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Relation proximal point with some dynamical properties

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Abstract :

In this paper we discussed relation proximal points with many of dynamical properties through studied topological transformation group , and it will given necessary condition for proximal relation to be minimal set ,and introduce new define replete set and semi-replete set by using concept of the replete set and semi-replete set as well as we introduce that many of new relations and theorem.

Key words: Proximal point, replete proximal point, syndetic set, semi-replete set, minimal set, almost periodic point .

Introduction :

The first has been introduced the proximal point in topological transformation group (Z, T, π) with compact hausdorff Z was Ellis R. and Gottschalk W.[4][5].We show relation proximal point with some dynamical properties (fixed point-minimal point-almost periodic),also we studied replete proximal and semi-replete proximal independed on replete set and semi-replete set ,and it will be given a necessary and sufficient condition for proximal and semi-replete proximal to be syndetic set ,and we studied the image of proximal points under epimomorphism ,we give some theorem about proximal points .We use symbol Δ to indication the end.

1.Preliminaries:

In this section we given important concepts that we needed in this work.

Definition (1-1) [4]:

A topological group is a set T with two structures:

1. T is a group
2. T is a topological space

Definition (1-2)[4]:

A subset ρ of T is said to be $\{left\}, \{right\}$ syndetic in T if and only if there exists a compact subset k of T such that $\rho k = k\rho = T$.

Definition (1-3) [4]

Let (Z, T) be topological group :

1. A subset w of T is said to be replete if for each compact set β of T there exist $t_1, t_2 \in T$ such that $t_1 \beta t_2 \subset w$.
2. A subset w of T is said to be semi-replete if for each compact set β of T there exist $t \in T$ such that $\beta t \subset w$.

Definition (1-4)[4]:

A right topological transformation group is a triple (Z, T, π) where Z is a topological space called the phase space , T is a topological group called the phase group $\pi: Z \times T \to Z, \pi(z, t) \to xt$ is a continuous mapping such that :

1. $ze = z$ $(z \in Z)$,where e is the identity of T
2. $(zt)b = z(tb)$ $(z \in Z, z, b \in T)$

Definition (1-5) [3]:

Let (Z, T, π)be a topological transformation group

1. A subset A ⊂Z is said to be invariant set if $AT = A$.
2. A non-empty closed invariant set A ⊂Z is said to be minimal set if it contains no non-empty ,proper, closed invariant subset .

Definition (1-6)[4]

Let (Z, T, π)be a topological transformation group ,then the set $zT = \{zt: t \in T\}$is called the orbit of z and the set $\overline{zT}$ the orbit closure of z.

Definition (1-7)

Let (Z, T, π) be a topological transformation group is said to be strongly effective if for each $z \in Z, t = e$ there exist $t \in T$ such that $zt = z$.

Definition (1-8) [5]

Let (Z, T, π) be a topological transformation group:

1. Let $z \in Z$, is said to be Fixed point under T if $zT = z$.
2. Let $z \in Z$, is said to be minimal points if closure orbit is minimal set.

3. Let $z \in Z$, is said to be almost periodic points if for each invariant neighborhood V of z there exist syndetic subset A of T such that $zA \subset V$.

Definition (1-9) [5]

Let (Z, T, π) and (H, G, σ) be a topological transformation group, and $\tau: Z \to H$ be continuous ,$\gamma: T \to G$ be continuous homomorphism then $(\tau, \gamma): (Z, T, \pi) \to (H, G, \sigma)$ is said to be homomorphism and $((z, t)\pi)\tau = \big((\mathrm{z})\tau, (t)\gamma\big)\sigma$.If for each τ, γ onto then homomorphism is said to be epimorphism .

Theorem (1- 10):

Let (Z, T, π)be a topological transformation group, wg be replete subset of T if and only if w is replete set.

Proof: Let wg be replete subset of T then for each compact subset β of T there exist $t_1, t_2 \in T$ such that $t_1 \beta t_2 \subset wg$, since T group there exist $g^{-1} \in T \in$ such that $t_1 \beta t = t_1 \beta t_2 g^{-1} \subset w$ for some $t \in T$ thus w is replete set. Same method proof part other.

Remark (1- 11)

Let (Z, T, π)be a topological transformation group , the following statements are valid.

1. if K compact subset of T then gK compact set for some $g \in T$.
2. if A syndetic subset of T then gA syndetic set for some $g \in T$.

Theorem (2-12)

Let (Z, T, π) be a topological transformation group , $(n, m) \in P$, then (Z, T, π) strongly effective.

Proof: Let n and m are proximal points then for each index φ in Z there exist a $t \in T$ such that $(n, m)t \in \varphi$,it is enough to show that $t = e$,since φ be invariant then $(n, m)t \in \varphi t$ by hypothesis there exist $t^{-1} \in T$,it follows that $(n, m)e \in \varphi$, and $(n, m)t \cap (n, m)e \neq \emptyset$ this lead to $(n, m)t \subset (n, m)e$ and $(nt, mt) = (ne, me)$.
Clearly $t = e$ therefore (Z, T, π) strongly effective. Δ

2.Main results

In this section , we introduce proximal point in topological transformation group and show that relation proximal point with dynamical properties

Definition (2-1) [1,2]

Let (Z,T,π)be a topological transformation group ,a two points n and m of Z are called proximal proved that for each index φ in Z there exist a $t \in T$ such that $(n,m)t \in \varphi$.The set of all proximal pairs are called the proximal relation and are denoted by $P(Z,T)$ or simply P.

Definition (2-2)

Let (Z,T,π)be a topological transformation group ,a two points n and m of Z are called replete proximal proved that for each index φ in Z there exist a a replete subset w of T such that $(n,m)w \subset \varphi$.The set of all replete proximal pairs are called the replete proximal relation and are denoted by $RP(Z,T)$ or simply RP.

Definition (2-3)

Let (Z,T,π)be a topological transformation group ,a two points n and m of Z are called semi-replete proximal proved that for each index φ in Z there exist a a semi-replete subset v of T such that $(n,m)v \subset \varphi$.The set of all semi-replete proximal pairs are called the semi-replete proximal relation and are denoted by $SRP(Z,T)$ or simply SRP.

Remark (2- 4)

Let (Z,T,π)be a topological transformation group then:

1. P(X,T) is invariant set.
2. P(X,T) is close set .

Theorem (2-5)

Let (Z,T,π) be a topological transformation abelain group,(n,m) are replete proximal points then φ is invariant .

Proof: We may assume that T be replete group ,let n and m are replete proximal points then for each index φ in Z there exist a replete subset w of T such that $(n,m)w \subset \varphi$ and $(n,m)wT \subset \varphi T$ so $(n,m)wT \cap (n,m)w \neq \emptyset$ thus $\varphi T \subset (n,m)w \subset \varphi$ since $\varphi e \subset \varphi T$,thus φ is invariant under T.

Theorem (2- 6)

Let (Z,T,π)be a topological transformation abelain group,(n,m) are proximal points if and only if (n,m) are replete proximal points.

Proof: Suppose that β be compact subset of T , since n and m are proximal points then for each index φ in Z there exist a $t \in T$ such that $(n,m)t \in \varphi$, by hypothesis there exist identity e in T such that $\beta e \subset T$ so $\beta g g^{-1} \subset T$ for all $g \in T$, $\beta g \subset Tg \subset T$ and $g_1 \beta g \subset g_1 T \subset T$ for all $g_1 \in T$ .Therefore T be replete and n and m are replete proximal points. Conversely let n and m are replete proximal points ,then for each index φ in Z there exist a replete subset w of T such that $(n,m)w \subset \varphi$, since w replete subset of T then for each compact set β of T there exist $t_1, t_2 \in T$ such that $t_1 \beta t_2 \subset w$, $(n,m)t_1 \beta t_2 \subset (n,m)w \subset \varphi$ so $(n,m)t_1 \beta t_2 T \subset \varphi T$ by theorem (2-5) we obtain $(n,m)t_1 \beta t_2 T \subset \varphi$ by hypothesis $(n,m)\beta t_1 t_2 T \cap (n,m)\beta T \neq \emptyset, thus$ $(n,m)\beta T \subset \varphi$ since T syndetic set and β compact set then $(n,m)T \subset \varphi$. Δ

Theorem (2-7)

Let (Z,T,π) be a topological transformation group,(n,m) are proximal points uf and only if (n,m) are semi-replete proximal points.

Proof: Suppose that μ be compact subset of T , since n and m are proximal points then for each index φ in Z there exist a $t \in T$ such that $(n,m)t \in \varphi$,we need to prove T be semi-replete , by hypothesis $\mu t \subset Tt \subset T$ .Then T be semi-replete proximal thus and n and m are proximal points. Conversely let n and m are semi-replete proximal points ,then for each index φ in Z there exist a semi- replete subset w of T such that $(n,m)w \subset \varphi$, since w semi-replete subset of T then for each compact set μ of T there exist $t_1 \in T$ such that $t_1 \mu \subset w$,and $(n,m)t_1 \mu \subset \varphi$ so $(n,m)t_1 \mu T \subset \varphi T$ by theorem (2-5) we obtain $(n,m)t_1 \mu T \subset \varphi$ by hypothesis $(n,m)\mu T \subset \varphi$ since T syndetic set and β compact set then $(n,m)T \subset \varphi$ (n,m) are proximal points . Δ

Theorem (2-8)

Let (Z,T,π) be a topological transformation abelain group,(n,m) are replete proximal points if and only if (n,m) are semi replete proximal points.

Proof: let n and m are replete proximal points ,then for each index φ in Z there exist a replete subset w of T such that $(n,m)w \subset \varphi$, since w replete subset of T then for each compact set β of T there exist $t_1, t_2 \in T$ such that $t_1 \beta t_2 \subset w$, $(n,m)t_1 \beta t_2 \subset (n,m)w \subset \varphi$ by hypothesis we get $\beta t \subset w$, for some $t \in$ T then w be semi- replete proximal subset of T thus$(n,m) \in SRP$. Conversely let n and m are semi-replete proximal points ,then for each index φ in Z there exist a semi- replete subset w of T such that $(n,m)w \subset \varphi$, since w semi-replete subset of T then for each compact set μ of T there exist $t_1 \in T$ such that $t_1 \mu \subset w$ so $t_1 \mu e = t_1 \mu\, t.t^{-1} \subset w$ for some $t^{-1} \in T$ and $t_1 \mu\, t \subset wt$ then wt replete subset of T from theorem (1-10) we get w be replete set and $(n,m) \in RP$. Δ

Theorem (2- 9)

Let (Z,T,π) be a topological transformation group,(n,m) are proximal points then (Z,T) are pointwise proximal points.

Proof: Let $(n,m)\in P$,it follows that $(n,m)T\subset PT$ clearly by remark (2-4) that $(n,m)T\subset P$ so $(n,m)TT\subset PT,(n,m)T^2\subset P$ again $(n,m)T^2T\subset PT,(n,m)T^3\subset P$ after n- time we get $(n,m)T^n\subset P$ then we obtain that $(Z\times Z,T)$ are proximal for each point in $Z\times Z$ therefore (Z,T) are pointwise proximal points. Δ

Theorem (2-10)

Let (Z,T,π) be a topological transformation abelain group , $F\subset T\ and\ (n,m)\in P$ $then\ F$ syndetic set .

Proof: Let F be subset of T ,since (n,m) are proximal points then for each index φ in Z there exist $t\in T$ such that $(n,m)t\in\varphi$ by theorem (2-5) we have $\varphi T=\varphi$ and $\varphi TF=\varphi F$ since T be syndetic there exist compact set β of T such that $\varphi TF\beta=\varphi F\,\beta$ since $T\ be\ abeline$ then $\varphi TF\cap\varphi F\beta\subset\varphi T\cap\varphi F\beta$ and $\varphi TF\cap\varphi F\beta\neq\emptyset$,Thus $\varphi T\cap\varphi F\beta\neq\emptyset$ so $T\subset F\beta$ by hypothesis $T=F\beta$ then F syndetic set.Δ

Theorem (2-11)

Let (Z,T,π) be a topological transformation abelain group , $F\subset T\ and\ (n,m)\in RP$ $then\ F$ syndetic set .

Proof: Let (n,m) are replete proximal points then for each index φ in Z there exist replete subset E of T such that $(n,m)E\subset\varphi$ since E replete set then for each compact set K in T there exist $g_1g_2\in T$ such that $g_1Kg_2\subset E$ since T by syndetic and K by remark (1-11 number (1)) we have $Tg_2\subset E$ and $Tg_2K\subset EK$ so $T\subset EK$ by hypothesis we obtain $T=EK$ therefore E be syndetic set. Δ

Remark (2-12)

Let (Z,T,π) be a topological transformation abelain group , $F\subset T\ ,if(n,m)\in SRP$ $then\ F$ syndetic set .

Theorem (2-13)

Let (Z,T,π) be a topological transformation group then P(X,T) is minimal set.

Proof: Let $(n,m)\in P$ and $(n,m)T\subset PT$ by remark (2-4 number (1)) we obtain $(n,m)T\in P$, so $\overline{(n,m)T}\subset\bar{P}$ then $\overline{(n,m)T}\subset P$ by remark (2-4 number (2)) since $\overline{(n,m)T}$ be least closed invariant subset $Z \times Z$ contain (n,m) therefore $P \subset \overline{(n,m)T}$,thus $P \subset \overline{(n,m)T}$ then P be minimal set and (n,m) be minimal points . Δ

Theorem (2-14)

Let (Z,T,π)be a topological transformation group then $(n,m)$is fixed points.

Theorem (2-15)

Let (Z,T,π) be a topological transformation group , $(n,m) \in P$, if and only if (n,m) is almost periodic points.

Proof: Assume that V be invariant neighborhood of (n,m) and A subset of T It is enough to show that A be syndetic set ,since (n,m) are proximal points then for each index φ in Z there exist $t \in T$ such that $(n,m)t \in \varphi$ and $(n,m)A \subset (n,m)T \subset \varphi$. It follows that A be syndetic set by hypothesis $(n,m)A \subset VA \subset VT \subset V$,thus (n,m) is almost periodic points .Conversely assume that $(n,m) \in \varphi$ since (n,m) almost periodic point then for each invariant neighborhood V of (n,m) there exist syndetic subset A of T such that $(n,m)A \subset V$, $(n,m)Ag \subset Vg = V$ for some $g \in$ T .It is enough to show that Ag be semi- replete set ,since A be syndetic set then Ag be syndetic set by remark (1-11 number (2))there exist compact subset K of T such that $AgK = T$,for each $t \in T$ there exist $a \in A, k \in K$ such that $agk =$ t so $agk = tk^{-1}g^{-1}$ for some $k^{-1}, g^{-1} \in T$,it follows that $tK \subset Ag$ thus Ag be semi-replete set by hypothesis it was found $(n,m)Ag \subset \varphi$, then (n,m) be semi-replete proximal points by theorem (2-7) we obtain $(n,m) \in P$. Δ

Theorem (2-16)

Let$(\tau,\gamma):(Z,T,\pi) \to (H,G,\sigma)$ epimorphism , (n,m) are proximal points under T then $((n)\tau,(m)\tau)$ are proximal points under G.

Proof: Let (n,m) are proximal points then for each index φ in Z there exist $t \in T$ such that $(n,m)t \in \varphi$.It follows that from define homomorphism we obtain $((n)\tau,(m)\tau)T\gamma \in (\varphi)\tau$, since γ be onto then we get $((n)\tau,(m)\tau)G \in (\varphi)\tau$ therefore
$((n)\tau,(m)\tau)G \subset (\varphi)\tau$ thus $((n)\tau,(m)\tau)$ are proximal points under G. Δ

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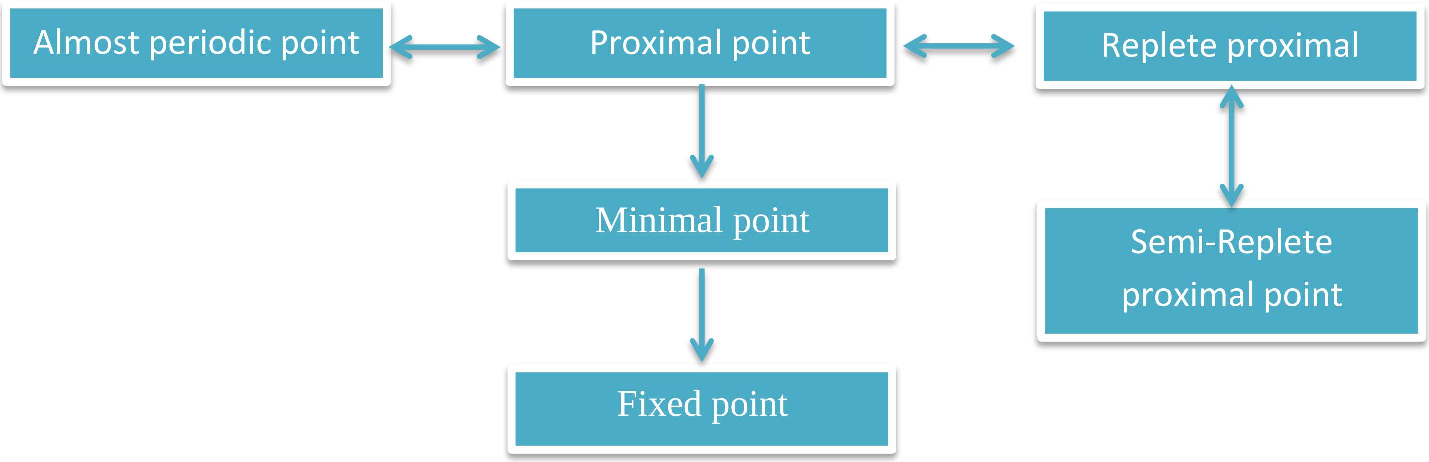

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