STUDY OF EXPANSION FOR I-FUNCTION OF MULTIVARIABLE AND ITS APPLICATION

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ABSTRACT

In the present paper few finite integrals involving product of Jacobi polynomials and multivariable I-function of generalized arguments have been evaluated. These integrals have been utilized to establish the expansion for I $\left[z_1,....z_r\right]$ in series involving product of Jacobi polynomials, since multivariable I-function is quite general function in nature. On specializing the parameters of the functions involved in the results, many new as well as known relations may be obtained in application section.

KEYWORDS : I-function of multivariable

MATHEMATICAL SUBJECT CLASSIFICATION : 2011, 33C50

1. INTRODUCTION : Y.N. Prasad [4], defined the I-function of multivariable. We have to use following notation which is more summarized & self explanation

$$\left[z_1,....z_r\right] = I^{o,n:\{m_i,n_i\}}_{p,q:\{p_i,q_i\}}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\middle| \begin{matrix} S:T \\ \\ S':T' \end{matrix}\right]$$

$$I\left[z_1,....z_r\right] = \frac{1}{(2\pi\omega)^r}\int_{L_1}\int_{L_r}\phi_i s_i \prod_{j=1}^{\infty}(\phi_i s_i) Z_i^{S_i} ds_i,....,ds_r$$

(1.1)

where

$$\omega = \sqrt{(-1)}$$

$$S = \left(a_{2_j};\alpha'_{2_j},\alpha''_{2_j}\right)_{1,p_2},.........\left(a_{r_j};\alpha'_{r_j},\alpha''_{r_j}\right)_{1,p_r}.$$

(1.2)

$$S' = \left(b_{2_j}; \beta'_{2_j}, \beta''_{2_j}\right)_{1,q_2}, \ldots\ldots\ldots \left(b_{r_j}; \beta'_{r_j}, \beta''_{r_j}\right)_{1,q_r}.$$

(1.3)

$$T = \left(a'_j, \alpha'_j\right)_{1,p}, \ldots\ldots\ldots \left(a_j^{(r)}, \alpha_j^{(r)}\right)_{1,p^{(r)}}.$$

(1.4)

$$T' = \left(b'_j, \beta'_j\right)_{1,q}, \ldots\ldots\ldots \left(b_j^{(r)}, \beta_j^{(r)}\right)_{1,q^{(r)}}.$$

(1.5)

$$\phi_i s_i = \frac{\prod_{j=1}^{m^{(i)}} \Gamma\left(b_j^{(i)} - \beta_j^{(i)} s_i\right)}{\prod_{j=m^{(i)}+1}^{q^{(i)}} \Gamma\left(1 - b_j^{(i)} - \beta_j^{(i)} s_i\right)} \times \frac{\prod_{j=1}^{n^{(i)}} \Gamma\left(1 - a_j^{(i)} + \alpha_j^{(i)} s_i\right)}{\prod_{j=n^{(i)}+1}^{p^{(i)}} \Gamma\left(a_j^{(i)} - \alpha_j^{(i)} s_i\right)}$$

(1.6)

$$\theta_i s_i = \frac{\prod_{j=1}^{n_2} \Gamma\left(1 - a_{2_j} + \sum_{i=1}^{2}\right)\left(\alpha_{2_j}^{(i)} s_i\right) \ldots\ldots \prod_{j=1}^{n_r} \Gamma\left(1 - a_{2_j} + \sum_{i=1}^{r}\right)\left(\alpha_{r_j}^{(i)} s_i\right)}{\prod_{j=n_2+1}^{p_2} \Gamma\left(a_{2_j} - \sum_{i=1}^{2}\right)\left(\alpha_{2_j}^{(i)} s_i\right) \ldots\ldots \prod_{j=n_r+1}^{p_r} \Gamma\left(a_{2_j} + \sum_{i=1}^{r}\right)\left(\alpha_{r_j}^{(i)} s_i\right)}$$

(1.7)

Where, $i \in \{1, \ldots\ldots, r\}$.

With all other conditions already detailed by Ronghe [6] and Prasad [8].

The following known results will be utilized in the [1] present paper.

(i) $$\int_{-1}^{1} x^{\lambda} (1-x)^{\alpha} (1+x)^{\sigma} \mathbf{P}_{n\,(x)}^{(\alpha,\beta)} dx =$$

$$\frac{(-1)^n 2^{\sigma+\alpha+1} \Gamma(\alpha+n+1)}{n!\Gamma(\sigma+\alpha+n+2)} \, {}_3F_2\left[\begin{matrix} -\lambda, \sigma-\beta+1, \sigma+1; \\ \sigma+\beta+n+1, \sigma+\alpha+n+2, \end{matrix} : 2\right]$$

(1.8)

(ii) $$\int_{-1}^{1} x^{\lambda} (1-x)^{\rho} (1+x)^{\beta} \mathbf{P}_{n\,(x)}^{(\alpha,\beta)} dx =$$

$$\frac{(-1)^n 2^{\rho+\beta+1}\Gamma(\beta+n+1)}{n!\Gamma(\rho+\beta+n+2)}\ {}_3F_2\left[\begin{matrix}-\lambda,\rho-\beta+1,\rho+1;\\ \rho+\alpha-n+1,\rho+\beta+n+2,\end{matrix}:2\right]$$

(1.9)

2. INTEGRATION :

In this part we evaluate four Jacobi polynomial involving multivariable I-function and using this in expansion of $I[z_1,....z_r]$

<u>First Integral</u> :

$$\int_{-1}^{1} x^{\lambda}(1-x)^{\alpha}(1+x)^{\rho} P_n^{(\alpha,\beta)}(x)\, I^{o,n_2:.......:o,n_r\{m_i,n_i\}}_{p_2,q_2:......:p_r,q_r\{p_i,q_i\}}\begin{bmatrix} z_1(1+x/x)^{\mu_1}\\ \vdots \\ z_r(1+x/x)^{\mu_r}\end{bmatrix}dx$$

$$=\frac{(-1)^n 2^{\sigma+\alpha+1}\Gamma(\alpha+n+1)}{n!}\sum_{k=0}^{\infty}\frac{2^k}{k!} I^{o,n+3:\{m_i,n_i\}}_{p+3,\,q+3:\{p_i,q_i\}}\left[\begin{matrix} z_1 2^{\mu_1}\\ \vdots\\ z_r 2^{\mu_r}\end{matrix}\middle|\begin{matrix}P,S:T\\ \\ Q,S':T'\end{matrix}\right]$$

(2.1)

Where P and Q are set of parameters, that are as follows :

$$P=\{(1+\lambda-k:\mu_1,......,\mu_r),(\beta-k-\sigma:\mu_1,......,\mu_r),(-\sigma-k:\mu_1,......,\mu_r)\},$$

$$Q=\{(1+\lambda:\mu_1,......,\mu_r),(\beta+n-\sigma:\mu_1,......,\mu_r),(-1-\sigma-\alpha-n-k:\mu_1,......,\mu_r)\},$$

Integral (2.1) is valid under the following condition provided $\mu_1,......,\mu_r$ are positive real numbers such that not all of them are zero.

$$\text{Re}(\lambda)>-1, \text{Re}(\alpha)>-1, |\arg(z_i)|>\frac{1}{2}\Omega_i\pi, \text{Re}[(\sigma+\mu_i\xi_i)]>0, \text{Re}(\lambda-\mu_j\xi_j)>0,$$

Where $\xi_j=\sum_{j=1}^{r}\min_{1\le\le M_r}\left(\text{Re}\left(d_j^{(r)}\big/\delta_j^{(r)}\right)\right), \forall\, i,k\in\{1,......,r\},$

<u>Second Integral</u> :

$$\int_{-1}^{1} x^{\lambda}(1-x)^{\alpha}(1+x)^{\sigma} P_n^{(\alpha,\beta)}(x)\, I^{o,n_2:.......:o,n_r\{m_i,n_i\}}_{p_2,q_2:......:p_r,q_r\{p_i,q_i\}}\begin{bmatrix} z_1 x^{\mu_1}(1-x)^{\delta_1}\\ \vdots \\ z_r x^{\mu_r}(1-x)^{\delta_r}\end{bmatrix}dx$$

$$=\frac{(-1)^n 2^{\sigma+\alpha+1}\Gamma(\alpha+n+1)}{n!}\sum_{k=0}^{\infty}\frac{(-1)^k 2^k}{k!} I_{p+3,\,q+3:\{p_i,q_i\}}^{o,n+3:\{m_i,n_i\}}\left[\begin{array}{c|c} z_1\, 2^{\delta_1} & P,S:T \\ \vdots & \\ z_r\, 2^{\delta_r} & Q,S':T' \end{array}\right]$$

(2.2)

Where P and Q are set of parameters, that are as follows :

$P=\{(-\lambda:\mu_1,......,\mu_r),(\beta-\sigma-k:\delta_1,......,\delta_r),(-k-\sigma:\delta_1,......,\delta_r)\},$

$Q=\{(k-\lambda:\mu_1,......,\mu_r),(\beta+n-\sigma-k:\delta_1,......,\delta_r),(-\ell-n-k-\alpha:\delta_1,......,\delta_r)\},$

Integral (2.2) is valid under the following condition provided $\mu_1,......,\mu_r$ and $\delta_1,......,\delta_r$ are positive real numbers such that not all of them are zero.

$$\operatorname{Re}(\lambda)>-1, |\arg(z_i)|>\frac{1}{2}\Omega_i\pi, \operatorname{Re}(\lambda)+-\mu_j\xi_j>0, \operatorname{Re}[(\sigma)+\delta_j\xi_j]>0,$$

Where $\xi_j=\sum_{j=1}^{r}\min_{1\le\le M_r}\left(\operatorname{Re}\left(d_j^{(r)}\big/\delta_j^{(r)}\right)\right), \forall\, i,j,k\in\{1,......,r\},$

Third Integral :

$$\int_{-1}^{1}x^{\lambda}(1-x)^{\rho}(1+x)^{\beta}P_n^{(\alpha,\beta)}(x) I_{p_2,q_2:......:p_r,q_r\{p_i,q_i\}}^{o,n_2:......:o,n_r\{m_i,n_i\}}\left[\begin{array}{c} z_1\, x^{\mu_1}(1+x)^{\delta_1} \\ \vdots \\ z_r\, x^{\mu_r}(1+x)^{\delta_r} \end{array}\right]dx$$

$$=\frac{(-1)^n 2^{\rho+\beta+1}\Gamma(\beta+n+1)}{n!}\sum_{k=0}^{\infty}\frac{(-1)^k 2^k}{k!} I_{p+3,\,q+3:\{p_i,q_i\}}^{o,n+3:\{m_i,n_i\}}\left[\begin{array}{c|c} z_1\, 2^{\delta_1} & P,S:T \\ \vdots & \\ z_r\, 2^{\delta_r} & Q,S':T' \end{array}\right]$$

(2.3)

Where P and Q are set of parameters, that are as follows :

$P=\{(-\lambda:\mu_1,......,\mu_r),(\alpha-k-\rho:\delta_1,......,\delta_r),(-k-\rho:\delta_1,......,\delta_r)\},$

$Q=\{(k-\lambda:\mu_1,......,\mu_r),(\alpha-n-\rho:\delta_1,......,\delta_r),(-1-\beta-n-\rho:\delta_1,......,\delta_r)\},$

Integral (2.3) is valid under the following condition provided $\mu_1,......,\mu_r$ and $\delta_1,......,\delta_r$ are positive real numbers such that not all of them are zero.

$$\operatorname{Re}(\beta)>-1, \operatorname{Re}(\lambda)>-1, |\arg(z_i)|>\frac{1}{2}\Omega_i\pi, \operatorname{Re}(\rho)+\delta_j\xi_j>0, \operatorname{Re}[(\lambda)+\mu_j\xi_j]>0,$$

Where $\xi_j = \sum_{j=1}^{r} \min_{1\le \le M_r} \left(\operatorname{Re}\left(d_j^{(r)} \big/ \delta_j^{(r)} \right)\right), \forall\, i, j, k \in \{1,\ldots\ldots,r\},$

Fourth Integral :

$$\int_{-1}^{1} x^{\lambda}(1-x)^{\rho}(1+x)^{\beta} P_n^{(\alpha,\beta)}(x)\, I^{o,n_2:\ldots\ldots:o,n_r\{m_i,n_i\}}_{p_2,q_2:\ldots\ldots:p_r,q_r\{p_i,q_i\}}\left[\begin{matrix} z_1\left(1-x\big/x\right)^{\mu_1} \\ \vdots \\ z_r\left(1-x\big/x\right)^{\mu_r}\end{matrix}\right]dx$$

$$=\frac{(-1)^n 2^{\rho+\beta+1}\Gamma(\beta+n+1)}{n!}\sum_{k=0}^{\infty}\frac{2^k}{k!} I^{o,n+3:\{m_i,n_i\}}_{p+3,\,q+3:\{p_i,q_i\}}\left[\begin{matrix} z_1\, 2^{\mu_1} \\ \vdots \\ z_r\, 2^{\mu_r}\end{matrix}\left|\begin{matrix} P, S:T \\ \\ Q, S':T'\end{matrix}\right.\right]$$

(2.4)

Where P and Q are set of parameters, that are as follows :

$P=\{(1+\lambda-k:\mu_1,\ldots\ldots,\mu_r),(\alpha-\rho-k:\mu_1,\ldots\ldots,\mu_r),(-\rho-k:\mu_1,\ldots\ldots,\mu_r)\},$

$Q=\{(1+\lambda:\mu_1,\ldots\ldots,\mu_r),(\alpha+n-k:\mu_1,\ldots\ldots,\mu_r),(-1-\rho-\beta-n-k:\mu_1,\ldots\ldots,\mu_r)\},$

Integral (2.4) is valid under the following condition provided $\mu_1,\ldots\ldots,\mu_r$ and $\delta_1,\ldots\ldots,\delta_r$ are positive real numbers such that not all of them are zero.

$$\operatorname{Re}(\lambda)>1,\ \operatorname{Re}(\beta)>-1,\ |\arg(z_i)|>\frac{1}{2}\Omega_i\pi,\ \operatorname{Re}(\rho)+\mu_j\xi_j>0,\ \operatorname{Re}[(\lambda)+\mu_j\xi_j]>0,$$

Where $\xi_j = \sum_{j=1}^{r} \min_{1\le \le M_r} \left(\operatorname{Re}\left(d_j^{(k)} \big/ \delta_j^{(k)} \right)\right), \forall\, i, j, k \in \{1,\ldots\ldots,r\},$

PROOF :-To establish (2.1) expressing the I-function of multivariable in term of Mellin-Barnes type contour integral, interchanging order of integration which is justifiable due to absolute converges of the integrals involving in the process we have,

$$\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi_i s_i \prod_{j=1}^{r}(\theta_i s_i) Z_i^{S_i}$$

$$\times\left\{\int_{-1}^{1} x^{\lambda-(\mu_1\delta_1+\ldots..+\mu_r\delta_r)}(1-x)^{\alpha}(1+x)^{\sigma+(\mu_1\delta_1+\ldots..+\mu_r\delta_r)} P_n^{(\alpha,\beta)}(x)\right\} ds_i,\ldots.,ds_r$$

Evaluating the inner integral with the help of (1.8) & (1.9) we have,

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \phi_i s_i \prod_{j=1}^{r} (\theta_i s_i) Z_i^{S_i} \frac{(-1)^n 2^{\sigma+\alpha+1} \sum_{n=1}^{r} \mu_n \delta_n \Gamma(\alpha+n+1)}{n!\Gamma(\sigma+\alpha+n+2) \sum_{n=1}^{r} \mu_n \delta_n}$$

$${}_3F_2\left[\begin{matrix} -D, E, F\,; \\ G, H\,, \end{matrix} : 2\right] ds_i, \ldots, ds_r$$

Where

$$D = \left\{\left(\lambda + \sum_{n=1}^{r} \mu_n \delta_n\right)\right\}, E\left\{\left(\sigma - \beta - 1 + \sum_{n=1}^{r} \mu_n \delta_n\right)\right\}$$

$$F = \left\{\left(1 + \sigma + \sum_{n=1}^{r} \mu_n \delta_n\right)\right\}, G\left\{\left(1 - \sigma + \beta + n + \sum_{n=1}^{r} \mu_n \delta_n\right)\right\}$$

$$H\left\{\left(2 + \alpha + n + \sigma + \sum_{n=1}^{r} \mu_n \delta_n\right)\right\}$$

Now expressing the hypergeometric function as series, chaning the order of summation and integration in the view of [4, p. 176 (75)] and using definition of multivariable I-function (1.1) we get r.h.s. of (2.1).

Similarly with the help of (1.8) and (1.9) we evaluated integral (2.2), (2.3) and (2.4).

3. EXPANSION :

In this section we discuss the expression (2.1) has been obtained by using (2.4), and evaluated 4 expansion formula.

First Expansion :

$$x^{\lambda} (1+x)^{\sigma}\, I^{o,n_2:\ldots\ldots:o,n_r\{m_i,n_i\}}_{p_2,q_2:\ldots\ldots:p_r,q_r\{p_i,q_i\}} \left[\begin{matrix} z_1 (1+x/x)^{\mu_1} \\ \vdots \\ z_r (1+x/x)^{\mu_r} \end{matrix}\right]$$

$$= 2^{\sigma} \sum_{i=0}^{\infty} \sum_{k=0}^{\infty} \frac{\Gamma(1+\alpha+\beta+\ell)(1+\alpha+\beta+2\ell)}{\Gamma(1+\beta+\ell)} P_{n\ (x)}^{(\alpha,\beta)} \frac{2^k}{k!} \times$$

$$I^{o,n+3:\{m_i,n_i\}}_{p+3,q+3:\{p_i,q_i\}}\left[\begin{array}{c|c} z_1\, 2^{\mu_1} & P, S:T \\ \vdots & \\ z_r\, 2^{\mu_r} & Q, S':T' \end{array}\right]$$

(3.1)

Where P and Q are set of parameters, that are as follows :

$$P=\{(1+\lambda-k:\mu_1,......,\mu_r),(-k-\sigma:\mu_1,......,\mu_r),(-\sigma+\beta-k:\mu_1,......,\mu_r)\},$$

$$Q=\{(1+\lambda:\mu_1,......,\mu_r),(-\ell-\sigma-k:\mu_1,......,\mu_r),(-1-\sigma-\alpha-\ell-k:\mu_1,......,\mu_r)\},$$

The set of condition mentioned with (2.1) are satisfied.

<u>Second Expansion :</u>

$$x^{\lambda}(1+x)^{\sigma}\, I^{o,n_2:......:o,n_r\{m_i,n_i\}}_{p_2,q_2:......:p_r,q_r\{p_i,q_i\}}\left[\begin{array}{c} z_1\, x^{\mu_1}(1-x)^{\delta_1} \\ \vdots \\ z_r\, x^{\mu_r}(1-x)^{\delta_r} \end{array}\right]$$

$$=2^{\sigma}\sum_{\ell=0}^{\infty}\sum_{k=0}^{\infty}\frac{(-1)^{\ell}\,\Gamma(1+\alpha+\beta+\ell)(1+\alpha+\beta+2\ell)}{\Gamma(1+\beta+\ell)}\,\frac{(-1)^k\,2^k}{k!}\,P_{n}^{(\alpha,\beta)}(x)\times$$

$$I^{o,n+3:\{m_i,n_i\}}_{p+3,q+3:\{p_i,q_i\}}\left[\begin{array}{c|c} z_1\, 2^{\mu_1} & P, S:T \\ \vdots & \\ z_r\, 2^{\mu_r} & Q, S':T' \end{array}\right]$$

(3.2)

Where P and Q are set of parameters, that are as follows :

$$P=\{(-\lambda:\mu_1,......,\mu_r),(-\sigma-k:\delta_1,......,\delta_r),(-k-\sigma-\beta:\delta_1,......,\delta_r)\},$$

$$Q=\{(k-\lambda:\mu_1,......,\mu_r),(\ell-\sigma-k:\delta_1,......,\delta_r),(-1-\ell-k-\alpha-\beta:\delta_1,......,\delta_r)\},$$

The set of condition mentioned with (2.2) are satisfied.

<u>Third Expansion :</u>

$$x^{\lambda}(1-x)^{\rho}\, I^{o,n_2:......:o,n_r\{m_i,n_i\}}_{p_2,q_2:......:p_r,q_r\{p_i,q_i\}}\left[\begin{array}{c} z_1\, x^{\mu_1}(1-x)^{\delta_1} \\ \vdots \\ z_r\, x^{\mu_r}(1-x)^{\delta_r} \end{array}\right]$$

$$=2^{\rho}\sum_{\ell=0}^{\infty}\sum_{k=0}^{\infty}\frac{(-1)^{\ell}\,\Gamma(\alpha+\beta+1)(1+\alpha+\beta+2\ell)}{\Gamma(\ell+\alpha+1)}\,\frac{(-1)^k\,2^k}{k!}\,P_{n}^{(\alpha,\beta)}(x)\times$$

$$I^{o,n+3:\{m_i,n_i\}}_{p+3,q+3:\{p_i,q_i\}}\left[\begin{matrix} z_1\, 2^{\mu_1} \\ \vdots \\ z_r\, 2^{\mu_r} \end{matrix}\middle| \begin{matrix} P,S:T \\ \\ Q,S':T' \end{matrix}\right]$$

(3.3)

Where P and Q are set of parameters, that are as follows :

$$P=\{(-\lambda:\mu_1,......,\mu_r),(-\sigma-k:\delta_1,......,\delta_r),(-k-\rho-1:\delta_1,......,\delta_r)\},$$

$$Q=\{(k-\lambda:\mu_1,......,\mu_r),(\ell-\rho-k:\delta_1,......,\delta_r),(1-\alpha-\beta-\ell-k-\rho:\delta_1,......,\delta_r)\},$$

The set of condition mentioned with (2.3) are satisfied.

Fourth Expansion :

$$x^{\lambda}(1-x)^{\rho}\, I^{o,n_2:......:o,n_r\{m_i,n_i\}}_{p_2,q_2:......:p_r,q_r\{p_i,q_i\}}\left[\begin{matrix} z_1\left(1-x/x\right)^{\mu_1} \\ \vdots \\ z_r\left(1-x/x\right)^{\mu_r} \end{matrix}\right]$$

$$=2^{\rho}\sum_{\ell=0}^{\infty}\sum_{k=0}^{\infty}\frac{(-1)^{\ell}\,\Gamma(1+\alpha+\beta+\ell)(1+\alpha+\beta+2\ell)}{\Gamma(1+\alpha+\ell)}\frac{2^k}{k!}P_{\ell}^{(\alpha,\beta)}(x)\times$$

$$I^{o,n+3:\{m_i,n_i\}}_{p+3,q+3:\{p_i,q_i\}}\left[\begin{matrix} z_1\, 2^{\mu_1} \\ \vdots \\ z_r\, 2^{\mu_r} \end{matrix}\middle| \begin{matrix} P,S:T \\ \\ Q,S':T' \end{matrix}\right]$$

(3.4)

Where P and Q are set of parameters, that are as follows :

$$P=\{(\ell+\lambda-k:\mu_1,......,\mu_r),(-\rho-k:\mu_1,......,\mu_r),(-\rho-\alpha-k:\mu_1,......,\mu_r)\},$$

$$Q=\{(1+\lambda:\mu_1,......,\mu_r),(\ell-k-\rho:\mu_1,......,\mu_r),(1-\alpha-\rho-k-\beta-\ell:\mu_1,......,\mu_r)\},$$

The set of condition mentioned with (2.4) are satisfied.

PROOF :-To establish (3.1) let,

$$x^{\lambda}(1-x)^{\rho}\, I^{o,n_2:......:o,n_r\{m_i,n_i\}}_{p_2,q_2:......:p_r,q_r\{p_i,q_i\}}\left[\begin{matrix} z_1\,(1+x/x)^{\mu_1} \\ \vdots \\ z_r\,(1+x/x)^{\mu_r} \end{matrix}\right]\sum_{\ell=0}^{\infty}M_{\ell}\,P_n^{(\alpha,\beta)}(x)$$

(3.5)

Where M_{ℓ} is constant to be determined.

Equation (3.5) is valid since the expression on the left hand side is continues and is of bounded variation in the open interval (-1, 1), multiplying both side of (3.5) by $(1-x)^{\alpha}(1+x)^{\beta} P_{n}^{(\alpha,\beta)}(x)$ and integrating w.r.t to x between the limits -1 to 1 and right hand side changing the order of summation and integration, using the orthogonal property for the Jacobi polynomials Erdeliy; A. [3, p. 285 (5 and 9)] i.e.

$$M_{\ell} \frac{(-1)^{n} 2^{\sigma}(\ell+\alpha+\beta+n)\Gamma(\ell+2n+\alpha)}{\Gamma(1+\beta+\ell)} I_{p+3,\,q+3:\{p_i,q_i\}}^{o,n+3:\{m_i,n_i\}}\left[\begin{matrix} z_1\, 2^{\mu_1} \\ \vdots \\ z_r\, 2^{\mu_r} \end{matrix}\middle| \begin{matrix} P, S:T \\ \\ Q, S':T' \end{matrix}\right]$$

(3.6)

Where P and Q are set of parameters, that are as follows :

$$P=\{(1+\lambda-k:\mu_1,......,\mu_r),(\beta-k-\sigma:\mu_1,......,\mu_r),(-\sigma-k:\mu_1,......,\mu_r)\},$$

$$Q=\{(1+\lambda:\mu_1,......,\mu_r),(\beta+\ell-\sigma:\mu_1,......,\mu_r),(1-\sigma-\ell-k:\mu_1,......,\mu_r)\},$$

Substituting the value of M_{ℓ} from (3.6) in (3.5) we get the result (3.1).

Similarly evaluated other expansions.

4. APPLICATIONS :

(1) If put r = 2 in first, second, third and fourth respectively then expansion reduces to result of Anandani and Singh [1].

(2) If again putting r = 1 in first, second, third and fourth respectively expansion then author gives old results [6].

Similarly many known and unknown results we may obtained.

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