Mathematical Theory and Modeling
ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)
Vol.5, No.8, 2015

A Mathematical Model to Study the Dynamics of Hazardous Substances in E-Waste on Ecosystem in Developing Countries

Shikaa S.,[*1] Taparki R.[1] and Waniyos H. U.[2]

1. Department of Mathematical Sciences, Taraba State University Jalingo, Taraba State – Nigeria.
2. Department of Statistics, Adamawa State Polytechnic, Yola, Adamawa State Nigeria.

Abstract

Environmental pollution has become a global concern several decades ago due to human activities particularly industrialization. Many pollutants have immediate effects while most pollutants like greenhouse gases, radioactive substances and other heavy metals for instance, have a generational consequence. It is against this background that, this research uses mathematical formulations and analysis to simulate the dynamics of hazardous substances in electronic and medical wastes. The system is compartmentalized in to five units namely: terrestrial habitats, aquatic habitats, terrestrial living organisms, aquatic living organisms and the human population. The system has an equilibrium point which is found uniformly and asymptotically stable.

1 Introduction

Apart from greenhouse gases, radioactive substances and other heavy metals, have a generational consequence. Electronic waste products become a challenge when added to the environment faster than expected. According to Tientenberg 2006, once contamination occurs, it is difficult or impossible to remove these pollutants from the environment. These wastes are generated by production processes, households, commercial activities, hospitals and other health providers. The problem of e-waste has become an immediate and long term concern as its unregulated accumulation and recycling can lead to major environmental problems endangering human health.

Both developed countries and developing countries like Nigeria face the problem of e-waste management. The rapid growth of technology, up gradation of technical innovations and a high rate of obsolescence in the electronics industry have led to one of the fastest growing waste streams in the world which consist of end of life electrical and electronic equipment products. It comprises a whole range of electrical and electronic items such as refrigerators, washing machines, computers and printers, televisions, mobiles, i-pods, etc., many of which contain toxic materials. Obsolete computers pose the most significant environmental and health hazard among the e-wastes (Neha, 2010). The enormity of electronic wastes in 500 million computers was published by Asha, 2003 as: Plastic-6.32 Billion Pounds, Lead-1.58 Billion Pounds, Cadmium-3 Million Pounds, Chromium-1.9 Million Pounds and Mercury-632,000 Pounds.

The challenge is that, since the markets in the West have matured; it is expected to account for only 2 per cent of the total solid waste generated in developed countries as of 2010 (Schwarzer *et al*, 2005). United Nations predicted that by 2020, e-waste from old computers would jump by 400 per cent on 2007 levels in China and by 500 per cent in India. Additionally, e-waste from discarded mobile phones would be about seven times higher than 2007 levels and, in India, 18 times higher by 2020 (Tom, 2010).

Such predictions highlight the urgent need to address the problem of e-waste in developing countries like Nigeria where the collection and management of e-waste and the recycling process is yet to be properly regulated.

Separate studies by Terada, 2012 and Puckett *et al.*, 2005 estimated 500 containers of used electronics are imported to Nigeria every month from Europe with each container holding 500 to 800 computers and monitors representing about 400,000 arrivals every month.

Here, we propose a mathematical model of the dynamics of the effect of hazardous substances in electronic waste. The system is partitioned into five compartments of the concentration of hazardous substances in the human population $x_1(t)$, the concentration of hazardous substances in aquatic living organisms $x_2(t)$, the concentration of hazardous substances in terrestrial living organisms $x_3(t)$, the concentration of hazardous substances in terrestrial habitats $x_4(t)$ and the concentration of hazardous substances in aquatic habitats $x_5(t)$.

It is assumed that, the hazardous substances are transferable from one level of food chain to another. Animals acquire the harmful substances by eating either plants or animals or drinking contaminated water in both habitats and inhaling. Plants acquire the hazardous substances by absorption and the growth of pollutant is directly proportional the human population growth.

2. Formulation of the Model Equations

The following diagram will be found useful in formulating the model equations

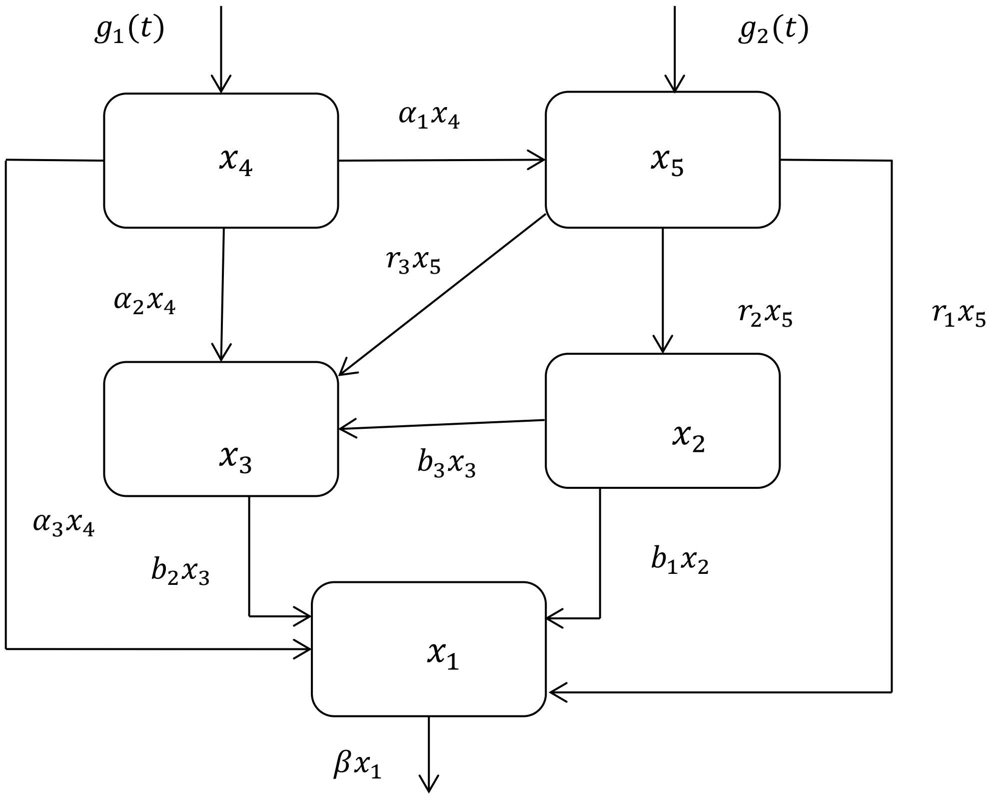

Fig. 1: A flow diagram of the dynamics of hazardous substance in e-waste.

Table 1: Model Parameters

x_1	The concentration of hazardous substances in the human population
x_2	The concentration of hazardous substances in aquatic living organisms
x_3	The concentration of hazardous substances in terrestrial living organisms
x_4	The concentration of hazardous substances in terrestrial habitats
x_5	The concentration of hazardous substances in aquatic habitats
α_1	The rate at which hazardous substances flow from terrestrial habitats to aquatic habitats
α_2	The rate at which terrestrial living organisms absorb hazardous substances from the habitat
β	The rate at which the human population takes in hazardous substances from various sources
b_2	The rate at which the human population consumes terrestrial living organisms contaminated with hazardous substances
b_3	The rate at which terrestrial living organisms consume contaminated aquatic living organisms and absorb or drink contaminated water
r_1	The rate at which the human population drinks water contaminated with hazardous substances
r_2	The rate at which the aquatic living organisms drink or absorb water contaminated with hazardous substances
r_3	The rate at which the terrestrial living organisms drink or absorb water contaminated with hazardous substances
$g_1(t)$	The rate of inflow of hazardous substances in form of electronic or medical wastes into the terrestrial habitats at time t
$g_2(t)$	The rate of inflow of hazardous substances in form of electronic or medical wastes into the aquatic habitats at time t

3. The Mathematical Model

From the assumptions and flow diagrams (Fig. 1), the following model equations are derived:

$$\frac{dx_1}{dt} = -\beta x_1 + b_1 x_2 + b_3 x_3 + \alpha_3 x_4 + r_1 x_5 \qquad (3.1)$$

$$\frac{dx_2}{dt} = -b_1x_2 - b_3x_3 + r_2x_5 \qquad (3.2)$$
$$\frac{dx_3}{dt} = -(b_2 - b_3)x_3 + \alpha_2x_4 + r_3x_5 \qquad (3.3)$$
$$\frac{dx_4}{dt} = -(\alpha_1 + \alpha_2 + \alpha_2)x_4 + g_1(t) \qquad (3.4)$$
$$\frac{dx_5}{dt} = \alpha_1x_4 - (r_1 + r_2 + r_3)x_5 + g_2(t) \qquad (3.5)$$

The matrix-vector equation of the above system is given by

$\boldsymbol{y}' = \boldsymbol{A}(t)\boldsymbol{y} + \boldsymbol{b}(t)$

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \\ x_5' \end{pmatrix} = \begin{pmatrix} -\beta & b_1 & b_3 & \alpha_3 & r_1 \\ 0 & -b_1 & -b_3 & 0 & r_2 \\ 0 & 0 & \varphi_1 & \alpha_2 & r_3 \\ 0 & 0 & 0 & \varphi_2 & 0 \\ 0 & 0 & 0 & \alpha_1 & \varphi_3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ g_1(t) \\ g_2(t) \end{pmatrix}. \qquad (3.6)$$

where

$\varphi_1 = -(b_2 - b_3)$

$\varphi_2 = -(\alpha_1 + \alpha_2 + \alpha_3)$

$\varphi_3 = -(r_1 + r_2 + r_3)$

$\alpha_1,\ \alpha_2,\ \alpha_3\ \beta,\ b_1,\ b_2,\ b_3,\ r_1,\ r_2,\ r_3 \geq 0$

4. Stability Analysis of the Equilibrium States of the Model

The equilibrium state exists at $x_1' = x_2' = x_3' = x_4' = x_5' = 0$. Equating the right hand sides of (3.1)-(3.5) to zero and solving

$$\begin{pmatrix} -\beta & b_1 & b_3 & \alpha_3 & r_1 \\ 0 & -b_1 & -b_3 & 0 & r_2 \\ 0 & 0 & \varphi_1 & \alpha_2 & r_3 \\ 0 & 0 & 0 & \varphi_2 & 0 \\ 0 & 0 & 0 & \alpha_1 & \varphi_3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ g_1(t) \\ g_2(t) \end{pmatrix} = 0$$

simultaneously, we got the equilibrium state of the system as follows:

$$x_1^* = -\frac{(r_1+r_2)(\varphi_2g_2^* - \alpha_1g_1^*) + \alpha_3g_1^*\varphi_3}{\beta\varphi_2\varphi_3} \qquad (3.7)$$
$$x_2^* = -\frac{b_3g_1^*(\alpha_2\varphi_3 - 2\alpha_1r_3) + r_2\varphi_1(\varphi_2g_2^* - \alpha_1g_1^*)}{b_1\varphi_1\varphi_2\varphi_3} \qquad (3.8)$$
$$x_3^* = \frac{\alpha_2g_1^*\varphi_3 + r_3\varphi_2g_2^* - r_3\alpha_1g_1^*}{\varphi_1\varphi_2\varphi_3} \qquad (3.9)$$
$$x_4^* = -\frac{g_1^*}{\varphi_2} \qquad (3.10)$$
$$x_5^* = -\frac{\varphi_2g_2^* - \alpha_1g_1^*}{\varphi_2\varphi_3} \qquad (3.11)$$

provided $b_2 > b_3$.

Which means that at this point, the rate of change concentration of hazardous substances from electronic and medical wastes remain constant irrespective of time.

Now, the stability analysis of the equilibrium state of the mathematical model is stated and proved in the follow theorem below:

5. Theorem 1

Given $\alpha_1,\ \alpha_2,\ \alpha_3\ \beta,\ b_1,\ b_2,\ b_3,\ r_1,\ r_2,\ r_3 > 0$ and $b_2 > b_3$ then, the equilibrium state (3.7)-(3.11) of the system (3.1)-(3.5) is uniformly and asymptotically stable UAS.

Proof:

The system (3.1)-(3.5) is nonhomogeneous linear systems with constant coefficients and the matrix of coefficients is given as

$$\boldsymbol{A} = \begin{pmatrix} -\beta & b_1 & b_3 & \alpha_3 & r_1 \\ 0 & -b_1 & -b_3 & 0 & r_2 \\ 0 & 0 & \varphi_1 & \alpha_2 & r_3 \\ 0 & 0 & 0 & \varphi_2 & 0 \\ 0 & 0 & 0 & \alpha_1 & \varphi_3 \end{pmatrix}.$$

We complete the proof by using a theorem which states that, if the matrix $\boldsymbol{A}$ is a constant matrix of $\boldsymbol{y}' = \boldsymbol{A}(t)\boldsymbol{y} + \boldsymbol{b}(t)$, with eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_n$ and $Re(\lambda_i) < 0$ for all $i = 1, 2, \dots, n$. Let $\boldsymbol{b}(t)$ be defined and continuous for all $t \geq t_0$ and satisfy $\frac{\|\boldsymbol{b}(t)\|}{|t|} \to 0$ as $t \to \infty$, then the equilibrium state of $\boldsymbol{y}' = \boldsymbol{A}(t)\boldsymbol{y} + \boldsymbol{b}(t)$ is uniformly and asymptotically stable whenever it exists (Grimshaw, 1990). Now, applying this result, we have

$$|\mathbf{A}-\lambda\mathbf{I}| = \begin{vmatrix} -\beta-\lambda & b_1 & b_3 & \alpha_3 & r_1 \\ 0 & -b_1-\lambda & -b_3 & 0 & r_2 \\ 0 & 0 & \varphi_1-\lambda & \alpha_2 & r_3 \\ 0 & 0 & 0 & \varphi_2-\lambda & 0 \\ 0 & 0 & 0 & \alpha_1 & \varphi_3-\lambda \end{vmatrix} = 0$$

where λ is the eigenvalue. Evaluating the determinant yields:

$$(-\beta-\lambda)(-b_1-\lambda)(\varphi_1-\lambda)(\varphi_2-\lambda)(\varphi_3-\lambda)=0 \qquad (3.12)$$

Solving equation (3.12) gives

$\lambda_1 = -\beta$
$\lambda_2 = -b_1$
$\lambda_3 = -(b_2 - b_3)$
$\lambda_4 = -(\alpha_1 + \alpha_2 + \alpha_3)$
$\lambda_5 = -(r_1 + r_2 + r_3)$

Since all the eigenvalues are negative, we then conclude that the equilibrium state of the model is uniformly and asymptotically stable (UAS) provided $\frac{\|\boldsymbol{b}(t)\|}{|t|} \to 0$ as $|t| \to \infty$.

5. Results and Discussions

In this research, we formulated and studied the mathematical model of the dynamics of hazardous substances in electronic waste. The major result of the study is found in theorem 1, where the stability of the system was treated. The eigenvalues $\lambda_1, \lambda_2, \lambda_4$ and λ_5 are permanently negative while

$\lambda_3 = -(b_2 - b_3)$

may not be always less than zero. From the right hand side of the expression for λ_3, we see that increasing the value of b_3 (i.e., increasing the rate at which terrestrial living organisms consume contaminated aquatic living organisms and absorb or drink contaminated water) such that $b_2 \leq b_3$ and subsequently $b_2 \geq b_3$ which makes the system unstable. This means that there will be great increase of harmful substances in the human population.

Acknowledgement: Special gratitude to the Management of Taraba State University via TETFund for success of this work.

Reference:

Amit J., (2008) 'Global e-waste growth' in Rakesh Johri, *E-waste: Implications, regulations and management in India and current global best practices*, TERI, New Delhi.
Asha K., (2003). 'Importing danger', *Frontline,* Vol. 20, Issue 25,
Neha L., (2010) 'E-scare', The *Times of India*.
Puckett, J., Westervelt, S.Gutierrez, R. and Takamiya, Y., (2005). The digital dump: Exporting re-use and abuse to Africa. The Basel Action Network, Seattle, USA, 85pp. www.ban.org/banreports/10-24-05/documents/thedigitaldump_print.pdf.
Schwarzer S, Bono A. D, Peduzzi P, Giuliani G and Kluser S (2005) E-waste, the hidden side of IT equipment's manufacturing and use. UNEP Early Warning on Emerging Environmental Threats No. 5. Switzerland: United Nations Environment Program.
Terada, C., (2012). Recycling electronic waste in Nigeria: Putting environmental and human rights at risk. NW. J. Int'l Human Rights 10(3) 154.
Tientenberg, T., (2006) Economics of pollution control. *Environmental and Natural Resource Economics*, 7th Edition, Pearson, Boston.
Tom Y., (2010) 'E-waste a growing problem for China and India'