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THE ACT OF AN OPERATOR AND ABSTRACT CAUCHY PROBLEMS

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Abstract

In this paper we will explain how we can introduce the solution of Abstract Cauchy Problem without need the initial value u_0 and time t by use the act of an operator V_T.

Key words: Abstract Cauchy Problem, C_0-Semigroup, the act of an operator.

1. Introduction and Definitions.

[1], [2], [3] shows the standard situation in which such operator semigroups naturally appear are so –called Abstract Cauchy Problems

$$\text{(ACP)} \qquad \begin{cases} u^{\cdot} = A\,u(t) & \text{for } t \geq 0, \\ U(0) = x, \end{cases}$$

Where A is a linear operator on a Banach space X. Here the problem consists in finding a differential function u on R_+ such that (ACP) holds .If for each initial value x$\in X$ a unique solution u (. , x) exists ,then T(t)x :=u(t, x),t≥0, x$\in X$, where T(t) is a linear operator on a vector space X , and the map describing the change of x ∈ X at t=0 into T(t)x at time t such that

T (t + s) = T (t) .T(s) and T (0) = id, for all t , s ≥0, and $(T(t)_{t\geq 0}$ is called a (one parameter) operator semigroup.

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Defines an operator semigroup. The relation between $((T(t)_{t\geq 0}$ and A is given by the formulas T (t) $=e^{tA}$ and A= $\frac{d}{dt}$ T (t) at t=0,

Definition 1.1 The (one –parameter) semigroup $(e^{tA}\ )_{t\geq 0}$generated by the matrix A $\in M_n(\mathbb{C})$.Where $M_n(\mathbb{C})$ is space of all complex n×n matrix.([1], p.37)

Definition 1.2: A family S=$\{s(t)_{t\geq 0}$ of bounded linear operator acting on a Banach space E is called a C_0-semigroup if the following properties are satisfied:

1. S (0) = I.
2. S (t).S(s) =S (t +s) for all t, s ≥ 0.
3. Lim t ↓ 0 ‖ $S(t)_x$ x ‖ = 0 for all x∈ E. **([2], p.90)**.

We have the following result from $(\mathbf{[1], p.18})$

Theorem 1.3: Every uniformly continuous semigroup $((S(t)_{t\geq 0}$ on a Banach space X is of the form S (t) = e^{tA} , t≥ 0, for some bounded operator A∈ $\mathcal{L}(x)$).

We study in (**[1] ,p.2)**,and(**[2],p.90)** ,the unique solution of (ACP) is given by u (t) =$e^{tA}u_0$, u_0 = u(0) = x ,t≥0

Where e^{tA} = $\sum_{n=0}^{\infty}\frac{t^nA^n}{n!}$.The matrices e^{tA} may be thought of as "solution operators" mapping the initial value u_0 to the solution $e^{tA}u_0$ at time t.Where A is a linear operator on a Banach space X.

2. Preliminaries.

The act of an operator have been introduce and studied, in [4] , we defined it as follows:

Let V be a Banach space over a field F, Let T be a bounded operator on V,

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and let $S = \{e^x : x \in R\}$ be the semigroup.

Define $\mu : S \times V \to V$ by $\mu(e^x, v) = e^T(v)$.

This function makes V a left S-act, denoted by V_T, is called the associated S-act of T. We gave some basic facts about the associated V_T , we proved:

1. For any operator T then the S-act V_T is faithful act, separated act, and torsion free act.
2. If T is onto and V_T is finitely generated, then V is finite dimensional.
3. For any operator T and V_T is singular S-act then V is generated by one element.
4. If V is a finite dimensional normed space and T is an operator on V, then V_T is Noetherian S-act.

The following proposition play a very important role in this search,

Proposition 1.1[4]: Let T and S be two operators on V, if S is similar to T then V_s is isomorphic to V_T.

3. Main Results.

Clearly e^{0A} =I , $e^{tA}\, e^{SA} = e^{SA}\, e^{tA}$ $=e^{(t+S)A}$ $also$

$t \mapsto e^{tA}$is continuous .We already know that the set of all solutions for(ACP) Equation formed a semi group.

The semi group over itself is an act denoted by V_A ([5],[6],[7])then by proposition (1.1) this act V_A is isomorphic to the act V_T ,thus [4] we get analysis properties the vector space and the operator which generates the set of all solution of (ACP).

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