Simultaneous Triple Series Equations Associated With Laguerre Polynomials With Matrix Argument

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Abstract

Integral and Series equations are very useful in the theory of elasticity, elastostatics , diffraction theory and acoustics. Particularly these equations are very much useful in finding the solution of crack problems of fracture mechanics. In this paper solution of simultaneous triple series equations associated with Laguerre polynomials with matrix argument has been obtained , which arises in the Crack problems of Fracture Machanics.

Keywords: Integral equations, Series equations , Laguerre polynomials, Matrix argument.

1. Introduction

In the present paper, we have considered the following simultaneous triple series equations of the form

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} a_{ij}\ c_{nj}\quad \Gamma_m\ (\alpha+\beta+ni)\ L_{ni}\ (\alpha:\mathrm{x})= f_i\,(\mathrm{x}),\ 0\leq x\leq D, \tag{1.1}$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij}\quad c_{nj}\quad .\ \frac{\Gamma_m(\alpha+ni+\frac{m+1}{2})\Gamma_m(\alpha+\beta+ni)}{II_m\,(\alpha+\beta+ni)}$$

$$L_{ni}\ (\alpha+\beta-\frac{m+1}{2},y)=g_i\,(x),D\leq\times\leq E, \tag{1.2}$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} c_{nj}\quad \Gamma_m\ (\alpha+\beta-\frac{m+1}{2})=L_{ni}\,(\alpha,x)=h_i\ (x),\ E\leq\times\leq\infty\,, \tag{1.3}$$

for $\quad \alpha+\beta>\frac{m+1}{2}\ -1,0\leq\beta\leq 1$

where,

$$L_n(\alpha,x)\ =\frac{II_m(\alpha+n)}{\Pi_m(\alpha)}\ 1F1\ (-n,\ \ \alpha+\frac{m+1}{2}),x)$$

is Laguerre polynomial of matrix argument ,

for $R\ (\alpha) > -1,$

$$R(n+\alpha) > 1, \quad and \quad II_m\ (a) = \Gamma_m\ (a+\frac{m+1}{2})$$

$$\Gamma_m(a) = \pi^{m(m-1)/4} \prod_{i=1}^{m} (a - \frac{i-1}{2})$$

n =0,1,2,……………………., J = 1,2,3,……………s ,

f(x), g(x) h(x) are known functions of non – singular matrix x of order m ; a_{ij} , b_{ij} and c_{ij} are known constant .By using multiplying factor technique [Srivastava], the unknown function C_{nj} is determined .

2 . Some Useful Results

(i) The following integrals are required from Erdelyi et al with matrix argument

$$\int_0^y |y|^{\alpha} |y-x|^{B-(m+1)/2} L_n(\alpha, x) dx$$

$$= \frac{\Gamma_m(\beta)\Gamma_m(\alpha+n+\frac{m+1}{2})}{\Gamma_m(\alpha+\beta+n+\frac{m+1}{2})} |y|^{\alpha+\beta} L_n(\alpha+\beta; y) \tag{2.1}$$

for $\alpha > -1,\ \beta > \frac{m+1}{2} - 1$ and

$$\int_y^{\infty} |x-y|^{-\beta}\ etr(-x)\ L_n(\alpha, x) dx$$

$$= \Gamma_m(\frac{m+1}{2} - \beta)\, etr(-y)\, L_n(\alpha+\beta - \frac{m+1}{2}; y) \tag{2.2}$$

for $\beta < \frac{m+1}{2}, \alpha+\beta > \frac{m+1}{2} - 1$.

(ii) The orthogonality relation for Laguerre polynomial with matrix argument

$$\int_{x>0} |x|^{\alpha}\, etr(-x)\, L_p(\alpha;x)\, L_q(\alpha;x)$$

$$=\frac{\Gamma_m(\alpha+\beta+\frac{m+1}{2})}{\Gamma_m(\text{p}+\frac{m+1}{2})}\delta_{pq} \quad (2.3)$$

for $\alpha > -1$ and δ_{pq} being the kronecker delta .

(iii) The differential formula with matrix argument due to Erdelyi et al

$$Dx[|x|^{\alpha} L_n(\alpha;x)]=|x|^{\alpha-\frac{m+1}{2}} L_n(\alpha-\frac{m+1}{2};x) \quad (2.4)$$

3. Solution

Multiply eq. (1.1) by $|x|^{\alpha}\,|y-x|^{\beta-(m+1)/2}$ and eq. (1.2) by $|x-y|^{-\beta}\, etr(-x)$ and then integrating w.r.t. x over the range $(0,y)$ and (y,∞) respectively, on using the result (2.1) and (2.2), we obtain

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} a_{ij}\; c_{nj}\frac{\Gamma_m(\alpha+ni+\frac{m+1}{2})\Gamma_m(\alpha+\beta+ni)}{II_m(\alpha+\beta+ni)}L_{ni}(\alpha+\beta;y)=$$

$$\frac{|y|^{-\alpha-\beta}}{\Gamma_m(\beta)}\int_0^y |x|^{\alpha}\,|y-x|^{\beta-(m+1)/2}\; f_i(x)dx\ , \quad (3.1)$$

for $\beta > \frac{m+1}{2},\ \alpha > -1,\ 0<y<D$

and

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij}\; c_{nj}\frac{\Gamma_m(\alpha+ni+\frac{m+1}{2})\;\Gamma_m(\alpha+\beta+ni)}{II_m(\alpha+\beta+ni)}\; L_{ni}(\alpha+\beta-\frac{m+1}{2};y)$$

$$=\frac{etr(y)}{\Gamma_m(\frac{m+1}{2}-\beta)}\int_y^{\infty} etr\;(-x)\;|x-y|^{-\beta}\; g_i(x)dx\ , \quad (3.2)$$

$$for\ \ \beta<\frac{m+1}{2},\ \ \alpha+\beta>\frac{m+1}{2}-1, D<y<\infty\,.$$

If we new multiply eq.(3.1) by by $|y|^{\alpha+\beta}$, differentiating w.r.t. Y and using the formula (2.4), we find

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij}\ c_{nj} \frac{\Gamma_m(\alpha+ni+\frac{m+1}{2})\Gamma_m(\alpha+\beta+ni)}{II_m(\alpha+\beta+ni)} \ .L_{ni}(\alpha+\beta-\frac{m+1}{2};y)=\sum_{j=1}^{s} e_{ij}\frac{|y|^{(\frac{m+1}{2})-\alpha-\beta}}{\Gamma_m(\beta)}.$$

$$Dy\int_0^y |x|^{\alpha}\,|y-x|^{\beta-(m+1)/2,}\ f_i(x)dx\ , \qquad (3.3)$$

for $0<y<D,\ \beta>\frac{m+1}{2}-1, \alpha>-1,$ and e_{ij} are the element of the matrix $[b_{ij}][a_{ij}]^{-1}$ and $i=1,2,.....s.$

The left- hand side of eq.. (3.2), (3.3) and (1.2) are identical and hence on using the orthogonality relation (2.3), we find the solution of eq. (1.1), (1.2) and (1.3) for

$$\alpha+\beta>\frac{m+1}{2}-1, 0<\beta<1$$

$$C_{nj}=\sum_{j=1}^{s} d_{ij}\frac{\Gamma_m(ni+\frac{m+1}{2})\Gamma_m(\alpha+\beta+ni)}{\Gamma_m(\alpha+ni+\frac{m+1}{2})[\Gamma_m(\alpha+\beta+ni)]^2} B_{ni}(\alpha+\beta;D) \qquad (3.4)$$

Where , $n=0,1,2.......; j=1,2,3,.......,\mathrm{s}$ and d_{ij} are the element of the matrix $[b_{ij}]^{-1}$ and

$$B_{ni}(\alpha,\beta;D)=\sum_{j=1}^{s} e_{ij}\frac{1}{\Gamma_m(\beta)}\int_0^D etr(-y)L_{ni}(\alpha+\beta-\tfrac{m+1}{2};y)\ .$$

$$F_i\,(y)dy+\int_D^E etr(-y)|y|^{\alpha+\beta-(m+1)/2}\,L_n(\alpha+\beta-\frac{m+1}{2},y)G_i(y)dy$$
$$+\frac{1}{\Gamma_m(\frac{m+1}{2}-\beta)}\int_E^{\infty}|y|^{\alpha+\beta-(m+1)/2}\,L_n(\alpha+\beta-\frac{m+1}{2};y)H(y)dy \qquad (3.5)$$

$$F_i(y)=Dy\int_0^y |x|^{\alpha}\,|y-x|^{\beta-(m+1)/2}\,f_i(x)dx \qquad (3.6)$$

$$G_i(y)=g_i(y)$$

$$H_i(y)=\int_y^{\infty}|x-y|^{-\beta}\,etr(-x)h_i(x)dx$$

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