Simultaneous Triple Series Equations

Associated With Laguerre Polynomials With Matrix Argument

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Abstract

Integral and Series equations are very useful in the theory of elasticity, elastostatics, diffraction theory and acoustics. Particularly these equations are very much useful in finding the solution of crack problems of fracture mechanics. In this paper solution of simultaneous triple series equations associated with Laguerre polynomials with matrix argument has been obtained, which arises in the Crack problems of Fracture Mechanics.

Keywords: Integral equations, Series equations, Laguerre polynomials, Matrix argument.

1. Introduction

In the present paper, we have considered the following simultaneous triple series equations of the form

\[ \sum_{n=0}^{\infty} \sum_{j=1}^{s} a_{nj} c_{nj} \Gamma_{m}(\alpha+\beta+n) L_{nj}(\alpha: x) = f_{j}(x), \quad 0 \leq x \leq D, \quad (1.1) \]

\[ \sum_{n=0}^{\infty} \sum_{j=1}^{s} b_{nj} c_{nj} \Gamma_{m}(\alpha+ni + \frac{m+1}{2}) \Gamma_{m}(\alpha+\beta+ni) \]

\[ L_{nj}(\alpha+\beta - \frac{m+1}{2}, y) = g_{j}(x), \quad D \leq x \leq E, \quad (1.2) \]

\[ \sum_{n=0}^{\infty} \sum_{j=1}^{s} c_{nj} \Gamma_{m}(\alpha+\beta - \frac{m+1}{2}) = L_{nj}(\alpha, x) = h_{j}(x), \quad E \leq x \leq \infty, \quad (1.3) \]

for \( \alpha + \beta > \frac{m+1}{2} - 1, 0 \leq \beta \leq 1 \)

where,

\[ L_{n}(\alpha, x) = \frac{\Pi_{m}(\alpha+n)}{\Pi_{m}(\alpha)} 1F1(-n, \alpha+\frac{m+1}{2}, x) \]
is Laguerre polynomial of matrix argument, 

for \( R(\alpha) > -1 \),

\[ R(n+\alpha) > 1, \quad \text{and} \quad \Pi_m(a) = \Gamma_m(a + \frac{m+1}{2}) \]

\[ \Gamma_m(a) = \pi^{m(m-1)/4} \prod_{i=1}^{m}(a - \frac{i-1}{2}) \]

\( n = 0, 1, 2, \ldots \ldots \quad J = 1, 2, 3, \ldots \ldots s \),

\( f(x), g(x), h(x) \) are known functions of non-singular matrix \( x \) of order \( m \); \( a_{ij}, b_{ij} \) and \( c_{ij} \) are known constant. By using multiplying factor technique [Srivastava], the unknown function \( C_{nj} \) is determined.

2. Some Useful Results

(i) The following integrals are required from Erdelyi et al. with matrix argument

\[ \int_{0}^{\infty} y^{\alpha} |y-x|^{\beta-(m+1)/2} L_n(\alpha, x) dx \]

\[ = \frac{\Gamma_m(\beta) \Gamma_m(\alpha+n+m+1/2)}{\Gamma_m(\alpha+\beta+n+m+1/2)} |y|^{\alpha+\beta} L_n(\alpha+\beta; y) \quad (2.1) \]

for \( \alpha > -1, \beta > \frac{m+1}{2} - 1 \) and

\[ \int_{y}^{\infty} |x-y|^{-\beta} etr(-x) L_n(\alpha, x) dx \]

\[ = \Gamma_m\left(\frac{m+1}{2} - \beta\right) etr(-y) L_n(\alpha + \beta - \frac{m+1}{2}; y) \quad (2.2) \]

for \( \beta < \frac{m+1}{2}, \alpha + \beta > \frac{m+1}{2} - 1 \).

(ii) The orthogonality relation for Laguerre polynomial with matrix argument
\[
\int_{x>0} \left| x \right|^\alpha \text{etr}(-x) L_p(\alpha;x) L_q(\alpha;x)
\]

\[
\Gamma_m(\alpha + \beta + \frac{m+1}{2}) = \frac{\Gamma_m(p + \frac{m+1}{2})}{\delta_{pq}} \tag{2.3}
\]

for \( \alpha > -1 \) and \( \delta_{pq} \) being the kronecker delta.

(iii) The differential formula with matrix argument due to Erdelyi et al

\[
Dx[[x]^\alpha L_n(\alpha;x)] = |x|^{\frac{m+1}{2} \alpha} L_n(\alpha - \frac{m+1}{2};x) \tag{2.4}
\]

3. Solution

Multiply eq. (1.1) by \( |x|^\alpha \left| y - x \right|^{\beta-(m+1)/2} \) and eq. (1.2) by \( |x - y|^\beta \text{etr}(-x) \) and then integrating w.r.t. \( x \) over the range \((0, y)\) and \((y, \infty)\) respectively, on using the result (2.1) and (2.2), we obtain

\[
\sum_{n=0}^\infty \sum_{j=1}^k a_{nj} c_{nj} \frac{\Gamma_m(\alpha + ni + \frac{m+1}{2}) \Gamma_m(\alpha + \beta + ni)}{\Gamma_m(\alpha + \beta + ni)} L_n(\alpha + \beta + y) =
\]

\[
|y|^{\alpha - \beta} \frac{1}{\Gamma_m(\beta)} \int_0^y |x|^\alpha \left| y - x \right|^{\beta-(m+1)/2} f(j)(x)dx , \tag{3.1}
\]

for \( \beta > \frac{m+1}{2} \), \( \alpha > -1 \), \( 0 < y < D \)

and

\[
\sum_{n=0}^\infty \sum_{j=1}^k b_{nj} c_{nj} \frac{\Gamma_m(\alpha + ni + \frac{m+1}{2}) \Gamma_m(\alpha + \beta + ni)}{\Gamma_m(\alpha + \beta + ni)} L_n(\alpha + \beta - \frac{m+1}{2};y) =
\]

\[
= \frac{\text{etr}(-y)}{\Gamma_m(\frac{m+1}{2} - \beta)} \int_0^y \text{etr}(-x) \left| x - y \right|^{\beta} g(j)(x)dx , \tag{3.2}
\]

for \( \beta < \frac{m+1}{2} \), \( \alpha + \beta > \frac{m+1}{2} - 1 \), \( D < y < \infty \).
If we multiply eq. (3.1) by \( |y|^{\alpha+\beta} \), differentiating w.r.t. \( Y \) and using the formula (2.4), we find

\[
\sum_{n=0}^{\infty} \sum_{i=1}^{s} b_{ij} c_{ij} \frac{\Gamma_m(\alpha + ni + \frac{m+1}{2}) \Gamma_m(\alpha + \beta + ni)}{\Pi_m(\alpha + \beta + ni)} . L_m(\alpha + \beta - \frac{m+1}{2}; y) = \sum_{j=1}^{s} e_{ij} \frac{y^{\frac{m+1}{2} - \alpha - \beta}}{\Gamma_m(\beta)} .
\]

for \( 0 < y < D, \beta > \frac{m+1}{2} - 1, \alpha > -1, \) and \( e_{ij} \) are the element of the matrix \([b_{ij}][a_{ij}]^{-1}\) and \( i = 1, 2, ..., s.\)

The left-hand side of eqs. (3.2), (3.3) and (1.2) are identical and hence on using the orthogonality relation (2.3), we find the solution of eq. (1.1), (1.2) and (1.3) for

\[
\alpha + \beta > \frac{m+1}{2} - 1, 0 < \beta < 1
\]

\[
C_{ij} = \sum_{j=1}^{s} d_{ij} \frac{\Gamma_m(\alpha + ni + \frac{m+1}{2}) \Gamma_m(\alpha + \beta + ni)}{\Pi_m(\alpha + \beta + ni)[\Gamma_m(\alpha + \beta + ni)]^2} B_{ij}(\alpha + \beta; D) \quad (3.4)
\]

Where, \( n = 0, 1, 2, ..., j = 1, 2, 3, ..., s \) and \( d_{ij} \) are the element of the matrix \([b_{ij}]^{-1}\) and

\[
B_{ij}(\alpha, \beta; D) = \sum_{j=1}^{s} e_{ij} \frac{1}{\Gamma_m(\beta)} \int_{0}^{D} \text{etr}(-y) L_m(\alpha + \beta - \frac{m+1}{2}; y) .
\]

\[
F_i(y)dy + \int_{D}^{\infty} \text{etr}(-y) |y|^{\alpha+\beta-(m+1)/2} L_n(\alpha + \beta - \frac{m+1}{2}, y) G_i(y)dy
\]

\[
+ \frac{1}{\Gamma_m(\alpha + \beta - \frac{m+1}{2})} \int_{0}^{\infty} |y|^{\alpha+\beta-(m+1)/2} L_n(\alpha + \beta - \frac{m+1}{2}, y) H(y)dy \quad (3.5)
\]

\[
F_i(y) = D_y \int_{0}^{y} |x|^{\alpha+\beta/2} |y-x|^{\beta-(m+1)/2} f_i(x)dx \quad (3.6)
\]

\[
G_i(y) = g_i(y)
\]

\[
H_i(y) = \int_{y}^{\infty} \left| x-y \right|^{\beta} \text{etr}(-x) h_i(x)dx
\]
References


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