Laplace Decomposition Method for the System of Linear and Non-Linear Ordinary Differential Equations

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Abstract

In this paper we use Modified form of Adomian's Decomposition Method Laplace, which is a mixture of Laplace transforms and Adomian's Decomposition Method called the Laplace Decomposition Method (LDM) to solve the system of ordinary differential equation of the first order and an ordinary differential equation of any order by converting it into a system of differential equation of order one. Some examples are presented to show the ability of the method for linear and non-linear systems of differential equations also present the comparison of their solution with the exact solution through graphically.

Keywords: Laplace Transformation, Adomian's Decomposition Method (ADM), System of differential equation, linear differential equation and non-linear ordinary differential equation.

1. Introduction

Adomian's Decomposition Method was introduced by Adomian in [1, 2] and used heavily in the literature to solve the wide class of natural and engineering problems [3-9]. Adomian's Decomposition Method has been applied to a vast wide variety of problems in Physics, Biology and Chemical reactions. This method was applied to Non-linear differential equation [10], Non-linear Dynamic system [11], The Heat equation [12, 13], The Wave equation [14] Coupled Non-Linear Partial differential equation [15, 16] Linear and Non-linear Integro-differential equation [17] and Airy's equation [18] successfully.

Laplace Adomian's Decomposition Method (LADM) was first introduced by Suheil A. Khuri [19, 20], and has been successfully used to find the solution of differential equations [21-26]. The Laplace Adomian's Decomposition Method is a combination of ADM and Laplace Transforms. This Method is successfully used to find the exact solution of the Bratu and Duffing equation in [27, 28]. The Significant advantage of this method is its capability of combining the two powerful methods to obtain exact solution for non-linear equation.

2. The System of Differential Equation

Consider the system of ordinary differential equations of the first order as follow [29]:

$$\begin{aligned} y_1' &= g_1\left(x, y_1, y_2, \cdots, y_n\right) \\ y_2' &= g_2\left(x, y_1, y_2, \cdots, y_n\right) \\ &\vdots \\ y_n' &= g_n\left(x, y_1, y_2, \cdots, y_n\right), \end{aligned} \tag{1}$$

Where each represents the derivative of first order of one of the unknown functions as a mapping depending on the independent variable *x*, and *n* unknown functions $g_1, g_2, \cdots, g_n$. Since every ordinary differential equations of *n* order can be written as a system consisting of *n* ordinary differential equation of order one, we restrict our study to a system of differential equation of the first order.

3. Analysis of Adomian's Decomposition Method

Consider the differential equation in the general form [30],

$$L\,y(x) + R\,y(x) + N\,y(x) = f\,(x), \tag{2}$$

where L is the linear operator of the highest-order derivative which is assume to be invertible easily, R is also a linear operator of order less than L, and $N\,y\,(x)$, indicate the non-linear term and f is the source term. Thus applying the inverse operator L^{-1}, to Eq. (2) on both sides, we get

$$y(x)=g_0+L^{-1}\left(f(x)-R\,y(x)-N\,y(x)\right), \tag{3}$$

where g_0, is the solution of the homogeneous equation,

$$L\,y(x)=0. \tag{4}$$

The constants of integration involved in the solution of homogeneous Eq. (4) are to be determined by the initial conditions, according to the problem, whether it is initial value problem or boundary value problem. According to ADM, the solution of the unknown function $y(x)$, can be expressed by an infinite series of the form

$$y(x)=\sum_{n=0}^{\infty} y_n(x). \tag{5}$$

and the non-linear term can be decomposed by the infinite series of the form

$$N\,y(x)=\sum_{n=0}^{\infty} A_n\,. \tag{6}$$

and $A_n s$, are called Adomian's Polynomials, which can be determined by the algorithm defined in [31, 32].

By substituting Eq. (5) and (6) into (3),

$$\sum_{n=0}^{\infty} y_n(x)=g_0+L^{-1}\left(f(x)-R\sum_{n=0}^{\infty} y_n(x)-\sum_{n=0}^{\infty} A_n\right),$$

where the components $y_0, y_1, y_2, \ldots,$ are determined by the recursive relation.

$$y_0=g_0,$$

$$y_{k+1}=-L^{-1}(R\,y_k)-L^{-1}(A_k),\ k\geq 0. \tag{7}$$

Hence the series solution from Eq. (4) can be obtained immediately.

4. Analysis of Laplace Decomposition Method (LDM)

We represent the system (1) by using the ith equation as [33]:

$$D[y_1]=g_1(x,y_1,y_2,\cdots,y_n),\ \ i=1,2,3,\cdots,n \tag{8}$$

where D is the linear differential operator. Appling Laplace Transform on both sides on Eq. (8), we get

$$L[y_i]=\frac{1}{s}y_i(0)+\frac{1}{s}L[g_i(x,y_1,y_2,\cdots,y_n)],\ i=1,2,3,\cdots,n \tag{9}$$

where L is a Laplace operator,

according to Adomian's Decomposition Method, the solution of Eq. (9) will be the some of the series:

$$y_i = \sum_{j=0}^{\infty} g_{i,j}, \tag{10}$$

and the non-linear terms can be written as the sum of the following series:

$$g_i(x, y_1, \cdots, y_n) = \sum_{j=0}^{\infty} A_{i,j}(g_{i,0}, g_{i,1}, \cdots, g_{i,j}), \tag{11}$$

where $A_{i,j}(g_{i,0}, g_{i,1}, \cdots, g_{i,j})$, are called Adomian's polynomials. Which can be determined by the algorithm described in [31, 32].

Now, substituting Eq. (10), Eq. (11) in Eq. (9), we get

$$L\left[\sum_{j=0}^{\infty} g_{i,j}\right] = \frac{1}{s} y_i(0) + \frac{1}{s} L\left[A_{i,j}(g_{i,0}, g_{i,1}, \cdots, g_{i,j})\right], \; i = 1, 2, 3, \cdots, n \tag{12}$$

taking the inverse Laplace Transform of Eq. (12), then we write its recurrence relations as:

$$g_{i,0} = L^{-1}\left[\frac{1}{s} y_i(0)\right],$$

$$g_{i,n+1} = L^{-1}\left[\frac{1}{s} L\left[A_{i,n}(g_{i,0}, g_{i,1}, \cdots, g_{i,n})\right]\right], \; n \geq 0, \tag{13}$$

Hence the series solution from Eq. (10) can be obtained immediately.

5. Numerical Applications

Example 5.1 Consider the following system of three differential equations of first order,

$$y_1' = y_3 - \cos x, \tag{14}$$

$$y_2' = y_3 - e^x, \tag{15}$$

$$y_3' = y_1 - y_2, \tag{16}$$

subjected to the initial conditions,

$$y_1(0) = 1, \quad y_2(0) = 0, \quad y_3(0) = 2.$$

Appling Laplace Transform on Eq. (14), (15) and (16), we get

$$L[y_1] = \frac{1}{s} - \frac{1}{s^2 + 1} + \frac{1}{s} L[y_3], \tag{17}$$

$$L[y_2] = \frac{1}{s} - \frac{1}{s - 1} + \frac{1}{s} L[y_3], \tag{18}$$

$$L[y_3] = \frac{2}{s} + \frac{1}{s} L[y_1 - y_2]. \tag{19}$$

According Adomian's Decomposition method, the solution of Eq. (17), (18) and (19) is

$$y_i(x)=\sum_{j=0}^{\infty} y_{i,j}(x),\ i=1,2,3. \tag{20}$$

Then, Eq. (17), (18), (19), becomes

$$L\left[\sum_{j=0}^{\infty} y_{1,j}(x)\right]=\frac{1}{s}-\frac{1}{s^2+1}+\frac{1}{s}L\left[\sum_{j=0}^{\infty} y_{3,j}(x)\right], \tag{21}$$

$$L\left[\sum_{j=0}^{\infty} y_{2,j}(x)\right]=\frac{1}{s}-\frac{1}{s-1}+\frac{1}{s}L\left[\sum_{j=0}^{\infty} y_{3,j}(x)\right], \tag{22}$$

$$L\left[\sum_{j=0}^{\infty} y_{3,j}(x)\right]=\frac{2}{s}+\frac{1}{s}L\left[\sum_{j=0}^{\infty} y_{1,j}(x)-\sum_{j=0}^{\infty} y_{2,j}(x)\right]. \tag{23}$$

Taking inverse Laplace transforms of Eq. (21), (22) and (23), we get

$$\sum_{j=0}^{\infty} y_{1,j}(x)=L^{-1}\left[\frac{1}{s}-\frac{1}{s^2+1}\right]+L^{-1}\left[\frac{1}{s}L\left[\sum_{j=0}^{\infty} y_{3,j}(x)\right]\right],$$

$$\sum_{j=0}^{\infty} y_{2,j}(x)=L^{-1}\left[\frac{1}{s}-\frac{1}{s-1}\right]+L^{-1}\left[\frac{1}{s}L\left[\sum_{j=0}^{\infty} y_{3,j}(x)\right]\right],$$

$$\sum_{j=0}^{\infty} y_{3,j}(x)=L^{-1}\left[\frac{2}{s}\right]+L^{-1}\left[\frac{1}{s}L\left[\sum_{j=0}^{\infty} y_{1,j}(x)-\sum_{j=0}^{\infty} y_{2,j}(x)\right]\right].$$

Its recurrence relation can be written as,

$$y_{1,0}(x)=L^{-1}\left[\frac{1}{s}-\frac{1}{s^2+1}\right],\qquad y_{1,n+1}(x)=L^{-1}\left[\frac{1}{s}L\left[y_{3,n}(x)\right]\right],\ n\geq 0 \tag{24}$$

$$y_{2,0}(x)=L^{-1}\left[\frac{1}{s}-\frac{1}{s-1}\right],\qquad y_{2,n+1}(x)=L^{-1}\left[\frac{1}{s}L\left[y_{3,n}(x)\right]\right],\ n\geq 0 \tag{25}$$

$$y_{3,0}(x)=L^{-1}\left[\frac{2}{s}\right],\qquad y_{3,n+1}(x)=L^{-1}\left[\frac{1}{s}L\left[y_{1,n}(x)-y_{2,n}(x)\right]\right].\ n\geq 0 \tag{26}$$

Thus, from Eq. (24), (25) and (26) we get

$$y_{1,0}(x)=1-\sin x,$$

$$y_{2,0}(x)=1-e^x,$$

$$y_{3,0}(x)=2,$$

for $n=0$, Eq. (24), (25) and (26) gives

$$y_{1,1}(x)=L^{-1}\left[\frac{1}{s}L[y_{3,0}(x)]\right]=2x,$$

$$y_{2,1}(x)=L^{-1}\left[\frac{1}{s}L[y_{3,0}(x)]\right]=2x,$$

$$y_{3,1}(x)=L^{-1}\left[\frac{1}{s}L[y_{1,0}(x)-y_{2,0}(x)]\right]=\cos x+e^{x}-2,$$

for $n=1$, Eq. (24), (25) and (26) becomes

$$y_{1,2}(x)=L^{-1}\left[\frac{1}{s}L[y_{3,1}(x)]\right]=\sin x+e^{x}-2x-1,$$

$$y_{2,2}(x)=L^{-1}\left[\frac{1}{s}L[y_{3,1}(x)]\right]=\sin x+e^{x}-2x-1,$$

$$y_{3,2}(x)=L^{-1}\left[\frac{1}{s}L[y_{1,1}(x)-y_{2,1}(x)]\right]=0$$

Now by putting all values in Eq. (20), we get the result

$$y_1=e^{x},$$

$$y_2=\sin x,$$

$$y_3=\cos x+e^{x}$$

Which the same result as obtained by ADM in [29].

Example 5.2 Consider the non-linear system of differential equations,

$$\frac{dy_1}{dx}=2y_2{}^{2}, \tag{27}$$

$$\frac{dy_2}{dx}=e^{-x}y_1, \tag{28}$$

$$\frac{dy_3}{dx}=y_2+y_3, \tag{29}$$

subjected to the initial conditions,

$$y_1(0)=1,\quad y_2(0)=1,\quad y_3(0)=0.$$

Its exact solutions are as follows, $y_1=e^{2x}$, $y_2=e^{x}$, and $y_3=xe^{x}$,

Appling Laplace Transform on Eq. (27), (28) and (29), we get

$$L[y_1]=\frac{y_1(0)}{s}+\frac{2}{s}L[y_2{}^2], \tag{30}$$

$$L[y_2]=\frac{y_2(0)}{s}+\frac{1}{s}L[e^{-x}y_1], \tag{31}$$

$$L[y_3]=\frac{y_3(0)}{s}+\frac{1}{s}L[y_2+y_3]. \tag{32}$$

According Adomian's Decomposition method, the solution of Eq. (30), (31) and (32) is

$$y_i(x)=\sum_{j=0}^{\infty}y_{i,j}(x),\ i=1,2,3. \tag{33}$$

And non-linear term $y_2{}^2(x)$, can be written as the sum of the series,

$$y_{2,j}{}^2(x)=\sum_{j=0}^{\infty}A_{2,j}(x), \tag{34}$$

where, $A_{2,j}$'s, are called Adomian's Polynomials, which can be determined by the algorithm defined in [31, 32].

Then, Eq. (30), (31) and (32), becomes

$$L\left[\sum_{j=0}^{\infty}y_{1,j}(x)\right]=\frac{y_1(0)}{s}+\frac{2}{s}L\left[\sum_{j=0}^{\infty}A_{2,j}(x)\right], \tag{35}$$

$$L\left[\sum_{j=0}^{\infty}y_{2,j}(x)\right]=\frac{y_2(0)}{s}+\frac{1}{s}L\left[e^{-x}\sum_{j=0}^{\infty}y_{1,j}(x)\right], \tag{36}$$

$$L\left[\sum_{j=0}^{\infty}y_{3,j}(x)\right]=\frac{y_3(0)}{s}+\frac{1}{s}L\left[\sum_{j=0}^{\infty}y_{2,j}(x)+\sum_{j=0}^{\infty}y_{3,j}(x)\right]. \tag{37}$$

Taking inverse Laplace transforms of Eq. (35), (36) and (37), we get

$$\sum_{j=0}^{\infty}y_{1,j}(x)=L^{-1}\left[\frac{y_1(0)}{s}\right]+L^{-1}\left[\frac{2}{s}L\left[\sum_{j=0}^{\infty}A_{2,j}(x)\right]\right], \tag{38}$$

$$\sum_{j=0}^{\infty}y_{2,j}(x)=L^{-1}\left[\frac{y_2(0)}{s}\right]+L^{-1}\left[\frac{1}{s}L\left[e^{-x}\sum_{j=0}^{\infty}y_{1,j}(x)\right]\right], \tag{39}$$

$$\sum_{j=0}^{\infty}y_{3,j}(x)=L^{-1}\left[\frac{y_3(0)}{s}\right]+L^{-1}\left[\frac{1}{s}L\left[\sum_{j=0}^{\infty}y_{2,j}(x)+\sum_{j=0}^{\infty}y_{3,j}(x)\right]\right]. \tag{40}$$

Its recurrence relation can be written as,

$$y_{1,0}(x)=L^{-1}\left[\frac{y_1(0)}{s}\right],\qquad y_{1,n+1}=L^{-1}\left[\frac{2}{s}L\left[\sum_{n=0}^{\infty}A_{2,n}(x)\right]\right],\ n\geq 0, \tag{41}$$

$$y_{2,0}(x)=L^{-1}\left[\frac{y_2(0)}{s}\right], \qquad y_{2,n+1}=L^{-1}\left[\frac{1}{s}L\left[e^{-x}\sum_{n=0}^{\infty}y_{1,n}(x)\right]\right],\ n\geq 0, \tag{42}$$

$$y_{3,0}(x)=L^{-1}\left[\frac{y_3(0)}{s}\right], \qquad y_{3,n+1}=L^{-1}\left[\frac{1}{s}L\left[\sum_{n=0}^{\infty}y_{2,n}(x)+\sum_{n=0}^{\infty}y_{3,n}(x)\right]\right],\ n\geq 0, \tag{43}$$

Thus, from Eq. (41), (42) and (43) we get

$$y_{1,0}(x)=1,$$

$$y_{2,0}(x)=1,$$

$$y_{3,0}(x)=0,$$

for $n=0$, Eq. (41), (42) and (43) gives

$$y_{1,1}=L^{-1}\left[\frac{2}{s}L[A_{2,0}(x)]\right]=2x,$$

$$y_{2,1}=L^{-1}\left[\frac{1}{s}L[e^{-x}y_{1,0}(x)]\right]=1-e^{-x},$$

$$y_{3,1}=L^{-1}\left[\frac{1}{s}L[y_{2,0}(x)+y_{3,0}(x)]\right]=x,$$

for $n=1$, Eq. (41), (42) and (43) gives

$$y_{1,2}=L^{-1}\left[\frac{2}{s}L[A_{2,1}(x)]\right]=-4+4x+4e^{-x},$$

$$y_{2,2}=L^{-1}\left[\frac{1}{s}L[e^{-x}y_{1,1}(x)]\right]=2-2e^{-2x}-2xe^{-x},$$

$$y_{3,2}=L^{-1}\left[\frac{1}{s}L[y_{2,1}(x)+y_{3,1}(x)]\right]=-1+e^{-x}+x+\frac{x^2}{2},$$

for $n=2$, Eq. (41), (42) and (43) gives

$$y_{1,3}=L^{-1}\left[\frac{2}{s}L[A_{2,2}(x)]\right]=-15+10x+12e^{-x}+3e^{-2x}+8xe^{-x},$$

$$y_{2,3}=L^{-1}\left[\frac{1}{s}L[e^{-x}y_{1,2}(x)]\right]=2-2e^{-2x}-4xe^{-x},$$

$$y_{3,3}=L^{-1}\left[\frac{1}{s}L\left[y_{2,2}(x)+y_{3,2}(x)\right]\right]=-2+2xe^{-x}+2xe^{-x}+e^{-2x}+x+\frac{x^2}{2}+\frac{x^3}{6},$$
$$\vdots$$

Approximations to the solutions with the five terms are as follows:

$$y_1(x)\approx 4(26x+31)e^{-x}+\left(-4x^2-6x+15\right)e^{-2x}-3.11111e^{-3x}+58.6666x-134.888$$

$$y_2(x)\approx -4(8x-7)e^{-x}-8(2x+5)e^{-2x}+0.111111(12x+13)e^{-3x}+11.5555,$$

$$y_3(x)\approx(12x+3)e^{-x}+(2x+7.5)e^{-2x}-0.111111e^{-3x}+0.0083333x^5+x^2+4.6666x-10.3888,$$

This is same as obtain by ADM in [29].

Some numerical values of these solution are presented in Table 1.

x_i	$y_1(x_i)$	$ey_1(x_i)$	$y_2(x_i)$	$ey_2(x_i)$	$y_3(x_i)$	$ey_3(x_i)$
0	1.00008	0	1	0	0	0
0.1	1.22132	1.6535E-5	1.10516	2.9323E-6	0.110517	0
0.2	1.49186	5.3375E-5	1.22139	1.1211E-5	0.244275	0
0.3	1.82161	5.9740E-4	1.34974	1.1407E-4	0.404906	5.1165E-5
0.4	2.22249	3.1315E-3	1.49125	5.7298E-4	0.594560	2.7328E-4
0.5	2.70702	1.1341E-2	1.64676	1.9591E-3	0.823372	9.8853E-4
0.6	3.28813	3.2068E-2	1.81686	5.2497E-3	1.090460	2.8076E-3
0.7	3.97860	7.6660E-2	2.00184	1.1909E-2	1.402860	6.7609E-3
0.8	4.79050	6.6253E-1	2.20161	2.3929E-2	1.766030	1.1440E-2
0.9	5.73528	3.1445E-1	2.41576	4.3841E-2	2.185680	2.7962E-2

Table 1: Numerical values of the solutions of problem 2

Graphical solutions are presented as:

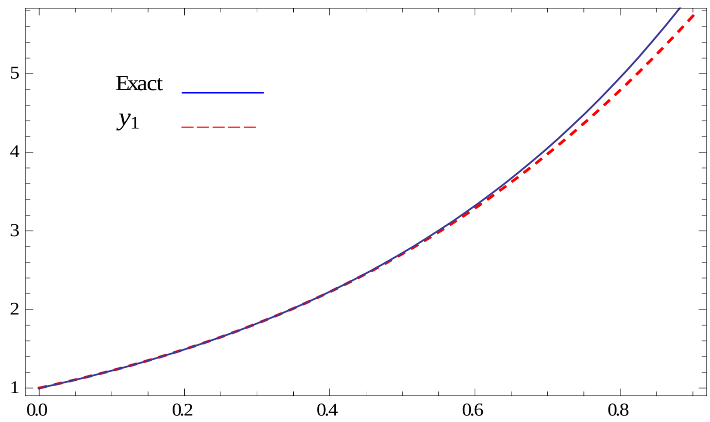

Fig. 1: Comparison of exact and approximate solution of $y_{1,}$

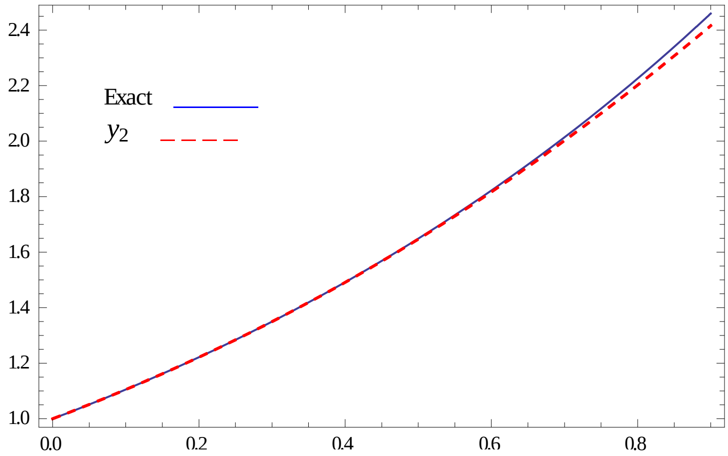

Fig. 2: Comparison of exact and approximate solution of $y_{2,}$

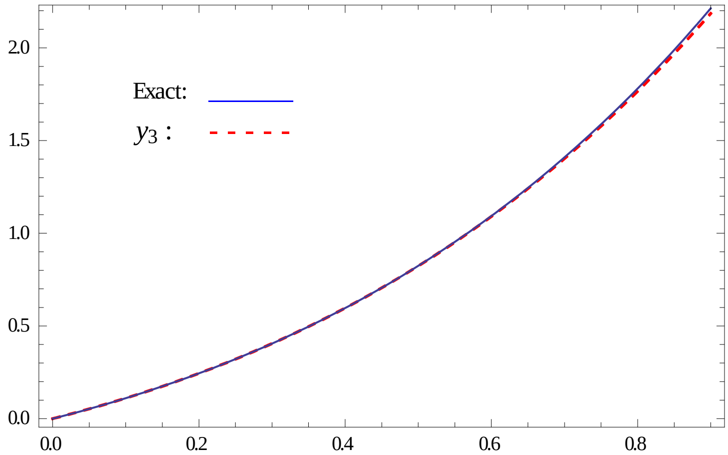

Fig. 3: Comparison of exact and approximate solution of y_3 ,

Example 5.3: Consider a non-linear ordinary differential equation of order 3,

$$\frac{d^3 y}{d x^3}=\frac{1}{x}y+\frac{d y}{d x}, \tag{44}$$

subjected to the boundary conditions,

$$y(0)=0, \qquad y'(0)=1, \qquad y''(0)=2,$$

and the exact solution is $y(x)=xe^x$,

Considering three functions, $y_1(x)=y(x)$, $y_2(x)=y'(x)$, and $y_3(x)=y''(x)$, we convert Eq. (44) in the system of non-linear of three differential equation of order one, i.e.

$$y_1'(x)=y_2(x), \tag{45}$$

$$y_2'(x)=y_3(x), \tag{46}$$

$$y_3'(x)=\frac{1}{x}y_1(x)+y_3(x), \tag{47}$$

Appling Laplace Transform on Eq. (45), (46) and (47), we get

$$L[y_1(x)]=\frac{1}{s}y_1(0)+\frac{1}{s}L[y_2(x)], \tag{48}$$

$$L[y_2(x)]=\frac{1}{s}y_2(0)+\frac{1}{s}L[y_3(x)], \tag{49}$$

$$L[y_3(x)]=\frac{1}{s}y_3(0)+\frac{1}{s}L\left[\frac{1}{x}y_1(x)+y_2\right], \tag{50}$$

According Adomian's Decomposition method, the solution of Eq. (48), (49) and (50) is

$$y_i(x)=\sum_{j=0}^{\infty}y_{i,j}(x),\ i=1,2,3. \tag{51}$$

Then, the recurrence relation of Eq. (48), (49) and (50), can be written as,

$$y_{1,0}(x)=L^{-1}\left[\frac{1}{s}y_1(0)\right], \qquad y_{1,n+1}=\frac{1}{s}L[y_{2,n}(x)], \tag{52}$$

$$y_{2,0}(x)=L^{-1}\left[\frac{1}{s}y_2(0)\right], \qquad y_{2,n+1}=\frac{1}{s}L[y_{3,n}(x)], \tag{53}$$

$$y_{3,0}(x)=L^{-1}\left[\frac{1}{s}y_3(0)\right], \qquad y_{3,n+1}=L^{-1}\left[\frac{1}{s}L\left[\frac{1}{x}y_{1,n}(x)+y_{2,n}\right]\right], \tag{54}$$

Therefore,

$$y_{1,0}(x)=0,$$

$$y_{2,0}(x)=1,$$

$$y_{3,0}(x)=2,$$

Let $y^r=y_{1,0}+y_{1,1}+y_{1,2}+\cdots+y_{1,r}$ is a notation for an approximation to the solution with $p+1$ term. Therefore, some computed approximations are as follows:

$$y^3=x\left(1+x+\frac{x^2}{3}\right),$$

$$y^4=x\left(1+x+\frac{x^2}{2}+\frac{x^3}{12}\right),$$

$$y^5=x\left(1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{60}\right),$$

$$y^6 = x\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{7x^4}{180} + \frac{x^5}{360}\right),$$
$$\vdots$$

Hence, we get the exact solution i.e. $y(x) = xe^x$,

This solution same as obtain by ADM in [29].

6. Conclusion

We conclude that, Laplace Decomposition Method is a reliable and a powerful tool to solve the system of ordinary differential equations. It is demonstrated that, this modified form of Adomian's Decomposition Method has the ability of solving system of both linear and non-linear differential equations.

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