

Generalised Ratio Estimators Using Conventional Location Parameters in Survey Sampling

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Abstract: The present paper concentrates on estimating the finite population mean by proposing the new generalised ratio type estimators in simple random sampling without replacement using linear combination of coefficient of variation and population deciles of auxiliary variable. The expressions for mean square error and bias were calculated and compared with the classical and existing estimators. Theoretical results are supported by numerical illustration.

Keywords: Coefficient of variation; deciles; ratio-type estimators; mean square error; bias; efficiency.

1. INTRODUCTION

History of using auxiliary information in sample surveys is as old as the history of the applications of survey sampling itself. The classical ratio estimator was given by Cochran (1940) for estimating population mean of a study variable which are based on some Prior information of corresponding population mean of an Auxiliary variable are well known in the sample survey. Consider a finite population $U = \{U_1, U_2, U_3, ..., U_N\}$ of N distinct and identifiable units. Let Y be the study variable with value Y_i measured of U_i, $i = 1,2,3,..., N$ giving a vector $Y = \{Y_1, Y_2, Y_3, ..., Y_N\}$. The objective is to estimate population mean $\overline{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$ on the basis of a random sample, the mean ratio estimator for estimating the Population mean, $\overline{Y}$, of the study variable Y is defined as

$$\hat{\overline{Y}}_r = \frac{\overline{y}}{\overline{x}}\overline{X}$$

The bias, related constant and the mean squared error (MSE) of the ratio estimator are respectively given by

$$B(\hat{\overline{Y}}_r) = \frac{(1-f)}{n}\frac{1}{\overline{X}}(RS_x^2 - \rho S_x S_Y) \quad R = \frac{\overline{Y}}{\overline{X}} \quad MSE(\hat{\overline{Y}}_r) = \frac{1-f}{n}(S_y^2 + R^2 S_x^2 - 2R\rho S_x S_Y)$$

Various authors such as Sisodia & Dwivedi (1981), Singh et al. (2004) and Subramani and Kumarpandiyan, (2012 a, b and c) utilized coefficient of variation of the auxiliary variable and other conventional location parameters as supplementary information in order to estimate the finite population mean of the study variable and the significance of the present paper stresses on making such new generalised ratio type estimators in the sampling literature in order to get estimates with more precision than the already existing estimators in the literature.

2. Existing Ratio Estimators

Kadilar and Cingi (2004) suggested below ratio type estimators for the population mean in the simple random sampling using some known auxiliary information on coefficient of kurtosis and coefficient of variation.

$$\hat{\bar{Y}}_1 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{\bar{x}}\bar{X}, \qquad \hat{\bar{Y}}_2 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+C_x)}(\bar{X}+C_x), \qquad \hat{\bar{Y}}_3 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\beta_2)}(\bar{X}+\beta_2),$$

$$\hat{\bar{Y}}_4 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+C_x)}(\bar{X}\beta_2+C_x), \qquad \hat{\bar{Y}}_5 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\beta_2)}(\bar{X}C_x+\beta_2),$$

Kadilar and Cingi (2006) developed some modified ratio estimators using known value of coefficient of correlation, kurtosis and coefficient of variation as follows:

$$\hat{\bar{Y}}_6 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\rho)}(\bar{X}+\rho), \quad \hat{\bar{Y}}_7 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\rho)}(\bar{X}C_x+\rho), \quad \hat{\bar{Y}}_8 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+C_x)}(\bar{X}\rho+C_x),$$

$$\hat{\bar{Y}}_9 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+\rho)}(\bar{X}\beta_2+\rho), \qquad \hat{\bar{Y}}_{10} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\beta_2)}(\bar{X}\rho+\beta_2).$$

3. Proposed Modified Ratio Estimator

Motivated by the mentioned estimators in Section 2, we propose new class of efficient ratio type estimators using the linear combination of coefficient of variation and population deciles.

$$\hat{\bar{Y}}_{p1} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+D_1)}(\bar{X}C_x+D_1). \qquad \hat{\bar{Y}}_{p2} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+D_2)}(\bar{X}C_x+D_2).$$

$$\hat{\bar{Y}}_{p3} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+D_3)}(\bar{X}C_x+D_3). \qquad \hat{\bar{Y}}_{p4} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+D_4)}(\bar{X}C_x+D_4).$$

$$\hat{\bar{Y}}_{p5} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+D_5)}(\bar{X}C_x+D_5). \qquad \hat{\bar{Y}}_{p6} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+D_6)}(\bar{X}C_x+D_6).$$

$$\hat{\bar{Y}}_{p7} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+D_7)}(\bar{X}C_x+D_7). \qquad \hat{\bar{Y}}_{p8} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+D_8)}(\bar{X}C_x+D_8).$$

$$\hat{\bar{Y}}_{p9} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+D_9)}(\bar{X}C_x+D_9). \qquad \hat{\bar{Y}}_{p10} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+D_{10})}(\bar{X}C_x+D_{10}).$$

The bias, related constant and the MSE for proposed estimator can be obtained as follows:

$$B(\hat{\bar{Y}}_{pj}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_j^2, \qquad R_j = \frac{\bar{X}C_x}{\bar{X}C_x+D_j} \qquad MSE(\hat{\bar{Y}}_{pj}) = \frac{(1-f)}{n}(R_j^2 S_x^2 + S_y^2(1-\rho^2)).$$

$j = 1,2,...,10$

4. Efficiency Comparisons

From the expressions of the MSE of the proposed estimators and the existing estimators, we have derived the conditions for which the proposed estimators are more efficient than the existing modified ratio estimators as follows:

$$MSE(\hat{\bar{Y}}_{pj}) \leq MSE(\hat{\bar{Y}}_i),$$

$$\frac{(1-f)}{n}(R_{pj}^2 S_x^2 + S_y^2(1-\rho^2)) \leq \frac{(1-f)}{n}(R_i^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$R_{pj}^2 S_x^2 \leq R_i^2 S_x^2,$$

$$R_{pj} \leq R_i,$$

Where $j = 1,2,....,10$ and $i = 1,2,....,10$

5. Empirical Study

The Population is taken from Singh and Chaudhary (1986).

$N = 34$ $n = 20$ $\bar{Y} = 856.4117$ $\bar{X} = 199.4412$ $\rho = 0.4453$

$S_y = 733.1407$ $C_y = 0.8561$ $S_x = 150.2150$ $C_x = 0.7531$

$\beta_2 = 1.0445$ $D_1 = 60.6000$ $D_2 = 83.0000$ $D_3 = 102.7000$

$D_4 = 111.2000$ $D_5 = 142.5000$ $D_6 = 210.2000$ $D_7 =$ 264.5000 $D_8 = 304.4000$ $D_9 = 373.2000$ $D_{10} = 634.0000$

Constants, Bias and MSE of the existing and the proposed estimators

Estimators	Constant	Bias	MSE
$\hat{\bar{Y}}_1$	4.294	10.002	17437.65
$\hat{\bar{Y}}_2$	4.278	9.927	17373.31
$\hat{\bar{Y}}_3$	4.272	9.898	17348.62
$\hat{\bar{Y}}_4$	4.279	9.930	17376.04
$\hat{\bar{Y}}_5$	4.264	9.865	17319.75
$\hat{\bar{Y}}_6$	4.285	9.957	17399.52
$\hat{\bar{Y}}_7$	4.281	9.943	17387.08

$\hat{\bar{Y}}_8$	4.258	9.834	17294.19
$\hat{\bar{Y}}_9$	4.285	9.960	17401.14
$\hat{\bar{Y}}_{10}$	4.244	9.771	17239.66
$\hat{\bar{Y}}_{p1}$	**0.712522584**	**0.274217484**	**9068.584621**
$\hat{\bar{Y}}_{p2}$	**0.644081062**	**0.224067533**	**9025.635617**
$\hat{\bar{Y}}_{p3}$	**0.593909300**	**0.190518926**	**8996.904197**
$\hat{\bar{Y}}_{p4}$	**0.574596962**	**0.178330047**	**8986.465498**
$\hat{\bar{Y}}_{p5}$	**0.513152015**	**0.142229587**	**8955.548642**
$\hat{\bar{Y}}_{p6}$	**0.416757809**	**0.093813505**	**8914.084543**
$\hat{\bar{Y}}_{p7}$	**0.362188254**	**0.070854362**	**8894.422065**
$\hat{\bar{Y}}_{p8}$	**0.330399126**	**0.058962467**	**8884.237707**
$\hat{\bar{Y}}_{p9}$	**0.286968678**	**0.044480213**	**8871.834935**
$\hat{\bar{Y}}_{p10}$	**0.191531914**	**0.019814365**	**8850.710814**

Percentage relative efficiency (PRE) of the proposed estimator with existing estimators

	$\hat{\bar{Y}}_{p1}$	$\hat{\bar{Y}}_{p2}$	$\hat{\bar{Y}}_{p3}$	$\hat{\bar{Y}}_{p4}$	$\hat{\bar{Y}}_{p5}$	$\hat{\bar{Y}}_{p6}$	$\hat{\bar{Y}}_{p7}$	$\hat{\bar{Y}}_{p8}$	$\hat{\bar{Y}}_{p9}$	$\hat{\bar{Y}}_{p10}$
$\hat{\bar{Y}}_1$	192.2863	193.2013	193.8183	194.0434	194.7133	195.6190	196.0515	196.2762	196.5506	197.0197
$\hat{\bar{Y}}_2$	191.5768	192.4884	193.1032	193.3275	193.9949	194.8972	195.3281	195.5520	195.8254	196.2928
$\hat{\bar{Y}}_3$	191.3046	192.2149	192.8287	193.0527	193.7192	194.6203	195.0505	195.2741	195.5471	196.0138
$\hat{\bar{Y}}_4$	191.6069	192.5187	193.1335	193.3578	194.0254	194.9279	195.3588	195.5827	195.8562	196.3236
$\hat{\bar{Y}}_5$	190.9862	191.8950	192.5078	192.7315	193.3968	194.2964	194.7259	194.9491	195.2217	195.6876
$\hat{\bar{Y}}_6$	191.8658	192.7788	193.3945	193.6191	194.2875	195.1913	195.6228	195.8470	196.1208	196.5889
$\hat{\bar{Y}}_7$	191.7287	192.6410	193.2562	193.4807	194.1486	195.0517	195.4829	195.7070	195.9806	196.4484

$\hat{\bar{Y}}_8$	190.7044	191.6118	192.2237	192.4470	193.1114	194.0097	194.4386	194.6614	194.9336	195.3988
$\hat{\bar{Y}}_9$	191.8837	192.7968	193.4125	193.6371	194.3056	195.2095	195.6410	195.8653	196.1391	196.6072
$\hat{\bar{Y}}_{10}$	190.1030	191.0077	191.6176	191.8402	192.5025	193.3979	193.8255	194.0477	194.3189	194.7827

Conclusion

From the above empirical study we found that our proposed estimators are more efficient than the classical and existing estimators as their MSE and bias is lower than classical and existing estimators and also by percent relative efficiency (PRE) criterion. Hence, thus preferred for practical applications.

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