6) to get $F(s)$

$$\frac{d}{ds} G(s) = c_1 - c_2 \int_s^{\infty} f(x) dx$$

$$\frac{d}{ds} G(s) = c_1 - c_2 [1 - F(s)]$$

At the turning point we have

$$c_1 - c_2 \int_s^{\infty} f(x) dx = 0 \tag{2.7}$$

By definition, we have: $\int_s^{\infty} f(x) dx = 1 - F(s)$ and $f(x) = \frac{d}{dx} F(x)$. Since $G(s)$ is continuous, and twice differentiable in s, we have $\frac{d^2}{ds^2} G(s) = f(s) > 0$. The turning point yields a minimum value since $\frac{d^2}{ds^2} G(s) = f(s) > 0$. This shows that (2.1) is properly formulated and (2.6) is well behaved. Since $f(x)$ is a probability density function we have:

$$\int_{-\infty}^{\infty} f(x) dx = \int_{\Omega} f(x) dx = 1 \tag{2.8}$$

where Ω is the probability space of the random variable X.

$$\text{Let } F(s) = \mathrm{P}(X \le s) = \int_{-\infty}^{s} f(x) dx \tag{2.9}$$

By (2.8) and (2.9) we have

$$\mathrm{P}(X > s) = \int_s^{\infty} f(x) dx = 1 - F(s) \tag{2.10}$$

Putting (2.10) into (2.7)

$$F(s) = \frac{c_2 - c_1}{c_2} = 1 - \frac{c_1}{c_2} \tag{2.11}$$

The function $F(s)$ is the cumulative probability that the demand for casual workers is less than the optimal number, s. Let $s^{\bullet}$ be the optimal solution of problem (2.11)

Thus $s^{\bullet} = F^{-1}\left(\frac{c_2 - c_1}{c_2}\right)$, Zipkin (2000), where F^{-1} denotes the inverse cumulative distribution function of the demand. To use (2.11), certain assumptions must be made. These are stated below:

Assumptions

i. If a casual worker is not available, paying a regular worker for overtime to do the job would make up the demand.

ii. The amount c_2 paid as overtime to a regular worker for doing the job of a casual worker is always greater than the amount c_1 that a casual worker is paid per day.

iii. There is no carry over of demand for casual worker from one day to another.

iv. The category of regular workers considered here are those entitled to daily overtime payment.

3.0: IMPLEMENTATION AND INTERPRETATION OF THE MODEL

Implementation: The data in Table 3.1 below were generated from Dangote Cement PLC, Ibese. We shall use the data to test the validity and applicability of the model.

Table 3.1: Distribution of demand x of casual workers for one year (365 days) with other relevant computations

casual workers demanded per – day	Frequency F(days)	Cumulative frequency	Relative Frequency	cumulative relative frequency	percentage of relative frequency	percentage of cumulative relative frequency	mid - point (X) of demand	Fx
0 – 4	38	38	0.10410959	0.09620253	10.4109589	9.620253	2	76
5 – 9	30	68	0.08219178	0.18630137	8.219178082	18.63013699	7	210
10 – 14	35	103	0.09589041	0.282191781	9.589041096	28.21917808	12	420
15 – 19	39	142	0.10684932	0.389041096	10.68493151	38.90410959	17	663
20 – 24	38	180	0.10410959	0.493150685	10.4109589	49.31506849	22	836
25 – 29	35	215	0.09589041	0.589041096	9.589041096	58.90410959	27	945
30 – 34	35	250	0.09589041	0.684931507	9.589041096	68.49315068	32	1120
35 – 39	40	290	0.10958904	0.794520548	10.95890411	79.45205479	37	1480
40 – 44	36	326	0.09863014	0.893150685	9.863013699	89.31506849	42	1512
45 – 49	39	365	0.10684932	1	10.68493151	100	47	1833
TOTAL	**365**		**1**		**100**			**9095**

To implement (2.11) we use the following average figures obtained from Dangote Cement PLC, Ibese. That is,

c_1 – N1100.00 per person per day

c_2 – N2500.00 per person per day

Therefore, by (2.11) we have:

$$F(s) = 1 - \frac{c_1}{c_2} = 1 - \frac{1100}{2500} = 0.56 = 56\% \qquad (3.1)$$

This shows that, for the 365 days, the cumulative probability of the demand x for casual workers being less than or equal to the optimal number, S, is 0.56. This implies that S is 56 percentile of the distribution; meaning that S lies in the class interval 5 – 9. This class interval shall henceforth be called optimal class.

To determine S, we shall use the following statistical relationship, a modification of Frank, H and Althoen, S, C. (1995).

$$S = u_b + \left(\frac{F(s) - c_l}{f_l} \right) \times h \qquad (3.2)$$

Where

u_b - Upper class limit of the class interval preceding the optimal class

c_l - Cumulative relative frequency of the class preceding the class containing the optimal class

f_l - Relative frequency of the optimal class and

h - Width of the Class interval

Therefore, $S = u_b + \left(\frac{F(s) - c_l}{f_l} \right) \times h$

$$S = u_b + \left(\frac{F(s) - c_l}{f_l} \right) \times h$$

$$S = 4 + \left(\frac{0.56 - 0.0962}{0.0821} \right) \times 5 = 32.2 \approx 33 \text{ persons} \qquad (3.3)$$

This implies that the optimal number of daily demand for casual workers is about 33 persons.

Next we shall find the average $\bar{x}$ of the daily demand.

Therefore, by table 3.1, we have

$$E(x) = \frac{\sum_{i=1}^{10} f_i x_i}{\sum_{i=1}^{10} f_i} = \frac{9095}{365} = 24.9 \approx 25 \text{ Persons} \qquad (3.4)$$

This means that the daily average demand of casual workers is about 25 persons. That is 25 persons is the average daily demand of casual workers over a number of days, say 365 days and not 31 persons which is a random quantity as mentioned. This institution is justified by the strong law of large numbers.

The strong law of large numbers guarantees that as n gets larger, the average is closer to the expected value. However, expected value is relevant only when the demand for casual workers is observed repeatedly over many periods. In this case, the optimal demand $s^{\bullet}$ should be used for every period, although actual demand for casual workers in each day fluctuates.

Interpretations

We shall us the following result to determine the fraction of the regular workers $W_r(s)$ that are used daily.

Proposition 3.1: Let $\phi(x)$ be as earlier defined. Let the fraction of the casual workers be given

$$W_c(s) = \frac{\int_s^{\infty} (x-s)\phi(x)dx}{\int_0^{\infty} x\phi(x)dx}, \text{ for every } x, s \in [0, \infty) \qquad (3.5)$$

Then the fraction of the regular workers is

$$W_r(s) = \frac{1}{E[x]}\left[\int_0^s x\phi(x)dx + s\frac{c_1}{c_2}\right] \qquad (3.6)$$

Now, to find $W_r(s)$ by using (3.6) we let

$$E(s) = \int_0^{\infty} x\phi(x)dx \qquad (3.8)$$

From table (3.1) we have

$E(x) = \dfrac{\sum_{i=1}^{i_s} f_i x_i}{\sum_{i=1}^{i_s} f_i}$. The symbol i_s is the i^{th} class interval where the s lies. Therefore,

$E(x) = \dfrac{210}{68} = 3.0882$. Next, we have $s\dfrac{c_1}{c_2} = 31 \times 0.44 = 13.64$

Therefore, $W_r(s) = \frac{1}{E[x]}\left[3.0882 + s\frac{c_1}{c_2}\right] = \frac{17.6282}{25} = 0.7051 = 70.51\%$

This implies that 70.51% of the casual workers used daily are regular workers. This shows that, out of the optimal number of 33 casual workers required daily, the regular workers comprise of between 23 and 24. Hence, the number of non-regular workers that is needed to be kept in the pool is between 9 and 10. It is this number that a retainance amount be worked out for, by the company's management.

4.0: CONCLUSION

i. It was observed that when another set of data (different from that in table 3.1) was obtained with different pattern, the optimal size was not significantly different (when the same rate of payments was used).

ii. By (2.11), if c_1 and c_2 are close to each other then the optimal size S, is small while it is large when they are far apart.

iii. From the average number of causal workers required and the optimal number, S obtained in the model, it is useful, in the sense that the use of the optimal number is more economical than that of the averages, which was the common practice. It is, therefore, hoped that the paper has contributed to the scientific advancement of managerial skills.

iv. The optimal size was found to be the same when the cumulative probability curve (ogive) was drawn. This implies that the graphical method could be used to obtain S.

5.0: RECCOMENDATION(S)

With the knowledge of the optimal number and particularly the non regular workers, a retainance amount could be determined by the company's management and implemented. This will go along way in ensuring the availability of casual workers. From the model it is advised that the difference in the amount paid to non-regular workers and regular ones be small. This will forestall a situation whereby regular workers will develop more interest in taking part-time duties. It eventually affects job performace adversely. The size of the optimal number in this model is a function of the amount paid to the two categories of casual workers. An improvement of the model could be investigated further, in other to build into the model other parameters that could measure "some other needs" of the company.

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