# FORMULATION OF CONCAVE-CONVEX FRACTIONAL PROGRAMMING MODEL FOR BANK PORTFOLIO SELECTIONS

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## ABSTRACT

In this paper, a concave-convex fractional programming model for bank portfolio selections is formulated. We have transformed the model into a concave quadratic programming problem and developed a technique for its solution. A real life application of the model is performed with twelve banks in Nigeria. The optimal solution determined by the proportion of investment to be made by an investor in each bank in order to maximize the expected returns at minimum risk is highlighted. However, the computational results show that the proposed model can generate a favourable portfolio strategy according to the investor's satisfactory degree. The trade-off curve also indicates the amount of risk that is commensurate with a particular expected return.

**Key words:** concave-convex, fractional programming problem, optimization, transformation

## 1.1 INTRODUCTION

The effectiveness/workability of a system is most times characterised by a ratio of technical and economic problems. Maximizing system efficiency gives rise to fractional programs. The frequently occurring objectives are maximization of productivity, maximization of return on investment, maximization of return/risk, minimization of cost/time and maximization of output/input. Other non-economic applications arise from information theory, applied mathematics and physics among others (Schaible (2000)). Most of these applications are on linear fractional programming where both the numerator and denominator of the objective function are linear. In real life situations linear fractional models arise in decision making such as construction planning, economic and commercial planning, production planning, financial and corporate planning, health care and hospital planning, bank balance sheet management, water resources management. Thus, mathematical models taking objective function as a ratio of two linear functions have many applications in financial planning. Indeed, in such situations, it is often a question of optimizing a ratio: debt/equity, output/employee, actual cost/standard cost, profit/cost, inventory/sales, risk asset/capital, student/cost, doctor/patient and so on subject to some constraints. If the constraints are linear, we obtain the linear fractional programming problem (LFPP) Pandian et al. (2013), Mehrjerdi (2010). Therefore, Linear Fractional programming problem deals with that class of mathematical programming problem in which the relation between the variables in the problem are linear, the constraint relation are in linear form and the objective function to be optimized is a ratio of two linear functions. Narayanamoorthy and Kalyani (2015). In the literature several methods have been recommended for the solution of LFPP. LFPP has drawn the interest of many researchers since it is widely applied in many important fields. In recent times, much has been done with respect to propounding optimal solution to fractional programming problems. Optimization of fractional programming problem involves several methods of transforming the problem to a linear or quadratic form in diverse areas of the application. This has been done using various techniques and by different researchers which include: Harvey (1968), Bitran and Novaes (1972), Charnes and Cooper (1973), Schaible (1981), Hasan and Acharjee (2011), Narayanamoorthy and Kalyani (2015), Schaible (1980), Singh (1981), Verna et al. (1990), Tantawy (2008), Xiao (2010), Penclaim and Jayalakshmi (2013), Lokhande et al (2013).

### 1.2 Non- linear fractional programming

In linear programming the aim is to maximize or minimize a linear function subject to linear constraints. In many interesting maximization and minimization problems the objective function may not be a linear function and some of the constraints may not be linear constraints. Such an optimization problem is called a Non-linear programming problem (NLP) Winston(1994). Literature on nonlinear fractional programming include the following: Sulaiman (2013), Abdulrahim (2014), Frag et al (2009), Shen et al. (2009),
Bisoi et al (2011), Sharma et.al. (2011), Abdulrahim (2014). In this work, we intend to look at the Non-linear fractional programming problem where the objective function is a rational function in which the numerator is linear and the denominator is quadratic with linear constraint. Portfolio selection application of maximization of return on risk which is expected to yield a global optimal solution is used.

## 2.0 GENERAL FRACTIONAL PROGRAMMING MODEL

The general fractional programming problem is of the form:

$$Max\ Z = \frac{f(x)}{g(x)}\ , x \in s \qquad (1)$$

$where$

$S = \{x \in R^n : hx \leq b;\ b \subset R^n\}$ ,

$g(x) \geq 0$

## 3.0 Formulation of the Model for Bank Portfolio Selections

### 3.1 Definition of variables and parameters

Let

$x_i =$ Proportion of investment made in each bank; i = 1, 2, . . . , 12

$L_i =$ Liquidity of the $ith$ bank

$E_i =$ Earnings per shares of the $ith$ bank

$R_i =$ Return on investment in the $ith$ bank

$L =$ The lowest acceptable expected Liquidity per unit money invested in the

entire portfolio.

$E =$ The lowest acceptable earnings per share per unit money invested in the entire portfolio

$\mu_i =$ Expected return on investment in the $i^{th}$ bank

$\sigma_{ij} =$ Covariance of the expected returns on security i and j

$R_p =$ Return for the portfolio

### 3.2 Formulation of the objective function

The return for the portfolio ($R_p$) is given by

$$R_t = \sum_{i=1}^{12} R_i x_i \tag{2}$$

Taking expectations of both sides of equation (2) we obtain

$$E[R_i] = \sum_{i=1}^{12} E[R_i] x_i$$

This may be rewritten as

$$\mu_{Rt} = \sum_{i=1}^{12} \mu_i x_i$$

 

$$f(x) = \sum_{i=1}^{12} \mu_i x_i \tag{3}$$

Where

$f(x) = \mathrm{E}[R]$ *and* $\mu_i = \mathrm{E}[R_i]$

The risk of the portfolio as measured by variance of the total returns is given by

$$Var(R) = \sum_{i=1}^{12} x_i^2 Var(R_i) + \sum_{i=1}^{12}\sum_{j=1}^{12} x_i x_j Cov(R_i, R_j) \tag{4}$$

This may be written in an expanded form as

$$\begin{aligned}
g(x) &= x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + x_3^2\sigma_3^2 + x_4^2\sigma_4^2 + x_5^2\sigma_5^2 + x_6^2\sigma_6^2 + x_7^2\sigma_7^2 + x_8^2\sigma_8^2 + x_9^2\sigma_9^2 + x_{10}^2\sigma_{10}^2 + x_{11}^2\sigma_{11}^2 + x_{12}^2\sigma_{12}^2 \\
&+ 2x_1x_2\sigma_{12} + 2x_1x_3\sigma_{13} + 2x_1x_4\sigma_{14} + 2x_1x_5\sigma_{15+} 2x_1x_6\sigma_{16} + 2x_1x_7\sigma_{17} + 2x_1x_8\sigma_{18} + 2x_1x_9\sigma_{19} + 2x_1x_{10}\sigma_{1,10} \\
&+ 2x_1x_{11}\sigma_{1,11} + 2x_1x_{12}\sigma_{1,12} + 2x_2x_3\sigma_{23} + 2x_2x_4\sigma_{24} + 2x_2x_5\sigma_{25} + 2x_2x_6\sigma_{26} + 2x_2x_7\sigma_{27} + 2x_2x_8\sigma_{28} \\
&+ 2x_2x_9\sigma_{29} + 2x_2x_{10}\sigma_{2,10} + 2x_2x_{11}\sigma_{2,11} + 2x_2x_{12}\sigma_{2,12} + 2x_3x_4\sigma_{34} + 2x_3x_5\sigma_{35} + 2x_3x_6\sigma_{36} + 2x_3x_7\sigma_{37} \\
&+ 2x_3x_8\sigma_{38} + 2x_3x_9\sigma_{39} + 2x_3x_{10}\sigma_{3,10} + 2x_3x_{11}\sigma_{3,11} + 2x_3x_{12}\sigma_{3,12} + 2x_4x_5\sigma_{45} + 2x_4x_6\sigma_{46} + 2x_4x_7\sigma_{47} \\
&+ 2x_4x_8\sigma_{48} + 2x_4x_9\sigma_{49} + 2x_4x_{10}\sigma_{4,10} + 2x_4x_{11}\sigma_{4,11} + 2x_4x_{12}\sigma_{4,12} + 2x_5x_6\sigma_{56} + 2x_5x_7\sigma_{57} + 2x_5x_8\sigma_{58} \\
&+ 2x_5x_9\sigma_{59} + 2x_5x_{10}\sigma_{5,10} + 2x_5x_{11}\sigma_{5,11} + 2x_5x_{12}\sigma_{5,12} + 2x_6x_7\sigma_{67} + 2x_6x_8\sigma_{68} + 2x_6x_9\sigma_{69} + 2x_6x_{10}\sigma_{6,10} \\
&+ 2x_6x_{11}\sigma_{6,11} + 2x_6x_{12}\sigma_{6,12} + 2x_7x_8\sigma_{78} + 2x_7x_9\sigma_{79} + 2x_7x_{10}\sigma_{7,10} + 2x_7x_{11}\sigma_{7,11} + 2x_7x_{12}\sigma_{7,12} + 2x_8x_9\sigma_{89} \\
&+ 2x_8x_{10}\sigma_{8,10} + 2x_8x_{11}\sigma_{8,11} + 2x_8x_{12}\sigma_{8,12} + 2x_9x_{10}\sigma_{9,10} + 2x_9x_{11}\sigma_{9,11} + 2x_9x_{12}\sigma_{9,12} + 2x_{10}x_{11}\sigma_{10,11} \\
&+ 2x_{10}x_{12}\sigma_{10,12} + 2x_{11}x_{12}\sigma_{11,12}
\end{aligned}$$

Therefore, the objective function of the portfolio selection model is formulated thus:

$$\mathrm{Z} = \frac{f(x)}{g(x)} \tag{5}$$

This objective function is to be maximized.

### 3.3 Formulation of the constraint

The constraints are liquidity, earnings per shares and total assets and are formulated thus:

The liquidity constraint is

 

$$\sum_{i=1}^{12} L_i x_i \leq L \qquad (6)$$

The earnings per share constraint is

$$\sum_{i=1}^{12} E_i x_i \leq E \qquad (7)$$

The total asset constraint is given by

$$\sum_{i=1}^{12} x_i = 1 \qquad (8)$$

Non-negativity constraints are

$$x_i \geq 0, \quad i = 1,2,\dots,12 \qquad (9)$$

The proportion must be nonnegative

**3.4 The complete model**

$$\text{Max Z} = \frac{f(x)}{g(x)} \qquad (10)$$

Subject to:

$$\sum_{i=1}^{12} L_i x_i \leq L \qquad (11)$$

$$\sum_{i=1}^{12} E_i x_i \leq E \qquad (12)$$

$$\sum_{i=1}^{12} x_i = 1 \qquad (13)$$

$$x_i \geq 0, \quad i = 1,2,\dots,12$$

**3.5 The model excluding the liquidity and earnings per share constraints**

$$\text{Max Z} = \frac{f(x)}{g(x)} \qquad (14)$$

Subject to:

$$\sum_{i=1}^{12} x_i = 1$$

$$x_i \geq 0,$$

### 3.6 The conventional portfolio selection model

$$MinZ = g(x) \tag{15}$$

Subject to:

$$\sum_{i=1}^{12} \mu_i x_i = \mu$$

$$\sum_{i=1}^{12} x_i = 1$$

$$x_i \geq 0$$

## 4.0 Model Solution

**Lemma 1:**

$$Max\left\{\frac{f(x)}{g(x)}\right\} \cong -Max\left\{\frac{-g(x)}{f(x)}\right\}$$

**Proof:**

$$Max\left\{\frac{1}{p(x)}\right\} \cong Min\{p(x)\} \cong -Max\{-p(x)\}$$

Now letting $p(x) = \frac{g(x)}{f(x)}$

we obtain

$$Max\left\{\frac{f(x)}{g(x)}\right\} \cong -Max\left\{\frac{-g(x)}{f(x)}\right\}$$

**Lemma 2:** If $g(x)$ is convex, then $\sqrt{g(x)}$ is also convex and vice versa.

**Lemma 3:** If $g(x)$ is convex, then $-g(x)$is concave.

**Theorem 1:** If f(x) is concave and g(x) is convex, then the concave-convex fractional programming model can be transformed to a concave programming model.

**Proof:**

By lemma (1), the concave-convex programming model (10-13) is equivalent to

$$\text{Max } Z = \frac{-g(x)}{f(x)} \tag{16}$$

Subject to:

$$\sum_{i=1}^{12} L_i x_i \le L$$

$$\sum_{i=1}^{12} x_i = 1$$

$$x_i \ge 0, \quad i = 1,2,\dots,12$$

By lemma 2, equation (10) is equivalent to

$$-Max\left\{ Z = \frac{-\sqrt{g(x)}}{f(x)} \right\} \tag{17}$$

Subject to:

$$\sum_{i=1}^{12} L_i x_i \le L$$

$$\sum_{i=1}^{12} E_i x_i \le E$$

$$\sum_{i=1}^{12} x_i = 1$$

$$x_i \ge 0, \quad i = 1,2,\dots,12$$

Let $f(x) = \sum_{j=1}^{12} \mu_j x_j = t$

$$g(x) = \sum_{i=1}^{12} \sum_{j=1}^{12} q_{ij} x_i x_j$$

$x_j = ty_j \Rightarrow \dfrac{x_j}{t} = y_j$ and $r = \dfrac{1}{t} \Rightarrow rt = 1$

Then the model becomes

$$-Max\left\{ Z' = -\left[ \sum_{i=1}^{m} \sum_{j=1}^{n} q_{ij} y_i y_j \right]^{\frac{1}{2}} \right\} \quad (18)$$

Subject to:

$$\sum_{j=1}^{12} L_j y_j - Lr \leq 0 \quad (19)$$

$$\sum_{j=1}^{12} E_j y_j - Er \leq 0 \quad (20)$$

$$\sum_{i=1}^{12} y_i - r \leq 0 \quad (21)$$

$$\sum_{j=1}^{12} \mu_j y_j = 1 \quad (22)$$

$$y_j \geq 0,\ j = 1,2,...,12 \quad (23)$$

Finally, by Lemma (3), the model becomes:

$$-Max\left\{ Z' = -\sum_{i=1}^{m} \sum_{j=1}^{n} q_{ij} y_i y_j \right\} \quad (24)$$

Subject to:

$$\sum_{j=1}^{12} L_j y_j - Lr \leq 0 \quad (25)$$

$$\sum_{j=1}^{12} E_j y_j - Er \leq 0 \quad (26)$$

$$\sum_{i=1}^{12} y_i - r \leq 0 \quad (27)$$

$$\sum_{j=1}^{12} \mu_j y_j = 1 \tag{28}$$

$$y_j \geq 0,\ j = 1,2,\ldots,12 \tag{29}$$

Equation (24) – (29) is a concave quadratic programming model.

### 4.3 The transformed model excluding the liquidity and earnings per share constraints

$$-Max\left\{ Z' = -\sum_{i=1}^{m}\sum_{j=1}^{n} q_{ij} y_i y_j \right\} \tag{30}$$

Subject to:

$$\sum_{i=1}^{12} y_i - r \leq 0$$

$$\sum_{j=1}^{12} \mu_j y_j = 1$$

$$y_j \geq 0,\ j = 1,2,\ldots,12$$

### 4.4 Solving the Model with real data

#### 4.4.1 Data

Based on the data collected from the Nigerian Stock Exchange on the annual financial statement of twelve banks, the mean, variance, covariance, Hessian matrix are determined. Optimal solution of the concave-convex fractional programming model, the transformed model and the conventional portfolio selection model is obtained. Tables and graph are used for the respective interpretations and are subsequently discussed.

The transformed portfolio selection model to concave quadratic programming problem is given by

 

$$
\begin{aligned}
MaxZ' = -(&0.00863y_1^2 + 0.06414y_2^2 + 0.09550y_3^2 + 0.00476y_4^2 + 0.00284y_5^2 + 0.27322y_6^2 \\
&+ 0.05560y_7^2 + 0.01578y_8^2 + 0.00352y_9^2 + 1.2962y_{10}^2 + 0.01892y_{11}^2 + 0.01578y_{12}^2 + 0.00769y_1y_2 \\
&- 0.00220y_1y_3 - 0.00030y_1y_4 + 0.00402y_1y_5 - 0.01012y_1y_6 - 0.01896y_1y_7 - 0.00331y_1y_8 \\
&- 0.00374y_1y_9 - 0.03624y_1y_{10} - 0.00885y_1y_{11} - 0.00329y_1y_{12} + 0.00614y_2y_3 + 0.00502y_2y_4 \\
&+ 0.003734y_2y_5 + 0.04720y_2y_6 - 0.00010y_2y_7 + 0.01676y_2y_8 + 0.00496y_2y_9 + 0.10220y_2y_{10} \\
&- 0.00403y_2y_{11} + 0.01441y_2y_{12} + 0.00212y_3y_4 + 0.04023y_3y_5 + 0.04998y_3y_6 + 0.01699y_3y_7 \\
&+ 0.02148y_3y_8 + 0.00604y_3y_9 + 0.21728y_3y_{10} + 0.00721y_3y_{11} + 0.01654y_3y_{12} + 0.00620y_4y_5 \\
&- 0.00846y_4y_6 + 0.00746y_4y_7 + 0.00722y_4y_8 + 0.00142y_4y_9 - 0.02439y_4y_{10} + 0.00638y_4y_{11} \\
&+ 0.00762y_4y_{12} - 0.00660y_5y_6 + 0.00236y_5y_7 + 0.01634y_5y_8 + 0.00229y_5y_9 + 0.07170y_5y_{10} \\
&+ 0.00349y_5y_{11} + 0.01510y_5y_{12} + 0.04084y_6y_7 - 0.00760y_6y_8 + 0.01631y_6y_9 + 0.02780y_6y_{10} \\
&- 0.00540y_6y_{11} - 0.01180y_6y_{12} + 0.01808y_7y_8 + 0.01174y_7y_9 + 0.00374y_7y_{10} + 0.00281y_7y_{11} \\
&+ 0.01796y_7y_{12} + 0.00428y_8y_9 + 0.03417y_8y_{10} + 0.01232y_8y_{11} + 0.01564y_8y_{12} + 0.02455y_9y_{10} \\
&+ 0.00419y_9y_{11} + 0.00430y_9y_{12} - 0.01035y_{10}y_{11} + 0.02481y_{10}y_{12} + 0.01238y_{11}y_{12})
\end{aligned}
$$

Subject to:

$$
\begin{aligned}
&0.1519y_1 + 0.2378y_2 + 0.1868y_3 + 0.3723y_4 + 0.0809y_5 + 0.1759y_6 + 0.1859y_7 + 0.2131y_8 \\
&+ 0.0388y_9 + 0.4346y_{10} + 0.1147y_{11} + 0.3813y_{12} - 30.888r \le 0
\end{aligned}
$$

$$
\begin{aligned}
&1.2517y_1 + 0.7267y_2 + 0.2017y_3 + 0.3100y_4 + 0.7067y_5 + 0.0700y_6 + 0.2550y_7 + 2.2233y_8 \\
&+ 0.7350y_9 + 0.9983y_{10} + 0.8900y_{11} + 1.9617y_{12} - 123.96r \le 0
\end{aligned}
$$

$$
y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9 + y_{10} + y_{11} + y_{12} - r \le 0
$$

$$
\begin{aligned}
&0.0209y_1 + 0.0080y_2 + 0.0170y_3 + 0.0097y_4 + 0.010y_5 + 0.0036y_6 + 0.0046y_7 + 0.0390y_8 \\
&+ 0.0092y_9 + 0.0474y_{10} + 0.0068y_{11} + 0.0241y_{12} - r = 1
\end{aligned}
$$

$$
y_j \ge 0,\ j = 1,2,\ldots,12
$$

The transformed model excluding the liquidity and earnings per share constraint is given by

 

$$MaxZ' = -(0.00863y_1^2 + 0.06414y_2^2 + 0.09550y_3^2 + 0.00476y_4^2 + 0.00284y_5^2 + 0.27322y_6^2$$
$$+ 0.05560y_7^2 + 0.01578y_8^2 + 0.00352y_9^2 + 1.2962y_{10}^2 + 0.01892y_{11}^2 + 0.01578y_{12}^2 + 0.00769y_1y_2$$
$$- 0.00220y_1y_3 - 0.00030y_1y_4 + 0.00402y_1y_5 - 0.01012y_1y_6 - 0.01896y_1y_7 - 0.00331y_1y_8$$
$$- 0.00374y_1y_9 - 0.03624y_1y_{10} - 0.00885y_1y_{11} - 0.00329y_1y_{12} + 0.00614y_2y_3 + 0.00502y_2y_4$$
$$+ 0.003734y_2y_5 + 0.04720y_2y_6 - 0.00010y_2y_7 + 0.01676y_2y_8 + 0.00496y_2y_9 + 0.10220y_2y_{10}$$
$$- 0.00403y_2y_{11} + 0.01441y_2y_{12} + 0.00212y_3y_4 + 0.04023y_3y_5 + 0.04998y_3y_6 + 0.01699y_3y_7$$
$$+ 0.02148y_3y_8 + 0.00604y_3y_9 + 0.21728y_3y_{10} + 0.00721y_3y_{11} + 0.01654y_3y_{12} + 0.00620y_4y_5$$
$$- 0.00846y_4y_6 + 0.00746y_4y_7 + 0.00722y_4y_8 + 0.00142y_4y_9 - 0.02439y_4y_{10} + 0.00638y_4y_{11}$$
$$+ 0.00762y_4y_{12} - 0.00660y_5y_6 + 0.00236y_5y_7 + 0.01634y_5y_8 + 0.00229y_5y_9 + 0.07170y_5y_{10}$$
$$+ 0.00349y_5y_{11} + 0.01510y_5y_{12} + 0.04084y_6y_7 - 0.00760y_6y_8 + 0.01631y_6y_9 + 0.02780y_6y_{10}$$
$$- 0.00540y_6y_{11} - 0.01180y_6y_{12} + 0.01808y_7y_8 + 0.01174y_7y_9 + 0.00374y_7y_{10} + 0.00281y_7y_{11}$$
$$+ 0.01796y_7y_{12} + 0.00428y_8y_9 + 0.03417y_8y_{10} + 0.01232y_8y_{11} + 0.01564y_8y_{12} + 0.02455y_9y_{10}$$
$$+ 0.00419y_9y_{11} + 0.00430y_9y_{12} - 0.01035y_{10}y_{11} + 0.02481y_{10}y_{12} + 0.01238y_{11}y_{12})$$

Subject to:

$$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9 + y_{10} + y_{11} + y_{12} - r \leq 0$$

$$0.0209y_1 + 0.0080y_2 + 0.0170y_3 + 0.0097y_4 + 0.010y_5 + 0.0036y_6 + 0.0046y_7 + 0.0390y_8$$
$$+ 0.0092y_9 + 0.0474y_{10} + 0.0068y_{11} + 0.0241y_{12} - r = 1$$

$$y_j \geq 0,\ j = 1,2,...,12$$

The conventional portfolio selection model is given as

$$MinZ = 0.00863x_1^2 + 0.06414x_2^2 + 0.09550x_3^2 + 0.00476x_4^2 + 0.00284x_5^2 + 0.27322x_6^2 + 0.05560x_7^2$$
$$+ 0.01578x_8^2 + 0.00352x_9^2 + 1.2962x_{10}^2 + 0.01892x_{11}^2 + 0.01578x_{12}^2 + 0.00769x_1x_2 - 0.00220x_1x_3$$
$$- 0.00030x_1x_4 + 0.00402x_1x_5 - 0.01012x_1x_6 - 0.01896x_1x_7 - 0.00331x_1x_8 - 0.00374x_1x_9$$
$$- 0.03624x_1x_{10} - 0.00885x_1x_{11} - 0.00329x_1x_{12} + 0.00614x_2x_3 + 0.00502x_2x_4 + 0.003734x_2x_5$$
$$+ 0.04720x_2x_6 - 0.00010x_2x_7 + 0.01676x_2x_8 + 0.00496x_2x_9 + 0.10220x_2x_{10} - 0.00403x_2x_{11}$$
$$+ 0.01441x_2x_{12} + 0.00212x_3x_4 + 0.04023x_3x_5 + 0.04998x_3x_6 + 0.01699x_3x_7 + 0.02148x_3x_8$$
$$+ 0.00604x_3x_9 + 0.21728x_3x_{10} + 0.00721x_3x_{11} + 0.01654x_3x_{12} + 0.00620x_4x_5 - 0.00846x_4x_6$$
$$+ 0.00746x_4x_7 + 0.00722x_4x_8 + 0.00142x_4x_9 - 0.02439x_4x_{10} + 0.00638x_4x_{11} + 0.00762x_4x_{12}$$
$$- 0.00660x_5x_6 + 0.00236x_5x_7 + 0.01634x_5x_8 + 0.00229x_5x_9 + 0.07170x_5x_{10} + 0.00349x_5x_{11}$$
$$+ 0.01510x_5x_{12} + 0.04084x_6x_7 - 0.00760x_6x_8 + 0.01631x_6x_9 + 0.02780x_6x_{10} - 0.00540x_6x_{11}$$
$$- 0.01180x_6x_{12} + 0.01808x_7x_8 + 0.01174x_7x_9 + 0.00374x_7x_{10} + 0.00281x_7x_{11} + 0.01796x_7x_{12}$$
$$+ 0.00428x_8x_9 + 0.03417x_8x_{10} + 0.01232x_8x_{11} + 0.01564x_8x_{12} + 0.02455x_9x_{10} + 0.00419x_9x_{11}$$
$$+ 0.00430x_9x_{12} - 0.01035x_{10}x_{11} + 0.02481x_{10}x_{12} + 0.01238x_{11}x_{12}$$

Subject to:

$$0.0209x_1 + 0.0080x_2 + 0.0170x_3 + 0.0097x_4 + 0.010x_5 + 0.0036\,x_6 + 0.0046x_7 + 0.0390x_8 + 0.0092x_9 + 0.0474x_{10} + 0.0068\,x_{11} + 0.0241x_{12}$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} = 1$$

$$x_i \geq 0\,, i = 1, 2, \ . \ . \ . \ , 12$$

The result of the transformed model to quadratic program is as shown in table 1

**TABLE 1**

**Optimal feasible solution to the transformed model**

| Banks | $y_1$ | $Z'$ | $r$ |
|---|---|---|---|
| 1 | 18.2234205 | 250.7913 | 45.7221903 |
| 2 | 0 | | |
| 3 | 0 | | |
| 4 | 0 | | |
| 5 | 0 | | |
| 6 | 0.18957332 | | |
| 7 | 0 | | |
| 8 | 11.1721626 | | |
| 9 | 13.6730391 | | |
| 10 | 0.14176062 | | |
| 11 | 0.33199689 | | |
| 12 | 1.9902372 | | |

## TABLE 2

### Optimal feasible solution to the original formulated portfolio selection model

| Banks | $x_1$ | $f(x)$ | $g(x)$ | $Z = \frac{f(x)}{g(x)}$ |
|---|---|---|---|---|
| 1 | 0.398568 | 0.021871 | 0.001199 | 18.24103 |
| 2 | 0 | | | |
| 3 | 0 | | | |
| 4 | 0 | | | |
| 5 | 0 | | | |
| 6 | 0.004146 | | | |
| 7 | 0 | | | |
| 8 | 0.244349 | | | |
| 9 | 0.299046 | | | |
| 10 | 0.0031 | | | |
| 11 | 0.007261 | | | |
| 12 | 0.043529 | | | |
| | 1 | | | |

## TABLE 3

### Optimal feasible solution to the transformed model excluding the liquidity and earnings per share constraint.

| Banks | $y_1$ | $Z'$ | $r$ |
|---|---|---|---|
| 1 | 18.22622 | 250.7913 | 45.72383 |
| 2 | 0 | | |
| 3 | 0 | | |
| 4 | 0 | | |
| 5 | 0 | | |
| 6 | 0.18992 | | |
| 7 | 0 | | |
| 8 | 11.17278 | | |
| 9 | 13.66782 | | |
| 10 | 0.141884 | | |
| 11 | 0.338548 | | |
| 12 | 1.986665 | | |

**TABLE 4**

**Optimal feasible solution to the original model excluding the liquidity and earnings per share constraints**

| Banks | $x_1$ | $f(x)$ | $g(x)$ | $Z = \frac{f(x)}{g(x)}$ |
|---|---|---|---|---|
| 1 | 0.398615 | 0.02187 | 0.001199 | 18.2402 |
| 2 | 0 | | | |
| 3 | 0 | | | |
| 4 | 0 | | | |
| 5 | 0 | | | |
| 6 | 0.004154 | | | |
| 7 | 0 | | | |
| 8 | 0.244353 | | | |
| 9 | 0.298921 | | | |
| 10 | 0.003103 | | | |
| 11 | 0.007404 | | | |
| 12 | 0.043449 | | | |
| | 1 | | | |

**TABLE 5**

**Table of values for the trade-off curve method**

| Return | Risk |
|---|---|
| **0.013** | **0.0013** |
| **0.02** | **0.002** |
| **0.022** | **0.0024** |
| **0.025** | **0.003219** |
| **0.028** | **0.004282** |
| **0.03** | **0.0052** |
| **0.035** | **0.009375** |
| **0.037** | **0.012223** |
| **0.04** | **0.034** |

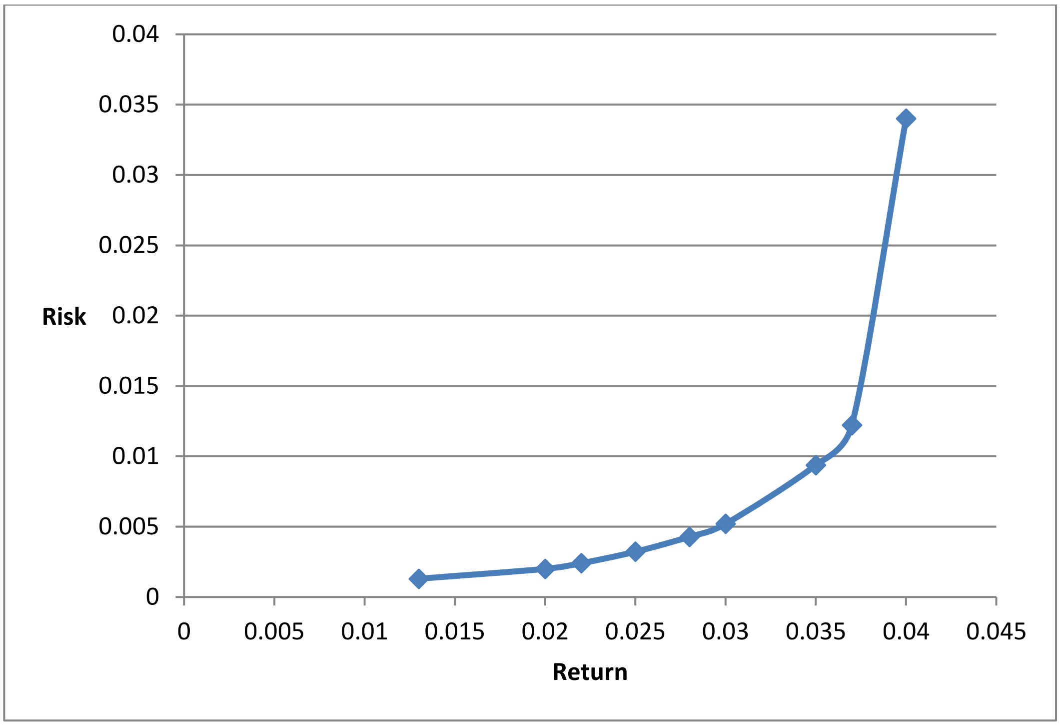


**FIGURE 1:** Trade off curve of risks versus returns

## 5.0 Conclusion

The portfolio selection model was formulated as a concave-convex fractional programming problem as shown in equation (10).The model was transformed into a concave quadratic programming problem and solved by the Frank Wolfe's modified algorithm in excel solver.

The optimal solution as shown in Table 2 determines the proportion of investments to be made by an investor in each bank in order to maximize the expected return at minimum risk. The second model without the liquidity and earnings per share constraint as shown in equation (30), indicated the same result in the analysis as the first model which shows that those two constraints are redundant. The trade-off curve as shown in fig 1, indicates the amount of risk to be taken for a particular expected return. Based on the solution of the analysis as shown in table 2 and table 4, it is seen that $x_2 = x_3 = x_4 = x_5 = x_7 = 0$, which implies that it is risky to invests in those Banks. It is safe for the investors to invest in each portfolio in the following proportion respectively: Bank A 39.86%, Bank F 0.44%, Bank H 24.44%, Bank I 29.91%, Bank J 0.31%, BANK K 0.73% and Bank L 4.35%. The computational results show that the proposed model can generate a favourite portfolio strategy according to the investor's satisfactory degree. The trade-off curve also indicates the amount of risk that is commensurate with a particular expected return.

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