

E- CONVEX FUNCTIONS

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Abstract
Youness introduced the concepts of E – convex sets and E – convex functions and studied their properties. Following this in this paper we further characterize E- convex functions.
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1. Introduction and Preliminaries

Youness introduced the concepts of E – convex sets and E – convex functions in 1999 which have applications in various branches of Mathematical Sciences. In this paper we further discuss some basic properties of E- convex functions.

Definition 1.1: Let E : $R^n \to R^n$ be a map . A set M ⊆ R^n is said to be E – convex if λ E(x) + (1 – λ)E(y) ϵ M for x, y ∈ M and 0 ≤ λ ≤1 [6]

E(M) ⊆ M whenever M is E-convex.

Definition 1.2: Let E : $R^n \to R^n$ be a map and let M ⊆ R^n be convex. A function f: $R^n \to$ R is said to be E – convex on M if

f(λ E(x) + (1 – λ)E(y)) ≤ $\lambda f(E(x)) + (1 - \lambda)\, f(E(y))$for x, y ϵ M and 0 ≤ λ ≤1 [6]

Definition 1.3:A function f on R^n *is said to be positively homogeneous* if for every x $f(\lambda x) = \lambda f(x), 0 < \lambda < \infty$. [1]

It can be proved that if f is positively homogeneous then $f(0) = 0$

Lemma 1.4: Let f : $R^n \to$ R , E : $R^n \to R^n$ *and* M ⊆ R^n *be an* $E-$ *convex set such that* $E(M)$ *is a convex set in* R^n . Then f is E- convex if and only if f(α_1E(x_1) +α_2E(x_2) +...+α_mE(x_m)) ≤ $\sum_{i=1}^{m} \alpha_i$f(E(x_i)) for every integer m ≥ 2 and all $x_i \in M$, $\alpha_i \geq 0$, i = 1,2, ... , m with $\sum_{i=1}^{m} \alpha_i = 1$ [7]

2 Properties

Theorem 2.1: Let f : $\boldsymbol{R^n} \to \mathbf{R}$, be any E convex function and α ∈ ($-\infty$, ∞), *the sets* (i)\{ x : $f(E(x)) < \alpha$ \} and (ii) \{ x : $f(E(x)) \leq \alpha$ \} are E – convex then It has equivalence property.

Reflexive Property : Let M = \{ x ∈ $\boldsymbol{R^n}$: $f(E(x)) < \alpha$ \}. Let $x \in M$, 0 ≤ λ ≤1 .

Then $f(E(x)) < \alpha$. Since f is E – convex, using Definition 1.2

we have $f(\lambda E(x) + (1-\lambda)E(x)) \leq \lambda f(E(x)) + (1-\lambda) f(E(x))$

$$< \lambda \alpha + (1-\lambda)\alpha$$

$$= \alpha$$

Symmetry Property :

we have $f(\lambda E(x) + (1-\lambda)E(y)) \leq \lambda f(E(x)) + (1-\lambda) f(E(y))$

$$< \lambda \alpha + (1-\lambda)\alpha$$

$$= \alpha$$

$f(\lambda E(y) + (1-\lambda)E(x)) \leq \lambda f(E(y)) + (1-\lambda) f(E(x))$

$< \lambda\alpha + (1-\lambda)\alpha$

$= \alpha$

Transitive Property : $f(\lambda E(x) + (1-\lambda)E(y)) < \alpha$

$f(\lambda E(y) + (1-\lambda)E(z)) < \alpha$

then $f(\lambda E(x) + (1-\lambda)E(z)) < \alpha$

The proof for (ii) is analog.

Lemma 1.5: Let $f : R^n \to R$, $E : R^n \to R^n$ and $M \subseteq R^n$ be an $E-$convex set such that $E(M)$ is a convex set in R^n. Then f is E- convex if and only if $f(\alpha_1 E(x_1) .\alpha_2 E(x_2) .....\alpha_m E(x_m)) \leq \prod_{i=1}^{m} \alpha_i f(E(x_i))$ for every integer $m \geq 2$ and all $x_i \in M$, $\alpha_i \geq 0$, $i = 1,2,\dots,m$ with $\prod_{i=1}^{m} \alpha_i = 1$

Lemma 1.6: Let $E : R^n \to R^n$ be linear. Let $C_1, C_2,\dots, C_m$ be the E- convex subsets of R^n. Then $\lambda_1 C_1 . \lambda_2 C_2 \dots . \lambda_m C_m$ is an E- convex set where $\lambda_i \in R$ for $i = 1,2,3,\dots,m$.

Theorem 2.2 : Let f be a positively homogeneous E – convex function on $M \subseteq R^n$ such that E(M) is convex in R^n. Then for $\lambda_1 > 0, \lambda_2 > 0, \dots, \lambda_m > 0$ we have

$f(\lambda_1 E(x_1) .\lambda_2 E(x_2) .....\lambda_m E(x_m)) \leq \prod_{i=1}^{m} \lambda_i f(E(x_i))$

Proof: Let $\lambda_1 > 0, \lambda_2 > 0, \dots, \lambda_m > 0$ and $\lambda_1 . \lambda_2 \dots . \lambda_m = \lambda^m$

Let $c_i = \frac{\lambda_i}{\lambda}$ for i = 1,2,..,m. Then since $c_1 . c_2 \dots c_m = 1$

using lemma 1.5 $f(c_1 E(x_1) .c_2 E(x_2) .....c_m E(x_m)) \leq \prod_{i=1}^{m} c_i f(E(x_i))$ and using Definition 1.3 we get

$$f(\lambda_1 E(x_1) . \lambda_2 E(x_2) ..... \lambda_m E(x_m)) \leq \prod_{i=1}^{m} \lambda_i f(E(x_i))$$

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