Generarlized Operations on Fuzzy Graphs

D.Venugopalam, Nagamurthi Kumar, M.vijaya kumar

Abstract
To discuss the Cartesian Product Composition, union and join on Interval-valued fuzzy graphs. We also introduce the notion of Interval-valued fuzzy complete graphs. Some properties of self complementary graph.

Key Words : Interval-valued fuzzy graph self complementary Interval valued fuzzy complete graphs

Mathematics Subject Classification 2000: 05C99

1.Introduction
It is quite well known that graphs are simply model of relations. A graphs is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and realtions by edges. When there is vagueness in the description of the objects or in its relationships or in both, it is natural that we need to design a Fuzzy Graph Model. Application of fuzzy relations are widespread and important; especially in the field of clustering analysis, neural networks, computer networks, pattern recognition, decision making and expert systems. In each of these the basic mathematical structure is that of a fuzzy graph.

We know that a graphs is a symmetric binary relation on a nonempty set V. Similary, a fuzzy graph is a symmetric binary fuzzy relation on a fuzzy subset. The first definition of a fuzzy graph was by Kaufmann [18] in 1973, based on Zadeh’s fuzzy relations [46]. But it was Azriel Rosenfeld [35] who considered fuzzy relations onfuzzy sets and developed the theory of fuzzy graphs in 1975. During the sam etime R.T.Yeh and S.Y.Bang [44] have also introduced various connectedness concepts in fuzzy graphs.

2 Preliminaries
Definition 2.1 : Let V be a nonempty set. A fuzzy graphs is a pair of functions. 
G : (σ, µ) where σ is a fuzzy subset of v and µ is a symmetric fuzzy relation on σ i.e. σ: V→[0, 1] and µ : V x V → [0, 1] such that µ(u, v) ≤ σ(u) Λ σ(v) for all u, v in V.

We denon the underlying (crisp) graph of G: (σ, µ) by G*: (σ*, µ*) where σ* is referred to as the (nonempty) set V of nodes and µ* = E ⊆ V x V. Note that the crisp graph (V, E) is a special case of a fuzzy graph with each vertex and edge of (V, E) having degree of membership 1. We need not consider loops and we assume that µ is reflexive. Also, the underlyign set V is assumed to be finite and σ can be chosen in any manner so as to satisfy the definition of a fuzzy graphs in all the examples.

Definition 2.2 : The fuzzy graph H: (τ, v) is called a partial fuzzy subgraph of G: (σ, µ) if τ ⊆ τ and v ⊆ µ. In particular, we call H: (τ, v) a fuzzy subgraph of G: (σ, µ) if τ(u) = σ(u) ∀ u, v ∈ V.

Definition 2.3 : For any fuzzy subset τ of V such that τ ⊆ σ, the partial fuzzy subgraph of (σ, µ) induced by τ is the maximal partial fuzzy subgraph of (σ, µ) that has fuzzy node set τ. This is the partial fuzzy subgraph (τ, v) where

T(u, v) = τ(u) Λ µ(u, v) for all u, v ∈ V.

Definition 2.4 : The fuzzy graph H: (τ, v) is called a fuzzy subgraph of G: (σ, µ) induced by P if P ⊆ V, τ(u) = σ(u) ∀ u, v ∈ P.

Definition 2.5 : A partial fuzzy subgraph (τ, v) spans the fuzzy graph (σ, µ) if σ = τ. In this case (τ, v) is called a partial fuzzy spanning subgraph of (σ, µ).

Next we introduce the concept of a fuzzy spanning subgraph as a special case of partial fuzzy spanning subgraph.

Operations 2.6 : Graphs g = (D, E) are simple : no multiple edges and no loops.

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Operations 2.8 : Graphs g = (D, E) are simple: no multiple edges and no loops.
An operation is a permutation on the set of graphs on \( D \):

\[ \alpha : g \rightarrow h \]

Let \( \langle \Gamma \rangle_D = \langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle_D \)

Be the subgroup of the symmetric group generated by \( \Gamma = \{ \alpha_1, \alpha_2, \ldots, \alpha_n \} \)

Transitivity 2.8: The problem setting: Given operations \( \Gamma = \{ \alpha_1, \alpha_2, \ldots, \alpha_n \} \)

And any two graphs \( g, h \) on \( D \) Does there exist a composition \( \alpha \in \langle \Gamma \rangle_D \)

\[ \alpha = \alpha_k, \alpha_{k-1}, \ldots, \alpha_1 \]

such that \( \alpha(g) = h \).

Complement 2.9: Let \( \begin{bmatrix} D \end{bmatrix} \) be the set of all 2-subsets \( \{ x, y \} \).

\[ \begin{bmatrix} = \text{Edges} \leftrightarrow \text{nonedges} \end{bmatrix} \]

Neighbours 2.10: Neighbours of \( x \) \( N_g(x) = \{ y \mid xy \in E \} \)

\( N'_g(x) = D \setminus (N_g(x) \cup \{ x \}) \)

Nonneighbours of \( x \)

Subgraphs 2.11: The symmetric difference: \( A \cup B = (A \setminus B) \cup (B \setminus A) \)

The subgraph of \( g \) induced by \( A \subseteq D : g[A] = \begin{bmatrix} A, E \cap \begin{bmatrix} 2 \end{bmatrix} \end{bmatrix} \)

Complementing Subgraphs 2.12: Denote by \( g \oplus A = \begin{bmatrix} D, E + \begin{bmatrix} 2 \end{bmatrix} \end{bmatrix} \)

3. Main Results

Theorem 3.1

Let \( G_1 = \langle V_1, E_1 \rangle \) and \( G_2 = \langle V_2, E_2 \rangle \) be two Interval Valued Fuzzy Graphs. Then

(i) \( \overline{G_1 + G_2} \cong \overline{G_1} \cup \overline{G_2} \)

(ii) \( G_1 \cup G_2 \cong \overline{G_1} + \overline{G_2} \)

Proof

Consider the identity map \( I : V_1 \cup V_2 \rightarrow V_1 \cup V_2 \),

To prove (i) it is enough to prove

(a) (i) \( \overline{\mu_i} \cup \overline{\mu_i} + \mu_i = \mu_i \cup \mu_i \)

(ii) \( \overline{\gamma_i + \gamma_i} = \overline{\gamma_i} \cup \overline{\gamma_i} \)

(b) (i) \( \overline{\mu_i} \cup \overline{\mu_i} + \mu_i \mu_j = \mu_i \mu_j \)

(ii) \( \overline{\gamma_i} \cup \overline{\gamma_i} + \gamma_i = \gamma_i \cup \gamma_i \)

(a) (i) \( \mu_i \mu_i + \mu_i \mu_i \)

\[ \begin{align*}
\mu_i(v_i) & \quad \text{if } v_i \in V_1 \\
\mu_i(v_i) & \quad \text{if } v_i \in V_2
\end{align*} \]

\[ = \begin{bmatrix} \overline{\mu_i(v_i)} \end{bmatrix} \]

\[ \begin{bmatrix} \overline{\mu_i(v_i)} \end{bmatrix} \]

\[ \begin{bmatrix} \overline{\mu_i(v_i)} \end{bmatrix} \]

\[ \begin{bmatrix} \overline{\mu_i(v_i)} \end{bmatrix} \]

(ii) \( \overline{\gamma_i} + \gamma_i \)

\[ \begin{bmatrix} \gamma_i(v_i) \end{bmatrix} \]

\[ \begin{bmatrix} \gamma_i(v_i) \end{bmatrix} \]

\[ \begin{bmatrix} \gamma_i(v_i) \end{bmatrix} \]

\[ \begin{bmatrix} \gamma_i(v_i) \end{bmatrix} \]
\[
\begin{align*}
\gamma_i(v_i) = \begin{cases} 
\gamma_1(v_i) & \text{if } v_i \in V_1 \\
\gamma_2(v_i) & \text{if } v_i \in V_2
\end{cases} \\
\bar{\gamma}_i(v_i) = \begin{cases} 
\bar{\gamma}_1(v_i) & \text{if } v_i \in V_1 \\
\bar{\gamma}_2(v_i) & \text{if } v_i \in V_2
\end{cases}
\end{align*}
\]

(b) (i) \[
\left(\mu_i + \mu_i^*\right)(v_i, v_j) = \left(\mu_i + \mu_i^*\right)(v_i)\left(\mu_i + \mu_i^*\right)(v_j) - \left(\mu_i + \mu_i^*\right)(v_i, v_j)
\]
\[
= \left(\mu_i \cup \mu_i^*\right)(v_i, v_j) - \left(\mu_i \cup \mu_i^*\right)(v_i)\left(\mu_i \cup \mu_i^*\right)(v_j) \quad \text{if } (v_i, v_j) \in E_1 \cup E_2
\]
\[
= \left(\mu_i \cup \mu_i^*\right)(v_i)\left(\mu_i \cup \mu_i^*\right)(v_j) - \mu_i(v_i)\mu_i^*(v_j) \quad \text{if } (v_i, v_j) \in E'
\]
\[
= \begin{cases} 
\mu_i(v_i, v_j) & \text{if } (v_i, v_j) \in E_1 \\
\mu_i(v_i)\mu_i^*(v_j) & \text{if } (v_i, v_j) \in E_2 \\
0 & \text{if } (v_i, v_j) \in E'
\end{cases}
\]
\[
= \left(\mu_i \cup \mu_i^*\right)(v_i, v_j)
\]

(b) (ii) \[
\left(\gamma_i + \gamma_i^*\right)(v_i, v_j) = \left(\gamma_i + \gamma_i^*\right)(v_i)\left(\gamma_i + \gamma_i^*\right)(v_j) - \left(\gamma_i + \gamma_i^*\right)(v_i, v_j)
\]
\[
= \left(\gamma_i \cup \gamma_i^*\right)(v_i, v_j) - \gamma_i(v_i)\gamma_i^*(v_j) \quad \text{if } (v_i, v_j) \in E_1 \cup E_2
\]
\[
= \left(\gamma_i \cup \gamma_i^*\right)(v_i)\left(\gamma_i \cup \gamma_i^*\right)(v_j) - \gamma_i(v_i)\gamma_i^*(v_j) \quad \text{if } (v_i, v_j) \in E'
\]
\[
= \begin{cases} 
\gamma_i(v_i, v_j) & \text{if } (v_i, v_j) \in E_1 \\
\gamma_i(v_i)\gamma_i^*(v_j) & \text{if } (v_i, v_j) \in E_2 \\
0 & \text{if } (v_i, v_j) \in E'
\end{cases}
\]
\[
= \left(\gamma_i \cup \gamma_i^*\right)(v_i, v_j)
\]

To prove (ii) it is enough to prove
(a) (i) \[
\left(\mu_i \cup \mu_i^*\right)(v_i) = \left(\mu_i \cup \mu_i^*\right)(v_i)
\]
(a) (ii) \[
\left(\gamma_i \cup \gamma_i^*\right)(v_i) = \left(\gamma_i + \gamma_i^*\right)(v_i)
\]
(b) (i) \[
\left(\mu_i \cup \mu_i^*\right)(v_i, v_j) = \left(\mu_i + \mu_i^*\right)(v_i, v_j)
\]
(b) (ii) \[
\left(\gamma_i \cup \gamma_i^*\right)(v_i, v_j) = \left(\gamma_i + \gamma_i^*\right)(v_i, v_j)
\]

Consider the identity map \( I : V_1 \cup V_2 \rightarrow V_1 \cup V_2 \)
(a) (i) \( (\mu_i \cup \mu_i')(v_i) = (\mu_i \cup \mu_i')(v_i) \)
\[
\begin{align*}
&= \begin{cases} 
\mu_i(v_i) & \text{if } v_i \in V_1 \\
\mu_i(v_i) & \text{if } v_i \in V_2 
\end{cases} 
\end{align*}
\]
\[= (\mu_i \cup \mu_i')(v_i) \]
(a) (ii) \( (\gamma_i \cup \gamma_i')(v_i) = (\gamma_i \cup \gamma_i')(v_i) \)
\[
\begin{align*}
&= \begin{cases} 
\gamma_i(v_i) & \text{if } v_i \in V_1 \\
\gamma_i'(v_i) & \text{if } v_i \in V_2 
\end{cases} 
\end{align*}
\[
= \begin{cases} 
\gamma_i(v_i) & \text{if } v_i \in V_1 \\
\gamma_i'(v_i) & \text{if } v_i \in V_2 
\end{cases}
\]
\[
= (\gamma_i \cup \gamma_i')(v_i) \]

(b) (i) \( (\mu_2 \cup \mu_2')(v_i, v_j) = (\mu_i \cup \mu_i')(v_i) (\mu_i \cup \mu_i')(v_j) \)
\[
\begin{align*}
&= \begin{cases} 
(\mu_i(v_i), \mu_i(v_i)) - (\mu_2(v_i, v_j)) & \text{if } (v_i, v_j) \in E_1 \\
(\mu_i(v_i), \mu_i(v_i)) - (\mu_2(v_i, v_j)) & \text{if } (v_i, v_j) \in E_2 \\
(\mu_i(v_i), \mu_i(v_i)) - 0 & \text{if } v_i, v_j \in V_2 
\end{cases} 
\end{align*}
\]
\[= (\gamma_i \cup \gamma_i')(v_i, v_j) \]
(b) (ii) \( (\gamma_2 \cup \gamma_2')(v_i, v_j) = (\gamma_i \cup \gamma_i')(v_i) (\gamma_i \cup \gamma_i')(v_j) \)
\[
\begin{align*}
&= \begin{cases} 
(\gamma_i(v_i), \gamma_i(v_i)) - (\gamma_2(v_i, v_j)) & \text{if } (v_i, v_j) \in E_1 \\
(\gamma_i(v_i), \gamma_i(v_i)) - (\gamma_2(v_i, v_j)) & \text{if } (v_i, v_j) \in E_2 \\
(\gamma_i(v_i), \gamma_i(v_i)) - 0 & \text{if } v_i, v_j \in V_2 
\end{cases} 
\end{align*}
\]
\[= (\gamma_i \cup \gamma_i')(v_i, v_j) \]
\[
\gamma'_2(u, v) = \gamma'_2(v, u)
\]

**Theorem 3.2**

Let \( G_1 = \langle V_1, E_1 \rangle \) and \( G_2 = \langle V_2, E_2 \rangle \) be two Interval Valued Fuzzy Graphs. Then \( G_1 \circ G_2 \) is a strong Interval Valued Fuzzy Graphs.

**Proof**

Let \( G_1 \circ G_2 = G = \langle V, E \rangle \) where \( V_1 \times V_2 \) and

\[
E = \{(u, u')(v_1, v_2) : u \in V_1, v_1, v_2 \in V_2 \} \cup \{(u, w) : \{v_1, w\} \in V_2, u, v_1 \in E_1 \}
\]

\( \cup \{(u_1, u_2)(v_1, v_2) : u_1, v_1, u_2 \neq v_2 \} \).

(i) \( \mu_2(u, u')(v_1, v_2) = \mu_1(u)\mu_1(v_2'), \) since \( G_2 \) is strong

\( = \mu_1(u)\mu_1(v_2') \), since \( G_2 \) is strong

\( = \mu_1(u)\mu_1(u)\mu_1(v_2) \)

\( = (\mu_1 \circ \mu_1)(u, u_2)(\mu_1 \circ \mu_1)(u, v_2) \)

\( \gamma'_2(u, u')(v_1, v_2) = \gamma'_1(u)\gamma'_2(v_1, v_2) \)

\( = \gamma'_1(u)\gamma'_1(u)\gamma'_1(v_2') \), since \( G_2 \) is strong

\( = \gamma'_1(u)\gamma'_1(u)\gamma'_1(u)\gamma'_1(v_2) \)

\( = \gamma'_1(u)\gamma'_1(u)\gamma'_1(u)\gamma'_1(v_2) \)

\( = (\gamma'_1 \circ \gamma'_1)(u, u_2)(\gamma'_1 \circ \gamma'_1)(u, v_2) \)

(ii) \( \mu_2((u_1, w)(v_1, v)) = \mu_1(w)\mu_1(v_1) \)

\( = \mu_1(w)\mu_1(v_1) \), since \( G_1 \) is strong

\( = \mu_1(w)\mu_1(v_1)\mu_1(v) \)

\( = (\mu_1 \circ \mu_1)(v_1, w)(\mu_1 \circ \mu_1)(v_1, v) \)

\( \gamma'_2((u_1, w)(v_1, v)) = \gamma'_1(w)\gamma'_2(v_1, v) \)

\( = \gamma'_1(w)\gamma'_1(v_1)\gamma'_1(v_2') \), since \( G_1 \) is strong

\( = \gamma'_1(v_1)\gamma'_1(v_2') \)

\( = \gamma'_1(v_1)\gamma'_1(v_2') \)

\( = (\gamma'_1 \circ \gamma'_1)(v_1, w)(\gamma'_1 \circ \gamma'_1)(v_1, v) \)

(iii) \( \mu_2(u_1, u_2)(v_1, v_2) = \mu_2(u_1, v_1)\mu_2(u_2, v_2) \)

\( = \mu_2(u_1, v_1)\mu_2(u_2, v_2) \), since \( G_1 \) is strong

\( = \mu_2(u_1, v_1)\mu_2(u_2, v_2) \)

\( = (\mu_2 \circ \mu_2)(u_1, u_2)(\mu_2 \circ \mu_2)(v_1, v_2) \)

\( \gamma'_2(u_1, u_2)(v_1, v_2) = \gamma'_2(u_1, v_1)\gamma'_2(u_2, v_2) \)

\( = \gamma'_2(u_1, v_1)\gamma'_2(u_2, v_2) \)

\( = \gamma'_2(u_1, v_1)\gamma'_2(u_2, v_2) \)

\( = \gamma'_2(u_1, v_1)\gamma'_2(u_2, v_2) \)

\( = (\gamma'_2 \circ \gamma'_2)(u_1, u_2)(\gamma'_2 \circ \gamma'_2)(v_1, v_2) \)

From (i), (ii), (iii), \( G \) is a strong Interval valued Fuzzy Graphs.

**References**


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