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The response of a vertical dipole above a conducting earth

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Abstract

An exact representation for the electromagnetic field above a conducting earth is investigated theoretically. The time domain response of a vertical magnetic dipole above a conducting half-space is of a crucial importance in the interpretation of air borne electromagnetic data. The classical approach in finding a solution is to apply Fourier inversion to the formal time harmonic solution, leading to a representation in the form of a double infinite integral.

Keywords: electromagnetic field, anisotropic and isotropic

1. Introduction

The pulsed electromagnetic radiation from a vertical magnetic dipole above a conducting earth is investigated theoretically. The propagation of electromagnetic waves over finitely conducting surfaces is considered to be well understood. Stratified anisotropic and isotropic models have been treated Wait (1986) and various mixed path geometries have been analyzed. Hill and Wait (1981), but with few acceptation Fischer (1964). The anisotropy is restricted to lateral uniformity. Subsequently, Kuster (1984) investigated the transient reflected field of a pulsed line source over a conducting half-space. Applying a method suggested by Dook (1952), he arrived at an exact solution in the form of a double finite integral. In this paper we study the electromagnetic field of a pulsed vertical magnetic dipole above a conducting earth. Abo Seliem(1998), theoretical study for computing the magnetic field from a Fitzgrald vector in the ionosphere is presented.

Zedan and Abo Seliem (2002), also the integral is evaluated by two dimensional methods. As our pulsed source is point source, this result is essentially more simple that Kuester's expression for the line source.

2. Formulation of the problem

To specify the position in the configuration, we employ the coordinates (x_1, x_2, x_3) with respect to d fixed. (Figure 1.), orthogonal, Cartesian reference frame. With origin O and three mutually perpendicular base vectors $\vec{i}_1, \vec{i}_2, \vec{i}_3$ of unit length each. In the indicated orders the tree base vectors form a right-handed system. In accordance with previous papers, $\vec{i}_3$ points vertically upward and the upper medium. The structure is shift invariant in the direction of $\vec{i}_2$ and the electromagnetic properties are characterized by their permittivity ε, permeability μ and the electrical conductivity σ, for the upper medium we have $\varepsilon = \varepsilon_o$, $\mu = \mu_o$ and $\sigma' = \sigma_0$. For the lower medium we have $\varepsilon = \varepsilon_1$, $\mu = \mu_o$ and $\sigma' = \sigma_0'$. At $x_1 = 0$, $x_2 = 0$ and $x_3 = h_T > 0$

A vertical magnetic dipole starts to radiate at the instant $\mathbf{t=0}$; it is assumed that prior to this instant all field quantities vanish identically. At $\mathbf{x_1 = d}$, $\mathbf{x_2 = 0}$ and $\mathbf{x_3 = h_R > 0}$ a receiver is located.

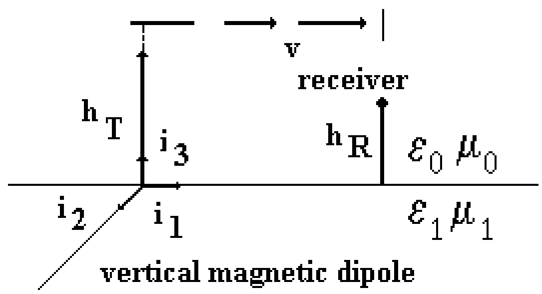

Fig.1: Geometric of the problem.

3. Method of solution

The electromagnetic field generated magnetic dipole can be derived from a magnetic Hertizan vector $\boldsymbol{\pi}^*$ of which only the $\mathbf{x_3}$ component is different from zero. $\mathbf{E}$ and $\mathbf{H}$ are expressed in terms of $\boldsymbol{\pi}^*$ through the relations:

$$\mathbf{E} = -\mu_o \ \mathbf{Curl}\left(\frac{\partial \pi^*}{\partial t}\right) \tag{1}$$

$$\mathbf{H} = \mathbf{grad\,div}\,\pi^* - \mu_o\varepsilon_o \frac{\partial^2 \pi^*}{\partial t^2} - \mu_o \sigma \frac{\partial\ \pi^*}{\partial t} \tag{2}$$

In the region $\mathbf{x_3 > 0}$:

$$\boldsymbol{\pi}^* = (\mathbf{u^i + u^r})\mathbf{i_3} \tag{3}$$

In the region $\mathbf{x_3 < 0}$:

$$\boldsymbol{\pi}^* = \mathbf{u^t i_3} \tag{4}$$

Where $\mathbf{u}^{i}$ yields the incident wave while $\mathbf{u}^{r}$ accounts for the reflection of the incident wave against the interface and also $\mathbf{u}^{t}$ denotes the transmitted field. The wave equation:

$$\nabla^{2}\mathbf{u}^{i}-\frac{1}{\mathbf{v}_{1}^{j}}\frac{\partial^{2}\mathbf{u}^{i}}{\partial \mathbf{t}^{2}}=\begin{cases}0 & \mathbf{i}=\mathbf{r} \quad 0<\mathbf{x}\\ \sigma\mu_{o}\dfrac{\partial \mathbf{u}^{i}}{\partial \mathbf{t}} & \mathbf{i}=\mathbf{t} \quad \mathbf{x}<0\end{cases} \qquad (5)$$

Where $\mathbf{v}_{j}^{-1}=\sqrt{\varepsilon_{j}\mu_{o}}$ and $\mathbf{j}=0\ ,\ 1$ are the velocities of propagation in the upper and lower medium respectively.

The field $\mathbf{u}^{i}$ is given by Stratton (1941)as :

$$\mathbf{u}^{i}=\frac{\mathbf{f}(\mathbf{t}-\mathbf{R}\mathbf{v}_{0}^{-1})}{4\pi\mathbf{R}} \qquad (6)$$

Where

$$\mathbf{R}=\sqrt{\mathbf{x}_{1}^{2}+\mathbf{x}_{2}^{2}+(\mathbf{x}_{3}-\mathbf{h}_{T})^{2}} \qquad (7)$$

The functions f (t) determinates the magnetic moment of the dipole as a function of time. The continuity of $\mathbf{E}_{1}$, $\mathbf{E}_{2}$, $\mathbf{H}_{1}$, and $\mathbf{H}_{2}$ in the interface is guaranteed if following boundary conditions are satisfied:

$$\lim_{\mathbf{x}_{3}\to 0^{-}}(\mathbf{u}^{i}+\mathbf{u}^{r})=\lim_{\mathbf{x}_{3}\to 0^{+}}(\mathbf{u}^{t}) \qquad (8)$$

$$\lim_{\mathbf{x}_{3}\to 0^{-}}\left(\frac{\partial \mathbf{u}^{i}}{\partial \mathbf{x}_{3}}+\frac{\partial \mathbf{u}^{r}}{\partial \mathbf{x}_{3}}\right)=\lim_{\mathbf{x}_{3}\to 0^{+}}\left(\frac{\partial \mathbf{u}^{t}}{\partial \mathbf{x}_{3}}\right) \qquad (9)$$

Next, we subject the wave equations (5) and (6) together with the boundary conditions (8) and (9) to a Laplace transformation with respect to time according to:

$$\overline{\mathbf{u}}(\mathbf{r},\mathbf{s})=\int_{\tau=0}^{\infty}\mathbf{u}(\tau)\exp(-\mathbf{s}\tau)\mathbf{d}\tau \qquad (10)$$

And a spatial Fourier transformation with respect to $\mathbf{x}_{1}$, and $\mathbf{x}_{2}$ according to

$$\overline{\mathbf{u}}(\alpha,\beta,\mathbf{x}_{3};\mathbf{s})=\int_{-\infty}^{\infty}\mathbf{d}\mathbf{x}_{2}\int_{-\infty}^{\infty}\exp(\mathbf{s}(\alpha\mathbf{x}_{1}+\beta\mathbf{x}_{2}))\tilde{\mathbf{u}}(\mathbf{r},\mathbf{s})\,\mathbf{d}\mathbf{x}_{1},\quad \alpha,\ \beta\in\oint \qquad (11)$$

In which $\oint$ denotes the imaginary axis. The corresponding transform-domain expression for $\overline{\mathbf{u}}(\alpha,\beta,\mathbf{x}_{3};\mathbf{s})$ is found:

$$\bar{u}^{i}(\alpha,\beta,x_3;s)=\frac{f(s)}{2s\gamma_0}\exp(-s\gamma_0|x_3-h_T| \quad (12)$$

In which

$$\gamma_0=\sqrt{v_0^{-2}-\alpha^2-\beta^2} \qquad \Re e(\gamma_0)\geq 0 \quad (13)$$

For the wave of (5) in the transform domain. We are left with two ordinary second-order differential equations for $\bar{u}^r$ and $\bar{u}^t$ with x_3 as independent variable the solutions that remain bounded as $|x_3|\rightarrow\infty$ can be written as

$$\bar{u}^{r}(\alpha,\beta,x_3;s)=\frac{f(s)}{2s\gamma_0}\left\{\frac{\gamma_0-\gamma_1^L}{\gamma_0+\gamma_1}\right\}\exp(-s\gamma_0(x_3+h_T)) \quad (14)$$

$$\bar{u}^{t}(\alpha,\beta,x_3;s)=\frac{f(s)}{s}\left\{\frac{1}{\gamma_0+\gamma_1^L}\right\}\exp(-s(\gamma_1^2x_3-\gamma_0h_T)) \quad (15)$$

In which

$$\gamma_1^L=\sqrt{\gamma_1^2+\sigma\mu_0 s^{-1}} \qquad \Re e(|\gamma_1^2|)\geq 0 \quad (16)$$

$$\gamma_1=\sqrt{v_1^{-2}-\alpha^2-\beta^2} \quad (17)$$

We multiply both the numerator and denominator of (14) with $\gamma_0-\gamma_1^L$, resulting to the form

$$\bar{u}^{r}(\alpha,\beta,x_3;s)=\frac{-f(s)}{2s\gamma_0}\left(\frac{\gamma_0^2-\gamma_1^2}{\gamma_1^2-\gamma_0^2}\right)[1-\sigma\mu_0\left(\frac{(\gamma_1^2-\gamma_0^2)^{-1}-(\gamma_1^2+\gamma_0^2)^{-1}}{s+\sigma\mu_0(\gamma_1^2-\gamma_0^2)^{-1}}\right)]\exp(-s\gamma_0(x_3+h_T))+$$
$$\frac{f(s)}{\gamma_1^2}\left(\frac{\gamma_1^2}{s(\gamma_1^2-\gamma_0^2)}\right)[1-\sigma\mu_0\left(\frac{(\gamma_1^2-\gamma_0^2)^{-1}-\gamma_1^{-2})}{s+\sigma\mu_0\gamma_1^2-\gamma_0^2)^{-1}}\right)]\exp(-s\gamma_0(x_3+h_T)) \quad (18)$$

The factor γ_1/γ_1^L can be recognized as the Laplace transform, see Hladik (1941)

$$\gamma_1/\gamma_1^L=1-\int_{k=0}^{\infty}a(I_o(ak)-I_1(ak))\exp(-(a+s)k)dk \quad (19)$$

Where

$$a=\frac{\sigma\mu_o}{2\gamma_1^2} \quad (20)$$

And I_o , I_1 denote the modified Bessel functions of the Kind of order zero and one respectively, substitution of (19) in (18). Using the fact that $\gamma_1^2-\gamma_0^2=\mu_0(\varepsilon_1-\varepsilon_0)$ yields

$$\bar{u}^{r}(\alpha,\beta,x_3;s)=f(s)\{[\omega_1+\frac{\beta\omega_2}{s+\beta}]\frac{\exp(-s\gamma_0(x_3+h_r))}{2s\gamma_0}+\int_{k=0}^{\infty}[\omega_3+\frac{\beta\omega_4}{s+\beta}]\frac{\exp(-s(\gamma_0(x_3+h_r)+k))}{2s\gamma_0}dk\} \quad (21)$$

Where

$$\beta=\frac{\sigma}{\varepsilon_1-\varepsilon_0} \quad , \qquad \beta>0 \quad (22)$$

$$\omega_1(\alpha,\beta)=\frac{\gamma_0-\gamma_1}{\gamma_1+\gamma_0} \quad (23)$$

$$\omega_2(\alpha,\beta)=\frac{2\gamma_0}{\gamma_1}-\frac{\gamma_0-\gamma_1}{\gamma_1+\gamma_0}-1 \quad (24)$$

$$\omega_3(\alpha,\beta,k)=-\beta\frac{\gamma_0}{\gamma_1}[I_0(\frac{\sigma\mu_0 k}{2\gamma_1^2})-I_1(\frac{\sigma\mu_0 k}{2\gamma_1^2})]\exp(-\frac{\sigma\mu_0 k}{2\gamma_1^2}) \quad (25)$$

$$\omega_4(\alpha,\beta,k)=-\beta(\frac{\gamma_0}{\gamma_1})^3[I_0(\frac{\sigma\mu_0 k}{2\gamma_1^2})-I_1(\frac{\sigma\mu_0 k}{2\gamma_1^2})]\exp(-\frac{\sigma\mu_0 k}{2\gamma_1^2}) \quad (26)$$

In which it is to be noted that all the function ω_1, ω_2, ω_3, ω_4 are independent of the Laplace parameters. Applying the inverse Fourier transformation with respect to x_1, and x_2 to (21), we obtain the corresponding expression, which is of the form:

$$\bar{u}^{r}(r,s)=sf(s)\bar{g}(r,s) \quad (27)$$

In which

$$\bar{g}(r,s)=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int[\frac{\exp(-s(\alpha x_1+\beta x_2+\gamma_0(x_3+h_T)))}{2\gamma_0}[\omega_1+\frac{\beta\omega_2}{s+\beta}]+$$
$$\int_{k=0}^{\infty}[\omega_3+\frac{\beta\omega_4}{s+\beta}]\exp(-sk)dk]\,d\alpha d\beta \quad (28)$$

In the next section it is shown that the integrals at the right-hand side of (28) can be transformed into

$$\bar{g}(r,s)=\int_{\tau=\frac{R}{v_0}}^{\infty}\exp(-s\tau)g(r,t)d\tau \quad (29)$$

Where

$$R=\sqrt{x_1^2+x_2^2+(x_3+h_T)^2} \quad (30)$$

And only real values of τ occur in the integration and where R is the distance from the image of the magnetic dipole source to the point of observation. For the field $\mathbf{u}^{\mathbf{r}}$ we finally end up with

$$u^r(r,t) = \begin{cases} 0 & 0 < t < R/v_0 \\ \int_{R/v_0}^{t} \frac{\partial}{\partial t} f(t-\tau) g(r,\tau) d\tau & R/v_0 < t < \infty \end{cases}$$

4. Conclusion

A theoretical study for computing the electromagnetic field by a vertical magnetic dipole above conducting earth. The problem of communication in the sea has been considered by many writers, the propagation of radio waves in the sea is of great importance in many practical applications. Considerable effort and speculation have thus been devoted to establish the theoretical fundamental for such problem.

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