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# Approach to Two-Phase Flow in Gas Transmission Pipeline Network System

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## Abstract

Flow of natural gas in a gas pipeline is not entirely single phase flow situation. Even from the well or treatment plant the gas stream might carry along with it appreciable amount of dirts, debris, water or corrosion products from the walls of the pipes, resulting in two or three phase flow situation [1]. Oftentimes the gas might be transported above or below the earth surface or below the sea bed through places of diverse temperature gradients, giving rise to certain undesirable problems. Condensates and hydrates formation are such problems. The condensates so formed accumulate at low points in the pipes, thereby obstructing flow of gas. Hydrates do form around valves resulting in total blockage or freeze up of the valves. A worst case situation is total obstruction of flow. The end result is increased pressure drop along the pipeline. The approach in the two-phase flow analysis in gas transmission line is to separate the liquid phase from the dry gas phase and develop or deduce the governing expressions for friction factor. The friction factor for two phases so deduced can be injected into any appropriate flow equations to determine the various pressure drop components, overall pressure drop, line throughput or any other flow variables of interest. Two-phase flow analysis approach in this work has enabled the deduction of mathematical models that would be more precise and accurate in predicting flow situations.

**Keywords:** Mean Flow Velocity; Compressibility Factor; Friction Factor; Liquid Holdup; Gas Holdup; Liquid Velocity Gradient; Gas Velocity Gradient; Liquid and Gas Acceleration Gradient; No Slip; Equivalent length; Two Phase; Pipeline Efficiency; Equilibrium Constant.

## 1. Introduction

Natural gas from oil wells or Gas/condensate wells or associated gas from oil wells separates into two-phase on entering surface facilities by virtue of temperature changes and pressure drops [1, 2, 3]. Hence flow of natural gas in gas pipeline networks is not entirely a single-phase flow situation. Condensates formation hydrates formation, particulates in the gas stream and particulate erosion might result in two or three phase flow conditions.

Deviations from single-phase flow situation may lead to increased pressure drop along the line. This may also reduce the throughput of the line per unit time. The cost of investment on equipment such as compressors, pumps, line valves, line pipes, separators, manifolds, etc would also be higher. The reason being that sizing these facilities is pressure and flow rates dependent. Even the effectiveness of calibration of a flow line depends on accurate prediction and determination of pressure drops and other flow variables. It is worthy to note that Flow-pressure conditions in a pipeline are determined by parameters which form the basis for economic design of the pipeline network, cost of investment in pipeline assets and facilities, efficient utilization and deployment of assets, and cost of construction, maintenance and operation.

Formation of condensates is one of such problems. Condensates are formed when the temperature of the gas flowing is less than the hydrocarbons and water dew points temperatures. The condensates so formed accumulate at low points in the pipes, thereby obstructing flow of gas. The end result is increased pressure drop along the pipeline. Condensates that are formed react with pipe wall materials to form corrosion products. This way the pipe walls are ultimately eaten up leading to line depressurization. All these factors will culminate in the reduction of the pipeline efficiency. More so, the cost of operation and maintenance of the line vis-à-vis the cost of investment on equipment will be higher.

To understand and tackle single phase or two phase flow problems many correlations have been developed. In recent time, the flow correlation equations used in gas transmission industries are Weymouth, Panhandle A and Modified Panandle B equations [4]. The flow friction factor for any particular situation is parameter of great concern, since it ultimately governs the overall line pressure drop. Many implicit and explicit correlations have been formulated for flow friction faction. Such correlations are that of Drew, Koo and McAdams [5]; Nikuradse [6]; Gwo and Ghalamor [3]; Colebrook [7] and Jain [8] among others. These correlations are applicable to smooth or rough walled pipes in the turbulent flow regions.

## 2 Purpose and Significance

The wall of pipe conducting fluid stream might be smooth or rough. The friction factor correlations to handle the pipe conditions are as spelt out in the text. As required flow friction factor might be for single phase or two phase flow situation. In two phase flow friction factor could only be determine after analyzing flow to separate the liquid holdup and gas holdup. The adequate account of the two phase flow friction factor can be gotten. The value the friction can be used in all pressure and flow capacity related problems. This is to enable adequate determination of the driving force or power required to conduct or transport a fluid stream.

## 3 Mathematical Model for Two-Phase Flow

### 3.1 Deduction of Friction Factor for Two Phase Flow

In real flow situation, flow of gas through a piping network is not entirely single-phase flow problem. Condensation of gas when dew point is attained often result in two-phase flow regime. In a two phase flow situation the main controlling factor is the flow friction factor, f. In gas pipeline let $R_L$, be the volume fraction of the liquid in the pipeline in the form of condensates and water vapour and $R_G$ the volume fraction of dry gas in the pipeline. Let $Q_L$ and $Q_G$ be the volumetric flow rates of the liquid and gas in the pipeline. The mixture flow rate, $Q_M$ is defined as [5]:

$$Q_M=Q_L+Q_G \tag{1}$$

Expressing the phase velocity in terms of the known volumetric flow rates, $Q_L$ and $Q_G$ and the pipe area of cross section, $A_P$.

*The average of liquid flowing,*

$$\overline{V}_L=\frac{Q_L}{\overline{R}_L A_P} \tag{2}$$

*The avarage velocity of gas flowing,*

$$\overline{V}_G=\frac{Q_G}{\overline{R}_G A_P} \tag{3}$$

*If* $\overline{V}_M$ *is the mixture velocity,*

$$V_M=\frac{Q_L+Q_G}{A_P} \tag{4}$$

The volume fraction of the gas flowing, λ is expressed as:

$$\lambda=\frac{Q_G}{Q_M} \tag{5}$$

Flow Reynolds number,

$$\mathrm{Re}_{TP}=DV_M\left[\frac{\rho_L\frac{(1-\lambda)^2}{\overline{R}_L}+\rho_G\frac{\lambda^2}{\overline{R}_G}C_1}{\mu_L(1-\lambda)+\mu_G\lambda C_2}\right] \tag{6}$$

$$C_1=\frac{R_G}{R_L}\cdot\frac{\overline{R}_L}{\overline{R}_G}\cdot\frac{\overline{V}_L^2}{\overline{V}_G^2}\cdot\frac{V_G}{V_L}\cdot\frac{\partial V_G/\partial Z}{\partial V_L/\partial Z} \tag{7}$$

$$C_2=\frac{R_G}{R_L}\cdot\frac{\overline{R}_L}{\overline{R}_G}\cdot\frac{\overline{V}_L}{\overline{V}_G}\cdot\frac{\partial^2 V_G/\partial n^2}{\partial^2 V_L/\partial n^2} \tag{8}$$

$\overline{\overline{R}}_L, \overline{R}_G$ – –*Volume fraction of liquid and gas in the pipeline averaged over entire length of the pipe*

$V_L, V_G$ – Liquid and gas local velocities
$\overline{V}_L, \overline{V}_G$ – Liquid and gas average velocities
$\partial V_L / \partial Z,\ \partial V_G / \partial Z$ – Liquid and gas velocity gradients along the pipes'axis
$\partial^2 V_L / \partial n^2,\ \partial^2 V_G / \partial n^2$ – Liquid and gas acceleration gradient perpendicular to the axis of the pipe

Reynolds number for two phase flow, $\mathrm{Re}_{TP}$:

$$\mathrm{Re}_{TP} = \frac{D\overline{V}_M \rho_{TP}}{\mu_{TP}} \tag{9}$$

*Where,*

$$\rho_{TP} = \rho_L \frac{(1-\lambda)^2}{R_L} + \rho_G \frac{\lambda^2}{\overline{R}_G} C_1 \tag{10}$$

$$\mu_{TP} = \mu_L (1-\lambda) + \mu_G (\lambda) C_2 \tag{11}$$

$\rho_{TP}$ – *Density for two phase flow*
$\mu_{TP}$ – *Absolute viscosity for two phase flow*

The C terms in equations 10 and 11 can be determined subject to the following conditions:

No Slip or Homogeneous Flow

For this condition to hold the flow velocity must be very high. Since the flow is homogeneous:

$$V_L = V_G,\ \overline{V}_L = \overline{V}_G,\ R_L = \overline{R}_L,\ R_G = \overline{R}_G$$

Velocities change for the gas and liquid in the two coordinates Z and n must be the same, since they have the same local and absolute velocities.

$$\frac{\partial V_G}{\partial Z} = \frac{\partial V_L}{\partial Z}, \qquad \frac{\partial^2 V_G}{\partial n^2} = \frac{\partial V_L}{\partial n^2}$$

Substituting these conditions in equations 7 and 8

$$C_1 = C_2 = 1 \tag{12}$$

$$1 - \lambda = \overline{R}_L \ \ and \ \ \lambda = \overline{R}_G \tag{13}$$

Substituting equations 12 and 13 in equations 10 to 11

$$\mathrm{Re}_{NS} = \frac{D\bar{V}_M \rho_{NS}}{\mu_{NS}} \tag{14}$$

$$\rho_{NS} = \rho_L(1-\lambda) + \rho_G\lambda \tag{15}$$

$$\mu_{NS} = \mu_L(1-\lambda) + \mu_G\lambda \tag{16}$$

$\mathrm{Re}_{NS}$ – Reynolds number at no slip condition

$\rho_{NS}$ – Density at no slip condition

$\mu_{NS}$ – Absolute viscosity no slip condition

Friction factor for two phase flow, $f_{TP}$, is deduced as follows:

$$\frac{f_{TP}}{f_0} = 1 + \frac{-(\ln\lambda)}{S} \tag{17}$$

$$f_0 = 0.00140 + \frac{0.125}{\mathrm{Re}_{TP}^{0.32}} = 0.00140 + \frac{0.125}{(\rho_{NS}\bar{V}_M D/\mu_{NS})^{0.32}}$$

$$S = 1.281 - 0.478(-\ln\lambda) + 0.444(-\ln\lambda)^2 - 0.94(-\ln\lambda)^3 + 0.0843(-\ln\lambda)^4$$

$$\textit{From equation } 13,\ \lambda = 1 - R_L$$

$f_0$ is the friction factor at $\lambda=1$, that is, the value of "f "for single phase flow. So, with reference to equations 15 and 16,

$\rho_{NS}=\rho_G$ and $\mu_{NS}=\mu_G$

$$f_0 = 0.00140 + \frac{0.125}{(\rho_G\bar{V}_M D/\mu_G)^{0.32}} \tag{18}$$

$$f_{TP} = \left[0.00140 + \frac{0.125}{(\rho_G\bar{V}_M D/\mu_G)^{0.32}}\right] \times \left[1 + \frac{-\ln(1-R_L)}{S}\right] \tag{19}$$

Where:

$f_{TP}$—friction factor for two phase flow

### 3.2 Expressions for Certain Critical Parameters of Hydrocarbon Mixtures

Natural gas is not unique in composition; the apparent molecular weight of the gas on the basis of the mole fractions of the different constituents is expressed as [2]:

$$M_a = \sum_{i=1}^{N} n_i M_i \tag{20}$$

The gaseous mixture specific gravity is given as:

$$\gamma_G = \frac{M_a}{M_{air}} \tag{21}$$

In terms of the pseudo reduced properties, the critical pressure of the gaseous mixture is expressed as:

$$P_c = \sum_{i=1}^{N} n_i P_{Ci} \tag{22}$$

The critical temperature of the mixture is given as:

$$T_C = \sum_{i=1}^{N} n_i T_{Ci} \tag{23}$$

The reduced pressure and temperature are:

$$P_r = \frac{P}{P_C} \tag{24}$$

$$T_r = \frac{T}{T_C} \tag{25}$$

With reference to the generalized compressibility chart, using the results of equations 24 and 25, the compressibility factor for the gas can be evaluated.

Average gas pressure, P is:

$$P = \frac{2}{3}\left(\frac{P_1^3 - P_2^3}{P_1^2 - P_2^2}\right) \tag{26}$$

The absolute gas viscosity for the mixture of the hydrocarbon constituents of natural gas is expressed:

$$\mu_{GHC} = 1.02247 \times 10^{-5} \begin{bmatrix} 8.188 \times 10^{-3} - 6.15 \times 10^{-3} \log(\gamma_G) \\ + (1.709 \times 10^{-5} - 2.062 \times 10^{-6} \gamma_G)(1.8T + 0.27) \end{bmatrix} \tag{27}$$

The absolute viscosity of nitrogen is given as:

$$\mu_{GN_2} = 1.02247 \times 10^{-5} \left[9.59 \times 10^{-3} + 8.48 \times 10^{-3} \log(\gamma_G)\right] n_{N_2} \tag{28}$$

The absolute viscosity of carbon dioxide expressed as:

$$\mu_{GCO_2} = 1.02247 \times 10^{-5} \left[6.24 \times 10^{-3} + 9.08 \times 10^{-3} \log(\gamma_G)\right] n_{CO_2} \tag{29}$$

Absolute viscosity of hydrogen sulphide is given as:

$$\mu_{GH_2S} = 1.02247\times10^{-5}\left[3.73\times10^{-3} + 8.49\times10^{-3}\log(\gamma)\right]n_{H_2S} \qquad (30)$$

The viscosity of the gaseous mixture is:

$$\mu_G = \mu_{GHC} + \mu_{GN_2} + \mu_{GCO_2} + \mu_{GH_2S} \qquad (31)$$

Gas density is given as [6]:

$$\rho_G = \frac{P}{ZRT} \qquad (32)$$

### 3.3 Liquid Holdup and Dew Point of Hydrocarbon Mixture

Two-phase flow friction factor is expressed is expressed as in equation 19. There is need to evaluate the volume fraction, $R_L$, of liquid and condensates as well as the volume fraction, $R_G$, of gas in the line.

The relationship between the amount of gas flowing in equilibrium with the liquid and condensate flowing is expressed as:

$$y_i = K_i x_i \qquad (33)$$
$$x_i = y_i / K_i$$

$K_i$—equilibrium constant

$x_i$—mole fraction of the liquid components

$y_i$—mole fraction of the gaseous components

The equilibrium constant, K [9]$_i$, for each of the hydrocarbon components is determined at the bulk temperature of the flowing fluid stream using the chart of equilibrium constants for hydrocarbon at low temperature range. Liquid holdup, $R_L$, in the line is given by the expression:

$$R_L = \sum_i^N x_i \qquad (34)$$

Gas holdup, $R_G$, is given as:

$$R_G = 1 - R_L \qquad (35)$$

Employing the chart of equilibrium constants for hydrocarbons at low temperature range [9], the dew point of the hydrocarbon mixture is obtained by trial and error. For a particular set of equilibrium constants at a particular temperature, mole fraction, $x_i$, of the liquid components are determined. At a particular temperature where the liquid holdup, $R_L$, as expressed by equation 34 is one, that temperature is the exact dew point of the hydrocarbon mixture. At this temperature, the liquid and gas components in the stream are in a state of equilibrium.

The line flow capacity can be determined employing any of the common gas flow equations namely:

Weymouth equation

$$Q = K\left(\frac{T_b}{P_b}\right)\sqrt{\frac{1}{f_{TP}}}\left[\frac{P_1^2 - P_2^2}{\gamma_G \bar{T} Z L}\right]^{0.5} D^{2/3} E \qquad (36)$$

$$K = 78.85 m^{8/6}\ {}^{\mathrm{o}}K^{-1/2} s^{-1}$$

*Substituting equation* 19 *in* 36

$$Q = K\left(\frac{T_b}{P_b}\right)\sqrt{\frac{1}{\left[0.00140 + \frac{0.125}{\left(\rho_G \bar{V}_M D/\mu_G\right)^{0.32}}\right] \times \left[1 + \frac{-\ln\left(1 - R_L\right)}{S}\right]}}\left[\frac{P_1^2 - P_2^2}{\gamma_G \bar{T} Z L}\right]^{0.5} D^{2/3} E \qquad (37)$$

If the liquid holdup R$_L$=0, a condition applicable to single phase flow of gas through the network system, then:

$$Q = K\left(\frac{T_b}{P_b}\right)\sqrt{\frac{1}{\left[0.00140 + \frac{0.125}{\left(\rho_G \bar{V}_M D/\mu_G\right)^{0.32}}\right]}}\left[\frac{P_1^2 - P_2^2}{\gamma_G \bar{T} Z L}\right]^{0.5} D^{2/3} E \qquad (38)$$

Panhandle A Equation

$$Q = K\left(\frac{T_b}{P_b}\right)^{1.788}\left(\frac{1}{f_{TP}}\right)^{0.5394}\left[\frac{P_1^2 - P_2^2}{\bar{T} Z L}\right]^{0.5394}\left(\frac{1}{\gamma_G}\right)^{0.4606} D^{2.6182} E \qquad (39)$$

$$K = 82.81 m^{0.9}\ {}^{\mathrm{o}}K^{-0.54} s^{-1}$$

Substituting equation 19 in 39

$$Q = K\left(\frac{T_b}{P_b}\right)^{1.788}\left(\frac{1}{\left[0.00140 + \frac{0.125}{\left(\rho_G \bar{V}_M D/\mu_G\right)^{0.32}}\right] \times \left[1 + \frac{-\ln\left(1 - R_L\right)}{S}\right].}\right)^{0.5394}\left[\frac{P_1^2 - P_2^2}{\bar{T} Z L}\right]^{0.5394}\left(\frac{1}{\gamma_G}\right)^{0.4606} D^{2.6182} E \qquad (40)$$

Subject to the condition that the liquid holdup R$_L$=0,

$$Q = K\left(\frac{T_b}{P_b}\right)^{1.788}\left(\frac{1}{\left[0.00140 + \frac{0.125}{\left(\rho_G \bar{V}_M D/\mu_G\right)^{0.32}}\right]}\right)^{0.5394}\left[\frac{P_1^2 - P_2^2}{\bar{T} Z L}\right]^{0.5394}\left(\frac{1}{\gamma_G}\right)^{0.4606} D^{2.6182} E \qquad (41)$$

Modified Panhandle B Equation

$$Q = K\left(\frac{T_b}{P_b}\right)^{1.02}\left(\frac{1}{f_{TP}}\right)^{0.51}\left[\frac{P_1^2 - P_2^2}{\bar{T}ZL\gamma_G^{\ 0.961}}\right]^{0.51} D^{2.53}E \qquad (42)$$

$$K = 45.03m^{-0.04}\ {}^{\otimes}K^{-0.49}$$

Substituting equation 19 in 42

$$Q = K\left(\frac{T_b}{P_b}\right)^{1.02}\left(\frac{1}{\left[0.00140 + \frac{0.125}{\left(\rho_G\bar{V}_M D/\mu_G\right)^{0.32}}\right]\times\left[1 + \frac{-\ln\left(1 - R_L\right)}{S}\right]}\right)^{0.51}\left[\frac{P_1^2 - P_2^2}{\bar{T}ZL\gamma_G^{\ 0.961}}\right]^{0.51} D^{2.53}E \qquad (43)$$

If the liquid holdup $R_L$=0,

$$Q = K\left(\frac{T_b}{P_b}\right)^{1.02}\left(\frac{1}{\left[0.00140 + \frac{0.125}{\left(\rho_G\bar{V}_M D/\mu_G\right)^{0.32}}\right]}\right)^{0.51}\left[\frac{P_1^2 - P_2^2}{\bar{T}ZL\gamma_G^{\ 0.961}}\right]^{0.51} D^{2.53}E \qquad (44)$$

## 4 Model Application

The set of mathematical models in equations 1 to 38 can be applied to single and two phase of gas in gas transmission lines. To these effects the line flow throughput could be monitored closely even the compression power of the line for continuous transmission of gas.

## 5 Recommendation for Future Research

Computer simulation of the mathematical models should be written to determine suitability of any particular flow equations (36 to 38) subject to certain flow conditions. The friction factor, dew point temperature, pressure drop components, overall pressure drop and flow capacity of complex hydrocarbons mixtures should be computed for different line configurations.

## 6 Conclusions

Previous studies adequately indicated that gas transmission in pipelines is not entirely single phase flow situation; three phase flow problems might also be prevalent. Accurate determination of the flow friction factor is a governing factor fur exact determination of the line pressure and the line throughput to ascertain the driving force of flowing fluid stream. The liquid and gas holdups $R_L$ and $R_G$ of an hydrocarbon mixture could be computed to enable the determination of two phase flow friction factor. The various pressure drop components could be calculated to precisely determine the flow capacity of a line. Ultimately this will facilitate the precision of designing and constructing, operation and maintenance of natural gas piping network.

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