Robustness of the Job-Finding, Job Loss (JFJL) Model in Modelling the Employment and Unemployment Rates of Ghana

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Abstract
The issues of employment and unemployment have become major macroeconomic factors that determine growth patterns of the modern Ghanaian economy. Periods of economic boom or growth of output can be associated with high rates of employment while recession periods correlate positively with woeful rates of unemployment. This undoubtedly suggests an inverse association between high rates of employment and recession; and high rates of unemployment and economic boom. This paper evaluates the robustness of the Job-Finding, Job Loss (JFJL) model in modelling the employment and unemployment rates in Ghana. It uses the job-finding and separation parameters as bases to model the employment and unemployment rates of Ghana in the form of simple Non-Homogenous First Order Ordinary Differential Equations. The resulting model is obtained by solving the differential equations via the Method of Variation of Parameters (MVP). The JFJL model suggests an environment in which labour force is allowed to vary with time. It assumes a stable state equilibrium condition of the labour market which assisted in obtaining the same expressions as those for the natural rates of employment and unemployment. The predictive ability of the models is ascertained with real data obtained from the Ministry of Labour and Employment, which served as the inputs of simple input/output functions written in Microsoft Excel. The data cover labour force, employment, employment rates, unemployment and unemployment rates from the year 2000 through to 2014. The results evince the closeness of the predicted values or rates to the actual values or rates of employment and unemployment. In fact, at certain points in time especially getting to the end of the period (2013, 2014 and 2015), the model predicted approximately the same values and rates as the actual values and rates of employment and unemployment. Thus, the robustness of the JFJL model in predicting the employment and unemployment rates in the Ghanaian economy is established.

Keywords: Employment and unemployment rates, Job-Loss, Job Finding model, Non-homogeneous first order ordinary differential equations, Method of variation of constants, Ministry of Labour and Employment and Stable state equilibrium of the labour market.

1. Introduction
The role of employment and unemployment at the center of modern growth patterns of the Ghanaian economy has inevitably become increasingly important. Periods of economic boom or growth of output can be associated with high rates of employment while recession periods correlate positively with woeful rates
of unemployment. This undoubtedly suggests an inverse association between high rates of employment and recession; and high rates of unemployment and economic boom. The significance of these two macroeconomic variables as the determinants of outstanding growth and development has driven researchers in the developed economies to come out with models that correctly describe, explain and estimate the employment and unemployment rates of their economies. Earlier models have paid attention to gross flow of workers and jobs while recent models emphasize the significance of the transition rates (the rate at which workers become unemployed and the rate at which unemployed individuals find job) faced by individual workers. We will refer to the latter as ‘Job Finding, Job Loss (JFJL)’ Model. The JFJL model uses job finding and separation rates to explain fluctuations of unemployment around the business cycle. Hall (2005) alluded variations in unemployment rates to the job-finding rates. He contended that separations are not responsible for rising unemployment in recession. Hall explained further that the pronounced rates of unemployment in recessions are as a result of the difficulty in finding jobs. Fujita and Ramey (2008) also argued that higher unemployment during recession might have been pushed by higher separation rates which generates series of job loss while an initial stage of low job-finding rates may drive unemployment upward. They conclusively established that both job-finding and separation rates play vanguard roles in explaining variations in unemployment. Last but not least, Shimer (2012) shared view with Hall by establishing that job finding probabilities are responsible for about 90 percent of the fluctuations in unemployment which contradicts the findings of Derby et al. (1985), Blanchard and Diamond (1990), Davis and Haltiwanger (1992) – who contended that recessions are periods characterized by high separation rates from employment.

An interesting question arises as regard the efficacy of the job-finding and separation rates in explaining employment and unemployment rates in most of the developing economies especially Sub-Saharan African economies. Our objective in this paper is to use the job-finding and separation rates as bases to explain a simple time-dependent JFJL model for the Ghanaian economy. The JFJL model considers the net effects of the job-finding and separation parameters on total employment and total unemployment, which can adequately be modelled as non-homogeneous first order ordinary differential equations and solution presented via MVP. Our version of the JFJL model is a modification of Shimer’s. Shimer (2012) introduced time-dependent job-finding and separation probabilities which he derived from an ordinary differential equation model of unemployment. By assuming a fixed labour force, Shimer was able to derive the natural rate of unemployment as an expression involving the job-finding and separation rates. In addition to demonstrating and explaining the development of differential equation models for employment and unemployment, we shall prove in this paper that by allowing labour force to vary, the natural rates of employment and unemployment remain unaffected. We will also estimate the job-finding and separation rates for the Ghanaian economy using available date and examine the relative significance of these parameters in accounting for rates of employment and unemployment in Ghanaian. This will enable us establish a close relationship of our suggested model with any of the models described for the U. S. economy. We bring to the notice of readers that the JFJL model described for the U. S. economy assumed that within every month, a certain proportion of the employed will lose their jobs (job separation rate) while a given proportion of the unemployed will find jobs (job-finding rate). We will make adjustment in this because of the constraint imposed by unavailability of monthly data on unemployment in Ghana currently. We assumed that every year a given proportion of the employed will lose their jobs (job separation rate) while a certain proportion of the unemployed will find jobs (job-finding rate).
The study will be significant because it can help policy makers in Ghana formulate appropriate employment policies to help curb the rampant rate of unemployment in the Ghanaian economy to near zero – a perceivably important poverty reduction strategy. Another importance can be traced to effective inflationary and interest rates policies and their subsequent reduction to single digits in the Ghanaian economy. This is possible because the Phillips’s Curve postulates an inverse relationship between inflation rate and unemployment rate while other economic theories such as the Fisher effects have identified a direct association between the nominal rate of inflation and interest rates. Last but not least, a very proactive exchange rate policy can also be achieved indirectly by making reference to the relationship between unemployment and exchange rate which we believe can be derived from the association between unemployment and inflation rates.

2. Review of Methods

We devote this section to describing the relevant materials and methods used in developing the JFJL model for the employment and unemployment rates in the Ghanaian economy. The job-finding and separation rates are used as bases to develop the JFJL model for the economy of Ghana. Let us devote some time to reconsider the basic assumptions underpinning the simple JFJL model in the U. S. (Derby et al., 1985; Blanchard and Diamond, 1990; Davis and Haltiwanger, 1992; Hall, 2005; Fujita and Ramey, 2008) and its development thereof.

The following assumptions were considered in the simple JFJL model:

i. Labour force ($L$) is fixed and is defined as the sum of the employed ($E$) and unemployed ($U$)

ii. In every month a given month proportion ($\varphi$) of the employed will lose their jobs while a given proportion ($\tau$) of the unemployed will find jobs. This rates are constants.

iii. Job-loss and separation are only considered as transitions from employment to unemployment or from unemployment to employment respectively but not as movements in and out of labour force.

iv. If the labour market is in a steady state equilibrium, unemployment remains static (neither falls nor increases) and the number of people finding jobs equals the number of people losing jobs.

v. Labour force is homogeneous in terms of losing and finding jobs—that is all employed persons are equally likely to lose their jobs and in the same fashion, all unemployed persons are equally likely to find jobs.
Fig. 1 shows a diagrammatic representation of the transition between employment and unemployment.

![Diagram of Employment and Unemployment Transition](image)

**Fig. 1: The Transition between Employment and Unemployment.**

From assumptions \((ii)\) and \((iv)\), we obtain

\[
\tau U = \phi E \tag{1}
\]

From assumption \((i)\), we obtain the relationship between \(L, E\) and \(U\) as follows:

\[
L = E + U \tag{2}
\]

The interest is to obtain an expression for the natural rate of unemployment in terms of \(\tau\) and \(\phi\) so we express \(E\) in terms of \(L\) and \(U\) as follows:

\[
E = L - U \tag{3}
\]

Substitute (3) into (1), we have

\[
\tau U = \phi (L - U) \\
U(\tau + \phi) = \phi L \\
\frac{U}{L} = \frac{\phi}{\tau + \phi} \tag{4}
\]

(4) is the natural rate of unemployment.

Similarly, we can obtain an expression for the natural rate of employment in terms of \(\tau\) and \(\phi\) as thus:
From (2),

\[ U = L - E \]  

(5)

We substitute (5) into (1) and simplify to obtain the equation below; which is the natural rate of employment.

\[ \frac{E}{L} = \frac{\tau}{\tau + \varphi} \]  

(6)

3. Model Development

In our version of the JFJT model, we relax the restriction that labour force is fixed and allow it to vary – that is labour force is time dependent. We will further assume that labour force is known priori. In order to demonstrate that this relaxation has little or no effect on the natural rates of employment and unemployment insofar as job-finding and separation rates are concerned, we will also reconsider the assumption (i). We will do this by considering the stable state equilibrium of the labour market and observe what happens to the natural rates of employment and unemployment in both cases. We will ignore short – term unemployment since the phenomenon is not very common Ghana and data are not readily available. By Shimer (2012), short-term unemployment refers to the class of labour force who were not employed at the beginning of time \( t \) but secured job at the end of time \( t \). We reiterate the fact that our model defines the job – finding and separation rates as averages or means of the annual rates of job-finding and separation rates over a period \( t \in (0, 1, 2, ..., n) \).

Let

- \( E \) represents the total number of employed labour force at time \( t \) years
- \( U \) be the total number of unemployed labour force at time \( t \) years
- \( \tau \in [0, 1) \) denotes the proportion of the unemployed labour force that find job at time \( t \) years
- \( \varphi \in [0, 1) \) signifies the proportion of the employed labour force that lose (or go out of) job at time \( t \) years

The number of unemployed labour force that finds job at time \( t \) years is given by \( \tau U \) while that of the employed labour force that exits employment is given by \( \varphi E \).

3.1 The Employment Model

It can be deduced from the above parameters and variables that the total number of employed labour force will vary directly as the number of the unemployed labour force that finds job (that is \( E \propto \tau U \)) but inversely as the number of employed labour force that exits employment (that is \( E \propto -\varphi E \)). Thus, at any time \( t \) years the change in the number of employed labour force with respect to time, denoted as \( \frac{dE}{dt} \), is given by...
\[
\frac{dE}{dt} = \tau U - \varphi E
\]  

(7)

The total number of labour force is the sum of employed and the unemployed labour force. We obtain as thus:

\[L = E + U\]  

(8)

We want to express (7) in terms of \(E\) only so we obtain from (8) as follows:

\[U = L - E\]  

(9)

Substitute (9) into (7) to obtain (4) as follows:

\[
\frac{dE}{dt} = \tau(L - E) - \varphi E
\]

\[
\frac{dE}{dt} = \tau L - \tau E - \varphi E
\]

\[
\frac{dE}{dt} = \tau L - (\tau + \varphi)E
\]

\[
\frac{dE}{dt} = -(\tau + \varphi)E + \tau L
\]  

(10)

(10) is a Non-Homogeneous First Order Ordinary Linear Differential Equation in \(E\) which can be reduced to the standard form as thus:

\[
\frac{dE(t)}{dt} = p(t)E(t) + q(t)
\]  

(11)

Where

\[p(t) = -(\tau + \varphi)\]

\[q(t) = \tau L(t)\]

We solve (10) using the method of variation of parameters (MVP). Here, we first assume that (10) is homogeneous, that is \(\tau L = 0; L \neq 0\). Accordingly we obtain as demonstrated below:

\[
\frac{dE}{dt} = -(\tau + \varphi)E
\]

By separating variables and integrating both sides, we obtain

\[
\int \frac{dE}{E} = -\int (\tau + \varphi)dt
\]

\[\ln[E(t)] = -(\tau + \varphi)t + k, \text{ (} k = \text{constant) }\]

\[E(t) = e^{-(\tau+\varphi)t+k}\]
\[ E(t) = K(t)e^{-(\tau + \varphi)t} ; \ K(t) = e^k \] (12)

We assume the constant to be a function of \( t \) as required by the MVP. We differentiate \( (12) \) to obtain as follows:

\[ \frac{dE}{dt} = K'(t)e^{-(\tau + \varphi)t} - (\tau + \varphi)K(t)e^{-(\tau + \varphi)t} \] (13)

Substitute \( (12) \) and \( (13) \) into \( (10) \) to obtain \( (14) \) as illustrated below:

\[
K'(t)e^{-(\tau + \varphi)t} - (\tau + \varphi)K(t)e^{-(\tau + \varphi)t} = -(\tau + \varphi)K(t)e^{-(\tau + \varphi)t} + \tau L(t)
\]

\[
K'(t)e^{-(\tau + \varphi)t} = \tau L(t)
\]

\[
K'(t) = \tau L(t)e^{(\tau + \varphi)t}
\]

\[
K(t) = \int \tau L(t)e^{(\tau + \varphi)t} dt
\]

\[
K(t) = \frac{\tau L(t)}{\tau + \varphi}e^{(\tau + \varphi)t} + A, \ (A = \text{constant})
\] (14)

Substituting \( (14) \) into \( (12) \), we obtain as thus:

\[
E(t) = \left[ \frac{\tau L(t)}{\tau + \varphi}e^{(\tau + \varphi)t} + A \right] e^{-(\tau + \varphi)t}
\]

\[
E(t) = \frac{\tau L(t)}{\tau + \varphi} + A e^{-(\tau + \varphi)t}
\] (15)

From \( (15) \), we obtain the value of the constant \( A \) by assuming the initial condition \( t = t_0 = 0 \). Thus \( E(t_0) = E(0), L(0) = L_0 \), which represent total number of employed labour force during the year 2000. To apply \( E(t_0) = E(0) \) to \( (15) \), do:

\[
E(t_0) = \frac{\tau L(0)}{\tau + \varphi} + A e^{-(\tau + \varphi)t_0}
\]

\[
E(0) = \frac{\tau L(0)}{\tau + \varphi} + A e^{-(\tau + \varphi)0}
\]

\[
E(0) = \frac{\tau L(0)}{\tau + \varphi} + A
\]

\[
A = E(0) - \frac{\tau L_0}{\tau + \varphi}
\] (16)

Now, put \( (16) \) into \( (15) \) so as to obtain \( (17) \).

\[
E(t) = \frac{\tau L}{\tau + \varphi} + \left( E(0) - \frac{\tau L(0)}{\tau + \varphi} \right) e^{-(\tau + \varphi)t}
\] (17)
Accordingly, (17) represents employment model.

The employment rate is defined as the percentage ratio of the total number of employed labour force to the total number of labour force. This can be mathematically expressed as thus:

$$E_r(t) = \frac{\frac{\tau L}{t+\varphi} + \left(E(0) - \frac{\tau L_0}{t+\varphi}\right)e^{-(\tau+\varphi)t}}{L} \times 100$$

$$\Rightarrow E_r(t) = \frac{100}{L}E(t) = \frac{1}{L}[100E(t)] \quad (18)$$

(18) represents the unemployment rate model.

3.2 The Unemployment Model

We also analyze that the number of unemployed at time $t$ years has an inverse or negative relationship with the number of unemployed that find job but directly related to the number of employed labour force that exit or loose job at time $t$ years. Hence at time $t$ years, the change in the number of unemployed labour force with respect to time $t$ can be denoted by $\frac{dU}{dt}$. This can be expressed as follows:

$$\frac{dU}{dt} = -\tau U + \varphi E \quad (19)$$

We express (19) in terms of $U$ only. From (2), we obtain an expression for $E$ in terms of $U$ and $L$.

$$E = L - U \quad (20)$$

Substituting (20) into (19) give rise to (21) as demonstrated below:

$$\frac{dU}{dt} = -\tau U + \varphi(L - U)$$

$$\frac{dU}{dt} = -\tau U + \varphi L - \varphi U$$

$$\frac{dU}{dt} = -(\tau + \varphi)U + \varphi L \quad (21)$$

(21) is a Non-Homogeneous First Order Linear Ordinary Differential Equation of the form

$$\frac{dU(t)}{dt} = p(t)U(t) + q(t) \quad (22)$$

Where
We solve (21) by first assuming that it is homogeneous using the MVP. This will help obtain a general solution. We do as follows:

\[
\frac{dU}{dt} = -(\tau + \varphi)U
\]

Separate variables and integrate both sides as illustrated below:

\[
\int \frac{dU}{U} = -\int (\tau + \varphi) dt
\]

\[
\ln[U(t)] = -(\tau + \varphi)t + c, (c = constant)
\]

\[
U(t) = e^{-(\tau+\varphi)t+c}
\]

\[
U(t) = C(t)e^{-(\tau+\varphi)t}
\]

\[
C(t) = e^c
\]

We assume the constant \( C \) to be a variable in \( t \) as required by the MVP.

If we differentiate (23), we obtain (24) accordingly.

\[
\frac{dU}{dt} = C'(t)e^{-(\tau+\varphi)t} - (\tau + \varphi)C(t)e^{-(\tau+\varphi)t}
\]

We substitute (23) and (24) into (21) as follows:

\[
C'(t)e^{-(\tau+\varphi)t} - (\tau + \varphi)C(t)e^{-(\tau+\varphi)t} = -(\tau + \varphi)C(t)e^{-(\tau+\varphi)t} + \varphi L(t)
\]

\[
C'(t)e^{-(\tau+\varphi)t} = \varphi L(t)
\]

\[
C'(t) = \varphi L(t) e^{(\tau+\varphi)t}
\]

\[
C(t) = \int \varphi L(t) e^{(\tau+\varphi)t} dt
\]

\[
C(t) = \frac{\varphi L}{\tau + \varphi} e^{(\tau+\varphi)t} + B, (B = constant)
\]

We substitute (25) into (23) to get the following equation:

\[
U(t) = \frac{\varphi L(t)}{\tau + \varphi} e^{(\tau+\varphi)t} + B e^{-(\tau+\varphi)t}
\]

\[
U(t) = \frac{\varphi L(t)}{\tau + \varphi} + Be^{-(\tau+\varphi)t}
\]
We assume an initial value condition \( t = t_0 = 0 \). Hence \( L(t_0) = L(0), U(t_0) = U(0) \), where \( L(0) \) and \( U(0) \) represent the total number of labourforce and unemployed in 2000. We apply the initial value condition to (26) and obtain as illustrated:

\[
\begin{align*}
U(t_0) &= \frac{\varphi L(0)}{\tau + \varphi} + B e^{-(\tau + \varphi)t_0} \\
U(0) &= \frac{\varphi L(0)}{\tau + \varphi} + B e^{-(\tau + \varphi)0} \\
U(0) &= \frac{\varphi L(0)}{\tau + \varphi} + B \\
B &= U(0) - \frac{\varphi L(0)}{\tau + \varphi} \\
\end{align*}
\]

We put (27) into (26) to obtain (28).

\[
U(t) = \frac{\varphi L(t)}{\tau + \varphi} + \left( U(0) - \frac{\varphi L(0)}{\tau + \varphi} \right) e^{-(\tau + \varphi)t} \tag{28}
\]

(28) represents the unemployment model.

Unemployment rate is defined as the percentage ratio of the total number of unemployed labour force to the total number of the labour force. Mathematically, the unemployment rate \( U_r(t) \) can be expressed as thus:

\[
U_r(t) = \frac{\frac{\varphi L(t)}{\tau + \varphi} + \left( U(0) - \frac{\varphi L(0)}{\tau + \varphi} \right) e^{-(\tau + \varphi)t}}{L(t)} \times 100
\]

\[
\Rightarrow U_r(t) = \frac{100}{L(t)} [U(t)] \tag{29}
\]

Thus (29) is the unemployment rate model.

3.3 Model Verification

If we consider the definition of stable state equilibrium of the labour market, then we have

\[
\lim_{t \to \infty} [E(t)] = E(t)
\]

\[
\Rightarrow \lim_{t \to \infty} [E(t)] = \lim_{t \to \infty} \left[ \frac{\tau L(t)}{\tau + \varphi} + \left( E(0) - \frac{\varphi L(0)}{\tau + \varphi} \right) e^{-(\tau + \varphi)t} \right]
\]
\[ E(t) = \frac{\tau L(t)}{\tau + \phi} \Rightarrow E(t) = \frac{\tau}{\tau + \phi} \]

Similarly,

\[ \lim_{t \to \infty} [U(t)] = U(t) \]

\[ \Rightarrow \lim_{t \to \infty} [U(t)] = \lim_{t \to \infty} \left[ \frac{\phi L(t)}{\tau + \phi} + \left( U(0) - \frac{\phi L(0)}{\tau + \phi} \right) e^{-(\tau + \phi) t} \right] \]

\[ U(t) = \frac{\phi L(t)}{\tau + \phi} \Rightarrow \frac{U(t)}{L(t)} = \frac{\phi}{\tau + \phi} \]

Thus, we have succeeded in deriving the natural rates of employment and unemployment by allowing for variations in labour force. Our duty here is to demonstrate that the relaxation of the assumption of fixed labour force has no effect on the natural rates of employment and unemployment.

If we go by the restriction of fixed labour force, we obtain the following time-dependent equations for employment and unemployment as derived (the later) by Shimer (2012):

\[ E(t) = \frac{\tau L(t)}{\tau + \phi} \left( 1 - e^{-(\tau + \phi) t} \right) + E(0)e^{-(\tau + \phi) t} \]

(30)

And

\[ U(t) = \frac{\phi L(t)}{\tau + \phi} \left( 1 - e^{-(\tau + \phi) t} \right) + U(0)e^{-(\tau + \phi) t} \]

(31)

With the assumption of a stable state equilibrium at the labour market, we demonstrate as thus:

\[ \lim_{t \to \infty} [E(t)] = \lim_{t \to \infty} \left[ \frac{\tau L(t)}{\tau + \phi} \left( 1 - e^{-(\tau + \phi) t} \right) + E(0)e^{-(\tau + \phi) t} \right] \]

\[ \Rightarrow \frac{E(t)}{L(t)} = \frac{\tau}{\tau + \phi} \]

Similarly, we have

\[ \lim_{t \to \infty} [U(t)] = \lim_{t \to \infty} \left[ \frac{\phi L(t)}{\tau + \phi} \left( 1 - e^{-(\tau + \phi) t} \right) + U(0)e^{-(\tau + \phi) t} \right] \]

\[ \Rightarrow \frac{U(t)}{L(t)} = \frac{\phi}{\tau + \phi} \]
We have just been able to demonstrate that by allowing for the more realistic assumption of variable labour force, the natural rates of employment and unemployment remain unaffected.

4. Discussion of Results

Tables 1 and 2 illustrate, respectively, the computations of the number of employed and unemployed labour force; and the employment and unemployment rates using the models. They were executed on a Toshiba Laptop Computer with a Hard Disc Drive of 500GB and a RAM of 4GB. The results (including the values of $\tau$ and $\varphi$) were computed using the Microsoft Excel functions:

$$E(t) = \frac{(\tau \times L)}{(\tau + \varphi)} + \frac{(E(0) - ((L(0) \times \tau)/(\tau + \varphi))) \times EXP(-(\tau + \varphi) \times (t))}{\tau + \varphi}$$  \hspace{1cm} (32)

$$U(t) = \frac{(\varphi \times L)}{(\tau + \varphi)} + \frac{(U(0) - ((L(0) \times \varphi)/(\tau + \varphi))) \times EXP(-(\tau + \varphi) \times (t))}{\tau + \varphi}$$  \hspace{1cm} (33)

$$E_r(t) = \frac{(100 \times E(t))}{L}$$  \hspace{1cm} (34)

$$U_r(t) = \frac{(100 \times U(t))}{L}$$  \hspace{1cm} (35)

Where $\tau = 0.975$, $\varphi = 0.025$, $L = L(t)$ (that is labour force at time $t$), $L(0) = 8820$, $E(0) = 7903$, $U(0) = 917$ and $t = 0, 1, 2, \ldots, 14$; with 0 corresponding to the year 2000, 1 corresponding to 2001, etc.

Figures 1, 2, 3 and 4 also give visual impressions of the nature of the computations of total employment, unemployment, employment rates and unemployment rates respectively.

Like the actual values, the predicted values consistently increase throughout the whole period. The rate of increase in the values of total employed labour force fluctuates. The differences between current and previous values of the predicted number of employed increase for the periods: 200 – 2001, 2006 – 2008, 2010 – 2012 and 2013 – 2014 while they decrease for the periods: 2001 – 2006, 2008 – 2010 and 2012 – 2013. This explains precisely the employment and unemployment situations of the Ghanaian economy especially for the periods between 2008 and 2010, when there was a policy, not to employ people for two years.

The predicted values of employment are very close to the actual values. For the initial period, the predicted and actual values are equal, but from 2001 to 2011, the predicted values lie slightly above the actual values; the values became equal in 2012, the predicted values of employment rose slightly above the actual values in 2013 but eventually became equal in 2014 (see trend from Fig. 1). Observe from Table 1 and Figure 1 that the difference between the predicted and actual values of employment for each year is very small. This difference decreases progressively and consistently throughout the years until it irons out in 2012 and 2014, where the values of the predicted and actual employment are equal. The correlation coefficient between the actual and our predicted values of employment is very high, implying a very strong positive relationship
between the actual and predicted values of employment which is also significant as evinced in Table 3. Thus the JFJL model is efficacious in explaining the employment situation in Ghana.

The predicted values of unemployment equal the actual values in 2000 but continue to decrease to about half the actual values of unemployment in 2001, less than half from 2002 to 2010, more than half in 2011 and approximately equal from 2012 to 2014. Thus the difference between the actual and predicted values for each year was very wide from the beginning but this bridges out progressively until the actual and predicted values of unemployment became equal (observe from Table 1 and Fig. 2). It is obvious from Table 4 that the correlation between the actual and predicted values of unemployment is significant though it appears to be weak. Thus the performance of the JFJL model in explaining unemployment in the Ghanaian economy improves with time and will do better predictions of future rates of unemployment.

Table 2 reveals attributes about the values of predicted employment rates that share similar characteristics as that of predicted employment. The initial rates are approximately equal. The rates of actual and predicted employments differ, inclusively, for the period 2001 – 2011 (verify also from Fig. 3) where the predicted rates of employment consistently lie above the actual rates of employment. These differences are however very small attributable to the fact that the predicted values are very close to the actual values of employment rates. The difference between the actual and predicted rates eventually reduces to zero percent or close from 2012 through to 2014. The correlation coefficient between the actual and predicted rates of employment is approximately 0.5, which implies a moderate but significant positive correlation between the actual and predicted rates of employment (see Table 5).

Furthermore, we also observe the synonymous behaviour of the predicted values of unemployment and that of predicted rates of unemployment. The initial rates are approximately equal. The actual and predicted rates of unemployment are equal in 2000. However, from 2001 through to 2011 predicted rates consistently lie below the predicted rates of unemployment, with high differences from the initial stages which eventually reduces to zero or approximately close to zero from 2012 through to 2014. Table 6 displays an analysis of the strength of association between our predicted and the actual rates of unemployment. This appears to be moderately positive but very significant. This shows how potent the job – finding and separation rates are in describing, explaining and predicting the unemployment rates in the Ghanaian economy.

Imperatively, we establish that the JFJL model can reliably predict the employment and unemployment and their respective rates in the Ghanaian economy.

The roles that the job-finding and separation rates, \( \tau \) and \( \varphi \) respectively, play in determining the behaviour of employment and unemployment in the Ghanaian economy cannot be underemphasized. (17) suggests that high values of \( \tau \) and small values of \( \varphi \) will give rise to high values of employment. On the contrary, small values of \( \tau \) and high values of \( \varphi \) will result in low values of employment. Thus, employment will be very high in Ghana if the job-finding rate is high while the job-separation rate is very low. Similarly, (29) suggests that lower values of \( \tau \) and larger values of \( \varphi \) will estimate the values of unemployment to be very high. On the other hand, the unemployment rate will be low if \( \tau \) has higher values and \( \varphi \) has lower values. Our model suggests that these rates must respectively be kept close to 97.5% and 2.5%.
5. Conclusion

Our suggested models for employment and unemployment and their respective rates predict values and rates that are very close to the actual values and rates of employment and unemployment. It is important to mention that the models for number employed and employment rate predicted better than that for number unemployed and unemployment rate. However, the poor performance of the unemployment (or unemployment rates) model in predicting the unemployment values (or rates) was improved as time grows and eventually predicted approximately equal values or rates that equated the actual and predicted values (or rates) of unemployment. The correlation analyses performed between the actual and predicted values (or rates) of employment and unemployment reveal a significantly very high positive correlation between the actual and predicted values of employment; and an otherwise weak positive correlation between the actual and predicted values of unemployment which is very significant. Similarly, the correlations between actual and predicted rates of employment; and that of unemployment are moderately positive and significant.

We have been able to establish, in one direction, that the job-finding rate $\tau$ is positively associated with the values and rates of employment but negatively associated with the values and rates of unemployment; while in the opposite direction, that the job separation rate $\varphi$ is negatively related with the values and rates of employment but positively related with the values and rates of unemployment. These values of the job-finding and separation rates were estimated to be 97.5% and 2.5% respectively for the Ghanaian economy, which is in accord with the findings of Shimer (2012). Though Fujita and Ramey (2008) advanced evidence in support of the fact that both the job-finding and separation rates play very significant roles in explaining high rates of unemployment around the business cycle, we, on the other hand, contend that if employment rates are already very high, then of what significance are the rates of unemployment? Thus to increase employment or reduce unemployment in Ghana, policy makers must implement proactive or very vibrant policies that will aim at achieving higher rates of employment, which automatically will reduce the unemployment rates to (or close to) zero.

In conclusion, we establish that the job-finding rate $\tau$ and job separation rate $\varphi$ are very robust in explaining and predicting the employment and unemployment and their respective rates in the Ghanaian economy. Thus, the JFJL model is very potent in predicting the employment and unemployment situations in the Ghanaian economy.

It is worthy to mention, in repeated times, that the behaviour of our predicted values (or rates) of employment and unemployment is highly dependent on the values of the job-finding and separation rates. If these values were approximately close to that which should have existed with regard to the employment and unemployment situations in Ghana, there would have been little or no difference between the actual and our predicted values of employment and unemployment.

We recommend that policy makers in Ghana should adopt and implement very proactive or vibrant policies that will increase employment. The Ministry of Labour and Employment is also recommended to improve upon its work by extending their records of data to include the number of employed that lose their jobs and that of unemployed that find jobs on monthly or yearly basis so as to assist in obtaining approximately accurate values of the job-finding and separation rates.
References


Table 1: Actual Versus Predicted Values of Employment and Unemployment

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Source: Results from analysis of data

Table 2: Actual Versus Predicted Rates of Employment and Unemployment

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### Table 3: Correlation Coefficient between Actual and Predicted Values of Employment

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Source: Result from analysis of data

Fig. 1: Actual versus Predicted Values of Employment
Fig. 2: Actual versus Predicted Values of Unemployment

Fig. 3: Actual versus Predicted Rates of Employment
Fig 4: Actual versus Predicted Rates of Unemployment