OF GHOST FIELDS, CELESTIAL MONSTERS AND HELLHOUNDS-A FORTY SEVEN STOREY MODEL

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ABSTRACT: We study a consolidated system of event; cause and n Qubit register which makes computation with n Qubits. Model extensively dilates upon systemic properties and analyses the systemic behaviour of the equations together with other concomitant properties. Inclusion of event and cause, we feel enhances the “Quantum ness” of the system holistically and brings out a relevance in the Quantum Computation on par with the classical system, in so far as the analysis is concerned. Additional VARIABLES OF Space Time provide bastion for the quantum space time studies.

INTRODUCTION:

EVENT AND ITS VINDICATION:

There definitely is a sense of compunction, contrition, hesitation, regret, remorse, hesitation and reservation to the acknowledgement of the fact that there is a personal relation to what happens to oneself. Louis de Broglie said that the events have already happened and it shall disclose to the people based on their level of consciousness. So there is destiny to start with! Say I am undergoing some seemingly insurmountable problem, which has hurt my sensitivities, susceptibilities and sentimentalities that I refuse to accept that that event was waiting for me to happen. In fact this is the statement of stoic philosophy which is referred to almost as bookish or abstract. Wound is there; it had to happen to me. So I was wounded. Stoics tell us that the wound existed before me; I was born to embody it. It is the question of consummation, consolidation, concretization, consubstantiation, that of this, that creates an “event” in us; thus you have become a quasi cause for this wound. For instance, my feeling to become an actor made me to behave with such perfectionism everywhere, that people’s expectations rose and when I did not come up to them I fell; thus the ‘wound’ was waiting for me and ‘I’ was waiting for the wound! One fellow professor used to say like you are searching for ides, ideas also searching for you. Thus the wound possesses in itself a nature which is “impersonal and preindividual” in character, beyond general and particular, the collective and the private. It is the question of becoming universalistic and holistic in your outlook. Unless this fate had not befallen you, the “grand design” would not have taken place in its entire entirety. It had to happen. And the concomitant ramifications and pernicious or positive implications, Everything is in order because the fate befell you. It is not as if the wound had to get something that is best from me or that I am a chosen by God to face the event. As said earlier “the grand design” would have been altered. And it cannot alter. You got to play your part and go; there is just no other way. The legacy must go on. You shall be torch bearer and you shall hand over the torch to somebody. This is the name of the game in totalistic and holistic way.

When it comes to ethics, I would say it makes no sense if any obstreperous, obstreperous, ululations, serenading, tintinnabulations are made for the event has happened to me. It means to say that you are unworthy of the fate that has befallen you. To feel that what happened to you was unwarranted and not autonomous, telling the world that you are aggressively iconoclastic, veritably resentful, and volitionally resentient, is choosing the cast of allegation aspersions and accusations at the Grand Design. What is
immoral is to invoke the name of god, because some event has **happened to** you. Cursing him is immoral. Realize that it is all "grand design" and you are playing a part. Resignation, renunciation, revocation is only one form of resentience. Willing the event is primarily to **release** the eternal truth; in fact you cannot release an event despite the fact everyone tries all ways and means they pray god; they prostrate for others destitution, poverty, penury, misery. But **releasing an** event is something like an "action at a distance" which only super natural power can do. Here we are face to face with volitional intuition and repetitive transmutation. Like a premeditated skirmisher, one **quarrel** with one self, with others, with god, and finally the accuser **leaves** this world in despair. Now look at this sentence which was quoted by I think Bousquet "if there is a **failure of will**", "I will **substitute a** longing for death" for that shall be apotheosis, a perpetual and progressive glorification of the will.

**EVENT AND SINGULARITIES IN QUANTUM SYSTEMS:**

What is an event? Or for that matter an ideal event? An event is a singularity or rather a set of singularities or set of singular points characterizing a mathematical curve, a physical state of affairs, a psychological person or a moral person. Singularities are turning points and points of inflection: they are bottle necks, foyers and centers; they are points of fusion; condensation and boiling; points of tears and joy; sickness and health; hope and anxiety; they are so to say "sensitive" points; such singularities should not be confused or confused, aggravated or exacerbated with personality of a system expressing itself; or the individuality and idiosyncrasies of a system which is designated with a proposition. They should also not be fused with the generalizational concept or universalistic axiomatic predications and postulation alcovishness, or the dipsomaniac flageolet dirge of a concept. Possible a concept could be signified by a figurative representation or a schematic configuration. "Singularity is essentially, pre individual, and has no personalized bias in it, or for that matter a prejudice or pre circumspection of a conceptual scheme. It is in this sense we can define a "singularity" as being neither affirmative nor non affirmative. It can be positive or negative; it can create or destroy. On the other hand it must be noted that singularity is different both in its thematic discursive from the run of the mill day to day musings and mundane drooling. They are in that sense "extra-ordinary".

Each singularity is a **source and resource**, the origin, reason and raison d'être of a mathematical series, it could be any series any type, and that is interpolated or extrapolated to the structural location of the destination of another singularity. This according to this standpoint, there are different. It can be positive or negative; it can create or destroy. On the other hand it must be noted that singularity is different both in its thematic discursive from the run of the mill day to day musings and mundane drooling. There are in that sense "extra-ordinary".

This according to the widely held standpoint, there are different, multifarious, myriad, series in a structure. In the eventuality of the fact that we conduct an unbiased and prudent examination of the series belonging to different "singularities" we can come to indubitable conclusions that the "singularity" of one system is different from the "other system" in the subterranean realm and ceratoid dualism of comparison and contrast

EPR experiment derived that there exists a communications between two particles. We go a further step to say that there exists a **channel** of communication however slovenly, inept, clumsy, between the two singularities. It is also possible the communication exchange could be one of belligerence, cantankerousness, tempestuousness, astutely truculent, with ensorcelled frenzy. That does not matter. All we are telling is that singularities communicate with each other.

Now, how do find the **reaction** of systems to these singularities. You do the same thing a boss does for you. "Problematize" the events and see how you behave. I will resort to "pressure tactics". "intimidation of deriding report", or "cut in the increment" to make you undergo trials, travails and tribulations. I am happy to see if you improve your work; but may or may not be sad if you succumb to it and hang yourself! We do the same thing with systems. systems show conducive response, felicitous reciprocation or behave erratically with inner soil, eponymous radicalism without and with blitzy conviction say like a solipsist nature of bellicose and blustering particles, or for that matter coruscation, trepidiational motion in fluid flows, or seemingly perfidious incendiaries in gormandizing fellow elementary particles, abnormal ebullitions, surcharges calumniations and unwarranted(you think so but the system does not!) unrighteous fulminations.

So the point that is made here is "like we problematize the "events" to understand the human behaviour we have to "problematize" the events of systems to understand their behaviour. This statement is made in connection to the fact that there shall be **creation or destruction** of particles or
complete obliteration of the system (blackhole evaporation) or obfuscation of results. Some systems are like "inside traders" they will not put signature at all! How do you find they did it? Anyway, there are possibilities of a CIA finding out as they recently did! So we can do the same thing with systems to. This is accentuation, corroboration, fortification, fomentatory notes to explain the various coefficients we have used in the model as also the dissipations called for.

In the Bank example we have clarified that various systems are individually conservative, and their conservativeness extends holistically too. that one law is universal does not mean there is complete adjudication of nonexistence of totality or global or holistic figure. Total always exists and “individual” systems always exist, if we do not bring Kant in to picture! For the time being let us not! Equations would become more eneuretic and frenzied...

Various, myriad, series in a structure. In the eventuality of the fact that we conduct an unbiased and prudent examination of the series belonging to different "singularities" we can come to indubitable conclusions that the "singularity" of one system is different from the “other system” in the subterranean realm and ceratoid dualism of comparison and contrast.

CONSERVATION LAWS:

Conservation laws bears ample testimony, infallible observatory, and impeccable demonstration to the fact that the essential predications, character constitutions, ontological consonances remain unchanged with evolution despite the system’s astute truculence, serenading whimsicality, assymmetric disposition or on the other hand anachronistic dispensation, eponymous radicality, entropic entrepotishness or the subdued, relationally contributive, diverse parametrisizational, conducive reciprocity to environment, unconventional behaviour, eneuretic nonlinear frenetic ness, ensorcelled frenzy, abnormal ebulliations, surcharged fulminations, or the inner roil. And that holds well with the evolution with time. We present a model of the generalizational conservation of the theories. A theory of all the conservation theories. That all conservation laws hold and there is no relationship between them is béte noir. We shall on this premise build a 36 storey model that deliberates on various issues, structural, dependent, thematic and discursive.

Note THAT The classification is executed on systemic properties and parameters. And everything that is known to us measurable. We do not know “intangible”. Nor we accept or acknowledge that. All laws of conservation must hold. Hence the holistic laws must hold. Towards that end, interrelationships must exist. All science like law wants evidence and here we shall provide one under the premise that for all conservations laws to hold each must be interrelated to the other, lest the very conception is a fricative contretemps. And we live in “Measurement” world.

QUANTUM REGISTER:

Devices that harness and explore the fundamental axiomatic predications of Physics has wide ranging amplitudinal ramification with its essence of locus and focus on information processing that outperforms their classical counterparts, and for unconditionally secure communication. However, in particular, implementations based on condensed-matter systems face the challenge of short coherence times. Carbon materials, particularly diamond, however, are suitable for hosting robust solid-state quantum registers, owing to their spin-free lattice and weak spin–orbit coupling. Studies with the structurally notched criticism and schizoid fragments of manifestations of historical perspective of diamond hosting quantum register have borne ample testimony and, and at differential and determinate levels have articulated the generalized significations and manifestations of quantum logic elements can be realized by exploring long-range magnetic dipolar coupling between individually addressable single electron spins associated with separate colour centres in diamond. The strong distance dependence of this coupling was used to characterize the separation of single qubits (98±3 Å) with accuracy close to the value of the crystal-lattice spacing. Coherent control over electron spins, conditional dynamics, selective readout as well as switchable interaction should rip open glittering façade for a prosperous and
scintillating irreducible affirmation of open the way towards a viable room-temperature solid-state quantum register. As both electron spins are optically addressable, this solid-state quantum device operating at ambient conditions provides a degree of control that is at present available only for a few systems at low temperature (See for instance P. Neumann, R. Kolesov, B. Naydenov, J. Bec F. Rempp, M. Steiner V. Jacques,, G. Balasubramanian,M, M. L. Markham,, D. J. Twitchen,, S. Pezzagna,, J. Meijer, J. Twamley, F. Jelezko & J. Wrachtrup).

CAUSE AND EVENT:

MODULE NUMBERED ONE

NOTATION :

\( G_{i3} \) : CATEGORY ONE OF CAUSE \\
\( G_{i4} \) : CATEGORY TWO OF CAUSE \\
\( G_{i5} \) : CATEGORY THREE OF CAUSE \\
\( T_{i3} \) : CATEGORY ONE OF EVENT \\
\( T_{i4} \) : CATEGORY TWO OF EVENT \\
\( T_{i5} \) : CATEGORY THREE OF EVENT \\

FIRST TWO CATEGORIES OF QUBIT COMPUTATION:

MODULE NUMBERED TWO:

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\( G_{16} \) : CATEGORY ONE OF FIRST SET OF QUBITS \\
\( G_{17} \) : CATEGORY TWO OF FIRST SET OF QUBITS \\
\( G_{18} \) : CATEGORY THREE OF FIRST SET OF QUBITS \\
\( T_{16} \) : CATEGORY ONE OF SECOND SET OF QUBITS \\
\( T_{17} \) : CATEGORY TWO OF SECOND SET OF QUBITS \\
\( T_{18} \) : CATEGORY THREE OF SECOND SET OF QUBITS

THIRD SET OF QUBITS AND FOURTH SET OF QUBITS:

MODULE NUMBERED THREE:

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\( G_{20} \) : CATEGORY ONE OF THIRD SET OF QUBITS \\
\( G_{21} \) : CATEGORY TWO OF THIRD SET OF QUBITS
\( G_{22} : \text{CATEGORY THREE OF THIRD SET OF QUBITS} \)
\( T_{20} : \text{CATEGORY ONE OF FOURTH SET OF QUBITS} \)
\( T_{21} : \text{CATEGORY TWO OF FOURTH SET OF QUBITS} \)
\( T_{22} : \text{CATEGORY THREE OF FOURTH SET OF QUBITS} \)

**FIFTH SET OF QUBITS AND SIXTH SET OF QUBITS**

- **MODULE NUMBERED FOUR**

\( G_{24} : \text{CATEGORY ONE OF FIFTH SET OF QUBITS} \)
\( G_{25} : \text{CATEGORY TWO OF FIFTH SET OF QUBITS} \)
\( G_{26} : \text{CATEGORY THREE OF FIFTH SET OF QUBITS} \)
\( T_{24} : \text{CATEGORY ONE OF SIXTH SET OF QUBITS} \)
\( T_{25} : \text{CATEGORY TWO OF SIXTH SET OF QUBITS} \)
\( T_{26} : \text{CATEGORY THREE OF SIXTH SET OF QUBITS} \)

**SEVENTH SET OF QUBITS AND EIGHTH SET OF QUBITS:**

- **MODULE NUMBERED FIVE**

\( G_{28} : \text{CATEGORY ONE OF SEVENTH SET OF QUBITS} \)
\( G_{29} : \text{CATEGORY TWO OF SEVENTH SET OF QUBITS} \)
\( G_{30} : \text{CATEGORY THREE OF SEVENTH SET OF QUBITS} \)
\( T_{28} : \text{CATEGORY ONE OF EIGHTH SET OF QUBITS} \)
\( T_{29} : \text{CATEGORY TWO OF EIGHTH SET OF QUBITS} \)
\( T_{30} : \text{CATEGORY THREE OF EIGHTH SET OF QUBITS} \)

\( (n-1)\text{TH SET OF QUBITS AND } n\text{TH SET OF QUBITS:**}

- **MODULE NUMBERED SIX**
\[ G_{32} : \text{CATEGORY ONE OF (n-1)TH SET OF QUBITS} \]

\[ G_{33} : \text{CATEGORY TWO OF (n-1)TH SET OF QUBITS} \]

\[ G_{34} : \text{CATEGORY THREE OF (N-1)TH SET OF QUBITS} \]

\[ T_{32} : \text{CATEGORY ONE OF n TH SET OF QUBITS} \]

\[ T_{33} : \text{CATEGORY TWO OF n TH SET OF QUBITS} \]

\[ T_{34} : \text{CATEGORY THREE OF n TH SET OF QUBITS} \]

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**GLOSSARY OF MODULE NUMBERED SEVEN**

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\[ G_{36} : \text{CATEGORY ONE OF TIME} \]

\[ G_{37} : \text{CATEGORY TWO OF TIME} \]

\[ G_{38} : \text{CATEGORY THREE OF TIME} \]

\[ T_{36} : \text{CATEGORY ONE OF SPACE} \]

\[ T_{37} : \text{CATEGORY TWO OF SPACE} \]

\[ T_{38} : \text{CATEGORY THREE OF SPACE} \]

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\[ \left( a_{13} \right)^{(1)}, \left( a_{14} \right)^{(1)}, \left( a_{15} \right)^{(1)}, \left( b_{13} \right)^{(1)}, \left( b_{14} \right)^{(1)}, \left( b_{15} \right)^{(1)}, \left( a_{16} \right)^{(2)}, \left( a_{17} \right)^{(2)}, \left( a_{18} \right)^{(2)} \]

\[ \left( b_{16} \right)^{(2)}, \left( b_{17} \right)^{(2)}, \left( b_{18} \right)^{(2)}, \left( a_{20} \right)^{(3)}, \left( a_{21} \right)^{(3)}, \left( a_{22} \right)^{(3)}, \left( b_{20} \right)^{(3)}, \left( b_{21} \right)^{(3)}, \left( b_{22} \right)^{(3)} \]

\[ \left( a_{24} \right)^{(4)}, \left( a_{25} \right)^{(4)}, \left( a_{26} \right)^{(4)}, \left( a_{27} \right)^{(4)}, \left( a_{28} \right)^{(5)}, \left( a_{29} \right)^{(5)}, \left( a_{30} \right)^{(5)} \]

\[ \left( a_{28} \right)^{(5)}, \left( a_{29} \right)^{(5)}, \left( a_{30} \right)^{(5)}, \left( a_{32} \right)^{(6)}, \left( a_{33} \right)^{(6)}, \left( a_{34} \right)^{(6)}, \left( b_{32} \right)^{(6)}, \left( b_{33} \right)^{(6)}, \left( b_{34} \right)^{(6)} \]

are Accentuation coefficients

\[ \left( a_{13}^{\prime} \right)^{(1)}, \left( a_{14}^{\prime} \right)^{(1)}, \left( a_{15}^{\prime} \right)^{(1)}, \left( b_{13}^{\prime} \right)^{(1)}, \left( b_{14}^{\prime} \right)^{(1)}, \left( b_{15}^{\prime} \right)^{(1)}, \left( a_{16}^{\prime} \right)^{(2)}, \left( a_{17}^{\prime} \right)^{(2)}, \left( a_{18}^{\prime} \right)^{(2)} \]

\[ \left( b_{16}^{\prime} \right)^{(2)}, \left( b_{17}^{\prime} \right)^{(2)}, \left( b_{18}^{\prime} \right)^{(2)}, \left( a_{20}^{\prime} \right)^{(3)}, \left( a_{21}^{\prime} \right)^{(3)}, \left( a_{22}^{\prime} \right)^{(3)}, \left( b_{20}^{\prime} \right)^{(3)}, \left( b_{21}^{\prime} \right)^{(3)}, \left( b_{22}^{\prime} \right)^{(3)} \]

\[ \left( a_{24}^{\prime} \right)^{(4)}, \left( a_{25}^{\prime} \right)^{(4)}, \left( a_{26}^{\prime} \right)^{(4)}, \left( a_{27}^{\prime} \right)^{(4)}, \left( a_{28}^{\prime} \right)^{(5)}, \left( a_{29}^{\prime} \right)^{(5)}, \left( a_{30}^{\prime} \right)^{(5)} \]

\[ \left( a_{28}^{\prime} \right)^{(5)}, \left( a_{29}^{\prime} \right)^{(5)}, \left( a_{30}^{\prime} \right)^{(5)}, \left( a_{32}^{\prime} \right)^{(6)}, \left( a_{33}^{\prime} \right)^{(6)}, \left( a_{34}^{\prime} \right)^{(6)}, \left( b_{32}^{\prime} \right)^{(6)}, \left( b_{33}^{\prime} \right)^{(6)}, \left( b_{34}^{\prime} \right)^{(6)} \]

are Dissipation coefficients

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**CAUSE AND EVENT:**

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**MODULE NUMBERED ONE**
The differential system of this model is now (Module Numbered one)

\[
\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{12}, t)\right]G_{13}
\]

\[
\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{12}, t)\right]G_{14}
\]

\[
\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{12}, t)\right]G_{15}
\]

\[
\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13}
\]

\[
\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14}
\]

\[
\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15}
\]

\[+(a'''_{13})^{(1)}(T_{14}, t) = \text{ First augmentation factor}\]

\[-(b'''_{13})^{(1)}(G, t) = \text{ First deteriorations factor}\]

**FIRST TWO CATEGORIES OF QUBITS COMPUTATION:**

**MODULE NUMBERED TWO:**

The differential system of this model is now (Module numbered two)

\[
\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16}
\]

\[
\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17}
\]

\[
\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18}
\]

\[
\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}, t)\right]T_{16}
\]

\[
\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}, t)\right]T_{17}
\]

\[
\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}, t)\right]T_{18}
\]

\[+(a'''_{16})^{(2)}(T_{17}, t) = \text{ First augmentation factor}\]

\[-(b'''_{16})^{(2)}(G_{19}, t) = \text{ First deteriorations factor}\]

**THIRD SET OF QUBITS AND FOURTH SET OF QUBITS:**

**MODULE NUMBERED THREE**

The differential system of this model is now (Module numbered three)
\[ \frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[ (a_{20}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t) \right] G_{20} \]
\[ \frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[ (a_{21}')^{(3)} + (a_{21}'')^{(3)}(T_{21}, t) \right] G_{21} \]
\[ \frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[ (a_{22}')^{(3)} + (a_{22}'')^{(3)}(T_{21}, t) \right] G_{22} \]
\[ \frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[ (b_{20}')^{(3)} - (b_{20}'')^{(3)}(G_{23}, t) \right] T_{20} \]
\[ \frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[ (b_{21}')^{(3)} - (b_{21}'')^{(3)}(G_{23}, t) \right] T_{21} \]
\[ \frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[ (b_{22}')^{(3)} - (b_{22}'')^{(3)}(G_{23}, t) \right] T_{22} \]

FIFTH SET OF QUBITS AND SIXTH SET OF QUBITS:

The differential system of this model is now (Module number four)

\[ \frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[ (a_{24}')^{(4)} + (a_{24}'')^{(4)}(T_{25}, t) \right] G_{24} \]
\[ \frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[ (a_{25}')^{(4)} + (a_{25}'')^{(4)}(T_{25}, t) \right] G_{25} \]
\[ \frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[ (a_{26}')^{(4)} + (a_{26}'')^{(4)}(T_{25}, t) \right] G_{26} \]
\[ \frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[ (b_{24}')^{(4)} - (b_{24}'')^{(4)}(G_{27}, t) \right] T_{24} \]
\[ \frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[ (b_{25}')^{(4)} - (b_{25}'')^{(4)}(G_{27}, t) \right] T_{25} \]
\[ \frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[ (b_{26}')^{(4)} - (b_{26}'')^{(4)}(G_{27}, t) \right] T_{26} \]

SEVENTH SET OF QUBITS AND EIGHTH SET OF QUBITS:

The differential system of this model is now (Module number five)

\[ \frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[ (a_{28}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t) \right] G_{28} \]
\[ \frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[ (a_{29}')^{(5)} + (a_{29}'')^{(5)}(T_{29}, t) \right] G_{29} \]
\[
\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30}
\]
\[
\frac{dG_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)\right]T_{28}
\]
\[
\frac{dG_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)\right]T_{29}
\]
\[
\frac{dG_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)\right]T_{30}
\]
\[
+(a''_{28})^{(5)}(T_{29}, t) = \text{First augmentation factor}
\]
\[
-(b''_{28})^{(5)}((G_{31}), t) = \text{First detritions factor}
\]

**n-1)TH SET OF QUBITS AND nTH SET OF QUBITS:**

**MODULE NUMBERED SIX:**

The differential system of this model is now (Module numbered Six)

\[
\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32}
\]
\[
\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33}
\]
\[
\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34}
\]
\[
\frac{dG_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)\right]T_{32}
\]
\[
\frac{dG_{33}}{dt} = (b_{33})^{(6)}T_{33} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{33}), t)\right]T_{33}
\]
\[
\frac{dG_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)\right]T_{34}
\]
\[
+(a''_{32})^{(6)}(T_{33}, t) = \text{First augmentation factor}
\]

**SPACE AND TIME: GOVERNING EQUATIONS:**

The differential system of this model is now (SEVENTH MODULE)

\[
\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36}
\]
\[
\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37}
\]
\[
\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[ (a_{38}')^{(7)} + (a_{38}''^{(7)}(T_{37}, t)) \right] G_{38}
\]

\[
\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[ (b_{36}')^{(7)} - (b_{36}''^{(7)}(G_{39}, t)) \right] T_{36}
\]

\[
\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[ (b_{37}')^{(7)} - (b_{37}''^{(7)}(G_{39}, t)) \right] T_{37}
\]

\[
\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[ (b_{38}')^{(7)} - (b_{38}''^{(7)}(G_{39}, t)) \right] T_{38}
\]

\[+(a_{38}''^{(7)}(T_{37}, t)) = \text{First augmentation factor} \]

\[-(b_{38}''^{(7)}(G_{39}, t)) = \text{First detritions factor} \]

**FIRST MODULE CONCATENATION:**

\[
\frac{dG_{13}}{dt} = (a_{13})^{(1)}(1)G_{14} - \left[ \left( a_{13}'^{(1)} + a_{13}''^{(1)}(T_{14}, t) + a_{13}'''^{(1)}(T_{15}, t) + a_{13}''''^{(1)}(T_{16}, t) \right) \right] G_{13}
\]

\[
\frac{dG_{14}}{dt} = (a_{14})^{(1)}(1)G_{13} - \left[ \left( a_{14}'^{(1)} + a_{14}''^{(1)}(T_{15}, t) + a_{14}'''^{(1)}(T_{16}, t) + a_{14}''''^{(1)}(T_{17}, t) \right) \right] G_{14}
\]

\[
\frac{dG_{15}}{dt} = (a_{15})^{(1)}(1)G_{14} - \left[ \left( a_{15}'^{(1)} + a_{15}''^{(1)}(T_{16}, t) + a_{15}'''^{(1)}(T_{17}, t) + a_{15}''''^{(1)}(T_{18}, t) \right) \right] G_{15}
\]

Where \([a_{13}']^{(1)}(T_{14}, t), [a_{13}''^{(1)}(T_{14}, t), [a_{13}''''^{(1)}(T_{14}, t)]\) are first augmentation coefficients for category 1, 2 and 3

\[+(a_{13}''^{(2,2)}(T_{15}, t), +a_{13}''^{(2,2)}(T_{16}, t), +a_{13}''^{(2,2)}(T_{17}, t)\) are second augmentation coefficient for category 1, 2 and 3

\[+(a_{13}'^{(3)}(T_{16}, t), +a_{13}^{(3)}(T_{17}, t), +a_{13}''^{(3)}(T_{18}, t)\) are third augmentation coefficient for category 1, 2 and 3

\[+(a_{13}'^{(4,4,4)}(T_{17}, t), +a_{13}^{(4,4,4)}(T_{18}, t), +a_{13}''^{(4,4,4)}(T_{19}, t)\) are fourth augmentation coefficient for category 1, 2 and 3

\[+(a_{13}'^{(5,5,5,5)}(T_{18}, t), +a_{13}^{(5,5,5,5)}(T_{19}, t), +a_{13}''^{(5,5,5,5)}(T_{20}, t)\) are fifth augmentation coefficient for category 1, 2 and 3

\[+(a_{13}'^{(6,6,6,6)}(T_{19}, t), +a_{13}^{(6,6,6,6)}(T_{20}, t), +a_{13}''^{(6,6,6,6)}(T_{21}, t)\) are sixth augmentation coefficient for category 1, 2 and 3

\[+(a_{13}'^{(7)}(T_{20}, t), +a_{13}^{(7)}(T_{21, t), +a_{13}''^{(7)}(T_{22, t)}\) are seventh augmentation coefficients

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\[
\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix}
(b_{13})^{(1)}(G_{t}) & -(b_{14}^{(1)}(G_{t})) & -(b_{15}^{(1)}G_{t}) & -(b_{30}^{(3,3)}(G_{399}, t)) & -(b_{31}^{(1)}(G_{399}, t)) & -(b_{32}^{(1)}(G_{399}, t))

(b_{24})^{(4,4,4,4)}(G_{27}, t) & -(b_{25}^{(5,5,5,5)}(G_{331}, t)) & -(b_{26}^{(6,6,6,6)}(G_{355}, t))

(b_{27}^{(2,2,2)}(G_{19}, t)) & -(b_{27}^{(3,3)}(G_{23}, t)) & -(b_{27}^{(5,5,5,5)}(G_{331}, t)) & -(b_{27}^{(6,6,6,6)}(G_{355}, t))

(b_{28}^{(4,4,4,4)}(G_{27}, t)) & -(b_{29}^{(5,5,5,5)}(G_{331}, t)) & -(b_{30}^{(6,6,6,6)}(G_{355}, t))

(b_{31}^{(7,7)}(G_{399}, t)) & -(b_{32}^{(7,7)}(G_{399}, t)) & -(b_{33}^{(7,7)}(G_{399}, t)) & -(b_{34}^{(7,7)}(G_{399}, t))
\end{bmatrix}
\]

\[
\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix}
(b_{14})^{(1)}(G_{t}) & -(b_{14}^{(1)}(G_{t})) & -(b_{15}^{(1)}G_{t}) & -(b_{16}^{(2,2,2)}(G_{19}, t)) & -(b_{22}^{(3,3)}(G_{23}, t)) & -(b_{34}^{(6,6,6,6)}(G_{355}, t))

(b_{25}^{(4,4,4,4)}(G_{27}, t)) & -(b_{26}^{(5,5,5,5)}(G_{331}, t)) & -(b_{27}^{(6,6,6,6)}(G_{355}, t))

(b_{27})^{(2,2,2)}(G_{19}, t)) & -(b_{27}^{(3,3)}(G_{23}, t)) & -(b_{27}^{(5,5,5,5)}(G_{331}, t)) & -(b_{27}^{(6,6,6,6)}(G_{355}, t))

(b_{28}^{(4,4,4,4)}(G_{27}, t)) & -(b_{29}^{(5,5,5,5)}(G_{331}, t)) & -(b_{30}^{(6,6,6,6)}(G_{355}, t))

(b_{31}^{(7,7)}(G_{399}, t)) & -(b_{32}^{(7,7)}(G_{399}, t)) & -(b_{33}^{(7,7)}(G_{399}, t)) & -(b_{34}^{(7,7)}(G_{399}, t))
\end{bmatrix}
\]

\[
\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix}
(b_{15})^{(1)}(G_{t}) & -(b_{14}^{(1)}(G_{t})) & -(b_{15}^{(1)}G_{t}) & -(b_{16}^{(2,2,2)}(G_{19}, t)) & -(b_{22}^{(3,3)}(G_{23}, t)) & -(b_{34}^{(6,6,6,6)}(G_{355}, t))

(b_{26}^{(4,4,4,4)}(G_{27}, t)) & -(b_{27}^{(5,5,5,5)}(G_{331}, t)) & -(b_{28}^{(6,6,6,6)}(G_{355}, t))

(b_{27})^{(2,2,2)}(G_{19}, t)) & -(b_{27}^{(3,3)}(G_{23}, t)) & -(b_{27}^{(5,5,5,5)}(G_{331}, t)) & -(b_{27}^{(6,6,6,6)}(G_{355}, t))

(b_{28}^{(4,4,4,4)}(G_{27}, t)) & -(b_{29}^{(5,5,5,5)}(G_{331}, t)) & -(b_{30}^{(6,6,6,6)}(G_{355}, t))

(b_{31}^{(7,7)}(G_{399}, t)) & -(b_{32}^{(7,7)}(G_{399}, t)) & -(b_{33}^{(7,7)}(G_{399}, t)) & -(b_{34}^{(7,7)}(G_{399}, t))
\end{bmatrix}
\]

Where \( -(b_{14}^{(1)}(G_{t})) \), \( -(b_{15}^{(1)}(G_{t})) \), \( -(b_{16}^{(1)}(G_{t})) \) are first detritions coefficients for category 1, 2 and 3

\( -(b_{16}^{(2,2,2)}(G_{19}, t)) \), \( -(b_{17}^{(3,3)}(G_{23}, t)) \) are second detritions coefficients for category 1, 2 and 3

\( -(b_{18}^{(4,4,4,4)}(G_{27}, t)) \), \( -(b_{19}^{(5,5,5,5)}(G_{331}, t)) \) are third detritions coefficients for category 1, 2 and 3

\( -(b_{20}^{(4,4,4,4)}(G_{27}, t)) \), \( -(b_{21}^{(5,5,5,5)}(G_{331}, t)) \) are fourth detritions coefficients for category 1, 2 and 3

\( -(b_{22}^{(5,5,5,5)}(G_{331}, t)) \) are fifth detritions coefficients for category 1, 2 and 3

\( -(b_{23}^{(6,6,6,6)}(G_{355}, t)) \) are sixth detritions coefficients for category 1, 2 and 3

\( -(b_{24}^{(7,7)}(G_{399}, t)) \), \( -(b_{25}^{(7,7)}(G_{399}, t)) \), \( -(b_{26}^{(7,7)}(G_{399}, t)) \) ARE SEVENTH DETRITION COEFFICIENTS
Where \((b_{15})^{(1)}(G, t)\), \((b_{15})^{(2,2)}(G, t)\), \((b_{15})^{(3,3)}(G, t)\) are first detription coefficients for category 1, 2 and 3. 

\((b_{26})^{(4,4,4)}(G, t)\), \((b_{26})^{(5,5,5)}(G, t)\), \((b_{26})^{(6,6,6)}(G, t)\) are second detritions coefficients for category 1, 2 and 3. 

\((b_{27})^{(4,4,4)}(G, t)\), \((b_{27})^{(5,5,5)}(G, t)\), \((b_{27})^{(6,6,6)}(G, t)\) are third detritions coefficients for category 1, 2 and 3. 

\((b_{28})^{(4,4,4)}(G, t)\), \((b_{28})^{(5,5,5)}(G, t)\), \((b_{28})^{(6,6,6)}(G, t)\) are fourth detritions coefficients for category 1, 2 and 3. 

\((b_{29})^{(5,5,5,5)}(G, t)\), \((b_{29})^{(5,5,5,5)}(G, t)\), \((b_{29})^{(5,5,5,5)}(G, t)\) are fifth detritions coefficients for category 1, 2 and 3. 

\((b_{30})^{(6,6,6,6)}(G, t)\), \((b_{30})^{(6,6,6,6)}(G, t)\), \((b_{30})^{(6,6,6,6)}(G, t)\) are sixth detritions coefficients for category 1, 2 and 3.
\[
\frac{dg_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[ (a_{16})^{(2)}(T_{17}, t) + (a_{17})^{(1,1)}(T_{14}, t) + (a_{20})^{(3,3)}(T_{21}, t) \right] G_{16}
\]
\[
\frac{dg_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[ (a_{17})^{(2)}(T_{17}, t) + (a_{17})^{(1,1)}(T_{14}, t) + (a_{21})^{(3,3)}(T_{21}, t) \right] G_{17}
\]
\[
\frac{dg_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[ (a_{18})^{(2)}(T_{17}, t) + (a_{18})^{(1,1)}(T_{14}, t) + (a_{22})^{(3,3)}(T_{21}, t) \right] G_{18}
\]

Where \( (a_{16})^{(2)}(T_{17}, t) \), \( (a_{17})^{(2)}(T_{17}, t) \), and \( (a_{18})^{(2)}(T_{17}, t) \) are first augmentation coefficients for category 1, 2 and 3,

\( (a_{16})^{(1,1)}(T_{14}, t) \), \( (a_{17})^{(1,1)}(T_{14}, t) \), and \( (a_{18})^{(1,1)}(T_{14}, t) \) are second augmentation coefficients for category 1, 2 and 3,

\( (a_{20})^{(3,3)}(T_{21}, t) \), \( (a_{21})^{(3,3)}(T_{21}, t) \), and \( (a_{22})^{(3,3)}(T_{21}, t) \) are third augmentation coefficients for category 1, 2 and 3,

\( (a_{20})^{(4,4,4,4)}(T_{25}, t) \), \( (a_{21})^{(4,4,4,4)}(T_{25}, t) \), and \( (a_{22})^{(4,4,4,4)}(T_{25}, t) \) are fourth augmentation coefficients for category 1, 2 and 3,

\( (a_{30})^{(5,5,5,5)}(T_{29}, t) \), \( (a_{31})^{(5,5,5,5)}(T_{29}, t) \), and \( (a_{32})^{(5,5,5,5)}(T_{29}, t) \) are fifth augmentation coefficients for category 1, 2 and 3,

\( (a_{36})^{(6,6,6,6)}(T_{33}, t) \), \( (a_{37})^{(6,6,6,6)}(T_{33}, t) \), and \( (a_{38})^{(6,6,6,6)}(T_{33}, t) \) are sixth augmentation coefficients for category 1, 2 and 3.

\( (a_{30})^{(7,7,7)}(T_{37}, t) \), \( (a_{37})^{(7,7,7)}(T_{37}, t) \), and \( (a_{38})^{(7,7,7)}(T_{37}, t) \) are seventh detrition coefficients.

\[
\frac{dv_{16}}{dt} = (b_{16})^{(2)}(G_{19}, t) - \left[ (b_{16})^{(2)}(G_{19}, t) - (b_{17})^{(1,1)}(G_{19}, t) - (b_{20})^{(3,3,3)}(G_{23}, t) \right] T_{16}
\]
\[
\frac{dv_{17}}{dt} = (b_{17})^{(2)}(G_{19}, t) - \left[ (b_{17})^{(2)}(G_{19}, t) - (b_{17})^{(1,1)}(G_{19}, t) - (b_{21})^{(3,3,3)}(G_{23}, t) \right] T_{17}
\]
\[
\frac{dv_{18}}{dt} = (b_{18})^{(2)}(G_{19}, t) - \left[ (b_{18})^{(2)}(G_{19}, t) - (b_{18})^{(1,1)}(G_{19}, t) - (b_{22})^{(3,3,3)}(G_{23}, t) \right] T_{18}
\]

where \( -(b_{16})^{(2)}(G_{19}, t) \), \( -(b_{17})^{(2)}(G_{19}, t) \), and \( -(b_{18})^{(2)}(G_{19}, t) \) are first detrition coefficients for category 1, 2 and 3,

\( -(b_{16})^{(1,1)}(G_{19}, t) \), \( -(b_{17})^{(1,1)}(G_{19}, t) \), \( -(b_{18})^{(1,1)}(G_{19}, t) \) are second detrition coefficients for category 1, 2 and 3.
\[-(b_{20}^{(3,3,3)})(G_{20}, t) \quad -(b_{21}^{(3,3,3)})(G_{21}, t) \quad -(b_{22}^{(3,3,3)})(G_{22}, t) \] are third detrition coefficients for category 1, 2 and 3

\[-(b_{20}^{(4,4,4,4)})(G_{20}, t) \quad -(b_{21}^{(4,4,4,4)})(G_{21}, t) \quad -(b_{22}^{(4,4,4,4)})(G_{22}, t) \] are fourth detritions coefficients for category 1, 2 and 3

\[-(b_{20}^{(5,5,5,5)})(G_{20}, t) \quad -(b_{21}^{(5,5,5,5)})(G_{21}, t) \quad -(b_{22}^{(5,5,5,5)})(G_{22}, t) \] are fifth detrtions coefficients for category 1, 2 and 3

\[-(b_{20}^{(6,6,6,6)})(G_{20}, t) \quad -(b_{21}^{(6,6,6,6)})(G_{21}, t) \quad -(b_{22}^{(6,6,6,6)})(G_{22}, t) \] are sixth detrtions coefficients for category 1, 2 and 3

\[-(b_{36}^{(7,7,7)})(G_{36}, t) \quad -(b_{36}^{(7,7,7)})(G_{36}, t) \quad -(b_{36}^{(7,7,7)})(G_{36}, t) \] are seventh detrtions coefficients

**THIRD MODULE CONCATENATION**

\[
\frac{da_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix}
(a_{20}^{(3)}) + (a_{20}^{(3)})(T_{21}, t) \\
+ (a_{20}^{(4,4,4,4)})(T_{25}, t) \\
+ (a_{20}^{(5,5,5,5)})(T_{29}, t) \\
+ (a_{20}^{(6,6,6,6)})(T_{33}, t)
\end{bmatrix} G_{20}
\]

\[
\frac{da_{21}}{dt} = (a_{21})^{(3)}G_{21} - \begin{bmatrix}
(a_{21}^{(3)}) + (a_{21}^{(3)})(T_{21}, t) \\
+ (a_{21}^{(4,4,4,4)})(T_{25}, t) \\
+ (a_{21}^{(5,5,5,5)})(T_{29}, t) \\
+ (a_{21}^{(6,6,6,6)})(T_{33}, t)
\end{bmatrix} G_{21}
\]

\[
\frac{da_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix}
(a_{22}^{(3)}) + (a_{22}^{(3)})(T_{21}, t) \\
+ (a_{22}^{(4,4,4,4)})(T_{25}, t) \\
+ (a_{22}^{(5,5,5,5)})(T_{29}, t) \\
+ (a_{22}^{(6,6,6,6)})(T_{33}, t)
\end{bmatrix} G_{21}
\]

\[+(a_{30}^{(1,1,1)})(T_{30}, t) + (a_{34}^{(1,1,1)})(T_{34}, t) \] are first augmentation coefficients for category 1, 2 and 3

\[+(a_{30}^{(2,2,2)})(T_{17}, t) + (a_{34}^{(2,2,2)})(T_{17}, t) \] are second augmentation coefficients for category 1, 2 and 3

\[+(a_{30}^{(1,1,1)})(T_{16}, t) + (a_{34}^{(1,1,1)})(T_{16}, t) \] are third augmentation coefficients for category 1, 2 and 3

\[+(a_{30}^{(4,4,4,4)})(T_{25}, t) + (a_{34}^{(4,4,4,4)})(T_{29}, t) + (a_{34}^{(4,4,4,4)})(T_{33}, t) \] are fourth augmentation coefficients for category 1, 2 and 3

\[+(a_{30}^{(5,5,5,5)})(T_{29}, t) + (a_{34}^{(5,5,5,5)})(T_{29}, t) + (a_{34}^{(5,5,5,5)})(T_{29}, t) \] are fifth augmentation coefficients for category 1, 2 and 3

\[+(a_{30}^{(6,6,6,6)})(T_{33}, t) + (a_{34}^{(6,6,6,6)})(T_{33}, t) + (a_{34}^{(6,6,6,6)})(T_{33}, t) \] are sixth augmentation coefficients for category 1, 2 and 3

\[+(a_{36}^{(7,7,7)})(T_{37}, t) + (a_{36}^{(7,7,7)})(T_{37}, t) + (a_{36}^{(7,7,7)})(T_{37}, t) \] are seventh augmentation coefficients.
\[
\frac{dT_20}{dt} = (b_{20})^3T_{21} - \left[ (b_{20}'^u)^{(3)}(G_{23}, t) \right. \\
\left. \quad - (b_{20}'^u)^{(3)}(G_{27}, t) \right] \\
\frac{dT_{21}}{dt} = (b_{21})^3T_{20} - \left[ (b_{21}'^u)^{(3)}(G_{23}, t) \right. \\
\left. \quad - (b_{21}'^u)^{(3)}(G_{27}, t) \right] \\
\frac{dT_{22}}{dt} = (b_{22})^3T_{21} - \left[ (b_{22}'^u)^{(3)}(G_{23}, t) \right. \\
\left. \quad - (b_{22}'^u)^{(3)}(G_{27}, t) \right]
\]

- \((b_{20}')^{(1,1,1)}(G_{23}, t)\) - \((b_{20}')^{(1,1,1)}(G_{27}, t)\) are first detriments coefficients for category 1, 2 and 3

- \((b_{21}')^{(2,2,2)}(G_{19}, t)\) - \((b_{21}')^{(2,2,2)}(G_{29}, t)\) are second detriments coefficients for category 1, 2 and 3

- \((b_{22}')^{(1,1,1)}(G_{23}, t)\) - \((b_{22}')^{(1,1,1)}(G_{27}, t)\) are third detriments coefficients for category 1, 2 and 3

- \((b_{20}'\prime^{4,4,4,4,4})(G_{22}, t)\) - \((b_{20}'\prime^{4,4,4,4,4})(G_{27}, t)\) are fourth detriments coefficients for category 1, 2 and 3

- \((b_{21}'\prime^{5,5,5,5,5})(G_{31}, t)\) - \((b_{21}'\prime^{5,5,5,5,5})(G_{37}, t)\) are fifth detriments coefficients for category 1, 2 and 3

- \((b_{22}'\prime^{6,6,6,6,6})(G_{35}, t)\) - \((b_{22}'\prime^{6,6,6,6,6})(G_{37}, t)\) are sixth detriments coefficients for category 1, 2 and 3

- \((b_{20}''\prime^{7,7,7,7})(G_{39}, t)\) - \((b_{20}''\prime^{7,7,7,7})(G_{39}, t)\) are seventh detriments coefficients

FOURTH MODULE CONCATENATION:

\[
\frac{dG_{24}}{dt} = (a_{24})^4G_{25} - \left[ (a_{24}'^u)^{(4)}(T_{25}, t) \right. \\
\left. \quad + (a_{24}'u)^{(4,5.5)}(T_{25}, t) \right] \\
\frac{dG_{25}}{dt} = (a_{25})^4G_{24} - \left[ (a_{25}'^u)^{(4)}(T_{25}, t) \right. \\
\left. \quad + (a_{25}'u)^{(4,5.5)}(T_{25}, t) \right]
\]

\[\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{26} - \left[ (a_{26})^{(4)} + (a_{26}')^{(4)}(T_{25}, t) + (a_{30})^{(5.5)}(T_{29}, t) + (a_{34})^{(6.6)}(T_{33}, t) + (a_{40})^{(1,1,1,1)}(T_{14}, t) + (a_{46})^{(2,2,2,2)}(T_{17}, t) + (a_{52})^{(3,3,3,3)}(T_{21}, t) + (a_{58})^{(7,7,7,7)}(T_{37}, t) \right] G_{26}\]

Where \( (a_{26})^{(4)}(T_{25}, t) \), \( (a_{30})^{(5.5)}(T_{29}, t) \), \( (a_{34})^{(6.6)}(T_{33}, t) \) are first augmentation coefficients for category 1, 2 and 3.

\[\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[ (b_{24})^{(4)} - (b_{24}')^{(4)}(G_{27}, t) - (b_{16})^{(2,2,2,2)}(G_{19}, t) - (b_{20})^{(3,3,3,3)}(G_{29}, t) - (b_{30})^{(5,5,5,5)}(G_{31}, t) - (b_{34})^{(6,6,6,6)}(G_{35}, t) - (b_{40})^{(1,1,1,1)}(G_{14}, t) - (b_{46})^{(2,2,2,2)}(G_{17}, t) - (b_{52})^{(3,3,3,3)}(G_{21}, t) - (b_{58})^{(7,7,7,7)}(G_{37}, t) \right] T_{24}\]

\[\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[ (b_{25})^{(4)} - (b_{25}')^{(4)}(G_{27}, t) - (b_{16})^{(2,2,2,2)}(G_{19}, t) - (b_{20})^{(3,3,3,3)}(G_{29}, t) - (b_{30})^{(5,5,5,5)}(G_{31}, t) - (b_{34})^{(6,6,6,6)}(G_{35}, t) - (b_{40})^{(1,1,1,1)}(G_{14}, t) - (b_{46})^{(2,2,2,2)}(G_{17}, t) - (b_{52})^{(3,3,3,3)}(G_{21}, t) - (b_{58})^{(7,7,7,7)}(G_{37}, t) \right] T_{25}\]

\[\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[ (b_{26})^{(4)} - (b_{26}')^{(4)}(G_{27}, t) - (b_{16})^{(2,2,2,2)}(G_{19}, t) - (b_{20})^{(3,3,3,3)}(G_{29}, t) - (b_{30})^{(5,5,5,5)}(G_{31}, t) - (b_{34})^{(6,6,6,6)}(G_{35}, t) - (b_{40})^{(1,1,1,1)}(G_{14}, t) - (b_{46})^{(2,2,2,2)}(G_{17}, t) - (b_{52})^{(3,3,3,3)}(G_{21}, t) - (b_{58})^{(7,7,7,7)}(G_{37}, t) \right] T_{26}\]

Where \(- (b_{24})^{(4)}(G_{27}, t)\), \(- (b_{24}')^{(4)}(G_{27}, t)\), \(- (b_{16})^{(2,2,2,2)}(G_{19}, t)\), \(- (b_{20})^{(3,3,3,3)}(G_{29}, t)\), \(- (b_{30})^{(5,5,5,5)}(G_{31}, t)\), \(- (b_{34})^{(6,6,6,6)}(G_{35}, t)\) are first detrition coefficients for category 1, 2 and 3.
are fifth detrition coefficients for category 1, 2 and 3

\[-(b^{(1,1,1,1)}_{25}(G_{25}, t)) - (b^{(1,1,1,1)}_{26}(G_{26}, t)) - (b^{(1,1,1,1)}_{27}(G_{27}, t))\]

are sixth detrition coefficients for category 1, 2 and 3

\[-(b^{(7,7,7,7,7)}_{31}(G_{31}, t)) - (b^{(7,7,7,7,7)}_{32}(G_{32}, t)) - (b^{(7,7,7,7,7)}_{33}(G_{33}, t))\]

**ARE SEVENTH DETRITION COEFFICIENTS**

**FIFTH MODULE CONCATENATION:**

\[
\frac{dg_{28}}{dt} = (a_{28}(^{(5)}G_{28} - \begin{bmatrix}
(a^{(5)}_{28}, T_{28}, t) + (a^{(4,4)}_{28}, T_{28}, t) + (a^{(6,6,6)}_{28}, T_{33}, t) \\
+ (a^{(1,1,1,1)}_{13}, T_{14}, t) + (a^{(2,2,2,2)}_{17}, T_{17}, t) + (a^{(3,3,3,3)}_{23}, T_{23}, t)
\end{bmatrix} ) G_{28} \\

\frac{dg_{29}}{dt} = (a_{29}(^{(5)}G_{29} - \begin{bmatrix}
(a^{(5)}_{29}, T_{29}, t) + (a^{(4,4)}_{29}, T_{29}, t) + (a^{(6,6,6)}_{29}, T_{33}, t) \\
+ (a^{(1,1,1,1)}_{14}, T_{14}, t) + (a^{(2,2,2,2)}_{17}, T_{17}, t) + (a^{(3,3,3,3)}_{23}, T_{23}, t)
\end{bmatrix} ) G_{29} \\

\frac{dg_{30}}{dt} = (a_{30}(^{(5)}G_{30} - \begin{bmatrix}
(a^{(5)}_{30}, T_{29}, t) + (a^{(4,4)}_{30}, T_{29}, t) + (a^{(6,6,6)}_{30}, T_{33}, t) \\
+ (a^{(1,1,1,1)}_{15}, T_{14}, t) + (a^{(2,2,2,2)}_{17}, T_{17}, t) + (a^{(3,3,3,3)}_{23}, T_{23}, t)
\end{bmatrix} ) G_{30}

Where \[+(a^{(4,4)}_{28}, T_{28}, t), +(a^{(5)}_{29}, T_{29}, t), +(a^{(5)}_{30}, T_{29}, t)\] are first augmentation coefficients for category 1, 2 and 3

\[+(a^{(1,1,1,1)}_{13}, T_{14}, t), +(a^{(2,2,2,2)}_{17}, T_{17}, t), +(a^{(3,3,3,3)}_{23}, T_{23}, t)\]

are second augmentation coefficient for category 1, 2 at

\[+(a^{(6,6,6)}_{28}, T_{28}, t), +(a^{(4,4,4)}_{29}, T_{28}, t), +(a^{(4,4,4)}_{30}, T_{29}, t)\]

are third augmentation coefficient for category 1, 2 and 3

\[+(a^{(1,1,1,1)}_{13}, T_{14}, t), +(a^{(1,1,1,1)}_{14}, T_{14}, t), +(a^{(1,1,1,1)}_{15}, T_{14}, t)\]

are fourth augmentation coefficients for category 1, 2, and 3

\[+(a^{(2,2,2,2)}_{17}, T_{17}, t), +(a^{(2,2,2,2)}_{17}, T_{17}, t), +(a^{(2,2,2,2)}_{17}, T_{17}, t)\]

are fifth augmentation coefficients for category 1, 2, and 3

\[+(a^{(3,3,3,3)}_{23}, T_{23}, t), +(a^{(3,3,3,3)}_{23}, T_{23}, t), +(a^{(3,3,3,3)}_{23}, T_{23}, t)\]

are sixth augmentation coefficients for category 1, 2, and 3

\[
\frac{dT_{28}}{dt} = (b^{(5)}_{28})T_{28} - \begin{bmatrix}
(b^{(5)}_{28}, G_{31}, t) - (b^{(4,4)}_{28}, G_{23}, t) - (b^{(6,6,6)}_{28}, G_{35}, t) \\
- (b^{(1,1,1,1)}_{13}, G_{14}, t) - (b^{(2,2,2,2)}_{17}, G_{19}, t) - (b^{(3,3,3,3)}_{23}, G_{23}, t)
\end{bmatrix} T_{28}
\]

\[
\frac{dT_{29}}{dt} = (b^{(5)}_{29})T_{29} - \begin{bmatrix}
(b^{(5)}_{29}, G_{31}, t) - (b^{(4,4)}_{29}, G_{23}, t) - (b^{(6,6,6)}_{29}, G_{35}, t) \\
- (b^{(1,1,1,1)}_{14}, G_{14}, t) - (b^{(2,2,2,2)}_{17}, G_{19}, t) - (b^{(3,3,3,3)}_{23}, G_{23}, t)
\end{bmatrix} T_{29}
\]
\[
\frac{dT_{30}}{dt} = (b_{30})^{(5)} T_{29} - \left[ \begin{array}{c}
(b_{15}^{(5)})(G_{31}, t) \\
(b_{13}^{(5)})(G_{31}, t) \\
(b_{26}^{(4,4)})(G_{27}, t) \\
(b_{25}^{(6,6,6)})(G_{35}, t) \\
(b_{12}^{(1,1,1,1)})(G_{19}, t) \\
(b_{10}^{(2,2,2,2,2)})(G_{19}, t) \\
(b_{22}^{(3,3,3,3,3)})(G_{23}, t) \\
(b_{20}^{(7,7,7,7,7,7)})(G_{25}, t)
\end{array} \right] T_{30}
\]

where \(- (b_{30}^{(5)})(G_{32}, t), -(b_{20}^{(5)})(G_{32}, t), -(b_{10}^{(5)})(G_{32}, t)\) are first detrition coefficients for category 1, 2, and 3.

\(- (b_{25}^{(4,4)})(G_{27}, t), -(b_{15}^{(4,4)})(G_{27}, t), -(b_{10}^{(4,4)})(G_{27}, t)\) are second detrition coefficients for category 1, 2, and 3.

\(- (b_{26}^{(6,6,6)})(G_{35}, t), -(b_{16}^{(6,6,6)})(G_{35}, t), -(b_{10}^{(6,6,6)})(G_{35}, t)\) are third detrition coefficients for category 1, 2, and 3.

\(- (b_{14}^{(1,1,1,1,1)})(G_{19}, t), -(b_{12}^{(1,1,1,1,1)})(G_{19}, t), -(b_{10}^{(1,1,1,1,1)})(G_{19}, t)\) are fourth detrition coefficients for category 1, 2, and 3.

\(- (b_{16}^{(1,2,2,2,2,2)})(G_{19}, t), -(b_{14}^{(1,2,2,2,2,2)})(G_{19}, t), -(b_{12}^{(1,2,2,2,2,2)})(G_{19}, t)\) are fifth detrition coefficients for category 1, 2, and 3.

\(- (b_{28}^{(1,1,1,1,1,1)})(G_{25}, t), -(b_{26}^{(1,1,1,1,1,1)})(G_{25}, t), -(b_{24}^{(1,1,1,1,1,1)})(G_{25}, t)\) are sixth detrition coefficients for category 1, 2, and 3.

\section*{SIXTH MODULE CONCATENATION}

\[
\frac{dG_{32}}{dt} = (a_{32})^{(6)} G_{33} - \left[ \begin{array}{c}
(a_{32}^{(6)})(T_{33}, t) + (a_{32}^{(5,5,5)})(T_{33}, t) + (a_{24}^{(4,4,4)})(T_{33}, t) \\
+(a_{13}^{(1,1,1,1,1)})(T_{14}, t) + (a_{16}^{(2,2,2,2,2)})(T_{14}, t) + (a_{26}^{(3,3,3,3,3)})(T_{21}, t) \\
+a_{36}^{(7,7,7,7,7,7)}(T_{37}, t)
\end{array} \right] G_{32}
\]

\[
\frac{dG_{33}}{dt} = (a_{33})^{(6)} G_{33} - \left[ \begin{array}{c}
(a_{33}^{(6)})(T_{33}, t) + (a_{33}^{(5,5,5)})(T_{33}, t) + (a_{26}^{(4,4,4)})(T_{21}, t) \\
+(a_{14}^{(1,1,1,1,1,1)})(T_{14}, t) + (a_{17}^{(2,2,2,2,2,2)})(T_{14}, t) + (a_{21}^{(3,3,3,3,3)})(T_{21}, t) \\
+a_{37}^{(7,7,7,7,7,7)}(T_{37}, t)
\end{array} \right] G_{33}
\]

\[
\frac{dG_{34}}{dt} = (a_{34})^{(6)} G_{33} - \left[ \begin{array}{c}
(a_{34}^{(6)})(T_{33}, t) + (a_{34}^{(5,5,5)})(T_{33}, t) + (a_{22}^{(4,4,4)})(T_{21}, t) \\
+(a_{15}^{(1,1,1,1,1,1)})(T_{16}, t) + (a_{18}^{(2,2,2,2,2,2,2)})(T_{17}, t) + (a_{22}^{(3,3,3,3,3,3)})(T_{21}, t) \\
+a_{38}^{(7,7,7,7,7,7,7)}(T_{37}, t)
\end{array} \right] G_{33}
\]

\(+ (a_{30}^{(5)})(T_{35}, t) + (a_{30}^{(5)})(T_{35}, t) + (a_{30}^{(5)})(T_{35}, t)\) are first augmentation coefficients for category 1, 2, and 3.

\(+ (a_{25}^{(5,5,5)})(T_{27}, t) + (a_{25}^{(5,5,5)})(T_{27}, t) + (a_{25}^{(5,5,5)})(T_{27}, t)\) are second augmentation coefficients for category 1, 2, and 3.

\(+ (a_{26}^{(4,4,4)})(T_{27}, t) + (a_{26}^{(4,4,4)})(T_{27}, t) + (a_{26}^{(4,4,4)})(T_{27}, t)\) are third augmentation coefficients for category 1, 2, and 3.

\(+ (a_{15}^{(1,1,1,1,1,1)})(T_{16}, t) + (a_{15}^{(1,1,1,1,1,1)})(T_{16}, t) + (a_{15}^{(1,1,1,1,1,1)})(T_{16}, t)\) - fourth augmentation coefficients.

\(+ (a_{16}^{(1,2,2,2,2,2,2)})(T_{17}, t) + (a_{16}^{(1,2,2,2,2,2,2)})(T_{17}, t) + (a_{16}^{(1,2,2,2,2,2,2)})(T_{17}, t)\) - fifth augmentation coefficients.

\(+ (a_{20}^{(1,3,3,3,3,3)})(T_{23}, t) + (a_{20}^{(1,3,3,3,3,3)})(T_{23}, t) + (a_{20}^{(1,3,3,3,3,3)})(T_{23}, t)\) - sixth augmentation coefficients.

\(+ (a_{36}^{(7,7,7,7,7,7)})(T_{37}, t) + (a_{36}^{(7,7,7,7,7,7)})(T_{37}, t) + (a_{36}^{(7,7,7,7,7,7)})(T_{37}, t)\) ARE SIXTH
AUGMENTATION COEFFICIENTS

\[
\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix}
(b_{12})^{(6)}(G_{35}, t) & -(b_{29}''')^{(5,5,5)}(G_{31}, t) & -(b_{24}''')^{(4,4,4)}(G_{27}, t) \\
-(b_{13})^{(1,1,1,1,1)}(G, t) & -(b_{16})^{(2,2,2,2)}(G_{19}, t) & -(b_{20}''')^{(3,3,3,3)}(G_{23}, t) \\
-(b_{14})^{(1,1,1,1,1)}(G, t) & -(b_{17})^{(2,2,2,2,2)}(G_{19}, t) & -(b_{23}''')^{(3,3,3,3)}(G_{23}, t)
\end{bmatrix} T_{32}
\]

\[
\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix}
(b_{13})^{(6)}(G_{35}, t) & -(b_{29}''')^{(5,5,5)}(G_{31}, t) & -(b_{25}''')^{(4,4,4)}(G_{27}, t) \\
-(b_{14})^{(1,1,1,1,1)}(G, t) & -(b_{17})^{(2,2,2,2)}(G_{19}, t) & -(b_{22}''')^{(3,3,3,3)}(G_{23}, t) \\
-(b_{15})^{(1,1,1,1,1)}(G, t) & -(b_{18})^{(2,2,2,2,2)}(G_{19}, t) & -(b_{23}''')^{(3,3,3,3)}(G_{23}, t)
\end{bmatrix} T_{33}
\]

\[
\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix}
(b_{14})^{(6)}(G_{35}, t) & -(b_{30}''')^{(5,5,5)}(G_{31}, t) & -(b_{26}''')^{(4,4,4)}(G_{27}, t) \\
-(b_{15})^{(1,1,1,1,1)}(G, t) & -(b_{18})^{(2,2,2,2,2)}(G_{19}, t) & -(b_{22}''')^{(3,3,3,3)}(G_{23}, t) \\
-(b_{16})^{(1,1,1,1,1)}(G, t) & -(b_{19})^{(2,2,2,2,2)}(G_{19}, t) & -(b_{23}''')^{(3,3,3,3)}(G_{23}, t)
\end{bmatrix} T_{34}
\]

\[
-(b_{20}''')^{(5,5,5)}(G_{25}, t) - (b_{23}''')^{(5,5,5)}(G_{35}, t) - (b_{26}''')^{(5,5,5)}(G_{35}, t)
\]

are first detrition coefficients for category 1, 2, and 3

\[
-(b_{29}''')^{(5,5,5)}(G_{21}, t) - (b_{30}''')^{(5,5,5)}(G_{31}, t) - (b_{26}''')^{(5,5,5)}(G_{31}, t)
\]

are second detrition coefficients for category 1, 2, and 3

\[
-(b_{25}''')^{(4,4,4)}(G_{25}, t) - (b_{26}''')^{(4,4,4)}(G_{25}, t)
\]

are third detrition coefficients for category 1, 2, and 3

\[
-(b_{14}''')^{(1,1,1,1,1)}(G, t) - (b_{17}''')^{(1,1,1,1,1)}(G, t)
\]

are fourth detrition coefficients for category 1, 2, and 3

\[
-(b_{15}''')^{(2,2,2,2,2)}(G_{19}, t) - (b_{16}''')^{(2,2,2,2,2)}(G_{19}, t)
\]

are fifth detrition coefficients for category 1, 2, and 3

\[
-(b_{16}''')^{(3,3,3,3,3)}(G_{23}, t) - (b_{17}''')^{(3,3,3,3,3)}(G_{23}, t) - (b_{18}''')^{(3,3,3,3,3)}(G_{23}, t)
\]

are sixth detrition coefficients for category 1, 2, and 3

\[
-(b_{20}''')^{(7,7,7,7,7)}(G_{39}, t) - (b_{23}''')^{(7,7,7,7,7)}(G_{39}, t) - (b_{26}''')^{(7,7,7,7,7)}(G_{39}, t) - (b_{29}''')^{(7,7,7,7,7)}(G_{39}, t)
\]

ARE SEVENTH DETRITION COEFFICIENTS

SEVENTH MODULE CONCATENATION:

\[
\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37}
\]

\[
\begin{bmatrix}
(a_{36}''')^{(7)}(T_{37}, t) + (a_{36}''')^{(7)}(T_{37}, t) + (a_{36}''')^{(7)}(T_{17}, t) + (a_{36}''')^{(7)}(T_{17}, t) + (a_{26}''')^{(7)}(T_{23}, t) + (a_{26}''')^{(7)}(T_{23}, t)
\end{bmatrix} G_{36}
\]

\[
\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36}
\]

\[
\begin{bmatrix}
(a_{37}''')^{(7)}(T_{17}, t) + (a_{37}''')^{(7)}(T_{17}, t) + (a_{37}''')^{(7)}(T_{17}, t) + (a_{14}''')^{(7)}(T_{14}, t) + (a_{14}''')^{(7)}(T_{14}, t) + (a_{21}''')^{(7)}(T_{21}, t) + (a_{21}''')^{(7)}(T_{21}, t)
\end{bmatrix} G_{37}
\]
\[
\frac{dG_{38}}{dt} = \frac{\partial G_{37}}{\partial (a_{38}(t))} - \left[ a_{26}'(t) + a_{26}''(t)T_{37}, t \right] + \left[ a_{34}'(t) + a_{34}''(t)T_{14'}, t \right] + \left[ a_{22}'(t) + a_{22}''(t)T_{2}, t \right] + \left[ a_{28}'(t) + a_{28}''(t)T_{17}, t \right] + \left[ a_{25}'(t) + a_{25}''(t)T_{25}, t \right] + \left[ a_{23}'(t) + a_{23}''(t)T_{29}, t \right] G_{38}
\]

\[
\frac{dT_{36}}{dt} = \frac{\partial T_{37}}{\partial (b_{36}(t))} - \left[ b_{36}'(t) - b_{36}''(t)(G_{38}, t) - b_{19}'(t)(G_{19}, t) - b_{15}'(t)(G_{15}, t) \right] - \left[ b_{24}'(t)(G_{27}, t) - b_{24}''(t)(G_{31}, t) \right] T_{36}
\]

\[
\frac{dT_{37}}{dt} = \frac{\partial T_{36}}{\partial (b_{37}(t))} - \left[ b_{37}'(t) - b_{37}''(t)(G_{38}, t) - b_{19}'(t)(G_{19}, t) - b_{15}'(t)(G_{15}, t) \right] - \left[ b_{24}'(t)(G_{27}, t) - b_{24}''(t)(G_{31}, t) \right] T_{37}
\]

Where we suppose

(A) \( (a_i)^{(1)}, (a_j)^{(1)}, (a_{ij}''^{(1)}), (b_i)^{(1)}, (b_j)^{(1)}, (b_{ij}''^{(1)}), i, j = 13, 14, 15 > 0, \)

(B) The functions \( (a_{ij}''^{(1)}), (b_{ij}''^{(1)}) \) are positive continuous increasing and bounded.

Definition of \( (p_i)^{(1)}, (r_i)^{(1)}): \n\]

\[
(a_i''^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (A_{13})^{(1)}
\]

\[
(b_i''^{(1)}(G, t) \leq (r_i)^{(1)} \leq (B_{13})^{(1)}
\]

(C) \( \lim_{T_{14} \to \infty} (a_{ij}''^{(1)}(T_{14}, t) = (p_i)^{(1)} \)

\[
\lim_{G \to \infty} (b_{ij}''^{(1)}(G, t) = (r_i)^{(1)}
\]

Definition of \( (A_{13})^{(1)}, (B_{13})^{(1)}): \n\]

Where \( (A_{13})^{(1)}, (B_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)} \) are positive constants

and \( i = 13, 14, 15 \)

They satisfy Lipschitz condition:

\[
|(a_{ij}''^{(1)}(T_{14}, t) - (a_{ij}''^{(1)}(T_{14}, t)| \leq (A_{13})^{(1)}T_{14} - T_{14}'\epsilon^{(A_{13})^{(1)}t}
\]

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With the Lipschitz condition, we place a restriction on the behavior of functions \( (a'_i)^{(1)}(T_{14}, t) \) and \( (a''_i)^{(1)}(T_{14}, t) \). \( (T'_{14}, t) \) and \( (T_{14}, t) \) are points belonging to the interval \( [(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}] \). It is to be noted that \( (a''_i)^{(1)}(T_{14}, t) \) is uniformly continuous. In the eventuality of the fact, that if \( (\hat{M}_{13})^{(1)} = 1 \) then the function \( (a''_i)^{(1)}(T_{14}, t) \), the first augmentation coefficient attributable to terrestrial organisms, would be absolutely continuous.

**Definition of** \((\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}\):

(D) \quad (\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, \text{ are positive constants} \]

\[
\frac{(a_j)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_{j})^{(1)}}{(\hat{M}_{13})^{(1)}} < 1
\]

**Definition of** \((\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}\):

(E) \quad \text{There exists two constants } (\hat{P}_{13})^{(1)} \text{ and } (\hat{Q}_{13})^{(1)} \text{ which together with } (\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (A_{13})^{(1)} \text{ and } (B_{13})^{(1)} \text{ and the constants } (a_i)^{(1)}, (a_j)^{(1)}, (b_i)^{(1)}, (b_j)^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13, 14, 15, \text{ satisfy the inequalities}

\[
\frac{1}{(\hat{M}_{13})^{(1)}} [(a_i)^{(1)} + (a_j)^{(1)} + (A_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1
\]

\[
\frac{1}{(\hat{M}_{13})^{(1)}} [(b_i)^{(1)} + (b_j)^{(1)} + (B_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1
\]
\[ \frac{d\tau_h}{dt} = \left( b_{38}''(T_{37}) \right) - \left( b_{36}'(T_{37}) \right) - \left( b_{34}''(G_{32}, t) \right) = \frac{1}{T_{38}} \]

\[ + (a_{i,j}'')''(T_{37}, t) = \text{First augmentation factor} \]

\[ (1)(a_i)''(2), (a_j)''(2), (b_i)''(2), (b_j)''(2), (a_{i,j}'')''(2) > 0, \quad i, j = 16, 17, 18 \]

\[(F) \quad \text{The functions } (a_{i,j}'')''(2), (b_{i,j}'')''(2) \text{ are positive continuous increasing and bounded.}\]

\[ \text{Definition of } (p_i)''(2), (r_i)''(2): \]

\[ (a_i)''(2)(T_{17}, t) \leq (p_i)''(2) \leq (\hat{A}_{16})''(2) \]

\[ (b_i)''(2)(G_{19}, t) \leq (r_i)''(2) \leq (\hat{B}_{16})''(2) \]

\[ (G) \quad \text{lim}_{t \to 0} (a_i)''(2)(T_{17}, t) = (p_i)''(2) \]

\[ \text{lim}_{t \to 0} (b_i)''(2)(G_{19}, t) = (r_i)''(2) \]

\[ \text{Definition of } (\hat{A}_{16})''(2), (\hat{B}_{16})''(2) : \]

\[ \text{Where } (\hat{A}_{16})''(2), (\hat{B}_{16})''(2), (p_i)''(2), (r_i)''(2) \text{ are positive constants and } i = 16, 17, 18 \]

They satisfy Lipschitz condition:

\[ |(a_i)''(2)(T_{17}, t) - (a_i)''(2)(T_{17}, t)| \leq (\hat{A}_{16})''(2)|T_{17} - T_{17}|e^{-(\hat{A}_{16})''(2)t} \]

\[ |(b_i)''(2)(G_{19}, t) - (b_i)''(2)(G_{19}, t)| \leq (\hat{B}_{16})''(2)|G_{19} - G_{19}|e^{-(\hat{B}_{16})''(2)t} \]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_{i,j}'')''(2)(T_{17}, t)\) and\((a_{i,j}'')''(2)(T_{17}, t) : (T_{17}, t) \text{ And } (T_{17}, t) \text{ are points belonging to the interval } [(\hat{k}_{16})''(2), (\hat{M}_{16})''(2)].\) It is to be noted that if \((\hat{M}_{16})''(2) = 1\) then the function \((a_{i,j}'')''(2)(T_{17}, t)\), the SECOND augmentation coefficient would be absolutely continuous.

\[ \text{Definition of } (\hat{M}_{16})''(2), (\hat{k}_{16})''(2) : \]

\[ (H) \quad (\hat{M}_{16})''(2), (\hat{k}_{16})''(2), \text{ are positive constants} \]

\[ \frac{(a_i)''(2)}{(\hat{M}_{16})''(2)}, \frac{(b_i)''(2)}{(\hat{B}_{16})''(2)} < 1 \]

\[ \text{Definition of } (\hat{P}_{13})''(2), (\hat{Q}_{13})''(2) : \]

There exists two constants \((\hat{P}_{16})''(2)\) and \((\hat{Q}_{16})''(2)\) which together with \((\hat{M}_{16})''(2), (\hat{k}_{16})''(2), (\hat{A}_{16})''(2)\) and \((\hat{B}_{16})''(2)\) and the constants \((a_i)''(2), (a_j)''(2), (b_i)''(2), (b_j)''(2), (p_i)''(2), (r_i)''(2), i = 16, 17, 18, \]
satisfy the inequalities
\[
\frac{1}{(\hat{M}\varepsilon_5)^2} \left[ (a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{B}_{16})^{(2)} (\hat{k}_{16})^{(2)} \right] < 1
\]
\[
\frac{1}{(\hat{M}\varepsilon_5)^2} \left[ (b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)} \right] < 1
\]
Where we suppose

(i) \((a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \ i, j = 20,21,22\)
The functions \((a_i'')^{(3)}, (b_i'')^{(3)}\) are positive continuous increasing and bounded.

**Definition of** \((p_i)^{(3)}, (r_i)^{(3)}:\)

\[
(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}
\]
\[
(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (\hat{B}_{20})^{(3)}
\]

\[
\lim_{T_{21} \to 0}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)}
\]
\[
\lim_{G_{23} \to 0}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}
\]

**Definition of** \((\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}:\)
Where \([\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}\] are positive constants and \(I = 20,21,22\)

They satisfy Lipschitz condition:

\[
|(a_i')^{(3)}(T_{21}', t) - (a_i')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21}' - T_{21}|e^{-(\hat{M}_{20})^{(3)}t}
\]
\[
|(b_i')^{(3)}(G_{23}', t) - (b_i')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}|G_{23}' - G_{23}|e^{-(\hat{M}_{20})^{(3)}t}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i'')^{(3)}(T_{21}', t)\) and\((a_i'')^{(3)}(T_{21}, t)\). And \((T_{21}, t)\) are points belonging to the interval \([\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\] . It is to be noted that \((a_i'')^{(3)}(T_{21}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\hat{M}_{20})^{(3)} = 1\) then the function \((a_i'')^{(3)}(T_{23}, t)\), the THIRD augmentation coefficient, would be absolutely continuous.

**Definition of** \((\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}:\)

(J) \((\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, \) are positive constants

\[
\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{k}_{20})^{(3)}} < 1
\]

There exists two constants \((\hat{P}_{20})^{(3)}\) and \((\hat{Q}_{20})^{(3)}\) which together with \((\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)}\) and \((\hat{B}_{20})^{(3)}\) and the constants \((a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22\), satisfy the inequalities

\[
\frac{1}{(\hat{M}_{20})^{(3)}} \left[ (a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)} \right] < 1
\]
\[
\frac{1}{(\hat{M}_{20})^{(3)}} \left[ (b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)} \right] < 1
\]
Where we suppose

\[ (a_i^{(4)}, a_i^{(4)}, a_i^{(4)}(a_i^{(4)}), (b_i^{(4)}), (b_i^{(4)}), (b_i^{(4)}), (b_i^{(4)}), (b_i^{(4)})) > 0, \quad i, j = 24, 25, 26 \]

(L) The functions \( (a_i^{(4)}), (b_i^{(4)}) \) are positive continuous increasing and bounded.

**Definition of** \((p_1^{(4)}), (r_1^{(4)})\):

\[
(a_i^{(4)})(T_{25}, t) \leq (p_1^{(4)}), (T_{25}, t) \leq (A_{24}^{(4)})
\]

\[
(b_i^{(4)})(G_{27}, t) \leq (r_i^{(4)}), (G_{27}, t) \leq (B_{24}^{(4)})
\]

(M) \( \lim_{T_{25} \to 0}(a_i^{(4)})(T_{25}, t) = (p_1^{(4)}), \quad \lim_{G \to 0}(b_i^{(4)})(G_{27}, t) = (r_1^{(4)}) \)

**Definition of** \((A_{24}^{(4)}), (B_{24}^{(4)})\):

Where \((A_{24}^{(4)}), (B_{24}^{(4)}), (p_1^{(4)}), (r_1^{(4)})\) are positive constants and \( i = 24, 25, 26 \)

They satisfy Lipschitz condition:

\[
|(a_i^{(4)})(T_{25}', t) - (a_i^{(4)})(T_{25}, t)| \leq (k_{24}^{(4)})(T_{25} - T_{25}')e^{-\beta_{24}^{(4)}t}
\]

\[
|(b_i^{(4)})(G_{27}', t) - (b_i^{(4)})(G_{27}, t)| < (k_{24}^{(4)})(G_{27} - G_{27}')e^{-\beta_{24}^{(4)}t}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i^{(4)})(T_{25}, t)\) and \((a_i^{(4)})(T_{25}', t)\) And \((T_{25}, t)\) and \((T_{25}', t)\) are points belonging to the interval \((\tilde{k}_{24}^{(4)}), (\tilde{M}_{24}^{(4)})\). It is to be noted that \((a_i^{(4)})(T_{25}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\tilde{M}_{24}^{(4)}) = 4\) then the function \((a_i^{(4)})(T_{25}, t)\), the FOURTH augmentation coefficient WOULD be absolutely continuous.

**Definition of** \((\tilde{M}_{24}^{(4)}), (\tilde{k}_{24}^{(4)})\):

\[
(a_i^{(4)}), (\tilde{M}_{24}^{(4)}), (\tilde{k}_{24}^{(4)}), \quad \text{are positive constants}
\]

\[
\frac{(a_i^{(4)})(\tilde{M}_{24}^{(4)}), (\tilde{k}_{24}^{(4)})(b_i^{(4)})}{(\tilde{M}_{24}^{(4)}), (\tilde{k}_{24}^{(4)})} < 1
\]

**Definition of** \((\tilde{P}_{24}^{(4)}), (\tilde{Q}_{24}^{(4)})\):

(P) \( \exists \) two constants \((\tilde{P}_{24}^{(4)}), (\tilde{Q}_{24}^{(4)})\) which together with \((\tilde{M}_{24}^{(4)}), (\tilde{k}_{24}^{(4)}), (\tilde{A}_{24}^{(4)}), (\tilde{B}_{24}^{(4)})\) and \((\tilde{P}_{24}^{(4)}), (\tilde{Q}_{24}^{(4)})\) and the constants \((a_i^{(4)}), (a_i^{(4)}), (b_i^{(4)}), (b_i^{(4)}), (p_i^{(4)}), (r_i^{(4)})\), \( i = 24, 25, 26 \), satisfy the inequalities

\[
\frac{1}{(\tilde{M}_{24}^{(4)}), (\tilde{Q}_{24}^{(4)})}[a_i^{(4)}(a_i^{(4)}) + (\tilde{A}_{24}^{(4)}), (\tilde{P}_{24}^{(4)})(\tilde{k}_{24}^{(4)})] < 1
\]

\[
\frac{1}{(\tilde{M}_{24}^{(4)}), (\tilde{Q}_{24}^{(4)})}[b_i^{(4)}(b_i^{(4)}) + (\tilde{B}_{24}^{(4)}), (\tilde{Q}_{24}^{(4)})(\tilde{k}_{24}^{(4)})] < 1
\]
Where we suppose

\[(a_i^{(5)}(t), a_i''^{(5)}(t), b_i^{(5)}(t), b_i''^{(5)}(t)) > 0, \quad i, j = 28, 29, 30\]

\[(R)\] The functions \((a_i''^{(5)}), (b_i''^{(5)})\) are positive continuous increasing and bounded.

**Definition of** \((p_i^{(5)}), (r_i^{(5)})\):

\[(a_i''^{(5)}(T_{29}, t) \leq (p_i^{(5)}) \leq (\hat{A}_{28})^{(5)})\]

\[(b_i''^{(5)}((G_{31}), t) \leq (r_i^{(5)}) \leq (b_i^{(5)}) \leq (\hat{B}_{28})^{(5)})\]

\[(S)\] \(\lim_{T_{29} \to 0}(a_i''^{(5)}(T_{29}, t) = (p_i^{(5)})\)

\[\lim_{G_{31} \to 0}(b_i''^{(5)}((G_{31}), t) = (r_i^{(5)})\)

**Definition of** \((\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}):\)

Where \(\{\hat{A}_{28}^{(5)}, \hat{B}_{28}^{(5)}, (p_i^{(5)}), (r_i^{(5)})\}\) are positive constants and \(\epsilon = 28, 29, 30\)

They satisfy Lipschitz condition:

\[\left| (a_i''^{(5)}(T_{29}, t) - (a_i''^{(5)}(T_{29}, t)) \right| \leq (\hat{K}_{28})^{(5)}|T_{29} - T_{29}'|e^{-((\hat{M}_{28})^{(5)})t}\]

\[\left| (b_i''^{(5)}((G_{31}), t) - (b_i''^{(5)}((G_{31}), t)) \right| < (\hat{K}_{31})^{(5)}|[G_{31}] - (G_{31})'|e^{-((\hat{M}_{28})^{(5)})t}\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i''^{(5)}(T_{29}, t)\)

and\((a_i''^{(5)}(T_{29}, t) \cdot (T_{29}, t)\) and \(T_{29}\) are points belonging to the interval \([\hat{K}_{28}^{(5)}, \hat{M}_{28}^{(5)}]\). It is to be noted that \((a_i''^{(5)}(T_{29}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\hat{M}_{28})^{(5)} = 5\) then the function \((a_i''^{(5)}(T_{29}, t)\), the FIFTH augmentation coefficient attributable would be absolutely continuous.

**Definition of** \((\hat{M}_{28})^{(5)}, (\hat{K}_{28})^{(5)}:\)

\[(\hat{M}_{28})^{(5)}, (\hat{K}_{28})^{(5)}, (a_i^{(5)}), (b_i^{(5)})\]

\[(\hat{M}_{28})^{(5)} < 1\]

**Definition of** \((\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}:\)

There exists two constants \((\hat{P}_{28})^{(5)}\) and \((\hat{Q}_{28})^{(5})\) which together with \((\hat{M}_{28})^{(5)}, (\hat{K}_{28})^{(5)}, (\hat{A}_{28})^{(5)}\) and \((\hat{B}_{28})^{(5)}\) and the constants \((a_i^{(5)}), (b_i^{(5)}), (a_i''^{(5)}), (b_i''^{(5)}), (p_i^{(5)}), (r_i^{(5)})\), \(i = 28, 29, 30\), satisfy the inequalities

\[
\frac{1}{(\hat{M}_{28})^{(5)}}[ (a_i^{(5)}) + (a_i''^{(5)}) + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{K}_{28})^{(5)}] < 1
\]

\[
\frac{1}{(\hat{M}_{28})^{(5)}}[ (b_i^{(5)}) + (b_i''^{(5)}) + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{K}_{28})^{(5)}] < 1
\]

Where we suppose

\[(a_i^{(6)}), (a_i''^{(6)}), (b_i^{(6)}), (b_i''^{(6)}), (b_i''^{(6)}) > 0, \quad i, j = 32, 33, 34\]

\[(S)\] The functions \((a_i''^{(6)}), (b_i''^{(6)})\) are positive continuous increasing and bounded.

**Definition of** \((p_i^{(6)}), (r_i^{(6)})\):
\[(a_{i}^{(6)}) (T_{33}, t) \leq (p_{i})^{(6)} \leq (\hat{A}_{32})^{(6)}\]
\[(b_{i}^{(6)}) (G_{33}, t) \leq (r_{i})^{(6)} \leq (b_{i}')^{(6)} \leq (\hat{B}_{32})^{(6)}\]

\[\lim_{T_{3} \to \infty} (a_{i}^{(6)}) (T_{33}, t) = (p_{i})^{(6)}\]
\[\lim_{G_{3} \to \infty} (b_{i}^{(6)}) (G_{33}, t) = (r_{i})^{(6)}\]

**Definition of** \((\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}\):

Where \((\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_{i})^{(6)}, (r_{i})^{(6)}\) are positive constants and \(i = 32,33,34\)

They satisfy Lipschitz condition:

\[| (a_{i}^{(6)}) (T'_{33}, t) - (a_{i}^{(6)}) (T_{33}, t) | \leq (\hat{K}_{32})^{(6)} | T_{33} - T'_{33} | e^{-\hat{B}_{32})^{(6)} t}\]
\[| (b_{i}^{(6)}) (G'_{33}, t) - (b_{i}^{(6)}) (G_{33}, t) | \leq (\hat{K}_{32})^{(6)} | (G_{35}) - (G'_{35}) | e^{-\hat{B}_{32})^{(6)} t}\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_{i}^{(6)}) (T_{33}, t), (a_{i}^{(6)}) (T_{33}, t), (T_{33}, t), (T_{33}, t)\) and \((T_{33}, t)\) and \((T_{33}, t)\) are points belonging to the interval \([\hat{K}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]\). It is to be noted that \((a_{i}^{(6)}) (T_{33}, t)\) is uniformly continuous. In the eventuality of the fact, if \((\hat{M}_{32})^{(6)} = 6\) then the function \((a_{i}^{(6)}) (T_{33}, t)\), the SIXTH augmentation coefficient would be absolutely continuous.

**Definition of** \((\bar{M}_{32})^{(6)}, (\bar{K}_{32})^{(6)}\):

\((\bar{M}_{32})^{(6)}, (\bar{K}_{32})^{(6)}\), are positive constants \[\frac{b_{i}^{(6)}}{(\bar{M}_{32})^{(6)}} \leq 1\]

**Definition of** \((\bar{P}_{32})^{(6)}, (\bar{Q}_{32})^{(6)}\):

There exists two constants \((\bar{P}_{32})^{(6)}\) and \((\bar{Q}_{32})^{(6)}\) which together with \((\bar{M}_{32})^{(6)}, (\bar{K}_{32})^{(6)}, (\bar{A}_{32})^{(6)}\) and \((\bar{B}_{32})^{(6)}\) and the constants \((a_{i})^{(6)}, (a_{i}^{(6)}), (b_{i})^{(6)}, (b_{i}')^{(6)}, (p_{i})^{(6)}, (r_{i})^{(6)}, i = 32,33,34,\), satisfy the inequalities

\[\frac{1}{(\bar{M}_{32})^{(6)}} [(a_{i})^{(6)} + (a_{i}')^{(6)}] + (\bar{A}_{32})^{(6)} + (\bar{P}_{32})^{(6)} (\bar{K}_{32})^{(6)} | < 1\]
\[\frac{1}{(\bar{M}_{32})^{(6)}} [(b_{i})^{(6)} + (b_{i}')^{(6)}] + (\bar{B}_{32})^{(6)} + (\bar{Q}_{32})^{(6)} (\bar{K}_{32})^{(6)} | < 1\]

**Theorem 1:** if the conditions IN THE FOREGOING above are fulfilled, there exists a solution satisfying the conditions

**Definition of** \(G_{i}(0), T_{i}(0)\):

\[G_{i}(t) \leq (\bar{P}_{13})^{(1)} e^{(\bar{M}_{13})^{(1)} t}, \quad G_{i}(0) = G_{i}^{0} > 0\]
\[T_{i}(t) \leq (\bar{Q}_{13})^{(1)} e^{(\bar{M}_{13})^{(1)} t}, \quad T_{i}(0) = T_{i}^{0} > 0\]
Definition of $G_1(0), T_1(0)$:

$G_i(t) \leq (\hat{\beta}_{16})^{(2)} e^{(R_{16})^{(2)}} t$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(R_{16})^{(2)}} t$, $T_i(0) = T_i^0 > 0$

$G_i(t) \leq (\hat{\beta}_{20})^{(3)} e^{(R_{20})^{(3)}} t$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(R_{20})^{(3)}} t$, $T_i(0) = T_i^0 > 0$

Definition of $G_2(0), T_2(0)$:

$G_i(t) \leq (\hat{\beta}_{24})^{(4)} e^{(R_{24})^{(4)}} t$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(R_{24})^{(4)}} t$, $T_i(0) = T_i^0 > 0$

Definition of $G_3(0), T_3(0)$:

$G_i(t) \leq (\hat{\beta}_{28})^{(5)} e^{(R_{28})^{(5)}} t$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(R_{28})^{(5)}} t$, $T_i(0) = T_i^0 > 0$

Definition of $G_4(0), T_4(0)$:

$G_i(t) \leq (\hat{\beta}_{32})^{(6)} e^{(R_{32})^{(6)}} t$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(R_{32})^{(6)}} t$, $T_i(0) = T_i^0 > 0$

Definition of $G_5(0), T_5(0)$:

$G_i(t) \leq (\hat{\beta}_{36})^{(7)} e^{(R_{36})^{(7)}} t$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(R_{36})^{(7)}} t$, $T_i(0) = T_i^0 > 0$
Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

\[
G_i(0) = G_i^0, \quad T_i(0) = T_i^0, \quad G_i^0 \leq (\hat{P}_{13})^{(1)}(t), \quad T_i^0 \leq (\hat{Q}_{13})^{(1)},
\]

\[
0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)}(t)e^{(\hat{A}_{13})^{(1)}t},
\]

\[
0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)}(t)e^{(\hat{A}_{13})^{(1)}t},
\]

By

\[
\tilde{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{13} (s_{13}) - \left( (a'_{13})^{(1)} + a''_{13}^{(1)}(T_{14}(s_{13}), s_{13}) \right) G_{13} (s_{13}) \right] ds_{13}
\]

\[
\tilde{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13} (s_{13}) - \left( (a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{13}), s_{13}) \right) G_{14} (s_{13}) \right] ds_{13}
\]

\[
\tilde{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14} (s_{13}) - \left( (a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{13}), s_{13}) \right) G_{15} (s_{13}) \right] ds_{13}
\]

\[
\tilde{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14} (s_{13}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{13}), s_{13}) \right) T_{13} (s_{13}) \right] ds_{13}
\]

\[
\tilde{T}_{14}(t) = T_{14}^0 + \int_0^t \left[ (b_{14})^{(1)} T_{13} (s_{13}) - \left( (b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{13}), s_{13}) \right) T_{14} (s_{13}) \right] ds_{13}
\]

\[
\tilde{T}_{15}(t) = T_{15}^0 + \int_0^t \left[ (b_{15})^{(1)} T_{14} (s_{13}) - \left( (b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{13}), s_{13}) \right) T_{15} (s_{13}) \right] ds_{13}
\]

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

if the conditions IN THE FOREGOING above are fulfilled, there exists a solution satisfying the conditions

\[
\text{Definition of } G_i(0), T_i(0):
\]

\[
G_i(0) \leq (\hat{P}_{36})^{(7)} e^{(\hat{A}_{36})^{(7)}t}, \quad G_i(0) = G_i^0 > 0
\]

\[
T_i(0) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{A}_{36})^{(7)}t}, \quad T_i(0) = T_i^0 > 0
\]

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

\[
G_i(0) = G_i^0, \quad T_i(0) = T_i^0, \quad G_i^0 \leq (\hat{P}_{36})^{(7)}, \quad T_i^0 \leq (\hat{Q}_{36})^{(7)},
\]

\[
0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\hat{A}_{36})^{(7)}t},
\]

\[
0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\hat{A}_{36})^{(7)}t}
\]
By

\[ \tilde{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)}G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36}^{(7)}(T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)} \]

\[ \tilde{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)}G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + a''_{37}^{(7)}(T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)} \]

\[ \tilde{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)}G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + a''_{38}^{(7)}(T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)} \]

\[ \tilde{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)}T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)}(G(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)} \]

\[ \tilde{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)}T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)}(G(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)} \]

\[ \tilde{T}_{38}(t) = T_{38}^0 + \int_0^t \left[ (b_{38})^{(7)}T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)}(G(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)} \]

Where \( s_{(36)} \) is the integrand that is integrated over an interval \((0, t)\).
Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, \quad T_i(0) = T_i^0, \quad G_i^0 \leq (\hat{P}_{16})^{(2)}, \quad T_i^0 \leq (\hat{Q}_{16})^{(2)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)}e^{(\mathcal{A}_{16})^{(2)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)}e^{(\mathcal{A}_{16})^{(2)}t}$$

By

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t (a_{16}^{(2)}G_{17}(s_{16}) - (a_{16}^{(2)} + a_{16}^{(2)}(T_{17}(s_{16}), s_{16}))) ds_{16}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t (a_{17}^{(2)}G_{16}(s_{16}) - (a_{17}^{(2)} + a_{17}^{(2)}(T_{16}(s_{16}), s_{16}))) ds_{16}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t (a_{18}^{(2)}G_{17}(s_{16}) - (a_{18}^{(2)} + a_{18}^{(2)}(T_{17}(s_{16}), s_{16}))) ds_{16}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t (b_{16}^{(2)}T_{17}(s_{16}) - (b_{16}^{(2)} - b_{16}^{(2)}(G(s_{16}), s_{16}))) ds_{16}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t (b_{17}^{(2)}T_{16}(s_{16}) - (b_{17}^{(2)} - b_{17}^{(2)}(G(s_{16}), s_{16}))) ds_{16}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t (b_{18}^{(2)}T_{17}(s_{16}) - (b_{18}^{(2)} - b_{18}^{(2)}(G(s_{16}), s_{16}))) ds_{16}$$

Where $s_{16}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, \quad T_i(0) = T_i^0, \quad G_i^0 \leq (\hat{P}_{20})^{(3)}, \quad T_i^0 \leq (\hat{Q}_{20})^{(3)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)}e^{(\mathcal{A}_{20})^{(3)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)}e^{(\mathcal{A}_{20})^{(3)}t}$$

By

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t (a_{20}^{(3)}G_{21}(s_{20}) - (a_{20}^{(3)} + a_{20}^{(3)}(T_{21}(s_{20}), s_{20}))) ds_{20}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t (a_{21}^{(3)}G_{20}(s_{20}) - (a_{21}^{(3)} + a_{21}^{(3)}(T_{20}(s_{20}), s_{20}))) ds_{20}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t (a_{22}^{(3)}G_{21}(s_{20}) - (a_{22}^{(3)} + a_{22}^{(3)}(T_{21}(s_{20}), s_{20}))) ds_{20}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t (b_{20}^{(3)}T_{21}(s_{20}) - (b_{20}^{(3)} - b_{20}^{(3)}(G(s_{20}), s_{20}))) ds_{20}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t (b_{21}^{(3)}T_{20}(s_{20}) - (b_{21}^{(3)} - b_{21}^{(3)}(G(s_{20}), s_{20}))) ds_{20}$$
\[ T_{22}(t) = T_{22}^0 + \int_0^t \left[ (b_{22})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{22})^{(3)} - (b''_{22})^{(3)}(G(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)} \]

Where \( s_{(20)} \) is the integrand that is integrated over an interval \((0, t)\)

Consider operator \( A^{(4)} \) defined on the space of sextuples of continuous functions \( G, T_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) which satisfy

\[ G_i(0) = G_i^0, \quad T_i(0) = T_i^0, \quad G_i^0 \leq (\bar{P}_{24})^{(4)}, \quad T_i^0 \leq (\bar{Q}_{24})^{(4)}, \]

\[ 0 \leq G_i(t) - G_i^0 \leq (\bar{P}_{24})^{(4)} e^{(\theta_{24})^{(4)} t} \]

\[ 0 \leq T_i(t) - T_i^0 \leq (\bar{Q}_{24})^{(4)} e^{(\theta_{24})^{(4)} t} \]

By

\[ \bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[ (b_{24})^{(4)} T_{25}(s_{(24)}) - \left( (b'_{24})^{(4)} - (b''_{24})^{(4)}(G(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)} \]

\[ \bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[ (a_{25})^{(4)} G_{26}(s_{(24)}) - \left( (a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)} \]

\[ \bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[ (a_{26})^{(4)} G_{25}(s_{(24)}) - \left( (a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)} \]

\[ \bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[ (b_{25})^{(4)} T_{24}(s_{(24)}) - \left( (b'_{25})^{(4)} - (b''_{25})^{(4)}(G(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)} \]

Where \( s_{(24)} \) is the integrand that is integrated over an interval \((0, t)\)

Consider operator \( A^{(5)} \) defined on the space of sextuples of continuous functions \( G, T_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) which satisfy

\[ G_i(0) = G_i^0, \quad T_i(0) = T_i^0, \quad G_i^0 \leq (\bar{P}_{28})^{(5)}, \quad T_i^0 \leq (\bar{Q}_{28})^{(5)}, \]

\[ 0 \leq G_i(t) - G_i^0 \leq (\bar{P}_{28})^{(5)} e^{(\theta_{28})^{(5)} t} \]

\[ 0 \leq T_i(t) - T_i^0 \leq (\bar{Q}_{28})^{(5)} e^{(\theta_{28})^{(5)} t} \]

By

\[ \bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[ (a_{28})^{(5)} G_{29}(s_{(28)}) - \left( (a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)} \]

\[ \bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[ (a_{29})^{(5)} G_{28}(s_{(28)}) - \left( (a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)} \]

\[ \bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[ (a_{30})^{(5)} G_{29}(s_{(28)}) - \left( (a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)} \]

\[ \bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[ (b_{28})^{(5)} T_{29}(s_{(28)}) - \left( (b'_{28})^{(5)} - (b''_{28})^{(5)}(G(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)} \]
\[
\tilde{T}_{29}(t) = T_{29}^0 + \int_0^t \left[ (b_{29}^{(5)} T_{29}(s_{28}) - (b_{29}^{(5)}) \left( G(s_{28}), s_{28} \right) \right] T_{29}(s_{28}) ds_{28}
\]

\[
\tilde{T}_{30}(t) = T_{30}^0 + \int_0^t \left[ (b_{30}^{(5)} T_{29}(s_{28}) - (b_{30}^{(5)}) \left( G(s_{28}), s_{28} \right) \right] T_{30}(s_{28}) ds_{28}
\]

Where \( s_{28} \) is the integrand that is integrated over an interval \((0, t)\)

Consider operator \( \mathcal{A}^{(6)} \) defined on the space of sextuples of continuous functions \( G_i, T_i : \mathbb{R}_+ \to \mathbb{R}_+ \) which satisfy

\[
G_i(0) = G_i^0, \quad T_i(0) = T_i^0, \quad G_i^0 \leq (\tilde{P}_{32})^{(6)}, \quad T_i^0 \leq (\tilde{Q}_{32})^{(6)},
\]

\[
0 \leq G_i(t) - G_i^0 \leq (\tilde{P}_{32})^{(6)} e^{(\tilde{\theta}_{32})^{(6)} t}
\]

\[
0 \leq T_i(t) - T_i^0 \leq (\tilde{Q}_{32})^{(6)} e^{(\tilde{\theta}_{32})^{(6)} t}
\]

By

\[
\tilde{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32}^{(6)} G_{33}(s_{32}) - (a_{32}^{(6)})^2 \left( T_{33}(s_{32}), s_{32} \right) \right] G_{32}(s_{32}) ds_{32}
\]

\[
\tilde{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33}^{(6)} G_{32}(s_{32}) - (a_{33}^{(6)})^2 \left( T_{32}(s_{32}), s_{32} \right) \right] G_{33}(s_{32}) ds_{32}
\]

\[
\tilde{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34}^{(6)} G_{33}(s_{32}) - (a_{34}^{(6)})^2 \left( T_{33}(s_{32}), s_{32} \right) \right] G_{34}(s_{32}) ds_{32}
\]

\[
\tilde{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32}^{(6)} T_{33}(s_{32}) - (b_{32}^{(6)})^2 \left( G(s_{32}), s_{32} \right) \right] T_{32}(s_{32}) ds_{32}
\]

\[
\tilde{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33}^{(6)} T_{32}(s_{32}) - (b_{33}^{(6)})^2 \left( G(s_{32}), s_{32} \right) \right] T_{33}(s_{32}) ds_{32}
\]

\[
\tilde{T}_{34}(t) = T_{34}^0 + \int_0^t \left[ (b_{34}^{(6)} T_{33}(s_{32}) - (b_{34}^{(6)})^2 \left( G(s_{32}), s_{32} \right) \right] T_{34}(s_{32}) ds_{32}
\]

Where \( s_{32} \) is the integrand that is integrated over an interval \((0, t)\)

(a) The operator \( \mathcal{A}^{(1)} \) maps the space of functions satisfying GLOBAL EQUATIONS into itself.

Indeed it is obvious that

\[
G_{13}(t) \leq G_{13}^0 + \int_0^t \left[ (a_{13}^{(1)} G_{14}^0 + (\tilde{P}_{13})^{(1)} e^{(\tilde{\theta}_{13})^{(1)} t}) \right] ds_{13} =
\]

\[
(1 + (a_{13}^{(1)}) t) G_{14}^0 + \frac{a_{13}^{(1)}(\tilde{P}_{13})^{(1)} e^{(\tilde{\theta}_{13})^{(1)} t}}{a_{13}^{(1)}} (e^{(\tilde{\theta}_{13})^{(1)} t} - 1)
\]

From which it follows that

\[
(G_{13}(t) - G_{13}^0) e^{-(\tilde{\theta}_{13})^{(1)} t} \leq \left( \frac{a_{13}^{(1)}}{(\tilde{\theta}_{13})^{(1)}} \right) \left( \tilde{P}_{13}^{(1)} + G_{14}^0 \right) e^{\frac{(\tilde{P}_{13})^{(1)} + G_{14}^0}{a_{13}^{(1)}}} + (\tilde{P}_{13})^{(1)}
\]

\((G_{13}^0)\) is as defined in the statement of theorem 1

Analogous inequalities hold also for \( G_{14}, G_{15}, T_{13}, T_{14}, T_{15} \)
(b) The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself.
Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[ \left( a_{16}^{(2)} \right) \left( \int (G_{17}^0 + \left( \hat{\mathcal{G}}_{16}^{(2)} \right) e^{(\hat{\mathcal{G}}_{16}^{(2)} t)} ) \right) \right] dS_{(16)} = (1 + (a_{16}^{(2)} t) G_{17}^0 + \frac{\left( \hat{\mathcal{G}}_{16}^{(2)} \right)^2}{(M_{16})^2} \left( e^{(\hat{\mathcal{G}}_{16}^{(2)} t)} - 1 \right)$$

From which it follows that

$$\left( G_{16}(t) - G_{16}^0 \right) e^{-(\hat{\mathcal{G}}_{16}^{(2)} t)} \leq \frac{\left( \hat{\mathcal{G}}_{16}^{(2)} \right)^2}{(M_{16})^2} \left[ \left( \hat{\mathcal{G}}_{16}^{(2)} \right)^2 + G_{17}^0 e^{\left( \frac{(\hat{\mathcal{G}}_{16}^{(2)} t)^2 + G_{17}^0}{M_{16}^2} \right)} + (\hat{\mathcal{G}}_{16}^{(2)}) \right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

(a) The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself.
Indeed it is obvious that

$$G_{20}(t) \leq G_{20}^0 + \int_0^t \left[ \left( a_{20}^{(3)} \right) \left( \int (G_{21}^0 + \left( \hat{\mathcal{G}}_{20}^{(3)} \right) e^{(\hat{\mathcal{G}}_{20}^{(3)} t)} ) \right) \right] dS_{(20)} = (1 + (a_{20}^{(3)} t) G_{21}^0 + \frac{\left( \hat{\mathcal{G}}_{20}^{(3)} \right)^3}{(M_{20})^3} \left( e^{(\hat{\mathcal{G}}_{20}^{(3)} t)} - 1 \right)$$

From which it follows that

$$\left( G_{20}(t) - G_{20}^0 \right) e^{-(\hat{\mathcal{G}}_{20}^{(3)} t)} \leq \frac{\left( \hat{\mathcal{G}}_{20}^{(3)} \right)^3}{(M_{20})^3} \left[ \left( \hat{\mathcal{G}}_{20}^{(3)} \right)^3 + G_{21}^0 e^{\left( \frac{(\hat{\mathcal{G}}_{20}^{(3)} t)^3 + G_{21}^0}{M_{20}^3} \right)} + (\hat{\mathcal{G}}_{20}^{(3)}) \right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

(b) The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself.
Indeed it is obvious that

$$G_{24}(t) \leq G_{24}^0 + \int_0^t \left[ \left( a_{24}^{(4)} \right) \left( \int (G_{25}^0 + \left( \hat{\mathcal{G}}_{24}^{(4)} \right) e^{(\hat{\mathcal{G}}_{24}^{(4)} t)} ) \right) \right] dS_{(24)} = (1 + (a_{24}^{(4)} t) G_{25}^0 + \frac{\left( \hat{\mathcal{G}}_{24}^{(4)} \right)^4}{(M_{24})^4} \left( e^{(\hat{\mathcal{G}}_{24}^{(4)} t)} - 1 \right)$$

From which it follows that

$$\left( G_{24}(t) - G_{24}^0 \right) e^{-(\hat{\mathcal{G}}_{24}^{(4)} t)} \leq \frac{\left( \hat{\mathcal{G}}_{24}^{(4)} \right)^4}{(M_{24})^4} \left[ \left( \hat{\mathcal{G}}_{24}^{(4)} \right)^4 + G_{25}^0 e^{\left( \frac{(\hat{\mathcal{G}}_{24}^{(4)} t)^4 + G_{25}^0}{M_{24}^4} \right)} + (\hat{\mathcal{G}}_{24}^{(4)}) \right]$$

$$(G_{24}^0)$$ is as defined in the statement of theorem 1

(c) The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself.
Indeed it is obvious that

$$G_{28}(t) \leq G_{28}^0 + \int_0^t \left[ \left( a_{28}^{(5)} \right) \left( \int (G_{29}^0 + \left( \hat{\mathcal{G}}_{28}^{(5)} \right) e^{(\hat{\mathcal{G}}_{28}^{(5)} t)} ) \right) \right] dS_{(28)} = (1 + (a_{28}^{(5)} t) G_{29}^0 + \frac{\left( \hat{\mathcal{G}}_{28}^{(5)} \right)^5}{(M_{28})^5} \left( e^{(\hat{\mathcal{G}}_{28}^{(5)} t)} - 1 \right)$$

From which it follows that
$$(G_{28}(t) - G_{28}^0)e^{-\left(\frac{G_{28}^0}{G_{28}}\right)^5} \leq \frac{G_{28}^0}{G_{28}} \left[ \left( \frac{G_{28}^0}{G_{28}} \right)^5 + G_{29}^0 e^{-\left(\frac{G_{28}^0}{G_{29}}\right)^5} \right]

(G_0^0) \text{ is as defined in the statement of theorem 1}

(d) The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that

$$G_{32}(t) \leq G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} \left( G_{33}^0 + (\hat{P}_{32})^{(6)} e^{\left(\frac{G_{32}^0}{G_{32}}\right)^6} \right) \right] \, ds_{(32)} = \left( 1 + (a_{32})^{(6)} t \right) G_{33}^0 + \frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(G_{32})^{(6)}} \left( e^{\left(\frac{G_{32}^0}{G_{32}}\right)^6} - 1 \right)$$

From which it follows that

$$G_{32}(t) - G_{32}^0 e^{-\left(\frac{G_{32}^0}{G_{32}}\right)^6} \leq \frac{(a_{32})^{(6)}}{(G_{32})^{(6)}} \left[ \left( \frac{G_{32}^0}{G_{32}} \right)^6 + G_{33}^0 e^{-\left(\frac{G_{32}^0}{G_{33}}\right)^6} \right] + (\hat{P}_{32})^{(6)}$$

$(G_0^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

It is now sufficient to take $\frac{(a_{13})^{(1)}}{(M_{13})^{(1)}}$, $\frac{(b_{13})^{(1)}}{(M_{13})^{(1)}} < 1$ and to choose

$$(\hat{P}_{13})^{(1)} \text{ and } (\hat{Q}_{13})^{(1)} \text{ large to have}$$

$$\left[ \left( \frac{(a_{13})^{(1)}}{(M_{13})^{(1)}} \right) \left( \frac{(b_{13})^{(1)}}{(M_{13})^{(1)}} \right) \right] \left[ (\hat{P}_{13})^{(1)} + \left( (\hat{P}_{13})^{(1)} + G_{13}^0 e^{-\left(\frac{G_{13}^0}{G_{13}}\right)^6} \right) \right] \leq (\hat{P}_{13})^{(1)}$$

$$\left[ \left( \frac{(a_{13})^{(1)}}{(M_{13})^{(1)}} \right) \left( \frac{(b_{13})^{(1)}}{(M_{13})^{(1)}} \right) \right] \left[ (\hat{Q}_{13})^{(1)} + T_j^0 \right] e^{-\left(\frac{(Q_{13})^{(1)}}{(T_j)} + T_j^0 \right)^6} \leq (\hat{P}_{13})^{(1)}$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying GLOBAL EQUATIONS into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric

$$d\left( (G^{(1)}, T^{(1)}), (G^{(2)}, T^{(2)}) \right) = \sup_{t \in \mathbb{R}^+} \left[ \max_i \left| G_i^{(1)}(t) - G_i^{(2)}(t) \right| e^{-\left(\frac{M_{13}^{(1)}}{M_{13}^{(2)}}\right)^6}, \max_i \left| T_i^{(1)}(t) - T_i^{(2)}(t) \right| e^{-\left(\frac{M_{13}^{(1)}}{M_{13}^{(2)}}\right)^6} \right]$$

Indeed if we denote

**Definition of $\hat{G}, \hat{T}$:**
\[ (\mathcal{G}, \mathcal{T}) = \mathcal{A}^{(1)}(\mathcal{G}, \mathcal{T}) \]

It results

\[
\begin{align*}
|\hat{G}^{(1)}_{i3} - \hat{G}^{(2)}_{i3}| & \leq \int_0^t (a_{13}^{(1)})^{(i)} |G_{i4}^{(1)} - G_{i4}^{(2)}| e^{-\left(R_{i3}^{(1)}(s_{(13)})\right)} e^{\left(R_{i3}^{(2)}(s_{(13)})\right)} ds_{(13)} + \\
\int_0^t (a_{13}^{(2)})^{(i)} |G_{i3}^{(1)} - G_{i3}^{(2)}| e^{-\left(R_{i3}^{(1)}(s_{(13)})\right)} e^{\left(R_{i3}^{(2)}(s_{(13)})\right)} ds_{(13)} + \\
(a_{13}^{(1)})^{(i)} (T_{i4}^{(1)}, s_{(13)}) |G_{i3}^{(1)} - G_{i3}^{(2)}| e^{-\left(R_{i3}^{(1)}(s_{(13)})\right)} e^{\left(R_{i3}^{(2)}(s_{(13)})\right)} ds_{(13)} + \\
G_{i3}^{(2)} (a_{13}^{(2)})^{(i)} (T_{i4}^{(2)}, s_{(13)}) - (a_{13}^{(2)})^{(i)} (T_{i4}^{(2)}, s_{(13)}) | e^{-\left(R_{i3}^{(1)}(s_{(13)})\right)} e^{\left(R_{i3}^{(2)}(s_{(13)})\right)} ds_{(13)}
\end{align*}
\]

Where \( s_{(13)} \) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows

\[
\begin{align*}
|G^{(1)} - G^{(2)}| e^{-\left(R_{i3}^{(1)}(t)\right)} \leq \\
\frac{1}{(\mathcal{M}_{i3})^{(1)}} (a_{13}^{(1)} + a_{13}^{(2)} + (\mathcal{A}_{i3})^{(1)} + (\mathcal{P}_{i3})^{(1)} (R_{i3})^{(1)}) d \left( (G^{(1)}, T^{(1)}), (G^{(2)}, T^{(2)}) \right)
\end{align*}
\]

And analogous inequalities for \( G_i \) and \( T_i \). Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed \((a_{i3}^{(r)})^{(1)}(b_{i3}^{(r)})^{(1)}\) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((\mathcal{P}_{i3})^{(1)} e^{(R_{i3})^{(1)}}\) and \((\mathcal{Q}_{i3})^{(1)} e^{(R_{i3})^{(1)}}\) respectively of \( \mathbb{R}_+ \).

If instead of proving the existence of the solution on \( \mathbb{R}_+ \), we have to prove it only on a compact then it suffices to consider that \((a_{i3}^{(r)})^{(1)}(b_{i3}^{(r)})^{(1)}, i = 13, 14, 15\) depend only on \( T_{i4} \) and respectively on \( G(\text{and not } t) \) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \( t \) where \( G_i(t) = 0 \) and \( T_i(t) = 0 \)

From 19 to 24 it results

\[
\begin{align*}
G_i(t) & \geq G_i^0 e^{-\int_0^t [(a_{i4}^{(1)} + (a_{i4}^{(2)})(t_{i4}(s_{(13)}), s_{(13)})]ds_{(13)}]} \geq 0 \\
T_i(t) & \geq T_i^0 e^{-\left(b_{i4}^{(1)}(t)\right)} > 0 \quad \text{for } t > 0
\end{align*}
\]

**Definition of** \((\mathcal{M}_{i3})^{(1)}, \text{ and } (\mathcal{M}_{i3})^{(1)} \) : 

**Remark 3:** If \( G_{i3} \) is bounded, the same property have also \( G_{i4} \) and \( G_{i5} \). indeed if

\[
G_{i3} < (\mathcal{M}_{i3})^{(1)} \text{ it follows } \frac{dG_{i4}}{dt} \leq \left( (\mathcal{M}_{i3})^{(1)} \right)_1 - (a_{i4}^{(1)}) G_{i4} \text{ and by integrating}
\]

\[
G_{i4} \leq \left( (\mathcal{M}_{i3})^{(1)} \right)_2 = G_{i4}^0 + 2(a_{i4}^{(1)}) \left( (\mathcal{M}_{i3})^{(1)} \right)_1 / (a_{i4}^{(1)})
\]

In the same way , one can obtain

\[
G_{i5} \leq \left( (\mathcal{M}_{i3})^{(1)} \right)_3 = G_{i5}^0 + 2(a_{i5}^{(1)}) \left( (\mathcal{M}_{i3})^{(1)} \right)_2 / (a_{i5}^{(1)})
\]

If \( G_{i4} \) or \( G_{i5} \) is bounded, the same property follows for \( G_{i3}, G_{i5} \) and \( G_{i3}, G_{i4} \) respectively.

**Remark 4:** If \( G_{i3} \) is bounded, from below, the same property holds for \( G_{i4} \) and \( G_{i5} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{i4} \) is bounded from below.
Remark 5: If $T_{13}$ is bounded from below and $\lim_{t \to \infty} ((b''_t)^{(1)} (G(t), t)) = (b''_{14})^{(1)}$ then $T_{14} \to \infty$.

Definition of $(m)^{(1)}$ and $\varepsilon_1$:

Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} (t) - (b''_{13})^{(1)}(G(t), t) < \varepsilon_1, T_{13} (t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right) (1 - e^{-\varepsilon_1 t} + T_{14}^0 e^{-\varepsilon_1 t}$$

If we take $t$ such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right), t = log \frac{2}{\varepsilon_1}$$

By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded. The same property holds for $T_{15}$ if $\lim_{t \to \infty} (b''_{15})^{(1)} (G(t), t) = (b''_{15})^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take $\frac{(a_{1})^{(2)}}{(\mathcal{M}_{16})^{(2)}}, \frac{(b_{1})^{(2)}}{(\mathcal{M}_{16})^{(2)}} < 1$ and to choose

$$\left(\begin{array}{c} P_{16} \end{array}\right)^{(2)} \text{ and } \left(\begin{array}{c} Q_{16} \end{array}\right)^{(2)}$$

large to have

$$\left(\frac{(a_{1})^{(2)}}{(\mathcal{M}_{16})^{(2)}}\right) \left(\begin{array}{c} (P_{16})^{(2)} \end{array}\right)^{(2)} + \left(\begin{array}{c} (P_{16})^{(2)} \end{array}\right)^{(2)} + G_{j}^{0} e^{-\left(\frac{\left(\begin{array}{c} P_{16} \end{array}\right)^{(2)} + G_{j}^{0}}{\mathcal{M}_{16}}\right)} \leq \left(\begin{array}{c} (P_{16}) \end{array}\right)^{(2)}$$

$$\left(\frac{(b_{1})^{(2)}}{(\mathcal{M}_{16})^{(2)}}\right) \left(\begin{array}{c} \left(\begin{array}{c} Q_{16} \end{array}\right)^{(2)} \end{array}\right)^{(2)} + \left(\begin{array}{c} \left(\begin{array}{c} Q_{16} \end{array}\right)^{(2)} \end{array}\right)^{(2)} + r_{j}^{0} e^{-\left(\frac{\left(\begin{array}{c} Q_{16} \end{array}\right)^{(2)} + r_{j}^{0}}{r_{j}}\right)} + \left(\begin{array}{c} (Q_{16}) \end{array}\right)^{(2)} \leq \left(\begin{array}{c} (Q_{16}) \end{array}\right)^{(2)}$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric

$$d \left(\left(\begin{array}{c} (G_{19}) \end{array}\right)^{(1)}, (T_{19})^{(1)}, \left(\begin{array}{c} (G_{19}) \end{array}\right)^{(2)}, (T_{19})^{(2)}\right) =$$

$$\sup_{t \in \mathbb{R}^{+}} \max_{i} \left|G_{i}^{1}(t) - G_{i}^{2}(t)\right| e^{-\left(\mathcal{M}_{16}\right)^{(1)} t}, \max_{t \in \mathbb{R}^{+}} \left|T_{1i}^{1}(t) - T_{1i}^{2}(t)\right| e^{-\left(\mathcal{M}_{16}\right)^{(2)} t}$$

Indeed if we denote

Definition of $\tilde{G}_{19}, \tilde{T}_{19}$:

It results

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_{16}^{(2)}\right| \leq f_{0}^{(a_{16})^{(2)}} \left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-\left(\mathcal{M}_{16}\right)^{(2)} s_{16}}, e^{\left(\mathcal{M}_{16}\right)^{(2)} s_{16}} ds_{16} +$$

$$f_{0}^{(a'_{16}^{(2)})} \left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-\left(\mathcal{M}_{16}\right)^{(2)} s_{16}} e^{-\left(\mathcal{M}_{16}\right)^{(2)} s_{16}} +$$

$$(a''_{16})^{(2)} \left(\begin{array}{c} T_{17} \end{array}\right)^{(1)}, s_{16}) \left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-\left(\mathcal{M}_{16}\right)^{(2)} s_{16}} e^{\left(\mathcal{M}_{16}\right)^{(2)} s_{16}} +$$

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\[ G_{16}^{(2)} \left[ (a_{16}'')^{(2)}(T_{17}^{(1)}, s_{16}(16)) - (a_{16}'')^{(2)}(T_{17}^{(2)}, s_{16}(16)) \right] e^{-(\bar{M}_{16}(2))s_{16}(16)} \int e^{(\bar{M}_{16}(2))s_{16}(16)} ds_{16} \]

Where \( s_{16} \) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows

\[
\frac{1}{(\bar{M}_{16}(2))} \left[ (a_{16}''')^{(2)} + (a_{16}')^{(2)} + (\bar{A}_{16})^{(2)} \right] d \left( \left( G_{19}^{(1)}(T_{19}^{(1)}), (G_{19}^{(2)}(T_{19}^{(2)}) \right) \right)
\]

And analogous inequalities for \( G_i \) and \( T_i \). Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed \((a_{16}'')^{(2)}(2)\) and \((b_{16}'')^{(2)}\) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \( (\bar{P}_{16}(2))e^{(\bar{M}_{16}(2)t)} \) and \( (\bar{Q}_{16}(2))e^{(\bar{M}_{16}(2)t)} \) respectively of \( \mathbb{R}_+ \).

If instead of proving the existence of the solution on \( \mathbb{R}_+ \), we have to prove it only on a compact then it suffices to consider that \((a_{16}'')^{(2)}(2)\) and \((b_{16}'')^{(2)}(2)\), \( i = 16, 17, 18 \) depend only on \( T_{17} \) and respectively on \( (G_{19})(t) \) and not on \( t \) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \( t \) where \( G_i(t) = 0 \) and \( T_i(t) = 0 \)

From 19 to 24 it results

\[
G_i(t) \geq G_i^0 e^{-\int_0^t (a_i''(2) - (a_i')(2)(T_{17}(s_{16}))) ds_{16}} \geq 0
\]

\[
T_i(t) \geq T_i^0 e^{-b_{17}'(2)} > 0 \quad \text{for } t > 0
\]

**Definition of** \((\bar{M}_{16}(2))_1\), \((\bar{M}_{16}(2))_2\) and \((\bar{M}_{16}(2))_3\):

**Remark 3:** If \( G_{16} \) is bounded, the same property have also \( G_{17} \) and \( G_{18} \). Indeed if

\[
G_{16} \leq (\bar{M}_{16}(2))_2
\]

it follows 

\[
\frac{dG_{17}}{dt} \leq \left( (\bar{M}_{16}(2))_1 - (a_{17})^{(2)} \right) G_{17} \]

and by integrating

\[
G_{17} \leq G_{17}^0 + 2(a_{17})^{(2)}((\bar{M}_{16}(2))_1)/(a_{17}'')^{(2)}
\]

In the same way, one can obtain

\[
G_{18} \leq G_{18}^0 + 2(a_{18})^{(2)}((\bar{M}_{16}(2))_2)/(a_{18}'')^{(2)}
\]

If \( G_{17} \) or \( G_{18} \) is bounded, the same property follows for \( G_{16} \), \( G_{17} \) and \( G_{18} \), respectively.

**Remark 4:** If \( G_{16} \) is bounded, from below, the same property holds for \( G_{17} \) and \( G_{18} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{17} \) is bounded from below.

**Remark 5:** If \( T_{16} \) is bounded from below and \( \lim_{t \to \infty} ((b_{17}'')^{(2)}(G_{19})(t), t) = (b_{17}''(2) \) then \( T_{17} \to \infty \).

**Definition of** \((m)^{(2)}\) and \( \varepsilon_2 \):

Indeed let \( t_2 \) be so that for \( t > t_2 \)

\[
(b_{17}'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}
\]
Then \( \frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17} \) which leads to

\[
T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right) \left(1 - e^{-\varepsilon_2 t}\right) + T_{17}^0 e^{-\varepsilon_2 t}
\]

If we take \( t \) such that \( e^{-\varepsilon_2 t} = \frac{1}{2} \) it results

\[
T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right) \cdot \frac{t}{2} = \log \frac{t}{\varepsilon_2} \quad \text{By taking now } \varepsilon_2 \quad \text{sufficiently small one sees that } \quad T_{17}
\]

is unbounded. The same property holds for \( T_{18} \) if \( \lim_{t \to \infty} (b_{18})^{(2)}(G_{18})(t, t) = (b_{18})^{(2)} \)

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take \( \left(\frac{a_{1j}}{(M_{20})^{(3)}}\right), \left(\frac{b_{1j}}{(M_{20})^{(3)}}\right) < 1 \) and to choose

\( (\tilde{P}_{20})^{(3)} \) and \( (\tilde{Q}_{20})^{(3)} \) large to have

\[
\left(\frac{a_{1j}}{(M_{20})^{(3)}}\right) \left(\tilde{P}_{20}^{(3)} + \left(\tilde{P}_{20}^{(3)} + \vartheta_j^{(1)}\right) e^{-\frac{(P_{20})^{(3)} + \vartheta_j^{(1)}}{\vartheta_j^{(1)}}}\right) \leq (\tilde{P}_{20})^{(3)}
\]

\[
\left(\frac{b_{1j}}{(M_{20})^{(3)}}\right) \left(\tilde{Q}_{20}^{(3)} + \vartheta_j^{(1)}\right) e^{-\frac{(Q_{20})^{(3)} + \vartheta_j^{(1)}}{\vartheta_j^{(1)}}} + (\tilde{Q}_{20})^{(3)} \leq (\tilde{Q}_{20})^{(3)}
\]

In order that the operator \( \mathcal{A}^{(3)} \) transforms the space of sextuples of functions \( G_i, T_i \) into itself

The operator \( \mathcal{A}^{(3)} \) is a contraction with respect to the metric

\[
d\left(\left(G_{23}, (T_{23})^{(1)}\right), \left(G_{23}, (T_{23})^{(2)}\right)\right) =
\sup \left\{ \max_t \left| G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-\left(\theta_{20}^{(3)}\right)t}, \max_t \left| T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-\left(\vartheta_{20}^{(3)}\right)t} \right\}
\]

Indeed if we denote

**Definition of** \( \tilde{G}_{23}, \tilde{T}_{23} : (G_{23}, (T_{23}) \) = \( \mathcal{A}^{(3)}\left(\left(G_{23}, (T_{23})\right)\right)\)

It results

\[
\left|\tilde{G}_{20}^{(1)}(t, \tilde{S}_{20}) - G_i^{(2)}(t)\right| \leq \int_0^t \left(\frac{a_{20}^{(1)}(t)}{(M_{20})^{(3)}}\right) \left| G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-\left(\theta_{20}^{(3)}\right)s(t)} e^{\left(\theta_{20}^{(3)}\right)s(t)} ds(t) +
\int_0^t \left(\frac{a_{20}^{(1)}(t)}{(M_{20})^{(3)}}\right) \left| G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-\left(\theta_{20}^{(3)}\right)s(t)} e^{\left(\theta_{20}^{(3)}\right)s(t)} +
\left(\frac{a_{20}^{(1)}(t)}{(M_{20})^{(3)}}\right) \left| T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-\left(\vartheta_{20}^{(3)}\right)s(t)} e^{\left(\vartheta_{20}^{(3)}\right)s(t)}
\]

\[
\left(\frac{a_{20}^{(1)}(t)}{(M_{20})^{(3)}}\right) \left| T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-\left(\vartheta_{20}^{(3)}\right)s(t)} e^{\left(\vartheta_{20}^{(3)}\right)s(t)} \left| G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-\left(\theta_{20}^{(3)}\right)s(t)} e^{\left(\theta_{20}^{(3)}\right)s(t)} ds(t)
\]

Where \( s(t) \) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows

\[
\left| G^{(1)} - G^{(2)}\right| e^{-\left(\theta_{20}^{(3)}\right)t} \leq \frac{1}{(M_{20})^{(3)}} \left(\frac{a_{20}^{(1)}(t)}{(M_{20})^{(3)}} + (a_{20}^{(1)}(t) + (\lambda_{20})^{(3)} +
\]

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\( (P_{20})^{(3)}(k_{20})^{(3)}d \left( ((G_{23})^{(1)},(T_{23})^{(1)}; (G_{23})^{(2)}, (T_{23})^{(2)}) \right) \)

And analogous inequalities for \( G_i \) and \( T_i \). Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed \((a_{i_{20}}^{(3)}))^{(3)}\) and \((b_{i_{20}}^{(3)}))^{(3)}\) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((P_{20})^{(3)}e^{(m)(3)t} \) and \((Q_{20})^{(3)}e^{(m)(3)t}\) respectively of \( \mathbb{R}^+ \).

If instead of proving the existence of the solution on \( \mathbb{R}^+ \), we have to prove it only on a compact then it suffices to consider that \((a_{i_{20}}^{(3))})^{(3)}\) and \((b_{i_{20}}^{(3))})^{(3)}\) depend only on \( T_{21} \) respectively and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \( t \) where \( G_i (t) = 0 \) and \( T_i (t) = 0 \).

From 19 to 24 it results

\[ G_i (t) \geq G_i \left( \int [a(t)]^{(3)}(t) - (a_{i_{20}}^{(3)}))^{(3)}(t,\xi_{20})]d\xi_{20} \right) \geq 0 \]

\[ T_i (t) \geq T_i e^{-(b_{i_{20}}^{(3))})^{(3)}t} > 0 \]

**Definition of \((\overline{M}_{20})^{(3)})_{1}, (\overline{M}_{20})^{(3)})_{2}\) and \((\overline{M}_{20})^{(3)})_{3}:**

**Remark 3:** If \( G_{20} \) is bounded, the same property have also \( G_{21} \) and \( G_{22}\). Indeed if \( G_{20} < (\overline{M}_{20})^{(3)}\) it follows \( \frac{dG_{21}}{dt} \leq \left( (\overline{M}_{20})^{(3)} \right)_{1} - (a_{21}^{(3)})^{(3)}G_{21} \) and by integrating

\[ G_{21} \leq \left( (\overline{M}_{20})^{(3)} \right)_{2} = G_{2}^{0} + 2(a_{21}^{(3)})^{(3)}((\overline{M}_{20})^{(3)})_{j}/(a_{21}^{(3)})^{(3)} \]

In the same way, one can obtain

\[ G_{22} \leq \left( (\overline{M}_{20})^{(3)} \right)_{3} = G_{2}^{0} + 2(a_{22}^{(3)})^{(3)}((\overline{M}_{20})^{(3)})_{j}/(a_{22}^{(3)})^{(3)} \]

If \( G_{21} \) or \( G_{22} \) is bounded, the same property follows for \( G_{20}, G_{22} \) and \( G_{20}, G_{21} \) respectively.

**Remark 4:** If \( \overline{M}_{20} \) is bounded, from below, the same property holds for \( G_{21} \) and \( G_{22} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{21} \) is bounded from below.

**Remark 5:** If \( T_{20} \) is bounded from below and \( \lim_{t \to \infty} ((b_{1_{20}}^{(3)})^{(3)}((G_{23})_{2}(t),t)) = (b_{1_{21}}^{(3)})^{(3)} \) then \( T_{21} \to \infty \).

**Definition of \((m)^{(3)}\) and \( \varepsilon_{3}:**

Indeed let \( t_{3} \) be so that for \( t > t_{3} \)

\[ (b_{21}^{(3)}) - (b_{1_{20}}^{(3)})((G_{23})_{2}(t),t) < \varepsilon_{3}, T_{20}(t) > (m)^{(3)} \]

Then \( \frac{dT_{21}}{dt} \geq (a_{21}^{(3)})^{(3)}(m)^{(3)} - \varepsilon_{3}T_{21} \) which leads to

\[ T_{21} \geq \frac{(a_{21}^{(3)})^{(3)}(m)^{(3)}}{\varepsilon_{3}}(1 - e^{-\varepsilon_{3}t}) + \frac{T_{20}^{0}e^{-\varepsilon_{3}t}}{\varepsilon_{3}} \]

If we take \( t \) such that \( e^{-\varepsilon_{3}t} = \frac{1}{2} \) it results

\[ T_{21} \geq \frac{(a_{22}^{(3)})^{(3)}(m)^{(3)}}{\varepsilon_{3}} \]

\[ t = \log \frac{2}{\varepsilon_{3}} \]

By taking now \( \varepsilon_{3} \) sufficiently small one sees that \( T_{21} \) is unbounded. The same property holds for \( T_{22} \) if \( \lim_{t \to \infty} ((b_{2_{20}}^{(3)})^{(3)}((G_{23})_{2}(t),t)) = (b_{2_{22}}^{(3)})^{(3)} \)

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We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take $(a_1^{(4)}), (b_0^{(4)}), (\bar{M}_{24})^{(4)} < 1$ and to choose $(\bar{P}_{24})^{(4)}$ and $(\bar{Q}_{24})^{(4)}$ large to have

\[
\left( \frac{(a_0)^{(4)}}{(M_{24})^{(4)}} \right) \left[ (\bar{P}_{24})^{(4)} + ((\bar{P}_{24})^{(4)} + G_0^{(4)} e^{-\frac{(P_{24})^{(4)} + G_0^{(4)}}{\bar{M}_{24}}}) \right] \leq (\bar{P}_{24})^{(4)}
\]

\[
\left( \frac{(b_0)^{(4)}}{(M_{24})^{(4)}} \right) \left[ ((\bar{Q}_{24})^{(4)} + T_j^0) e^{-\frac{(Q_{24})^{(4)} + T_j^0}{\bar{M}_{24}}} + (\bar{Q}_{24})^{(4)} \right] \leq (\bar{Q}_{24})^{(4)}
\]

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying IN to itself

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric

\[
d\left( (G_{27})^{(2)}, (T_{27})^{(2)} \right) = 
\sup_i \left( \left| G_i^{(1)}(t) - G_i^{(2)}(t) \right| e^{-((R_{24})^{(4)}) t}, \max_{t \in [0,1]} \left| T_i^{(1)}(t) - T_i^{(2)}(t) \right| e^{-((R_{24})^{(4)}) t} \right)
\]

Indeed if we denote

**Definition of $(\overline{G_{27}}, \overline{T_{27}})$: $(\overline{G_{27}}, \overline{T_{27}}) = \mathcal{A}^{(4)}((G_{27}), (T_{27}))$**

It results

\[
\| \bar{G}_i^{(4)} - \bar{G}_i^{(2)} \| \leq \int_0^t \left( a_{24}^{(4)} \right) \left| \int_{0}^{t} \left( a_{24}^{(4)} \right) G_{25}^{(1)} - G_{25}^{(2)} \right| e^{-((R_{24})^{(4)}) \bar{s}_{(24)}(t)} e^{((R_{24})^{(4)}) \bar{s}_{(24)}(t)} ds_{(24)} + 
\]

\[
\int_0^t \left( a_{24}^{(4)} \right) \left| \int_{0}^{t} \left( a_{24}^{(4)} \right) G_{24}^{(1)} - G_{24}^{(2)} \right| e^{-((R_{24})^{(4)}) \bar{s}_{(24)}(t)} e^{((R_{24})^{(4)}) \bar{s}_{(24)}(t)} + 
\]

\[
\left( a_{24}^{(4)} \right)(T_{25}^{(1)} , S_{(24)}) G_{24}^{(1)} - G_{24}^{(2)} | e^{-((R_{24})^{(4)}) \bar{s}_{(24)}(t)} e^{((R_{24})^{(4)}) \bar{s}_{(24)}(t)} + 
\]

\[
 G_{24}^{(2)} \left( a_{24}^{(4)} \right)(T_{25}^{(1)} , S_{(24)}) - \left( a_{24}^{(4)} \right)(T_{25}^{(2)} , S_{(24)}) \right| \ e^{-((R_{24})^{(4)}) \bar{s}_{(24)}(t)} e^{((R_{24})^{(4)}) \bar{s}_{(24)}(t)} ds_{(24)}
\]

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

\[
\left| (G_{27})^{(1)} - (G_{27})^{(2)} \right| e^{-((R_{24})^{(4)}) t} \leq 
\]

\[
\int \left( a_{24}^{(4)} \right) + \left( a_{24}^{(4)} \right) + (\overline{A_{24}})^{(4)} + 
\]

\[
(\bar{P}_{24})^{(4)}(\overline{K_{24}})^{(4)} d \left( ((G_{27})^{(1)}, (T_{27})^{(1)}; (G_{27})^{(2)}, (T_{27})^{(2)}) \right)
\]
And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on $t$ can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(P_{24})^{(4)}e^{(R_{24})^{(4)}t}$ and $(Q_{24})^{(4)}e^{(R_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_{25}^{(4)})$ and $(b_{25}^{(4)})$, $i = 24, 25, 26$ depend only on $T_{25}$ and respectively on $(G_{27})(and not on t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i (t) = 0$ and $T_i (t) = 0$. From GLOBAL EQUATIONS it results

$G_i (t) \geq G_0^{(4)} e^{-\int_0^t ((a''_i)^{(4)}(a''_i)) \tau(x_{24}))(dx_{24})} \geq 0$

$T_i (t) \geq T_0^{(4)} e^{-(b''_i)^{(4)}t} > 0$ for $t > 0$

**Definition of** $(\overline{M_{24}})^{(4)}$, $(\underline{M_{24}})^{(4)}$ and $(\overline{M_{25}})^{(4)}$.

**Remark 3:** If $G_{24}$ is bounded, the same property have also $G_{25}$ and $G_{26}$. indeed if

$G_{24} < (\overline{M_{24}})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq ((\overline{M_{24}})^{(4)})_1 - (a_{25})^{(4)}G_{25}$ and by integrating

$G_{25} \leq ((\overline{M_{24}})^{(4)})_1 = G_0^{(4)} + 2(a_{25})^{(4)}((\overline{M_{24}})^{(4)})_1/a_{25}^{(4)}$

In the same way, one can obtain

$G_{26} \leq ((\overline{M_{25}})^{(4)})_3 = G_0^{(4)} + 2(a_{26})^{(4)}((\overline{M_{24}})^{(4)})_3/a_{26}^{(4)}$

If $G_{25}$ or $G_{26}$ is bounded, the same property follows for $G_{24}$, $G_{26}$ and $G_{24}$, $G_{25}$ respectively.

**Remark 4:** If $G_{24}$ is bounded, from below, the same property holds for $G_{25}$ and $G_{26}$. The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below.

**Remark 5:** If $T_{24}$ is bounded from below and $\lim_{t \to \infty}((b''_i)^{(4)}((G_{27})(t), t)) = (b_{25})^{(4)}$ then $T_{25} \to \infty$.

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$:

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b''_i)^{(4)}((G_{27})(t), t) < \varepsilon_4 T_{24} (t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right) \left(1 - e^{-\varepsilon_4 t} + \tau_{25} e^{-\varepsilon_4 t}\right)$ If we take $t$ such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = \log_{\varepsilon_4} \frac{2}{\varepsilon_4}$

By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is
unbounded. The same property holds for 

We now state a more precise theorem about the behaviors at infinity of the solutions ANALOGOUS inequalities hold also for 

It is now sufficient to take 

and to choose 

In order that the operator \( \mathcal{A}(\vec{s}) \) transforms the space of sextuples of functions \( G_i, T_i \) into itself 

The operator \( \mathcal{A}(\vec{s}) \) is a contraction with respect to the metric 

Indeed if we denote 

**Definition of** \( \vec{s}(\vec{G}_3), \vec{s}(\vec{T}_3) : (G_{31}, T_{31}) \rightarrow \mathcal{A}(\vec{s})(G_{31}, T_{31}) \)

It results 

Where \( s_{(28)} \) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows 

\[ |(G_{31})^{(1)} - (G_{31})^{(2)}| e^{-\langle \vec{M}_{28} \rangle(\vec{s}_{(28)})} t \leq \]
\begin{align*}
\left( \frac{1}{(M_{28})^5} \right) (a_{28}^{(5)} + (a'_{28})^{(5)} + (A_{28})^{(5)} + \\
\left( P_{28} \right)^5 (k_{28}^{(5)}) d \left( (G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)} \right) \right)
\end{align*}

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis (35,35,36) the result follows

**Remark 1:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on $t$ can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(P_{28})^{(5)} e^{(M_{28})^{(5)}t}$ and $(Q_{28})^{(5)} e^{(M_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_{i})^{(5)}$ and $(b''_{i})^{(5)}$, $i = 28, 29, 30$ depend only on $T_{29}$ and respectively on $G_{31}$ (and not on $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i (t) = 0$ and $T_i (t) = 0$.

From GLOBAL EQUATIONS it results

\[
G_i (t) \geq G_{29}^0 e^{-\int_0^t [a''_{i}^{(5)}] (a''_{i}^{(5)}) (t_{29}, z_{29}) | dx_{29} ]} \geq 0
\]

\[
T_i (t) \geq T_{29}^0 e^{-\left( b''_{i}^{(5)} \right)} > 0 \quad \text{for } t > 0
\]

**Definition of** $(\overline{M}_{28})^{(5)}_{1}$, $(\overline{M}_{28})^{(5)}_{2}$ and $(\overline{M}_{28})^{(5)}_{3}$.

**Remark 3:** If $G_{28}$ is bounded, the same property have also $G_{29}$ and $G_{30}$.

Indeed let $a_{29}$ be so that for

\[
G_{29} \leq (\overline{M}_{28})^{(5)}_{2} = G_{29}^0 + 2(a_{29})^{(5)} (\overline{M}_{28})^{(5)}_{1}/(a'_{29})^{(5)}
\]

In the same way, one can obtain

\[
G_{30} \leq (\overline{M}_{28})^{(5)}_{3} = G_{30}^0 + 2(a_{30})^{(5)} (\overline{M}_{28})^{(5)}_{2}/(a'_{30})^{(5)}
\]

If $G_{29}$ or $G_{30}$ is bounded, the same property follows for $G_{28}$, $G_{30}$ and $G_{28}$, $G_{29}$ respectively.

**Remark 4:** If $G_{28}$ is bounded, from below, the same property holds for $G_{29}$ and $G_{30}$. The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below.

**Remark 5:** If $T_{28}$ is bounded from below and

\[
\lim_{T_{29} \to \infty} \left( (b''_{i}^{(5)}) ((G_{31})(t), t) \right) = (b''_{29})^{(5)}
\]

then $T_{29} \to \infty$.

**Definition of** $(m)^{(5)}$ and $\varepsilon_{5}$:

Indeed let $t_{5}$ be so that for $t > t_{5}$

\[
(b_{29})^{(5)} - (b''_{i}^{(5)}) ((G_{31})(t), t) < \varepsilon_{5}, T_{28} (t) > (m)^{(5)}
\]
Then \( \frac{dT_{29}}{dt} \geq (a_{29}(5)(m)^{(5)} - \varepsilon_5 T_{29} \) which leads to

\[
T_{29} \geq \left( \frac{(a_{29}(5)(m)^{(5)}}{\varepsilon_5} \right) (1 - e^{-\varepsilon_5 t}) + T_{29} e^{-\varepsilon_5 t}
\]

If we take \( t \) such that \( e^{-\varepsilon_5 t} = \frac{1}{2} \) it results

\[
T_{29} \geq \left( \frac{(a_{29}(5)(m)^{(5)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_5}
\]

By taking now \( \varepsilon_5 \) sufficiently small one sees that \( T_{29} \) is unbounded. The same property holds for \( T_{30} \) if \( \lim_{t \to \infty} (b_{30}^{(5)}) ((G_{31})(t), t) = (b_{30}^{(5)}) \)

We now state a more precise theorem about the behaviors at infinity of the solutions

Analogous inequalities hold also for \( G_{33}, G_{34}, T_{32}, T_{33}, T_{34} \)

It is now sufficient to take \( \left( \frac{a_i}{(M_{32})^{(6)}} \right), \frac{b_i}{(M_{32})^{(6)}} < 1 \) and to choose

\( (\bar{P}_{32})^{(6)} \) and \( (\bar{Q}_{32})^{(6)} \) large to have

\[
\left( \frac{a_0}{(M_{32})^{(6)}} \right) \left[ (\bar{P}_{32})^{(6)} + (\bar{P}_{32})^{(6)} + G_i^{(6)} e^{-\left( (\bar{P}_{32})^{(6)} + G_i^{(6)} \right)} \right] \leq (\bar{P}_{32})^{(6)}
\]

\[
\left( \frac{b_0}{(M_{32})^{(6)}} \right) \left[ (\bar{Q}_{32})^{(6)} + \tau_i^{(6)} e^{-\left( \tau_i^{(6)} \right)} \right] \leq (\bar{Q}_{32})^{(6)}
\]

In order that the operator \( \mathcal{A}^{(6)} \) transforms the space of sextuples of functions \( G_i, T_i \) into itself

The operator \( \mathcal{A}^{(6)} \) is a contraction with respect to the metric

\[
d\left( (G_{33}), (T_{33}) \right) = \sup_{i \in \mathbb{R}^+} \left[ |G_i^{(1)}(t) - G_i^{(2)}(t)| + \tau_i^{(6)} \right]
\]

Indeed if we denote

**Definition of** \( (\bar{G}_{33}, \bar{T}_{33}) : (G_{33}, T_{33}) \rightarrow \mathcal{A}^{(6)}((G_{33}), (T_{33})) \)

It results

\[
|\bar{G}_{32} - \bar{G}_{1}^{(2)}| \leq \int_0^t (a_{32})^{(6)} |G_{33}^{(1)} - G_{33}^{(2)}| \alpha^{(6)} x_{32} e^{(\bar{G}_{32})^{(6)} x_{32}} dS_{(32)} +
\]

\[
\int_0^t (a_{32}^{(6)}) |G_{32}^{(1)} - G_{32}^{(2)}| e^{-\bar{G}_{32}^{(6)} x_{32}} e^{(\bar{G}_{32})^{(6)} x_{32}} +
\]

\[
(a_{32}^{(6)})^{(6)} (T_{33}^{(1)}, s_{(32)}) |G_{32}^{(1)} - G_{32}^{(2)}| e^{-\bar{G}_{32}^{(6)} x_{32}} e^{(\bar{G}_{32})^{(6)} x_{32}} +
\]

\[
G_{32}^{(2)} ((a_{32}^{(6)})(T_{33}^{(1)}, s_{(32)})) e^{-\bar{G}_{32}^{(6)} x_{32}} e^{(\bar{G}_{32})^{(6)} x_{32}} dS_{(32)}
\]

Where \( s_{(32)} \) represents integrand that is integrated over the interval \([0, t] \)
From the hypotheses it follows

(1) \((a_i^{(1)}), (a_i^{(2)}), (b_i^{(1)}), (b_i^{(2)}), (b_i^{(3)}) > 0, i, j = 13, 14, 15\)

(2) The functions \((a_i^{(2)}), (b_i^{(2)})\) are positive continuous increasing and bounded.

**Definition of** \((p_i^{(1)}), (r_i^{(1)})\):

\(a_i^{(2)}(T_{14}, t) \leq (p_i^{(1)} \leq (\hat{A}_{13})^{(1)}\)

\(b_i^{(2)}(G, t) \leq (r_i^{(1)} \leq (\hat{B}_{13})^{(1)}\)

(3) \(\lim_{T_{14} \to 0} (a_i^{(2)}(T_{14}, t) = (p_i^{(1)}\)

\(\lim_{G \to 0} (b_i^{(2)}(G, t) = (r_i^{(1)}\)

**Definition of** \((\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}\):

Where \((\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i^{(1)}), (r_i^{(1)}\) are positive constants

and \([i = 13, 14, 15]\)

They satisfy Lipschitz condition:

\[|a_i^{(2)}(T_{14}, t) - (a_i^{(2)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{A}_{13})^{(1)}t}\]

\[|b_i^{(2)}(G', t) - (b_i^{(2)}(G, t)| \leq (\hat{k}_{13})^{(1)}|G - G'|e^{-(\hat{B}_{13})^{(1)}t}\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i^{(2)}(T_{14}, t)\) and \((b_i^{(2)}(T_{14}, t)\) . \((T_{14}, t)\) and \((T_{14}', t)\) are points belonging to the interval \([\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\). It is to be noted that \((a_i^{(2)}(T_{14}, t)\) is uniformly continuous. In the eventuality of the fact, that \((M_{13})^{(1)} = 1\) then the function \((a_i^{(2)}(T_{14}, t)\), the first augmentation coefficient attributable to terrestrial organisms, would be absolutely continuous.

**Definition of** \((\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}\):

(V) \((\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)},\) are positive constants

\[\frac{(a_i^{(1)})}{(\hat{M}_{13})^{(1)}}, \frac{(b_i^{(1)})}{(\hat{M}_{13})^{(1)}} < 1\]

**Definition of** \((\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}\):

(W) There exists two constants \((\hat{P}_{13})^{(1)}\) and \((\hat{Q}_{13})^{(1)}\) which together with

\((\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}\) and \((\hat{B}_{13})^{(1)}\) and the constants \((a_i^{(1)}), (a_i^{(2)}), (b_i^{(1)}), (b_i^{(2)}), (p_i^{(1)}), (r_i^{(1)}), i = 13, 14, 15,\)

satisfy the inequalities

\[\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i^{(1)} + (a_i^{(1)}) + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1\]

\[\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i^{(1)} + (b_i^{(1)}) + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1\]

\[|(G_{35})^{(1)} - (G_{35})^{(2)}|e^{-(\hat{M}_{32})^{(6)}}t \leq 368\]

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\[
\frac{1}{(32)^{6}} \left( (a'_{32})^{(6)} + (a'_{32})^{(6)} + (a_{32})^{(6)} + \right.
\left. (P_{32})^{(6)}(k_{32})^{(6)}d \left( (G_{35})^{(1)} , (T_{35})^{(1)} ; (G_{35})^{(2)} , (T_{35})^{(2)} \right) \right)
\]

And analogous inequalities for \( G_{i} \) and \( T_{i} \). Taking into account the hypothesis the result follows

NOTE: SIMILAR ANALYSIS FOLLOWS FOR MODULE SEVEN

Remark 1: The fact that we supposed \( (a'_{32})^{(6)} \) and \( (b'_{32})^{(6)} \) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \( (P_{32})^{(6)}e^{(32)^{6}t} \) and \( (Q_{32})^{(6)}e^{(32)^{6}t} \) respectively of \( \mathbb{R}_{+} \).

If instead of proving the existence of the solution on \( \mathbb{R}_{+} \), we have to prove it only on a compact then it suffices to consider that \( (a_{i}^{(6)}) \) and \( (b_{i}^{(6)}) \), \( i = 32,33,34 \) depend only on \( T_{33} \) and respectively on \( (G_{35}) \)(and not on \( t \)) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any \( t \) where \( G_{i}(t) = 0 \) and \( T_{i}(t) = 0 \)

From GLOBAL EQUATIONS it results

\[
G_{i}(t) \geq G_{0}^{i}e^{-\int_{0}^{t}(a'_{i})^{(6)}(a_{i})^{(6)}(r_{33}(s_{32}),s_{32}))ds_{32}} \geq 0
\]

\[
T_{i}(t) \geq T_{0}^{i}e^{-\left(b_{i}^{(6)}t\right)} > 0 \quad \text{for } t > 0
\]

Definition of \( \left( (M_{32})^{(6)} \right)^{1} , \left( (M_{32})^{(6)} \right)^{2} \) and \( \left( (M_{32})^{(6)} \right)^{3} \):

Remark 3: if \( G_{32} \) is bounded, the same property have also \( G_{33} \) and \( G_{34} \). indeed if

\[
G_{32} < \left( (M_{32})^{(6)} \right)^{1} \text{ it follows } \frac{dG_{33}}{dt} \leq \left( (M_{32})^{(6)} \right)^{1} - (a'_{33})^{(6)}G_{33} \text{ and by integrating }
\]

\[
G_{33} \leq \left( (M_{32})^{(6)} \right)^{2} = G_{0}^{33} + 2(a_{33})^{(6)}\left( (M_{32})^{(6)} \right)^{1}/(a'_{33})^{(6)}
\]

In the same way , one can obtain

\[
G_{34} \leq \left( (M_{32})^{(6)} \right)^{3} = G_{0}^{34} + 2(a_{34})^{(6)}\left( (M_{32})^{(6)} \right)^{2}/(a'_{34})^{(6)}
\]

If \( G_{33} \) or \( G_{34} \) is bounded, the same property follows for \( G_{32} , G_{33} \) and \( G_{32} , G_{33} \) respectively.

Remark 4: If \( G_{32} \) is bounded, from below, the same property holds for \( G_{33} \) and \( G_{34} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{33} \) is bounded from below.

Remark 5: If \( T_{32} \) is bounded from below and \( \lim_{t \to \infty}((b_{i}^{(6)})(G_{35}(t), t)) = (b'_{33})^{(6)} \) then \( T_{33} \to \infty \).

Definition of \( (m)^{(6)} \) and \( \varepsilon_{6} \):

Indeed let \( t_{6} \) be so that for \( t > t_{6} \)
\[(b_{33}^{(6)} - (b_i^{(6)}((G_{35})(t), t) < \varepsilon_6, T_{32}(t) > (m)^{(6)\

Then \[\frac{dT_{33}}{dt} \geq (a_{33}^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}\) which leads to

\[T_{33} \geq \left(\frac{(a_{33}^{(6)}(m)^{(6)})}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}\] If we take \(t\) such that \(e^{-\varepsilon_6 t} = \frac{1}{2}\) it results

\[T_{33} \geq \left(\frac{(a_{33}^{(6)}(m)^{(6)})}{\varepsilon_6}\right), \quad t = \log \frac{2}{\varepsilon_6}\] By taking now \(\varepsilon_6\) sufficiently small one sees that \(T_{33}\) is unbounded. The same property holds for \(T_{34}\) if \(\lim_{t \to -\infty} (b_{34}^{(6)}((G_{33})(t), t(t), t) = (b_{34}^{(6)})\

We now state a more precise theorem about the behaviors at infinity of the solutions

(c) The operator \(\mathcal{A}_{(7)}\) maps the space of functions satisfying 37,35,36 into itself. Indeed it is obvious that

\[G_{36}(t) \leq G_{36}^0 + \int_0^t \left(\left(\frac{(a_{36}^{(7)})}{(M_{36})^{(7)}}\right)(\hat{P}_{36})^{(7)}(\hat{Q}_{36})^{(7)}\right) J_{(36)} = \]

\[\left(1 + (a_{36}^{(7)}(t)G_{37}^0 + \frac{(a_{36}^{(7)})}{(M_{36})^{(7)}}(\hat{P}_{36})^{(7)}(\hat{Q}_{36})^{(7)}\right)(e^{(\hat{Q}_{36})^{(7)}t} - 1)\]

From which it follows that

\[(G_{36}(t) - G_{36}^0) e^{-(\hat{Q}_{36})^{(7)}t} \leq \left(\frac{(a_{36}^{(7)})}{(M_{36})^{(7)}}\right) \left(\left(\hat{P}_{36})^{(7)} + G_{37}^0 e^{\left(-\frac{(P_{36})^{(7)} + Q_{36}^0}{\hat{Q}_{36}}\right)} + (\hat{P}_{36})^{(7)}\right)\]

\((G_{36}^0)\) is as defined in the statement of theorem 1

Analogous inequalities hold also for \(G_{37}, G_{38}, T_{36}, T_{37}, T_{38}\)

It is now sufficient to take \(\frac{(a_i^{(7)})}{(M_{36})^{(7)}}, \frac{(b_i^{(7)})}{(M_{36})^{(7)}} < 1\) and to choose

\((\hat{P}_{36})^{(7)}\) and \((\hat{Q}_{36})^{(7)}\) large to have

\[\left(\frac{(a_i^{(7)})}{(M_{36})^{(7)}}\right) \left((\hat{P}_{36})^{(7)} + (G_{36}^0 + e^{-(\frac{(P_{36})^{(7)} + Q_{36}^0}{\hat{Q}_{36}})})\right) \leq (\hat{P}_{36})^{(7)}\]

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In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying GLOBAL EQUATIONS AND ITS CONCOMITANT CONDITIONALITIES into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric

$$d\left(\left(\left(G_{39}\right)^{(1)}, \left(T_{39}\right)^{(1)}\right), \left(\left(G_{39}\right)^{(2)}, \left(T_{39}\right)^{(2)}\right)\right) =$$

$$\max_i \left| \frac{\partial G_i}{\partial t} (t) - \frac{\partial G_i}{\partial t} (t) \right| e^{-\left(\frac{\partial T_i}{\partial t} (t) - \frac{\partial T_i}{\partial t} (t) \right)}$$

Indeed if we denote

**Definition of** $(\overline{G_{39}}), (\overline{T_{39}})$:

$$(\overline{G_{39}}), (\overline{T_{39}}) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$$

It results

$$\left| \frac{\partial G_i}{\partial t} - \frac{\partial G_i}{\partial t} \right| \leq \int_{0}^{t} \left( \int_{0}^{s} \left| a_{36}(s) \right| G_{36}^{(1)} - G_{36}^{(2)} \right| e^{-\left(\frac{\partial T_i}{\partial t} (s) - \frac{\partial T_i}{\partial t} (s) \right)} ds \right)$$

$$\int_{0}^{t} \left( \int_{0}^{s} \left| a_{36}(s) \right| G_{36}^{(1)} - G_{36}^{(2)} \right| e^{-\left(\frac{\partial T_i}{\partial t} (s) - \frac{\partial T_i}{\partial t} (s) \right)} ds \right)$$

$$\left| a_{36}(s) \right| \left| G_{36}^{(1)} - G_{36}^{(2)} \right| e^{-\left(\frac{\partial T_i}{\partial t} (s) - \frac{\partial T_i}{\partial t} (s) \right)} ds$$

$$\left| a_{36}(s) \right| \left| G_{36}^{(1)} - G_{36}^{(2)} \right| e^{-\left(\frac{\partial T_i}{\partial t} (s) - \frac{\partial T_i}{\partial t} (s) \right)} ds$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows
\[
\left| (G_{39})^{(1)} - (G_{39})^{(2)} \right| e^{-\left(\overline{R}_{36}\right)^{2}t} \leq
\frac{1}{\left(\overline{R}_{36}\right)^{2}} (a_{36}^{(7)}) + (a_{36}^{''})^{(7)} + (\overline{A}_{36})^{(7)} +
(\overline{P}_{36})^{(7)} (\overline{R}_{36})^{(7)} d \left( ((G_{39})^{(1)}, (T_{39})^{(1)}; (G_{39})^{(2)}, (T_{39})^{(2)}) \right)
\]

And analogous inequalities for \( G_i \) and \( T_i \). Taking into account the hypothesis (37,35,36) the result follows

**Remark 1:** The fact that we supposed \((a_{36}^{''})^{(7)}\) and \((b_{36}^{''})^{(7)}\) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((\overline{P}_{36})^{(7)} e^{\left(\overline{R}_{36}\right)^{2}t}\) and \((\overline{Q}_{36})^{(7)} e^{\left(\overline{R}_{36}\right)^{2}t}\) respectively of \( \mathbb{R}_+ \).

If instead of proving the existence of the solution on \( \mathbb{R}_+ \), we have to prove it only on a compact then it suffices to consider that \((a_{i}^{''})^{(7)}\) and \((b_{i}^{''})^{(7)}\), \( i = 36,37,38 \) depend only on \( T_{37} \) and respectively on \((G_{39})\) and not on \( t \) and hypothesis can replace by a usual Lipschitz condition.

**Remark 2:** There does not exist any \( t \) where \( G_i (t) = 0 \) and \( T_i (t) = 0 \)

From CONCATENATED GLOBAL EQUATIONS it results

\[
G_i (t) \geq G_i^{0} e^{-\int_{t_{0}}^{t} (a_{i}^{''})^{(7)} / (T_{37}(a_{36}^{''}))^{2} dt} \geq 0
\]

\[
T_i (t) \geq T_i^{0} e^{-(b_{i}^{''})^{(7)} t} > 0 \quad \text{for } t > 0
\]

**Definition of** \(( (\overline{M}_{36})^{(7)} )^{1} \), \(( (\overline{M}_{36})^{(7)} )^{2} \) and \(( (\overline{M}_{36})^{(7)} )^{3} \):

**Remark 3:** if \( G_{36} \) is bounded, the same property have also \( G_{37} \) and \( G_{38} \). Indeed if

\[
G_{36} < (\overline{M}_{36})^{(7)} \text{ it follows } \frac{d G_{37}}{dt} \leq ((\overline{M}_{36})^{(7)})^{1} - (a_{37}^{''})^{(7)} G_{37} \text{ and by integrating}
\]

\[
G_{37} \leq ((\overline{M}_{36})^{(7)})^{2} = G_{37}^{0} + 2(a_{37}^{''})^{(7)}((\overline{M}_{36})^{(7)})^{1}/(a_{37}^{''})^{(7)}
\]

In the same way, one can obtain

\[
G_{38} \leq ((\overline{M}_{36})^{(7)})^{3} = G_{38}^{0} + 2(a_{38}^{''})^{(7)}((\overline{M}_{36})^{(7)})^{2}/(a_{38}^{''})^{(7)}
\]
If $G_{37}$ or $G_{38}$ is bounded, the same property follows for $G_{36}, G_{38}$ and $G_{36}, G_{37}$ respectively.

**Remark 7:** If $G_{36}$ is bounded, from below, the same property holds for $G_{37}$ and $G_{38}$. The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below.

**Remark 5:** If $T_{36}$ is bounded from below and $\lim_{t \to \infty} (b_i(7)^{(7)}((G_{36})(t), t)) = (b_i(7)^{(7)}$ then $T_{37} \rightarrow \infty$.

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$:

Indeed let $t_7$ be so that for $t > t_7$

$$(b_{37}^{(7)}(t) - (b_i^{(7)}(7)((G_{36})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37}((m)^{(7)}) - \varepsilon_7 T_{37}$ which leads to

$$T_{37} \geq \left(\frac{(a_{37}((m)^{(7)}\varepsilon_7)}{2}\right)(1 - e^{-\varepsilon_7 t}) + T_{37} e^{-\varepsilon_7 t}$$

If we take $t$ such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$$T_{37} \geq \left(\frac{(a_{37}((m)^{(7)}\varepsilon_7)}{2}\right), \ t = log\frac{2 \varepsilon_7}{\varepsilon_7}$$

By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded. The same property holds for $T_{38}$ if $\lim_{t \to \infty} (b_{38}^{(7)}(7))((G_{39})(t), t) = (b_{38}^{(7)}(7)$

We now state a more precise theorem about the behaviors at infinity of the solutions

$-(\sigma_2)^{(2)} \leq -(a_{16})^{(2)} + (a_{17})^{(2)} - (a_{16})^{(2)}(T_{17}, t) + (a_{17})^{(2)}(T_{17}, t) \leq -(\sigma_1)^{(2)}$

$-(\tau_2)^{(2)} \leq -(b_{16})^{(2)} + (b_{17})^{(2)} - (b_{16})^{(2)}((G_{19}), t) - (b_{17})^{(2)}((G_{19}), t) \leq -(\tau_1)^{(2)}$

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$:

By $(\nu_1)^{(2)} > 0, (\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0, (u_2)^{(2)} < 0$ the roots

(a) of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}(\nu^{(2)} - (a_{16})^{(2)} = 0$ and

$(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}(u^{(2)} - (b_{16})^{(2)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(2)}, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$:

By $(\bar{\nu}_1)^{(2)} > 0, (\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0, (\bar{u}_2)^{(2)} < 0$ the roots of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_2)^{(2)}(\nu^{(2)} - (a_{16})^{(2)} = 0$

and $(b_{17})^{(2)}(u^{(2)})^2 + (\tau_2)^{(2)}(u^{(2)} - (b_{16})^{(2)} = 0$

**Definition of** $(\mu_1)^{(2)}, (\mu_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:
(b) If we define \((m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}\) by
\[
(m_2)^{(2)} = \left( v_0 \right)^{(2)}, \quad (\mu_1)^{(2)} = (v_1)^{(2)}, \quad \text{if } (v_0)^{(2)} < (v_1)^{(2)}
\]
and
\[
\left( v_0 \right)^{(2)} = \frac{g_0^{(2)}}{g_1^{(2)}}\]
\[
(m_2)^{(2)} = (v_1)^{(2)}, \quad (m_1)^{(2)} = (v_0)^{(2)}, \quad \text{if } (v_1)^{(2)} < (v_0)^{(2)} < (v_2)^{(2)}
\]
and analogously
\[
(\mu_2)^{(2)} = (u_0)^{(2)}, \quad (\mu_1)^{(2)} = (u_1)^{(2)}, \quad \text{if } (u_0)^{(2)} < (u_1)^{(2)}
\]
\[
(\mu_2)^{(2)} = (u_1)^{(2)}, \quad (\mu_1)^{(2)} = (u_0)^{(2)}, \quad \text{if } (u_1)^{(2)} < (u_0)^{(2)} < (u_2)^{(2)}
\]
and
\[
\left( u_0 \right)^{(2)} = \frac{T_0}{T_1^{(2)}}
\]
\[
(\mu_2)^{(2)} = (u_1)^{(2)}, \quad (\mu_1)^{(2)} = (u_0)^{(2)}, \quad \text{if } (u_1)^{(2)} < (u_0)^{(2)}
\]

Then the solution satisfies the inequalities
\[
G_{16}^0 e^{((S_1)^{\gamma}-(p_{16})^{(2)})t} \leq G_{16}^0 e^{(S_1)^{\gamma}t}
\]
\[
(p_1)^{(2)}
\]
is defined
\[
\frac{1}{(m_2)^{(2)}} G_{16}^0 e^{((S_1)^{\gamma}-(p_{16})^{(2)})t} \leq G_{17}^0 t \leq \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{\gamma}t}
\]
\[
\left( \frac{(a_{16})^{(2)} g_{16}^0}{(m_1)^{(2)}((S_1)^{(2)}-(p_{16})^{(2)})t} \right) \left[ e^{((S_1)^{(2)}-(p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t} \right] + G_{18}^0 e^{-(S_2)^{\gamma}t} \leq G_{19}^0 t \leq
\]
\[
\left( \frac{(a_{16})^{(2)} g_{16}^0}{(m_2)^{(2)}((S_1)^{(2)}-(a_{16})^{(2)})t} \right) \left[ e^{(S_1)^{\gamma}t} - e^{-(a_{16})^{(2)}t} \right] + G_{18}^0 e^{-(a_{16})^{(2)}t}
\]
\[
T_{16}^0 e^{(R_1)^{\gamma}t} \leq T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}^0 e^{(R_1)^{(2)}t}
\]
\[
\frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}^0 t \leq \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)}+\mu_2)^{(2)})t}
\]
\[
\left( \frac{(b_{16})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)}-(b_{16})^{(2)})t} \right) \left[ e^{(R_1)^{\gamma}t} - e^{-(b_{16})^{(2)}t} \right] + T_{16}^0 e^{-(b_{16})^{(2)}t} \leq T_{16}^0 t \leq
\]
\[
\left( \frac{(a_{16})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)}+(R_2)^{(2)}+\mu_2)^{(2)})t} \right) \left[ e^{((R_1)^{(2)}+(R_2)^{(2)}+\mu_2)^{\gamma}t} - e^{-(R_2)^{(2)}t} \right] + T_{16}^0 e^{-(R_2)^{(2)}t}
\]

**Definition of** \((S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}\):

Where \((S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a_{16})^{(2)}\)
\[(S_2)^{(2)} = (a_{16})^{(2)} - (p_{16})^{(2)}\]
\[(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(2)} - (b_{16})^{(2)}\]
\[(R_2)^{(2)} = (b_{16})^{(2)} - (r_{16})^{(2)}\]
Behavior of the solutions

If we denote and define

**Definition of** \( (\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)} \): 

(a) \( (\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)} \) four constants satisfying

\[ -(\sigma_2)^{(3)} \leq -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a'_{20})^{(3)}T_{21}, t \leq -(\sigma_1)^{(3)} \]
\[ -(\tau_2)^{(3)} \leq -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b'_{20})^{(3)}G, t \leq -(\tau_1)^{(3)} \]

**Definition of** \( (v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)} \):

(b) By \( (v_1)^{(3)} > 0, (v_2)^{(3)} < 0 \) and respectively \( (u_1)^{(3)} > 0, (u_2)^{(3)} < 0 \) the roots of the equations

\[ (\sigma_2)^{(3)}v^{(3)} + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0 \]
\[ (b_{21})^{(3)}u^{(3)} + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0 \]

By \( (\tilde{v}_1)^{(3)} > 0, (\tilde{v}_2)^{(3)} < 0 \) and respectively \( (\tilde{u}_1)^{(3)} > 0, (\tilde{u}_2)^{(3)} < 0 \) the roots of the equations

\[ (\sigma_2)^{(3)}v^{(3)} + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0 \]
\[ (b_{21})^{(3)}u^{(3)} + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0 \]

**Definition of** \( (m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)} \):

(c) If we define \( (m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)} \) by

\[ (m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}, \] if \( (v_0)^{(3)} < (v_1)^{(3)} \)
\[ (m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\tilde{v}_1)^{(3)}, \] if \( (v_1)^{(3)} < (v_0)^{(3)} < (\tilde{v}_1)^{(3)} \)

and

\[ (v_0)^{(3)} = \frac{\tilde{v}_{20}}{\tilde{v}_{21}} \]

\[ (m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}, \] if \( (\tilde{v}_1)^{(3)} < (v_0)^{(3)} \)

and analogously

\[ (\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}, \] if \( (u_0)^{(3)} < (u_1)^{(3)} \)
\[ (\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (\tilde{u}_1)^{(3)}, \] if \( (u_0)^{(3)} < (u_0)^{(3)} < (\tilde{u}_1)^{(3)} \)

\[ (\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}, \] if \( (\tilde{u}_1)^{(3)} < (u_0)^{(3)} \)

Then the solution satisfies the inequalities

\[ G_{20}e^{(S_2)^{(3)} - (p_{20})^{(3)}t} \leq G_{20} \leq G_{20}e^{(S_1)^{(3)}t} \]

\[ (p_2)^{(3)} \] is defined

\[ \frac{1}{(m_2)^{(3)}} G_{20}e^{(S_2)^{(3)} - (p_{20})^{(3)}t} \leq G_{21} \leq \frac{1}{(m_2)^{(3)}} G_{20}e^{(S_2)^{(3)}t} \]

\[ \frac{(a_{20})^{(3)}G_{20}}{(m_1)^{(3)}(S_2)^{(3)} - (p_{20})^{(3)}(S_2)^{(3)} - (S_3)^{(3)}t)} \left[ e^{((S_1)^{(3)} - (p_{20})^{(3)}t)} - e^{-(S_2)^{(3)}t} \right] + G_{22}e^{-(S_2)^{(3)}t} \leq G_{22} \leq \]
\[
\frac{(a_{22})^{(3)}}{(m_2^{(3)})^2} e^{(a_{22})^{(3)}t} - e^{-a_{22}^{(3)}t} + G_{22}^0 e^{-(a_{22})^{(3)}t} \\
T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}^0 (t) \leq T_{20}^0 e^{((R_1)^{(3)} + (r_{22})^{(3)}t)} \leq T_{22}^0 (t) \leq T_{22}^0 e^{-(R_2)^{(3)}t} + T_{22}^0 e^{-(R_2)^{(3)}t} \\
(\frac{(b_{22})^{(3)}}{(m_2^{(3)})^2} - b_{22}^{(3)}) e^{(R_1)^{(3)}t} - e^{-b_{22}^{(3)}t} + T_{22}^0 e^{-(b_{22})^{(3)}t} \leq T_{22}^0 (t) \leq T_{22}^0 e^{-(R_2)^{(3)}t} \\
(\frac{(a_{22})^{(3)}}{(m_2^{(3)})^2} (R_1)^{(3)} + (r_{22})^{(3)}t) - e^{-(R_2)^{(3)}t} + T_{22}^0 e^{-(R_2)^{(3)}t} \\
\]

Definition of \((S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}\):

Where \((S_1)^{(3)} = (a_{20})^{(3)} (m_2^{(3)}) - (a_{20}^{(3)})^{(3)}\)

\((S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}\)

\((R_1)^{(3)} = (b_{20})^{(3)} (\mu_2)^{(3)} - (b_{20})^{(3)}\)

\((R_2)^{(3)} = (b_{22})^{(3)} - (r_{22})^{(3)}\)

If we denote and define

Definition of \((\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (r_1)^{(4)}, (r_2)^{(4)}\):

(d) \((\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (r_1)^{(4)}, (r_2)^{(4)}\) four constants satisfying

\[-(\sigma_1)^{(4)} \leq -(a_{20})^{(4)} + (a_{20}^{(4)}) - (a_{20}^{(4)}) (T_{25}, t) + (a_{20}^{(4)}) (T_{25}, t) \leq -(\sigma_1)^{(4)}\]

\[-(r_2)^{(4)} \leq -(b_{24})^{(4)} + (b_{24})^{(4)} - (b_{24})^{(4)} (G_{27}, t) - (b_{24})^{(4)} (G_{27}, t) \leq -(r_2)^{(4)}\]

Definition of \((v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, (v')^{(4)}, (u')^{(4)}\):

(e) By \((v_1)^{(4)} > 0, (u_2)^{(4)} < 0\) and respectively \((u_1)^{(4)} > 0, (u_2)^{(4)} < 0\) the roots of the equations \((a_{20})^{(4)} (v')^{(4)} + (\sigma_1)^{(4)} (v')^{(4)} - (a_{20})^{(4)} = 0\)

and \((b_{25})^{(4)} (u')^{(4)} + (r_1)^{(4)} (u')^{(4)} - (b_{25})^{(4)} = 0\) and

Definition of \((\bar{v}_1)^{(4)}, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}\):

By \((\bar{v}_1)^{(4)} > 0, (\bar{v}_2)^{(4)} < 0\) and respectively \((\bar{u}_1)^{(4)} > 0, (\bar{u}_2)^{(4)} < 0\) the roots of the equations \((a_{25})^{(4)} (v')^{(4)} + (\sigma_2)^{(4)} (v')^{(4)} - (a_{25})^{(4)} = 0\)

and \((b_{25})^{(4)} (u')^{(4)} + (r_2)^{(4)} (u')^{(4)} - (b_{25})^{(4)} = 0\)

Definition of \((m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}\):

(f) If we define \((m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}\) by

\[(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}, if (v_0)^{(4)} < (v_1)^{(4)}\]

\[(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}, if (v_1)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}\]
and \( (v_{0})^{(4)} = \frac{\zeta_{0}}{\zeta_{2}} \).

\( (m_{2})^{(4)} = (v_{4})^{(4)}, (m_{1})^{(4)} = (v_{0})^{(4)}, \text{ if } (\bar{v}_{4})^{(4)} < (v_{0})^{(4)} \)

and analogously

\( (\mu_{2})^{(4)} = (u_{0})^{(4)}, (\mu_{1})^{(4)} = (u_{1})^{(4)}, \text{ if } (\bar{u}_{0})^{(4)} < (u_{0})^{(4)} \)

\( (\mu_{2})^{(4)} = (u_{2})^{(4)}, (\mu_{1})^{(4)} = (\bar{u}_{1})^{(4)}, \text{ if } (u_{2})^{(4)} < (u_{0})^{(4)} < (\bar{u}_{1})^{(4)} \),

\( (u_{0})^{(4)} = \frac{\theta_{0}^{p}}{\theta_{2}} \)

\( (\mu_{2})^{(4)} = (u_{1})^{(4)}, (\mu_{1})^{(4)} = (u_{0})^{(4)}, \text{ if } (\bar{u}_{1})^{(4)} < (u_{0})^{(4)} \) where \( (u_{1})^{(4)}, (\bar{u}_{1})^{(4)} \)

are defined respectively

Then the solution satisfies the inequalities

\( G_{24}e^{((S_{2})^{(4)}-(p_{24})^{(4)})t} \leq G_{24}(t) \leq G_{24}e^{(S_{1})^{(4)}t} \)

where \( (p_{2})^{(4)} \) is defined

\[
\frac{1}{(m_{2})^{(4)}}G_{24}^{0}((S_{2})^{(4)}-(p_{24})^{(4)})t \leq G_{24}(t) \leq \frac{1}{(m_{2})^{(4)}}G_{24}^{0}(S_{1})^{(4)}t
\]

\[
\left(\frac{G_{24}^{0}e^{((S_{2})^{(4)}-(p_{24})^{(4)})t} - e^{-(S_{2})^{(4)}t}}{G_{24}^{0}e^{-(S_{2})^{(4)}t} + G_{24}^{0}e^{-(a_{26})^{(4)}t}}\right) + \frac{G_{24}^{0}e^{-(S_{2})^{(4)}t} \leq G_{24}(t) \leq \frac{G_{24}^{0}e^{-(S_{2})^{(4)}t} + G_{24}^{0}e^{-(a_{26})^{(4)}t}}{G_{24}^{0}e^{-(S_{2})^{(4)}t} + G_{24}^{0}e^{-(a_{26})^{(4)}t}}
\]

\( \frac{G_{24}^{0}e^{(R_{1})^{(4)}t} \leq T_{24}(t) \leq \frac{G_{24}^{0}e^{((R_{1})^{(4)}+(r_{24})^{(4)})t}}{G_{24}^{0}e^{(R_{1})^{(4)}t}} \)

\[
\frac{1}{(a_{26})^{(4)}}T_{24}^{0}e^{(R_{2})^{(4)}t} \leq T_{24}(t) \leq \frac{1}{(a_{26})^{(4)}}T_{24}^{0}e^{((R_{2})^{(4)}+(r_{26})^{(4)})t}
\]

\[
\left(\frac{T_{24}^{0}e^{(R_{1})^{(4)}t} - e^{-(b_{26})^{(4)}t}}{T_{24}^{0}e^{-(b_{26})^{(4)}t} + T_{24}^{0}e^{-(b_{26})^{(4)}t}}\right) + \frac{T_{24}^{0}e^{-(b_{26})^{(4)}t} \leq T_{24}(t) \leq \frac{T_{24}^{0}e^{-(b_{26})^{(4)}t} + T_{24}^{0}e^{-(b_{26})^{(4)}t}}{T_{24}^{0}e^{-(b_{26})^{(4)}t} + T_{24}^{0}e^{-(b_{26})^{(4)}t}}
\]

**Definition of \( (S_{1})^{(4)}, (S_{2})^{(4)}, (R_{1})^{(4)}, (R_{2})^{(4)} \):**

Where \( (S_{1})^{(4)} = (a_{24})^{(4)}(m_{2})^{(4)} - (a_{24})^{(4)} \)

\( (S_{2})^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)} \)

\( (R_{1})^{(4)} = (b_{24})^{(4)}(\mu_{2})^{(4)} - (b_{24})^{(4)} \)

\( (R_{2})^{(4)} = (b_{26})^{(4)} - (r_{26})^{(4)} \)

**Behavior of the solutions**

If we denote and define

**Definition of \( (\sigma_{1})^{(5)}, (\sigma_{2})^{(5)}, (\tau_{1})^{(5)}, (\tau_{2})^{(5)} \):**
(g) \((\sigma_1^{(5)}, \sigma_2^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)})\) four constants satisfying

\[-(\sigma_2^{(5)}) \leq -(a_{28}^{(5)}t + a_{29}^{(5)} - (a_{28}^{(5)}T_29, t) + (a_{29}^{(5)}T_29, t) \leq -(\sigma_1^{(5)})\]

\[-(\tau_2^{(5)}) \leq -(b_{28}^{(5)}t + b_{29}^{(5)} - (b_{28}^{(5)}G_{31}, t) - (b_{29}^{(5)}G_{31}, t) \leq -(\tau_1^{(5)})\]

**Definition of** \((v_1^{(5)}, v_2^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)})\):

(h) By \((v_1^{(5)}) > 0, (v_2^{(5)}) < 0\) and respectively \((u_1)^{(5)}) > 0, (u_2)^{(5)}) < 0\) the roots of the equations \((a_{29}^{(5)}v^{(5)})^2 + (\sigma_1^{(5)}v^{(5)} - (a_{28})^{(5)} = 0\) and \((b_{29}^{(5)}u^{(5)})^2 + (\tau_1^{(5)}u^{(5)} - (b_{28})^{(5)} = 0\) respectively.

**Definition of** \((\bar{v}_1^{(5)}, \bar{v}_2^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)})\):

By \((\bar{v}_1^{(5)}) > 0, (\bar{v}_2^{(5)}) < 0\) and respectively \((\bar{u}_1)^{(5)}) > 0, (\bar{u}_2)^{(5)}) < 0\) the roots of the equations \((a_{29}^{(5)}(\bar{v}_1^{(5)})^2 + (\sigma_1^{(5)}\bar{v}_1^{(5)} - (a_{28})^{(5)} = 0\) and \((b_{29}^{(5)}(\bar{u}_1^{(5)})^2 + (\tau_1^{(5)}\bar{u}_1^{(5)} - (b_{28})^{(5)} = 0\)

**Definition of** \((m_2^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}):\)

(i) If we define \((m_2)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}\) by

\[(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}, \text{ if } (v_0)^{(5)} < (v_1)^{(5)}\]

\[(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}, \text{ if } (v_0)^{(5)} < (\bar{v}_1)^{(5)}\]

and

\[(v_0)^{(5)} = \frac{v_0^{(5)}}{-a_{28}^{(5)}}\]

\[(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (v_0)^{(5)}, \text{ if } (v_1)^{(5)} < (v_0)^{(5)}\]

and analogously

\[(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}, \text{ if } (u_0)^{(5)} < (u_1)^{(5)}\]

\[(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \text{ if } (u_0)^{(5)} < (\bar{u}_1)^{(5)}\]

and

\[(u_0)^{(5)} = \frac{u_0^{(5)}}{-b_{28}^{(5)}}\]

\[(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \text{ if } (u_1)^{(5)} < (u_0)^{(5)}\]

where \((u_0)^{(5)}\) are defined respectively.

Then the solution satisfies the inequalities

\[G_{28}^{0}e^{((s_1)^{(5)}-(p_{28})^{(5)}t) \leq G_{28}^{0}e^{(s_1)^{(5)}t} \leq G_{28}^{0}e^{(s_1)^{(5)}t}\]

where \((p_{28})^{(5)}\) is defined

\[
\frac{1}{(m_2)^{(5)}} G_{28}^{0}e^{((s_1)^{(5)}-(p_{28})^{(5)}t) \leq G_{29}^{0}e^{(s_1)^{(5)}t} \leq \frac{1}{(m_2)^{(5)}} G_{28}^{0}e^{(s_1)^{(5)}t}\]

\[
\frac{1}{(m_2)^{(5)}} G_{28}^{0}e^{((s_1)^{(5)}-(p_{28})^{(5)}-0)^{(5)}} \leq G_{30}^{0}e^{(s_2)^{(5)}t} \leq \frac{1}{(m_2)^{(5)}} G_{28}^{0}e^{(s_1)^{(5)}t}\]

\[
\frac{1}{(m_2)^{(5)}} G_{28}^{0}e^{((s_1)^{(5)}-(a_{50})^{(5)}^{(5)}) \leq G_{30}^{0}e^{(a_{50})^{(5)}t} + G_{30}^{0}e^{(a_{50})^{(5)}t}\]

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\[
\begin{align*}
T_{20}^2 e^{(R_1(5)t)} & \leq T_{28}^2 e^{((R_1(5)+(r_{20})5)t)} \quad 462 \\
\frac{1}{(\mu_2(5)^5)} \frac{T_{20}^2 e^{(R_1(5)t)}}{T_28 e^{((R_1(5)+(r_{20})5)t)}} & \leq \frac{1}{(\mu_2(5)^5)} T_{20} e^{((R_1(5)+(r_{20})5)t)} \quad 463 \\
T_{30} e^{-((R_1(5)+r_{20})5)t} & \leq T_{30}(t) \leq T_{30} e^{((R_1(5)+(r_{20})5)t)} \quad 464 \\
\text{Definition of } (S_1(5), (S_2(5), (R_1(5), (R_2(5))) := 465 \\
\text{Where } (S_1(5) = (a_{20}(5) m_2(5) - (\lambda_{20}(5)) \\
(S_2(5) = (a_{30}(5) - (p_{30}(5) \\
(R_1(5) = (b_{20}(5) \mu_2(5) - (b_{20}(5)) \\
(R_2(5) = (b_{30}(5) \lambda_4(5) - (r_{30}(5) \\
\text{Behavior of the solutions} 466 \\
\text{If we denote and define}\\
\text{Definition of } (\sigma_1(6), (\sigma_2(6), (\tau_1(6), (\tau_2(6) : 467 \\
(i) (\sigma_1(6), (\sigma_2(6), (\tau_1(6), (\tau_2(6) \quad \text{four constants satisfying} \\
-(\sigma_2)(6) \leq -(a_{32}(6) + (a_{33}(6) - (a_{32}(6)(T_{33}, t) + (a_{33}(6)(T_{33}, t) \leq -(\sigma_1)(6) \\
-(\tau_2)(6) \leq -(b_{32}(6) + (b_{33}(6) - (b_{32}(6)(T_{33}, t) - (b_{33}(6)(T_{33}, t) \leq -(\tau_1)(6) \\
\text{Definition of } (v_1(6), (v_2(6), (u_1(6), (u_2(6), (v(6), (u(6) : 468 \\
(k) By (v_1(6) > 0, (v_2(6) < 0 and respectively (u_1(6) > 0, (u_2(6) < 0 the roots of the equations \\
\text{and } (b_{33}(6)u(6) + (v_1(6) - (b_{32}(6) = 0 and \\
\text{Definition of } (\bar{v}_1(6), (\bar{v}_2(6), (\bar{u}_1(6), (\bar{u}_2(6) : 469 \\
By (\bar{v}_1(6) > 0, (\bar{v}_2(6) < 0 and respectively (\bar{u}_1(6) > 0, (\bar{u}_2(6) < 0 the roots of the equations \\
and (b_{33}(6)u(6) + (v_1(6)u(6) - (b_{32}(6) = 0 \\
\text{Definition of } (m_1(6), (m_2(6), (\mu_1(6), (\mu_2(6), (v(6) : 470 \\
(l) If we define (m_1(6), (m_2(6), (\mu_1(6), (\mu_2(6) by \\
(m_2(6) = (v_0(6), (m_1(6) = (v_1(6), (if (v)(6) < (v_1)(6) \\
(m_2(6) = (v_1(6), (m_1(6) = (v_0(6), (if (v)(6) < (v_0)(6) \\
(m_2(6) = (v_0(6), (m_1(6) = (v_0(6), (if (v)(6) < (v_0)(6) \\
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and analogously

\[(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \text{ if } (u_0)^{(6)} < (u_1)^{(6)}\]

\[(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \text{ if } (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}, \text{ and } \]
\[\frac{T_2}{T_{32}}\]

\[(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \text{ if } (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}\]

are defined respectively

Then the solution satisfies the inequalities

\[G_{32}^0 e^{((s_1)^{(6)}-p_{32})t} \leq G_{32}^0 (s_1)^{(6)}t \leq G_{32}^0 e^{((s_1)^{(6)}+p_{32})t}\]

where \((p_1)^{(6)}\) is defined

\[\frac{1}{(m_1)^{(6)}G_{32}^0 e^{((s_1)^{(6)}-p_{32})t}} \leq \frac{1}{(m_1)^{(6)}G_{32}^0 e^{((s_1)^{(6)}+p_{32})t}} \leq \frac{1}{(m_1)^{(6)}G_{32}^0 e^{((s_1)^{(6)}-p_{32})t}} \leq \frac{1}{(m_1)^{(6)}G_{32}^0 e^{((s_1)^{(6)}+p_{32})t}}\]

\[G_{34}^0 e^{-(s_2)^{(6)}t} \leq G_{34}^0 e^{((s_1)^{(6)}+(s_2)^{(6)}-p_{32})t} \leq G_{34}^0 e^{((s_1)^{(6)}+(s_2)^{(6)}+p_{32})t} \leq G_{34}^0 e^{-(s_2)^{(6)}t}\]

\[T_{32}^0 e^{(r_1)^{(6)}t} \leq T_{32}^0 e^{((r_1)^{(6)}+(r_2)^{(6)}-p_{32})t} \leq T_{32}^0 e^{((r_1)^{(6)}+(r_2)^{(6)}+p_{32})t} \leq T_{32}^0 e^{(r_1)^{(6)}t}\]

\[T_{34}^0 e^{-(r_2)^{(6)}t} \leq T_{34}^0 e^{((r_1)^{(6)}+(r_2)^{(6)}-p_{32})t} \leq T_{34}^0 e^{((r_1)^{(6)}+(r_2)^{(6)}+p_{32})t} \leq T_{34}^0 e^{-(r_2)^{(6)}t}\]

**Definition of** \((S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}\):-

Where \((S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a_{32}^{'})^{(6)}\)

\[(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}\]

\[(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b_{32}^{'})^{(6)}\]

\[(R_2)^{(6)} = (b_{34})^{(6)} - (r_{34})^{(6)}\]

If we denote and define

**Definition of** \((\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}\):

\[(m) (\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)} \text{ four constants satisfying}\]

\[-(\sigma_2)^{(7)} \leq -(a_{30}^{''})^{(7)} + (a_{37}^{''})^{(7)} - (a_{30}^{''})^{(7)}(T_{37}, t) + (a_{37}^{''})^{(7)}(T_{37}, t) \leq -(\sigma_1)^{(7)}\]

\[-(\tau_2)^{(7)} \leq -(b_{30}^{''})^{(7)} + (b_{37}^{''})^{(7)} - (b_{30}^{''})^{(7)}((G_{39}, t) - (b_{37}^{''})^{(7)}((G_{39}, t) \leq -(\tau_1)^{(7)}\]

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Definition of \((v_1)^{(n)}, (v_2)^{(n)}, (u_1)^{(n)}, (u_2)^{(n)}, v^{(n)}, u^{(n)}\):

(n) By \((v_1)^{(n)} > 0, (v_2)^{(n)} < 0\) and respectively \((u_1)^{(n)} > 0, (u_2)^{(n)} < 0\) the roots of the equations
\[
(a_{37})^{(n)}(v^{(n)})^2 + (\sigma_1)^{(n)}v^{(n)} - (a_{30})^{(n)} = 0
\]
and \((b_{37})^{(n)}u^{(n)} + (\tau_1)^{(n)}u^{(n)} - (b_{30})^{(n)} = 0\) and

Definition of \((\bar{v}_1)^{(n)}, (\bar{v}_2)^{(n)}, (\bar{u}_1)^{(n)}, (\bar{u}_2)^{(n)}\):

By \((\bar{v}_1)^{(n)} > 0, (\bar{v}_2)^{(n)} < 0\) and respectively \((\bar{u}_1)^{(n)} > 0, (\bar{u}_2)^{(n)} < 0\) the roots of the equations
\[
(a_{37})^{(n)}(\bar{v}^{(n)})^2 + (\sigma_2)^{(n)}\bar{v}^{(n)} - (a_{30})^{(n)} = 0
\]
and \((b_{37})^{(n)}u^{(n)} + (\tau_2)^{(n)}u^{(n)} - (b_{30})^{(n)} = 0\)

Definition of \((m_1)^{(n)}, (m_2)^{(n)}, (\mu_1)^{(n)}, (\mu_2)^{(n)}, (v_0)^{(n)}\):

(o) If we define \((m_1)^{(n)}, (m_2)^{(n)}, (\mu_1)^{(n)}, (\mu_2)^{(n)}\) by

\[
(m_2)^{(n)} = (v_0)^{(n)}, (m_1)^{(n)} = (v_1)^{(n)}, \text{ if } (v_0)^{(n)} < (v_1)^{(n)}
\]

\[
(m_2)^{(n)} = (v_1)^{(n)}, (m_1)^{(n)} = (\bar{v}_2)^{(n)}, \text{ if } (v_1)^{(n)} < (v_0)^{(n)} < (\bar{v}_1)^{(n)}
\]

and

\[
(v_0)^{(n)} = \frac{\phi_{36}}{\phi_{37}}
\]

\[
(m_2)^{(n)} = (v_1)^{(n)}, (m_1)^{(n)} = (v_0)^{(n)}, \text{ if } (v_1)^{(n)} < (v_0)^{(n)}
\]

and analogously

\[
(\mu_2)^{(n)} = (u_0)^{(n)}, (\mu_1)^{(n)} = (u_1)^{(n)}, \text{ if } (u_0)^{(n)} < (u_1)^{(n)}
\]

\[
(\mu_2)^{(n)} = (u_1)^{(n)}, (\mu_1)^{(n)} = (\bar{u}_1)^{(n)}, \text{ if } (u_1)^{(n)} < (u_0)^{(n)} < (\bar{u}_1)^{(n)}
\]

and

\[
(u_0)^{(n)} = \frac{\theta_{36}}{\theta_{37}}
\]

\[
(\mu_2)^{(n)} = (u_1)^{(n)}, (\mu_1)^{(n)} = (u_0)^{(n)}, \text{ if } (u_1)^{(n)} < (u_0)^{(n)} \text{ where } (u_1)^{(n)}, (\bar{u}_1)^{(n)}
\]

are defined respectively

Then the solution satisfies the inequalities

Then the solution satisfies the inequalities

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\[ G_{36}^0 e^{(S_1) - (p_{36}) t} \leq G_{36}^0 (t) \leq G_{36}^0 e^{(S_1) t} \]

where \((p_{1})^{(7)}\) is defined

\[
\frac{1}{(m_1)^{(7)}} G_{36}^0 e^{((S_1) - (p_{36}) t)} \leq G_{36}^0 (t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1) t}
\]

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\[
\left( \frac{(a_{36})^{(7)}}{(m_1)^{(7)}} e_{36}^0 \right) \left[ e^{((S_1) - (p_{36}) t)} - e^{-(S_2) t} \right] + G_{36}^0 e^{-(S_2) t} \leq G_{36}^0 (t) \leq \left( \frac{(a_{36})^{(7)}}{(m_2)^{(7)}} e_{36}^0 \right) \left[ e^{(S_1) t} - e^{-(a_{36})^{(7)} t} \right] + G_{36}^0 e^{-(a_{36})^{(7)} t}
\]

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\[
T_{36}^0 e^{(R_1) t} \leq T_{36}^0 (t) \leq T_{36}^0 e^{(R_1) + (r_{36}) t}
\]

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\[
\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1) t} \leq T_{36}^0 (t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{(R_1) + (r_{36}) t}
\]

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\[
\left( \frac{(p_{36})^{(7)}}{(\mu_1)^{(7)}} \right) \left[ e^{(R_1) t} - e^{-(p_{36})^{(7)} t} \right] + T_{36}^0 e^{-(p_{36})^{(7)} t} \leq T_{36}^0 (t) \leq \left( \frac{(a_{36})^{(7)}}{(\mu_2)^{(7)}} e_{36} \right) \left[ e^{((R_1) + (r_{36}) t)} - e^{-(R_2) t} \right] + T_{36}^0 e^{-(R_2) t}
\]

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Definition of \((S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}\)

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Where \((S_1)^{(7)} = (a_{36})^{(7)} (m_2) - (a_{36})^{(7)}\)
\[(S_2)^{(7)} = (a_{38}^{(7)}) - (p_{38}^{(7)})\]

\[(R_1)^{(7)} = (b_{36}^{(7)})(\mu_2^{(7)}) - (b_{36}^{'})^{(7)}\]

\[(R_2)^{(7)} = (b_{38}^{'})^{(7)} - (r_{38}^{(7)})\]

From GLOBAL EQUATIONS we obtain

\[
\frac{dv^{(7)}}{dt} = (a_{36}^{(7)}) - \left( (a'_{36})^{(7)} - (a_{37}^{(7)}) + (a''_{36})^{(7)}(T_{37}, t) \right) -
\]

\[ (a'_{37})^{(7)}(T_{37}, t)v^{(7)} - (a_{37}^{(7)})v^{(7)} \]

**Definition of** \(v^{(7)} :\)

\[ v^{(7)} = \frac{\alpha_{36}}{\alpha_{37}} \]

It follows

\[-\left( (a_{37}^{(7)})(v^{(7)})^2 + (\sigma_2^{(7)}v^{(7)} - (a_{36}^{(7)})) \right) \leq \frac{dv^{(7)}}{dt} \leq
\]

\[-\left( (a_{37}^{(7)})(v^{(7)})^2 + (\sigma_1^{(7)}v^{(7)} - (a_{36}^{(7)}) \right) \]

From which one obtains

**Definition of** \((\bar{v}_1)^{(7)}, (v_0)^{(7)} :\)

(a) For \( 0 < \frac{(v_0)^{(7)}}{\sigma_2^{(7)}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)} \)

\[ v^{(7)}(t) \geq \frac{(v_2)^{(7)}(v_2)^{(7)}e^{-[(a_{37})^{(7)}(v_1)^{(7)} - (v_0)^{(7)}]t}}{1 + (C)^{(7)}e^{-[(a_{37})^{(7)}(v_1)^{(7)} - (v_0)^{(7)}]t}}, \]

\[ (C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}(v_0)^{(7)} - (v_2)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}} \]
it follows \((v_0)^{(7)} \leq v^{(7)}(t) \leq (v_1)^{(7)}\)

In the same manner, we get

\[
v^{(7)}(t) \leq \frac{(v_2)^{(7)} + (\bar{C})^{(7)}(v_2)^{(7)}e^{[-(a_{32})^{(7)}(v_1)^{(7)} - (v_2)^{(7)}]} e^{[-(a_{32})^{(7)}(v_1)^{(7)} - (v_2)^{(7)}]} t} {1 + (\bar{C})^{(7)}e^{[-(a_{32})^{(7)}(v_1)^{(7)} - (v_2)^{(7)}]} t} \leq \frac{(v_3)^{(7)} - (v_0)^{(7)}\} {\bar{v}_{(7)}^{(7)}}.
\]

From which we deduce \((v_0)^{(7)} \leq v^{(7)}(t) \leq (\bar{v}_{(7)}^{(7)}\)

(b) If \(0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{\delta_{\bar{g}}}{\delta_{\bar{g}_2}} < (\bar{v}_{(7)}^{(7)}\) we find like in the previous case,

\[
(v_1)^{(7)} \leq \frac{(v_2)^{(7)} + (\bar{C})^{(7)}(v_2)^{(7)}e^{[-(a_{32})^{(7)}(v_1)^{(7)} - (v_2)^{(7)}]} e^{[-(a_{32})^{(7)}(v_1)^{(7)} - (v_2)^{(7)}]} t} {1 + (\bar{C})^{(7)}e^{[-(a_{32})^{(7)}(v_1)^{(7)} - (v_2)^{(7)}]} t} \leq v^{(7)}(t) \leq \frac{(v_3)^{(7)} - (v_0)^{(7)}\} {\bar{v}_{(7)}^{(7)}}
\]

(c) If \(0 < (v_2)^{(7)} \leq (\bar{v}_{(7)}^{(7)}\), we obtain

\[
(v_1)^{(7)} \leq v^{(7)}(t) \leq \frac{(v_2)^{(7)} + (\bar{C})^{(7)}(v_2)^{(7)}e^{[-(a_{32})^{(7)}(v_1)^{(7)} - (v_2)^{(7)}]} e^{[-(a_{32})^{(7)}(v_1)^{(7)} - (v_2)^{(7)}]} t} {1 + (\bar{C})^{(7)}e^{[-(a_{32})^{(7)}(v_1)^{(7)} - (v_2)^{(7)}]} t} \leq (v_0)^{(7)}
\]

And so with the notation of the first part of condition (c), we have

**Definition of** \(v^{(7)}(t) \) :-

\[
(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad v^{(7)}(t) = \frac{\delta_{\bar{g}_2}(t)}{\delta_{\bar{g}_2}(t)}
\]

In a completely analogous way, we obtain

**Definition of** \(u^{(7)}(t) \) :-
\[(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}\]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case:**

If \((a''_{4\gamma}^{(7)}) = (a''_{\gamma\gamma}^{(7)}), then (\sigma_1)^{(7)} = (\sigma_2)^{(7)} and in this case \((v_1)^{(7)} = (\bar{v}_1)^{(7)}\) if in addition \((v_0)^{(7)} = (v_1)^{(7)}\) and \(v^{(7)}(t) = (v_0)^{(7)}\) and as a consequence \(G_{36}(t) = (v_0)^{(7)}G_{37}(t)\) this also defines \((v_0)^{(7)}\) for the special case.

Analogously if \((b''_{36}^{(7)}) = (b''_{37}^{(7)}), then (\tau_1)^{(7)} = (\tau_2)^{(7)} and then \((u_1)^{(7)} = (\bar{u}_1)^{(7)}\) if in addition \((u_0)^{(7)} = (u_1)^{(7)}\) then \(T_{36}(t) = (u_0)^{(7)}T_{37}(t)\) This is an important consequence of the relation between \((v_1)^{(7)}\) and \((\bar{v}_1)^{(7)}\), and definition of \((u_0)^{(7)}\).

We can prove the following

*If \((a''^{(7)}) and (b''^{(7)}) are independent on \(t\), and the conditions*

\[a'_{36}^{(7)}(a'_{37}^{(7)} - (a_{36}^{(7)})(a_{37}^{(7)}) < 0\]

\[a'_{36}^{(7)}(a'_{37}^{(7)} - (a_{36}^{(7)})(a_{37}^{(7)}) + (a_{36}^{(7)})(p_{36}^{(7)}) + (a'_{37}^{(7)})(p_{37}^{(7)}) + (p_{36}^{(7)})(p_{37}^{(7)}) > 0\]

\[(b'_{36}^{(7)})(b'_{37}^{(7)}) - (b_{36}^{(7)})(b_{37}^{(7)}) > 0\]

\[(b'_{36}^{(7)})(b'_{37}^{(7)}) - (b_{36}^{(7)})(b_{37}^{(7)}) - (b'_{36}^{(7)})(r_{37}^{(7)}) - (b'_{37}^{(7)})(r_{37}^{(7)}) + (r_{36}^{(7)})(r_{37}^{(7)}) < 0\]

*with \((p_{36}^{(7)}), (r_{37}^{(7)})\) as defined are satisfied, then the system with the satisfaction of the following properties has a solution as derived below.*

*Particular case:*

If \((a''_{16}^{(2)}) = (a''_{1\gamma}^{(2)}), then (\sigma_1)^{(2)} = (\sigma_2)^{(2)} and in this case \((v_1)^{(2)} = (\bar{v}_1)^{(2)}\) if in addition
\((v_0)^{(2)} = (v_1)^{(2)}\) then \(v^{(2)}(t) = (v_0)^{(2)}\) and as a consequence \(G_{16}(t) = (v_0)^{(2)} G_{17}(t)\)

Analogously if \((b_1^{(2)} = (b_1^{(2)}\), then \((\tau_1)^{(2)} = (\tau_2)^{(2)}\) and then

\((u_1)^{(2)} = (u_1)^{(2)}\) if in addition \((u_0)^{(2)} = (u_1)^{(2)}\) then \(T_{16}(t) = (u_0)^{(2)} T_{17}(t)\) This is an important consequence of the relation between \((v_1)^{(2)}\) and \((v_1)^{(2)}\)

From GLOBAL EQUATIONS we obtain

\[ \frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left( (a_{21})^{(3)} - (a_{21})^{(3)} + (a_{20})^{(3)} - (a_{21})^{(3)} - (a_{21})^{(3)}(T_{21}, t) \right) - (a_{21})^{(3)}(T_{21}, t) v^{(3)} - (a_{21})^{(3)} v^{(3)} \]

**Definition of** \(v^{(3)} \): - \[ v^{(3)} = \frac{\sigma_{21}}{\sigma_{21}} \]

It follows

\[- \left( (a_{21})^{(3)} v^{(3)} + (a_{20})^{(3)} - (a_{21})^{(3)} \right) \leq \frac{dv^{(3)}}{dt} \leq - \left( (a_{21})^{(3)} v^{(3)} + (a_{21})^{(3)} v^{(3)} - (a_{21})^{(3)} \right) \]

From which one obtains

(a) For \(0 < (v_0)^{(3)} = \frac{a_{20}}{\sigma_{21}} < (v_1)^{(3)} < (v_1)^{(3)}\)

\[ v^{(3)}(t) \geq \frac{(v_1)^{(3)} - (v_0)^{(3)} t}{1 + (C)^{(3)} e^{-[(a_{21})^{(3)}(v_1)^{(3)} - (v_0)^{(3)}] t}} \]

\[ (C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)} - (v_0)^{(3)} - (v_1)^{(3)}}{(v_0)^{(3)} - (v_1)^{(3)}} \]

it follows \((v_0)^{(3)} \leq v^{(3)}(t) \leq (v_1)^{(3)}\)

In the same manner , we get

\[ v^{(3)}(t) \leq \frac{(v_1)^{(3)} - (v_0)^{(3)} t}{1 + (C)^{(3)} e^{-[(a_{21})^{(3)}(v_1)^{(3)} - (v_0)^{(3)}] t}} \]

\[ (C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)} - (v_0)^{(3)} - (v_1)^{(3)}}{(v_0)^{(3)} - (v_1)^{(3)}} \]

**Definition of** \((v_1)^{(3)} \): -

From which we deduce \((v_0)^{(3)} \leq v^{(3)}(t) \leq (v_1)^{(3)}\)

(b) If \(0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{a_{20}}{\sigma_{21}} < (v_1)^{(3)}\) we find like in the previous case,

\[ (v_1)^{(3)} \leq \frac{(v_1)^{(3)} - (v_0)^{(3)} t}{1 + (C)^{(3)} e^{-[(a_{21})^{(3)}(v_1)^{(3)} - (v_0)^{(3)}] t}} \leq v^{(3)}(t) \leq \frac{(v_1)^{(3)} - (v_0)^{(3)} t}{1 + (C)^{(3)} e^{-[(a_{21})^{(3)}(v_1)^{(3)} - (v_0)^{(3)}] t}} \leq (v_1)^{(3)} \]

(c) If \(0 < (v_1)^{(3)} \leq (v_1)^{(3)} \leq (v_0)^{(3)} = \frac{a_{20}}{\sigma_{21}}\), we obtain
\[(v_1)^{(3)} \leq v^{(3)}(t) \leq \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)} e^{-(a_{21})^{(3)}([v_1]^{(3)} - [v_2]^{(3)})t}}{1 + (C)^{(3)} e^{-(a_{21})^{(3)}([v_1]^{(3)} - [v_2]^{(3)})t}} \leq (v_0)^{(3)}\]

And so with the notation of the first part of condition (c), we have

**Definition of** \( v^{(3)}(t) \):

\[(m_2)^{(3)} \leq v^{(3)}(t) \leq (m_1)^{(3)}, \quad v^{(3)}(t) = \frac{g_{20}(t)}{g_{21}(t)}\]

In a completely analogous way, we obtain

**Definition of** \( u^{(3)}(t) \):

\[(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad u^{(3)}(t) = \frac{r_{20}(t)}{r_{21}(t)}\]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case**:

If \((a_{20})^{(3)} = (a_{21})^{(3)}, \text{then} \ (\sigma_1)^{(3)} = (\sigma_2)^{(3)} \text{ and in this case} \ (v_1)^{(3)} = (\bar{v}_1)^{(3)} \text{ if in addition} \ \ (v_0)^{(3)} = (v_1)^{(3)} \text{ then} \ v^{(3)}(t) = (v_0)^{(3)} \text{ and as a consequence} \ G_{20}(t) = (v_0)^{(3)} G_{21}(t)\]

Analogously if \((b_{20})^{(3)} = (b_{21})^{(3)}, \text{then} \ (\tau_1)^{(3)} = (\tau_2)^{(3)} \text{ and then} \ (u_1)^{(3)} = (\bar{u}_1)^{(3)} \text{ if in addition} \ (u_0)^{(3)} = (u_1)^{(3)} \text{ then} \ T_{20}(t) = (u_0)^{(3)} T_{21}(t)\] This is an important consequence of the relation between \((v_1)^{(3)}\) and \((\bar{v}_1)^{(3)}\)

: From GLOBAL EQUATIONS we obtain

\[
\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - ((a_{24}^{(4)}) - (a_{25}^{(4)}) + (a_{24}^{(4)}(T_{25}, t))) - (a_{25}^{(4)}(T_{25}, t))v^{(4)} - (a_{25})^{(4)}v^{(4)}
\]

**Definition of** \( v^{(4)} \):

\[v^{(4)} = \frac{\sigma_{24}}{\sigma_{25}}\]

It follows

\[-((a_{25})^{(4)}(v^{(4)})^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}) \leq \frac{dv^{(4)}}{dt} \leq -((a_{25})^{(4)}(v^{(4)})^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)})\]

From which one obtains

**Definition of** \((\bar{v}_1)^{(4)}, (v_0)^{(4)}\):

\[d(C)^{(4)} = \frac{(v_0)^{(4)} - (v_0)^{(4)}}{(v_0)^{(4)} - (v_0)^{(4)}}\]

it follows \((v_0)^{(4)} \leq v^{(4)}(t) \leq (v_1)^{(4)}\)

In the same manner, we get
\[
\psi(t) \leq \frac{(v_1(t) + (C_1(t)v_2(t) e^{-[\alpha_{22}(t)(v_1(t)) - \alpha_{22}(t)v_2(t)]} - [\alpha_{22}(t)(v_1(t)) - \alpha_{22}(t)v_2(t)]})}{1 + (C_1(t)v_2(t) e^{-[\alpha_{22}(t)(v_1(t)) - \alpha_{22}(t)v_2(t)]}} \cdot \frac{(v_2(t) - (v_0(t))}{(v_0(t)) - (v_2(t))}
\]

From which we deduce \((v_0)(t) \leq \psi(t) \leq (\tilde{v}_1)(t)\)

(e) If \(0 < (v_1)(t) < (v_0)(t) = \frac{624}{633} < (\tilde{v}_1)(t)\) we find like in the previous case,

\[
(v_1)(t) \leq \frac{(v_1(t) + (C_1(t)v_2(t) e^{-[\alpha_{22}(t)(v_1(t)) - \alpha_{22}(t)v_2(t)]})}{1 + (C_1(t)v_2(t) e^{-[\alpha_{22}(t)(v_1(t)) - \alpha_{22}(t)v_2(t)]}} \leq \psi(t) \leq (\tilde{v}_1)(t)
\]

(f) If \(0 < (v_1)(t) \leq (\tilde{v}_1)(t) \leq (v_0)(t) = \frac{624}{633} ,\) we obtain

\[
(v_1)(t) \leq \frac{(v_1(t) + (C_1(t)v_2(t) e^{-[\alpha_{22}(t)(v_1(t)) - \alpha_{22}(t)v_2(t)]})}{1 + (C_1(t)v_2(t) e^{-[\alpha_{22}(t)(v_1(t)) - \alpha_{22}(t)v_2(t)]}} \leq (v_0)(t)
\]

And so with the notation of the first part of condition (c), we have

**Definition of** \(\psi(t)\) :.

\[
(m_2)(t) \leq \psi(t) \leq (m_1)(t), \quad \psi(t) = \frac{\|G_2(t)\|}{\|G_2(t)\|}
\]

In a completely analogous way, we obtain

**Definition of** \(u(t)\) :.

\[
(\mu_2)(t) \leq u(t) \leq (\mu_1)(t), \quad u(t) = \frac{\|T_2(t)\|}{\|T_2(t)\|}
\]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case** :

If \((a_{y_2}^{(y)})(t) = (a_{y_2}^{(y)})(t)\), then \((\sigma_1)(t) = (\sigma_2)(t)\) and in this case \((v_1)(t) = (\tilde{v}_1)(t)\) if in addition \((v_0)(t) = (v_1)(t)\) then \(\psi(t) = (v_0)(t)\) and as a consequence \(G_2(t) = (v_0)(t)G_2(t)\) this also defines \((v_0)(t)\) for the special case.

Analogously if \((b_{y_2}^{(y)})(t) = (b_{y_2}^{(y)})(t)\), then \((\tau_1)(t) = (\tau_2)(t)\) and then \((u_1)(t) = (u_2)(t)\) if in addition \((u_0)(t) = (u_1)(t)\) then \(T_2(t) = (u_0)(t)T_2(t)\) This is an important consequence of the relation between \((v_1)(t)\) and \((\tilde{v}_1)(t)\), and **definition of** \((u_0)(t)\).

From GLOBAL EQUATIONS we obtain

\[
\frac{dv(t)}{dt} = (a_{28}(t)) - ((a_{28}(t) - (a_{28}(t)) + (a_{28}(t))(T_2(t) - (a_{28}(t))(T_2(t))) - (a_{28}(t))(T_2(t)v(t) - (a_{28}(t))(v(t))
\]

**Definition of** \(v(t)\) :.

\[
v(t) = \frac{\|G_2(t)\|}{\|G_2(t)\|}
\]

It follows

\[
-((a_{28}(t))(v(t))^2 + (\sigma_2)(t)v(t) - (a_{28}(t))) \leq \frac{dv(t)}{dt} \leq -((a_{28}(t))(v(t))^2 + (\sigma_1)(t)v(t) - (a_{28}(t)))
\]
From which one obtains

**Definition of** \((\bar{V}_1)^{(s)}, (v_0)^{(s)} :\)

(g) For \(0 < (v_0)^{(s)} = \frac{g_{28}}{g_{29}} < (v_1)^{(s)} < (\bar{V}_1)^{(s)}\)

\[ v^{(s)}(t) \geq \frac{(v_1)^{(s)}+(C)^{(s)}(v_2)^{(s)})e^{-[(a_{20})^{(s)}((v_1)^{(s)}-(v_0)^{(s)})]}(1+e^{-[(a_{20})^{(s)}((v_1)^{(s)}-(v_0)^{(s)})]}t)}{s^{(C)^{(s)}}e^{-(a_{20})^{(s)}((v_1)^{(s)}-(v_0)^{(s)})t}}, \quad (C)^{(s)} = \frac{(v_1)^{(s)}-(v_0)^{(s)}}{(v_0)^{(s)}-(v_2)^{(s)}} \]

it follows \((v_0)^{(s)} \leq v^{(s)}(t) \leq (v_1)^{(s)} \)

In the same manner, we get

\[ v^{(s)}(t) \leq \frac{(v_3)^{(s)}+(C)^{(s)}(v_2)^{(s)})e^{-[(a_{20})^{(s)}((v_3)^{(s)}-(v_2)^{(s)})]}(1+e^{-[(a_{20})^{(s)}((v_3)^{(s)}-(v_2)^{(s)})]}t)}{s^{(C)^{(s)}}e^{-(a_{20})^{(s)}((v_3)^{(s)}-(v_2)^{(s)})t}}, \quad (C)^{(s)} = \frac{(v_3)^{(s)}-(v_0)^{(s)}}{(v_0)^{(s)}-(v_2)^{(s)}} \]

From which we deduce \((v_0)^{(s)} \leq v^{(s)}(t) \leq (v_3)^{(s)} \)

(h) If \(0 < (v_1)^{(s)} < (v_0)^{(s)} = \frac{g_{28}}{g_{29}} < (\bar{V}_1)^{(s)}\) we find like in the previous case,

\[ (v_1)^{(s)} \leq \frac{(v_1)^{(s)}+(C)^{(s)}(v_2)^{(s)})e^{-[(a_{20})^{(s)}((v_1)^{(s)}-(v_2)^{(s)})]}(1+e^{-[(a_{20})^{(s)}((v_1)^{(s)}-(v_2)^{(s)})]}t)}{s^{(C)^{(s)}}e^{-(a_{20})^{(s)}((v_1)^{(s)}-(v_2)^{(s)})t}} \leq v^{(s)}(t) \leq \frac{(\bar{V}_1)^{(s)}+(C)^{(s)}(v_2)^{(s)})e^{-[(a_{20})^{(s)}((\bar{V}_1)^{(s)}-(v_2)^{(s)})]}(1+e^{-[(a_{20})^{(s)}((\bar{V}_1)^{(s)}-(v_2)^{(s)})]}t)}{s^{(C)^{(s)}}e^{-(a_{20})^{(s)}((\bar{V}_1)^{(s)}-(v_2)^{(s)})t}} \leq (v_3)^{(s)} \]

(i) If \(0 < (v_1)^{(s)} \leq (v_0)^{(s)} = \frac{g_{28}}{g_{29}}\), we obtain

\[ (v_1)^{(s)} \leq v^{(s)}(t) \leq \frac{(v_1)^{(s)}+(C)^{(s)}(v_2)^{(s)})e^{-[(a_{20})^{(s)}((v_1)^{(s)}-(v_2)^{(s)})]}(1+e^{-[(a_{20})^{(s)}((v_1)^{(s)}-(v_2)^{(s)})]}t)}{s^{(C)^{(s)}}e^{-(a_{20})^{(s)}((v_1)^{(s)}-(v_2)^{(s)})t}} \leq (v_0)^{(s)} \]

And so with the notation of the first part of condition (c), we have

**Definition of** \(v^{(s)}(t) :\)

\[(m_2)^{(s)} \leq v^{(s)}(t) \leq (m_1)^{(s)}, \quad (m_2)^{(s)} = \frac{g_{28}}{g_{29}}, \quad v^{(s)}(t) = \frac{g_{28}(t)}{g_{29}(t)} \]

In a completely analogous way, we obtain

**Definition of** \(u^{(s)}(t) :\)

\[(\mu_2)^{(s)} \leq u^{(s)}(t) \leq (\mu_1)^{(s)}, \quad (\mu_2)^{(s)} = \frac{g_{28}}{g_{29}}, \quad u^{(s)}(t) = \frac{g_{28}(t)}{g_{29}(t)} \]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case :**

If \((a_{20}^{(s)}) = (a_{20}^{(s)})\), then \((\sigma_1)^{(s)} = (\sigma_2)^{(s)}\) and in this case \((v_1)^{(s)} = (\bar{V}_1)^{(s)}\) if in addition \((v_0)^{(s)} = (v_1)^{(s)}\) then \(v^{(s)}(t) = (v_0)^{(s)}\) and as a consequence \(G_{28}(t) = (v_0)^{(s)}G_{29}(t)\) this also defines \((v_0)^{(s)}\) for the special case.
Analogously if \((b_3^{(5)}) = (b_2^{(5)})\), then \((\tau_1^{(5)}) = (\tau_2^{(5)})\) and then

\((u_1^{(5)}) = (\hat{u}^{(5)})\) if in addition \((u_0^{(5)}) = (u_1^{(5)})\) then \(T_{28}(t) = (u_0^{(5)}T_{29}(t)\). This is an important consequence of the relation between \((v_1^{(5)})\) and \((\bar{v}_1^{(5)})\) and definition of \((u_0^{(5)})\).

We obtain

\[
\frac{dv^{(6)}}{dt} = (a_{32}^{(6)}) - ((a^{(6)}_{32}) - (a^{(6)}_{33}) + (a^{(6)}_{32})(T_{33}, t)) - (a^{(6)}_{33})(T_{33}, t)v^{(6)} - (a^{(6)}_{33})v^{(6)}
\]

**Definition of \(v^{(6)}\):**

\[
v^{(6)} = \frac{a_{32}^{(6)}}{a_{33}^{(6)}}
\]

It follows

\[
-(a^{(6)}_{33})v^{(6)}^2 + (\sigma_2^{(6)})v^{(6)} - (a^{(6)}_{32}) \leq \frac{dv^{(6)}}{dt} \leq -(a^{(6)}_{33})v^{(6)}^2 + (\sigma_1^{(6)})v^{(6)} - (a^{(6)}_{32})
\]

From which one obtains

**Definition of \((\bar{v}_1^{(6)}), (v_0^{(6)})\):**

(j) For \(0 < \frac{a^{(6)}_{12}}{a^{(6)}_{13}} < (v_1^{(6)}) < (\bar{v}_1^{(6)})\)

\[
v^{(6)}(t) \geq \frac{(v_1^{(6)}) + (\bar{v}_1^{(6)})(v_2^{(6)}) e^{-(a^{(6)}_{33})(v_1^{(6)} - v_0^{(6)})t}}{1 + (\bar{c}^{(6)}) e^{-(a^{(6)}_{33})(v_1^{(6)} - v_0^{(6)})t}}
\]

(k) If \(0 < (v_1^{(6)}) < (v_0^{(6)}) = \frac{a^{(6)}_{12}}{a^{(6)}_{13}} < (\bar{v}_1^{(6)})\) we find like in the previous case,

\[
(v_1^{(6)}) \leq \frac{(v_1^{(6)}) + (\bar{v}_1^{(6)})(v_2^{(6)}) e^{-(a^{(6)}_{33})(v_1^{(6)} - v_2^{(6)})t}}{1 + (\bar{c}^{(6)}) e^{-(a^{(6)}_{33})(v_1^{(6)} - v_2^{(6)})t}} \leq v^{(6)}(t) \leq \frac{\bar{v}_1^{(6)} + (\bar{c}^{(6)})(v_2^{(6)}) e^{-(a^{(6)}_{33})(\bar{v}_1^{(6)} - v_2^{(6)})t}}{1 + (\bar{c}^{(6)}) e^{-(a^{(6)}_{33})(\bar{v}_1^{(6)} - v_2^{(6)})t}}
\]

(l) If \(0 < (v_1^{(6)}) \leq (\bar{v}_1^{(6)}) \leq \frac{a^{(6)}_{12}}{a^{(6)}_{13}}\), we obtain

\[
(v_1^{(6)}) \leq v^{(6)}(t) \leq \frac{(v_1^{(6)}) + (c^{(6)})(v_2^{(6)}) e^{-(a^{(6)}_{33})(v_1^{(6)} - v_2^{(6)})t}}{1 + (c^{(6)}) e^{-(a^{(6)}_{33})(v_1^{(6)} - v_2^{(6)})t}} \leq (v_0^{(6)})
\]

And so with the notation of the first part of condition (c), we have

**Definition of \(v^{(6)}(t)\):**

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Theorem 3

We can prove the following consequence of the relation between \( \frac{G_{32}(t)}{G_{33}(t)} \).

Analogously if \( b''_{i1}^{(6)} = b''_{i1}^{(6)} \), then \( (\tau_1)^{(6)} = (\tau_2)^{(6)} \) and then \( (\bar{u}_1)^{(6)} = (\bar{u}_2)^{(6)} \) if in addition \( (v_0)^{(6)} = (v_0)^{(6)} \) and as a consequence \( G_{32}(t) = (v_0)^{(6)} G_{32}(t) \) this also defines \( (v_0)^{(6)} \) for the special case.

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case:**

If \( a''_{i2}^{(6)} = a''_{i2}^{(6)} \), then \( (\sigma_1)^{(6)} = (\sigma_2)^{(6)} \) and in this case \( (v_1)^{(6)} = (v_2)^{(6)} \) if in addition \( (v_0)^{(6)} = (v_0)^{(6)} \) then \( v^{(6)}(t) = (v_0)^{(6)} \) and as a consequence \( G_{32}(t) = (v_0)^{(6)} G_{33}(t) \) this also defines \( (v_0)^{(6)} \) for the special case.

Analogously if \( b''_{i2}^{(6)} = b''_{i2}^{(6)} \), then \( (\tau_1)^{(6)} = (\tau_2)^{(6)} \) and then \( (u_1)^{(6)} = (u_2)^{(6)} \) if in addition \( (u_0)^{(6)} = (u_0)^{(6)} \) then \( T_{32}(t) = (u_0)^{(6)} T_{33}(t) \). This is an important consequence of the relation between \( (v_1)^{(6)} \) and \( (v_2)^{(6)} \), and definition of \( (u_0)^{(6)} \).

We can prove the following

**Theorem 3:** If \( (a''_{i1}^{(1)}) \) and \( (b''_{i1}^{(1)}) \) are independent on \( t \), and the conditions

\[
(a_{13}^{(1)})(a_{14}^{(1)}) - (a_{13}^{(1)})(a_{14}^{(1)}) < 0 \\
(a_{13}^{(1)})(a_{14}^{(1)}) - (a_{13}^{(1)})(a_{14}^{(1)}) + (a_{13}^{(1)})(a_{14}^{(1)}) + (a_{13}^{(1)})(a_{14}^{(1)}) + (a_{13}^{(1)})(a_{14}^{(1)}) + (a_{13}^{(1)})(a_{14}^{(1)}) > 0 \\
(b_{13}^{(1)})(b_{14}^{(1)}) - (b_{13}^{(1)})(b_{14}^{(1)}) < 0 , \\
(b_{13}^{(1)})(b_{14}^{(1)}) - (b_{13}^{(1)})(b_{14}^{(1)}) - (b_{13}^{(1)})(b_{14}^{(1)}) - (b_{13}^{(1)})(b_{14}^{(1)}) - (b_{13}^{(1)})(b_{14}^{(1)}) - (b_{13}^{(1)})(b_{14}^{(1)}) < 0
\]

with \( (p_{13}^{(1)}), (r_{14}^{(1)}) \) as defined, then the system

If \( (a''_{i2}^{(2)}) \) and \( (b''_{i2}^{(2)}) \) are independent on \( t \), and the conditions

\[
(a_{16}^{(2)})(a_{17}^{(2)}) - (a_{16}^{(2)})(a_{17}^{(2)}) < 0 \\
(a_{16}^{(2)})(a_{17}^{(2)}) - (a_{16}^{(2)})(a_{17}^{(2)}) + (a_{16}^{(2)})(a_{17}^{(2)}) + (a_{16}^{(2)})(a_{17}^{(2)}) + (a_{16}^{(2)})(a_{17}^{(2)}) + (a_{16}^{(2)})(a_{17}^{(2)}) > 0 \\
(b_{16}^{(2)})(b_{17}^{(2)}) - (b_{16}^{(2)})(b_{17}^{(2)}) < 0 , \\
(b_{16}^{(2)})(b_{17}^{(2)}) - (b_{16}^{(2)})(b_{17}^{(2)}) - (b_{16}^{(2)})(b_{17}^{(2)}) - (b_{16}^{(2)})(b_{17}^{(2)}) - (b_{16}^{(2)})(b_{17}^{(2)}) - (b_{16}^{(2)})(b_{17}^{(2)}) < 0
\]

with \( (p_{16}^{(2)}), (r_{17}^{(2)}) \) as defined are satisfied, then the system

If \( (a''_{i3}^{(3)}) \) and \( (b''_{i3}^{(3)}) \) are independent on \( t \), and the conditions

\[
(a_{20}^{(3)})(a_{21}^{(3)}) - (a_{20}^{(3)})(a_{21}^{(3)}) < 0 \\
(a_{20}^{(3)})(a_{21}^{(3)}) - (a_{20}^{(3)})(a_{21}^{(3)}) + (a_{20}^{(3)})(a_{21}^{(3)}) + (a_{20}^{(3)})(a_{21}^{(3)}) + (a_{20}^{(3)})(a_{21}^{(3)}) + (a_{20}^{(3)})(a_{21}^{(3)}) > 0 \\
(b_{20}^{(3)})(b_{21}^{(3)}) - (b_{20}^{(3)})(b_{21}^{(3)}) > 0 , \\
(b_{20}^{(3)})(b_{21}^{(3)}) - (b_{20}^{(3)})(b_{21}^{(3)}) > 0 .
\]
If \((a'_i)^{(4)}\) and \((b''_i)^{(4)}\) are independent on \(t\), and the conditions

\[
(a'_i)^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0
\]
\[
(a'_i)^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{25})^{(4)}(p_{24})^{(4)} + (p_{25})^{(4)}(p_{25})^{(4)} > 0
\]
\[
(b'_i)^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} < 0 ,
\]
\[
(b'_i)^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b_{24})^{(4)}(r_{25})^{(4)} - (b_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0
\]

with \((p_{24})^{(4)}\), \((r_{25})^{(4)}\) as defined are satisfied, then the system

If \((a'_i)^{(5)}\) and \((b''_i)^{(5)}\) are independent on \(t\), and the conditions

\[
(a'_i)^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0
\]
\[
(a'_i)^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a_{29})^{(5)}(p_{28})^{(5)} + (p_{28})^{(5)}(p_{28})^{(5)} > 0
\]
\[
(b'_i)^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} < 0 ,
\]
\[
(b'_i)^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b_{28})^{(5)}(r_{29})^{(5)} - (b_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0
\]

with \((p_{28})^{(5)}\), \((r_{29})^{(5)}\) as defined are satisfied, then the system

If \((a'_i)^{(6)}\) and \((b''_i)^{(6)}\) are independent on \(t\), and the conditions

\[
(a'_i)^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0
\]
\[
(a'_i)^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a_{33})^{(6)}(p_{32})^{(6)} + (p_{32})^{(6)}(p_{32})^{(6)} > 0
\]
\[
(b'_i)^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} < 0 ,
\]
\[
(b'_i)^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b_{32})^{(6)}(r_{33})^{(6)} - (b_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0
\]

with \((p_{32})^{(6)}\), \((r_{33})^{(6)}\) as defined are satisfied, then the system

\[
(a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})]G_{13} = 0
\]
\[
(a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})]G_{14} = 0
\]
\[
(a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})]G_{15} = 0
\]
\[
(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)]T_{13} = 0
\]
\[
(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)]T_{14} = 0
\]
\[
(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)]T_{15} = 0
\]

has a unique positive solution, which is an equilibrium solution for the system

\[
(a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})]G_{16} = 0
\]
\[(a_{17})^{(2)}G_{16} - [(a_{17})^{(2)} + (a_{17}')^{(2)}(T_{17})]G_{17} = 0\]

\[(a_{18})^{(2)}G_{17} - [(a_{18})^{(2)} + (a_{18}')^{(2)}(T_{17})]G_{18} = 0\]

\[(b_{16})^{(2)}T_{17} - [(b_{16}')^{(2)} - (b_{16}')^{(2)}(G_{19})]T_{16} = 0\]

\[(b_{17})^{(2)}T_{16} - [(b_{17}')^{(2)} - (b_{17}')^{(2)}(G_{19})]T_{17} = 0\]

\[(b_{18})^{(2)}T_{17} - [(b_{18}')^{(2)} - (b_{18}')^{(2)}(G_{19})]T_{18} = 0\]

has a unique positive solution, which is an equilibrium solution for the system

\[(a_{20})^{(3)}G_{21} - [(a_{20})^{(3)} + (a_{20}')^{(3)}(T_{21})]G_{20} = 0\]

\[(a_{21})^{(3)}G_{20} - [(a_{21})^{(3)} + (a_{21}')^{(3)}(T_{21})]G_{21} = 0\]

\[(a_{22})^{(3)}G_{21} - [(a_{22})^{(3)} + (a_{22}')^{(3)}(T_{21})]G_{22} = 0\]

\[(b_{20})^{(3)}T_{21} - [(b_{20}')^{(3)} - (b_{20}')^{(3)}(G_{23})]T_{20} = 0\]

\[(b_{21})^{(3)}T_{20} - [(b_{21}')^{(3)} - (b_{21}')^{(3)}(G_{23})]T_{21} = 0\]

\[(b_{22})^{(3)}T_{21} - [(b_{22}')^{(3)} - (b_{22}')^{(3)}(G_{23})]T_{22} = 0\]

has a unique positive solution, which is an equilibrium solution

\[(a_{24})^{(4)}G_{25} - [(a_{24})^{(4)} + (a_{24}')^{(4)}(T_{25})]G_{24} = 0\]

\[(a_{25})^{(4)}G_{24} - [(a_{25})^{(4)} + (a_{25}')^{(4)}(T_{25})]G_{25} = 0\]

\[(a_{26})^{(4)}G_{25} - [(a_{26})^{(4)} + (a_{26}')^{(4)}(T_{25})]G_{26} = 0\]

\[(b_{24})^{(4)}T_{25} - [(b_{24}')^{(4)} - (b_{24}')^{(4)}(G_{27})]T_{24} = 0\]

\[(b_{25})^{(4)}T_{24} - [(b_{25}')^{(4)} - (b_{25}')^{(4)}(G_{27})]T_{25} = 0\]

\[(b_{26})^{(4)}T_{25} - [(b_{26}')^{(4)} - (b_{26}')^{(4)}(G_{27})]T_{26} = 0\]

has a unique positive solution, which is an equilibrium solution for the system

\[(a_{28})^{(5)}G_{29} - [(a_{28})^{(5)} + (a_{28}')^{(5)}(T_{29})]G_{28} = 0\]

\[(a_{29})^{(5)}G_{28} - [(a_{29})^{(5)} + (a_{29}')^{(5)}(T_{29})]G_{29} = 0\]

\[(a_{30})^{(5)}G_{29} - [(a_{30})^{(5)} + (a_{30}')^{(5)}(T_{29})]G_{30} = 0\]

\[(b_{28})^{(5)}T_{29} - [(b_{28}')^{(5)} - (b_{28}')^{(5)}(G_{31})]T_{28} = 0\]

\[(b_{29})^{(5)}T_{28} - [(b_{29}')^{(5)} - (b_{29}')^{(5)}(G_{31})]T_{29} = 0\]
has a unique positive solution, which is an equilibrium solution for the system

\[(a_{32})^{(6)}G_{33} - [(a'_{32})^{(6)}(a''_{32})^{(6)}(T_{33})]G_{32} = 0\]

\[(a_{33})^{(6)}G_{32} - [(a'_{33})^{(6)}(a''_{33})^{(6)}(T_{33})]G_{33} = 0\]

\[(a_{34})^{(6)}G_{33} - [(a'_{34})^{(6)}(a''_{34})^{(6)}(T_{33})]G_{34} = 0\]

\[(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0\]

\[(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0\]

\[(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0\]

has a unique positive solution, which is an equilibrium solution for the system

\[(a_{36})^{(7)}G_{37} - [(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})]G_{36} = 0\]

\[(a_{37})^{(7)}G_{36} - [(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})]G_{37} = 0\]

\[(a_{38})^{(7)}G_{37} - [(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})]G_{38} = 0\]

\[(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0\]

\[(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})]T_{37} = 0\]

\[(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})]T_{38} = 0\]

has a unique positive solution, which is an equilibrium solution for the system (79 to 36)

(a) Indeed the first two equations have a nontrivial solution \(G_{36}, G_{37}\) if
Definition and uniqueness of $T_{37}^*$:

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i(T_{37}))$ being increasing, it follows that there exists a unique $T_{37}^*$ for which $f(T_{37}^*) = 0$. With this value, we obtain from the three first equations

\[ G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[a_{36}]^{(2)}+\theta(a_{36})^{(7)}]} , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[a_{38}]^{(2)}+\theta(a_{38})^{(7)}]} \]

By the same argument, the equations 92, 93 admit solutions $G_{36}, G_{37}$ if

\[ \varphi(G_{39}) = (b'_{36})^{(7)}(b_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - \]

\[ \left[(b_{36})^{(7)}(b_{37})^{(7)}(G_{39}) + (b_{37})^{(7)}(b_{38})^{(7)}(G_{39})\right] + (b_{38})^{(7)}(G_{39})^{(7)}(G_{39}) = 0 \]

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^*$ such that $\varphi(G^*) = 0$

Finally we obtain the unique solution of 89 to 97

\[ G_{37}^* \text{ given by } \varphi((G_{39})^*) = 0 \] \[ T_{37}^* \text{ given by } f(T_{37}^*) = 0 \] and

\[ G_{36}^* = \frac{(a_{36})^{(7)}G_{37}^*}{[a_{36}]^{(2)}+\theta(a_{36})^{(7)}]} , \quad G_{38}^* = \frac{(a_{38})^{(7)}G_{37}^*}{[a_{38}]^{(2)}+\theta(a_{38})^{(7)}]} \]

\[ T_{36}^* = \frac{(b_{36})^{(7)}T_{37}^*}{[b_{36}]^{(2)}-\theta(b_{36})^{(7)}]} , \quad T_{38}^* = \frac{(b_{38})^{(7)}T_{37}^*}{[b_{38}]^{(2)}-\theta(b_{38})^{(7)}]} \]

Definition and uniqueness of $T_{21}^*$:

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i(T_{21}))$ being increasing, it follows
that there exists a unique \( T^*_{21} \) for which \( f(T^*_{21}) = 0 \). With this value, we obtain from the three first equations

\[
G_{20} = \frac{(a_{20})^{(3)}a_{21}}{[(a_{20})^{(3)}+a_{20}^{(3)}(T_{21}^*)]^3}, \quad G_{22} = \frac{(a_{22})^{(3)}g_{22}}{[(a_{22})^{(3)}+a_{22}^{(3)}(T_{21}^*)]^3}
\]

**Definition and uniqueness of \( T^*_{25} \):**

After hypothesis \( f(0) < 0, f(\infty) > 0 \) and the functions \((a_i''')^{(4)}(T_{25})\) being increasing, it follows that there exists a unique \( T^*_{25} \) for which \( f(T^*_{25}) = 0 \). With this value, we obtain from the three first equations

\[
G_{24} = \frac{(a_{24})^{(4)}a_{25}}{[(a_{24})^{(4)}+a_{24}^{(4)}(T_{25}^*)]^4}, \quad G_{26} = \frac{(a_{26})^{(4)}g_{26}}{[(a_{26})^{(4)}+a_{26}^{(4)}(T_{25}^*)]^4}
\]

**Definition and uniqueness of \( T^*_{29} \):**

After hypothesis \( f(0) < 0, f(\infty) > 0 \) and the functions \((a_i''')^{(5)}(T_{29})\) being increasing, it follows that there exists a unique \( T^*_{29} \) for which \( f(T^*_{29}) = 0 \). With this value, we obtain from the three first equations

\[
G_{28} = \frac{(a_{28})^{(5)}a_{29}}{[(a_{28})^{(5)}+a_{28}^{(5)}(T_{29}^*)]^5}, \quad G_{30} = \frac{(a_{30})^{(5)}g_{30}}{[(a_{30})^{(5)}+a_{30}^{(5)}(T_{29}^*)]^5}
\]

**Definition and uniqueness of \( T^*_{33} \):**

After hypothesis \( f(0) < 0, f(\infty) > 0 \) and the functions \((a_i''')^{(6)}(T_{33})\) being increasing, it follows that there exists a unique \( T^*_{33} \) for which \( f(T^*_{33}) = 0 \). With this value, we obtain from the three first equations

\[
G_{32} = \frac{(a_{32})^{(6)}a_{33}}{[(a_{32})^{(6)}+a_{32}^{(6)}(T_{33}^*)]^6}, \quad G_{34} = \frac{(a_{34})^{(6)}g_{34}}{[(a_{34})^{(6)}+a_{34}^{(6)}(T_{33}^*)]^6}
\]

(g) By the same argument, the equations 92, 93 admit solutions \( G_{13}, G_{14} \) if

\[
q(G) = (b_{13}''(1))^{(1)}(b_{14}^{(1)})^{(1)} - (b_{13}^{(1)}(b_{14}^{(1)})^{(1)} -
\left[ (b_{13}^{(1)})^{(1)}(b_{14}^{(1)})^{(1)}(G) + (b_{14}^{(1)}(b_{13}^{(1)}(G) + (b_{13}^{(1)}(G)(b_{14}^{(1)}(G) = 0
\]

Where in \( G(G_{13}, G_{14}, G_{15}) \), \( G_{13}, G_{15} \) must be replaced by their values from 96. It is easy to see that \( q \) is a decreasing function in \( G_{14} \) taking into account the hypothesis \( q(0) > 0, q(\infty) < 0 \) it follows that there exists a unique \( G^*_{14} \) such that \( q(G^*_{14}) = 0 \)

(h) By the same argument, the equations 92, 93 admit solutions \( G_{16}, G_{17} \) if

\[
q(G_{19}) = (b_{16}^{(2)}(2)(b_{17}^{(2)})^{(2)} - (b_{16}^{(2)}(2)(b_{17}^{(2)})^{(2)} -
\left[ (b_{16}^{(2)}(2)(b_{17}^{(2)})^{(2)}(G_{19}) + (b_{17}^{(2)}(2)(b_{16}^{(2)})(G_{19}) + (b_{16}^{(2)}(2)(G_{19})(b_{17}^{(2)}(2)(G_{19}) = 0
\]

Where in \( G(G_{13}, G_{14}, G_{15}, G_{16}, G_{18}) \) must be replaced by their values from 96. It is easy to see that \( q \) is a decreasing function in \( G_{17} \) taking into account the hypothesis \( q(0) > 0, q(\infty) < 0 \) it follows that there exists a unique \( G^*_{14} \) such that \( q(G^*_{19}) = 0 \)

(i) By the same argument, the concatenated equations admit solutions \( G_{20}, G_{21} \) if
\[
\varphi(G_{23}) = (b_{20})^{(3)}(b_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - \\
[(b_{20})^{(3)}(b_{21})^{(3)}(G_{23}) + (b_{21})^{(3)}(b_{20})^{(3)}(G_{23})] + (b_{20})^{(3)}(G_{23}) = 0
\]

Where in \(G_{23}(G_{20}, G_{21}, G_{22})\), \(G_{20}, G_{22}\) must be replaced by their values from 96. It is easy to see that \(\varphi\) is a decreasing function in \(G_{21}\) taking into account the hypothesis \(\varphi(0) > 0, \varphi(\infty) < 0\) it follows that there exists a unique \(G_{21}\) such that \(\varphi((G_{23})') = 0\)

(j) By the same argument, the equations of modules admit solutions \(G_{24}, G_{25}\) if

\[
\varphi(G_{27}) = (b_{24})^{(4)}(b_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - \\
[(b_{24})^{(4)}(b_{25})^{(4)}(G_{27}) + (b_{25})^{(4)}(b_{24})^{(4)}(G_{27})] + (b_{24})^{(4)}(G_{27}) = 0
\]

Where in \((G_{27})(G_{20}, G_{21}, G_{26})\), \(G_{24}, G_{26}\) must be replaced by their values from 96. It is easy to see that \(\varphi\) is a decreasing function in \(G_{25}\) taking into account the hypothesis \(\varphi(0) > 0, \varphi(\infty) < 0\) it follows that there exists a unique \(G_{25}\) such that \(\varphi((G_{27})') = 0\)

(k) By the same argument, the equations (modules) admit solutions \(G_{30}, G_{29}\) if

\[
\varphi(G_{31}) = (b_{29})^{(5)}(b_{29})^{(5)} - (b_{29})^{(5)}(b_{29})^{(5)} - \\
[(b_{29})^{(5)}(b_{29})^{(5)}(G_{31}) + (b_{29})^{(5)}(b_{29})^{(5)}(G_{31})] + (b_{29})^{(5)}(G_{31}) = 0
\]

Where in \((G_{31})(G_{20}, G_{29}, G_{30})\), \(G_{29}, G_{30}\) must be replaced by their values from 96. It is easy to see that \(\varphi\) is a decreasing function in \(G_{29}\) taking into account the hypothesis \(\varphi(0) > 0, \varphi(\infty) < 0\) it follows that there exists a unique \(G_{29}\) such that \(\varphi((G_{31})') = 0\)

(l) By the same argument, the equations (modules) admit solutions \(G_{32}, G_{33}\) if

\[
\varphi(G_{33}) = (b_{32})^{(6)}(b_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - \\
[(b_{32})^{(6)}(b_{33})^{(6)}(G_{33}) + (b_{32})^{(6)}(b_{33})^{(6)}(G_{33})] + (b_{32})^{(6)}(G_{33}) = 0
\]

Where in \((G_{33})(G_{20}, G_{29}, G_{30})\), \(G_{32}, G_{30}\) must be replaced by their values It is easy to see that \(\varphi\) is a decreasing function in \(G_{33}\) taking into account the hypothesis \(\varphi(0) > 0, \varphi(\infty) < 0\) it follows that there exists a unique \(G_{33}\) such that \(\varphi(G^*) = 0\)

Finally we obtain the unique solution of 89 to 94

\[
G_{14} \text{ given by } \varphi(G^*) = 0, T^*_i \text{ given by } f(T^*_i) = 0 \text{ and }
\]

\[
G_{13} = \frac{(a_{13})^{(1)}g_{14}}{(a_{13})^{(1)} + (a_{13})^{(1)}(T^*_i)}, G_{15} = \frac{(a_{15})^{(1)}g_{14}}{(a_{15})^{(1)} + (a_{15})^{(1)}(T^*_i)}
\]

\[
T^*_j = \frac{(b_{20})^{(3)}(G^*)}{(b_{20})^{(3)} - (b_{20})^{(3)}(G^*)}, T^*_k = \frac{(b_{32})^{(6)}(G^*)}{(b_{32})^{(6)} - (b_{32})^{(6)}(G^*)}
\]

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution
G_{17} \text{ given by } \varphi(G_{17}) = 0, \quad T_{17} \text{ given by } f(T_{17}) = 0 \text{ and}

G_{16} = \frac{(a_{16})G_{17}}{[a_{16}]^2 + (a_{16})^2(T_{17})}, \quad \tilde{G}_{18} = \frac{(a_{18})G_{17}}{[a_{18}]^2 + (a_{18})^2(T_{17})}

T_{16} = \frac{(b_{16})T_{17}}{[b_{16}]^2 - (b_{16})^2(G_{17})}, \quad T_{19} = \frac{(b_{19})T_{17}}{[b_{19}]^2 - (b_{19})^2(G_{17})}

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{21} \text{ given by } \varphi(G_{21}) = 0, \quad T_{21} \text{ given by } f(T_{21}) = 0 \text{ and}

G_{20} = \frac{(a_{20})G_{21}}{[a_{20}]^2 + (a_{20})^2(T_{21})}, \quad \tilde{G}_{22} = \frac{(a_{22})G_{21}}{[a_{22}]^2 + (a_{22})^2(T_{21})}

T_{20} = \frac{(b_{20})T_{21}}{[b_{20}]^2 - (b_{20})^2(G_{21})}, \quad T_{22} = \frac{(b_{22})T_{21}}{[b_{22}]^2 - (b_{22})^2(G_{21})}

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{25} \text{ given by } \varphi(G_{25}) = 0, \quad T_{25} \text{ given by } f(T_{25}) = 0 \text{ and}

G_{24} = \frac{(a_{24})G_{25}}{[a_{24}]^2 + (a_{24})^2(T_{25})}, \quad \tilde{G}_{26} = \frac{(a_{26})G_{25}}{[a_{26}]^2 + (a_{26})^2(T_{25})}

T_{24} = \frac{(b_{24})T_{25}}{[b_{24}]^2 - (b_{24})^2(G_{25})}, \quad T_{26} = \frac{(b_{26})T_{25}}{[b_{26}]^2 - (b_{26})^2(G_{25})}

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{29} \text{ given by } \varphi(G_{29}) = 0, \quad T_{29} \text{ given by } f(T_{29}) = 0 \text{ and}

G_{28} = \frac{(a_{28})G_{29}}{[a_{28}]^2 + (a_{28})^2(T_{29})}, \quad \tilde{G}_{30} = \frac{(a_{30})G_{29}}{[a_{30}]^2 + (a_{30})^2(T_{29})}

T_{28} = \frac{(b_{28})T_{29}}{[b_{28}]^2 - (b_{28})^2(G_{29})}, \quad T_{30} = \frac{(b_{30})T_{29}}{[b_{30}]^2 - (b_{30})^2(G_{29})}

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{33} \text{ given by } \varphi(G_{33}) = 0, \quad T_{33} \text{ given by } f(T_{33}) = 0 \text{ and}

G_{32} = \frac{(a_{32})G_{33}}{[a_{32}]^2 + (a_{32})^2(T_{33})}, \quad \tilde{G}_{34} = \frac{(a_{34})G_{33}}{[a_{34}]^2 + (a_{34})^2(T_{33})}

T_{32} = \frac{(b_{32})T_{33}}{[b_{32}]^2 - (b_{32})^2(G_{33})}, \quad T_{34} = \frac{(b_{34})T_{33}}{[b_{34}]^2 - (b_{34})^2(G_{33})}

Obviously, these values represent an equilibrium solution
ASYMPTOTIC STABILITY ANALYSIS

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions \((a''_i(t))\) and \((b''_i(t))\) belong to \(C^1(\mathbb{R}+)\) then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** \(G_i, T_i \):-

\[ G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i \]

\[
\frac{\partial(a''_i(t))^{(1)}}{\partial r_{i4}}(T_i^*) = (q_{14})^{(1)} \quad \frac{\partial(b''_i(t))^{(1)}}{\partial G_j} (G^*) = s_{ij}
\]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
\frac{dG_i}{dt} = -((a''_i(t))^{(1)} + (p_{12})^{(1)}) G_{13} + (a_{13})^{(1)} G_{14} - (q_{13})^{(1)} G_{13}^* T_{14}
\]

\[
\frac{dG_i}{dt} = -((a''_1(t))^{(1)} + (p_{14})^{(1)}) G_{15} + (a_{14})^{(1)} G_{13} + (q_{14})^{(1)} G_{14}^* T_{14}
\]

\[
\frac{dG_i}{dt} = -((a''_i(t))^{(1)} + (p_{15})^{(1)}) G_{16} + (a_{15})^{(1)} G_{14} + (q_{15})^{(1)} G_{15}^* T_{14}
\]

\[
\frac{dT_i}{dt} = -((b''_i(t))^{(1)} - (r_{13})^{(1)}) T_{13} + (b_{13})^{(1)} T_{14} + \sum_{j=13}^{15} (s_{13}(j)) T_{14}^* G_j
\]

\[
\frac{dT_i}{dt} = -((b''_1(t))^{(1)} - (r_{14})^{(1)}) T_{14} + (b_{14})^{(1)} T_{13} + \sum_{j=13}^{15} (s_{14}(j)) T_{14}^* G_j
\]

\[
\frac{dT_i}{dt} = -((b''_i(t))^{(1)} - (r_{15})^{(1)}) T_{15} + (b_{15})^{(1)} T_{14} + \sum_{j=13}^{15} (s_{15}(j)) T_{14}^* G_j
\]

If the conditions of the previous theorem are satisfied and if the functions \((a''_i(t))^{(2)}\) and \((b''_i(t))^{(2)}\) belong to \(C^2(\mathbb{R}+)\) then the above equilibrium point is asymptotically stable.

Denote

**Definition of** \(G_i, T_i \):-

\[ G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i \]

\[
\frac{\partial(a''_l(t))^{(2)}}{\partial T_{17}} (T_i^*) = (q_{17})^{(2)} \quad \frac{\partial(b''_l(t))^{(2)}}{\partial G_j} (G_{19})^* = s_{ij}
\]

Taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
\frac{dG_i}{dt} = -((a''_i(t))^{(2)} + (p_{16})^{(2)}) G_{16} + (a_{16})^{(2)} G_{17} - (q_{16})^{(2)} G_{16}^* T_{17}
\]

\[
\frac{dG_i}{dt} = -((a''_i(t))^{(2)} + (p_{17})^{(2)}) G_{17} + (a_{17})^{(2)} G_{16} + (q_{17})^{(2)} G_{17}^* T_{17}
\]

\[
\frac{dG_i}{dt} = -((a''_i(t))^{(2)} + (p_{18})^{(2)}) G_{18} + (a_{18})^{(2)} G_{17} - (q_{18})^{(2)} G_{18}^* T_{17}
\]

\[
\frac{dT_i}{dt} = -((b''_i(t))^{(2)} - (r_{16})^{(2)}) T_{16} + (b_{16})^{(2)} T_{17} + \sum_{j=16}^{18} (s_{16}(j)) T_{17}^* G_j
\]

\[
\frac{dT_i}{dt} = -((b''_i(t))^{(2)} - (r_{17})^{(2)}) T_{17} + (b_{17})^{(2)} T_{16} + \sum_{j=16}^{18} (s_{17}(j)) T_{17}^* G_j
\]

\[
\frac{dT_i}{dt} = -((b''_i(t))^{(2)} - (r_{18})^{(2)}) T_{18} + (b_{18})^{(2)} T_{17} + \sum_{j=16}^{18} (s_{18}(j)) T_{17}^* G_j
\]
\[
\frac{dT_{18}}{dt} = -\left((b_{18}^{(2)}) - (r_{18})^{(2)}\right)T_{18} + (b_{18})^{(2)}T_{17} + \sum_{i=18}^{18} (s_{18}(j_i)T_{18}G_{ij})
\]

If the conditions of the previous theorem are satisfied and if the functions \((a_i^{(3)})\) and \((b_i^{(3)})\) Belong to \(C(\mathbb{R}_+)^{3}\) then the above equilibrium point is asymptotically stabl

Denote

**Definition of** \(G_i, T_i :\)

\[
G_i = G_i^* + G_i \quad , \quad T_i = T_i^* + T_i
\]

\[
\frac{\partial(a_i^{(3)})}{\partial T_{21}} (T_{21}^*) = (q_{21})^*, \quad \frac{\partial(b_i^{(3)})}{\partial G_j} (G_{22}^*) = s_{ij}
\]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
\frac{dG_{20}}{dt} = -\left((a_{20}^{(3)} + (p_{20})^{(3)})G_{20} + (a_{20})^{(3)}G_{21} - (q_{20})^{(3)}G_{20}^* T_{21}\right)
\]

\[
\frac{dG_{21}}{dt} = -\left((a_{21}^{(3)} + (p_{21})^{(3)})G_{21} + (a_{21})^{(3)}G_{20} - (q_{21})^{(3)}G_{21}^* T_{21}\right)
\]

\[
\frac{dG_{22}}{dt} = -\left((a_{22}^{(3)} + (p_{22})^{(3)})G_{22} + (a_{22})^{(3)}G_{21} - (q_{22})^{(3)}G_{22}^* T_{21}\right)
\]

\[
\frac{dT_{20}}{dt} = -\left((b_{20}^{(3)} - (r_{20})^{(3)})T_{20} + (b_{20})^{(3)}T_{21} + \sum_{j=20}^{22} (s_{20}(j)T_{20}G_{ij})\right)
\]

\[
\frac{dT_{21}}{dt} = -\left((b_{21}^{(3)} - (r_{21})^{(3)})T_{21} + (b_{21})^{(3)}T_{20} + \sum_{j=20}^{22} (s_{21}(j)T_{21}G_{ij})\right)
\]

\[
\frac{dT_{22}}{dt} = -\left((b_{22}^{(3)} - (r_{22})^{(3)})T_{22} + (b_{22})^{(3)}T_{21} + \sum_{j=20}^{22} (s_{22}(j)T_{22}G_{ij})\right)
\]

If the conditions of the previous theorem are satisfied and if the functions \((a_i^{(4)})\) and \((b_i^{(4)})\) Belong to \(C(\mathbb{R}_+)^{4}\) then the above equilibrium point is asymptotically stabl

Denote

**Definition of** \(G_i, T_i :\)

\[
G_i = G_i^* + G_i \quad , \quad T_i = T_i^* + T_i
\]

\[
\frac{\partial(a_i^{(4)})}{\partial T_{25}} (T_{25}^*) = (q_{25})^*, \quad \frac{\partial(b_i^{(4)})}{\partial G_j} (G_{25}^*) = s_{ij}
\]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
\frac{dG_{24}}{dt} = -\left((a_{24}^{(4)} + (p_{24})^{(4)})G_{24} + (a_{24})^{(4)}G_{25} - (q_{24})^{(4)}G_{24}^* T_{25}\right)
\]

\[
\frac{dG_{25}}{dt} = -\left((a_{25}^{(4)} + (p_{25})^{(4)})G_{25} + (a_{25})^{(4)}G_{24} - (q_{25})^{(4)}G_{25}^* T_{25}\right)
\]

\[
\frac{dG_{26}}{dt} = -\left((a_{26}^{(4)} + (p_{26})^{(4)})G_{26} + (a_{26})^{(4)}G_{25} - (q_{26})^{(4)}G_{26}^* T_{25}\right)
\]
If the conditions of the previous theorem are satisfied and if the functions \((a_i^{(5)})\) and \((b_i^{(5)})\)
Belong to \(C^{(5)}(\mathbb{R}_+}\) then the above equilibrium point is asymptotically stable

Denote

**Definition of \(G_i, T_i\):**

\[ G_i = G_i^* + G_i \quad , T_i = T_i^* + T_i \]

\[ \frac{\partial (a_i^{(5)})}{\partial T_2} (T_2) = (q_29)^{(5)} \quad , \quad \frac{\partial (b_i^{(5)})}{\partial T_2} (T_2) = s_{ij} \]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[ \frac{dG_{28}}{dt} = - ((a_{28}^{(5)}) + (p_{28}^{(5)})G_{28} + (a_{28}^{(5)})G_{29} - (q_{28}^{(5)})G_{28}^* T_{29} \]

\[ \frac{dG_{29}}{dt} = - ((a_{29}^{(5)}) + (p_{29}^{(5)})G_{29} + (a_{29}^{(5)})G_{28} - (q_{29}^{(5)})G_{29}^* T_{29} \]

\[ \frac{dG_{30}}{dt} = - ((a_{30}^{(5)}) + (p_{30}^{(5)})G_{30} + (a_{30}^{(5)})G_{29} - (q_{30}^{(5)})G_{30}^* T_{29} \]

\[ \frac{dG_{32}}{dt} = - ((a_{32}^{(5)}) + (p_{32}^{(5)})G_{32} + (a_{32}^{(5)})G_{33} - (q_{32}^{(5)})G_{32}^* T_{33} \]

If the conditions of the previous theorem are satisfied and if the functions \((a_i^{(6)})\) and \((b_i^{(6)})\)
Belong to \(C^{(6)}(\mathbb{R}_+)\) then the above equilibrium point is asymptotically stable

Denote

**Definition of \(G_i, T_i\):**

\[ G_i = G_i^* + G_i \quad , T_i = T_i^* + T_i \]

\[ \frac{\partial (a_i^{(6)})}{\partial T_3} (T_3) = (q_{33})^{(6)} \quad , \quad \frac{\partial (b_i^{(6)})}{\partial T_3} (T_3) = s_{ij} \]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[ \frac{dG_{32}}{dt} = - ((a_{32}^{(6)}) + (p_{32}^{(6)})G_{32} + (a_{32}^{(6)})G_{33} - (q_{32}^{(6)})G_{32}^* T_{33} \]
\[ \frac{dG_{33}}{dt} = -((a'_{33})^{(6)} + (p_{33})^{(6)})G_{33} + (a_{33})^{(6)}G_{32} - (q_{33})^{(6)}G_{33}^{*}T_{33} \]

\[ \frac{dG_{34}}{dt} = -((a'_{34})^{(6)} + (p_{34})^{(6)})G_{34} + (a_{34})^{(6)}G_{33} - (q_{34})^{(6)}G_{34}^{*}T_{33} \]

\[ \frac{dT_{32}}{dt} = -((b'_{32})^{(6)} - (r_{32})^{(6)})T_{32} + (b_{32})^{(6)}T_{33} + \sum_{j=32}^{34} s_{32}(j)T_{32}^{*}G_{j} \]

\[ \frac{dT_{33}}{dt} = -((b'_{33})^{(6)} - (r_{33})^{(6)})T_{33} + (b_{33})^{(6)}T_{33} + \sum_{j=32}^{34} s_{33}(j)T_{33}^{*}G_{j} \]

\[ \frac{dT_{34}}{dt} = -((b'_{34})^{(6)} - (r_{34})^{(6)})T_{34} + (b_{34})^{(6)}T_{33} + \sum_{j=32}^{34} s_{34}(j)T_{34}^{*}G_{j} \]

Obviously, these values represent an equilibrium solution of 79, 20, 36, 22, 23.

If the conditions of the previous theorem are satisfied and if the functions \((a''_{i})^{(7)}\) and \((b''_{i})^{(7)}\) Belong to \(C^{(7)}(\mathbb{R}_{+})\), then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** \(G_i, T_i \):-

\[ G_i = G'_i + G_i \quad , T_i = T'_i + T_i \]

\[ \frac{\partial (a''_{ij})^{(7)}}{\partial T_{37}} (T_{37}^{*}) = (q_{37})^{(7)} , \frac{\partial (b''_{ij})^{(7)}}{\partial G_j} (G_{39})^{*} = s_{ij} \]

Then taking into account equations(SOLUTIONAL) and neglecting the terms of power 2, we obtain

\[ \frac{dG_{36}}{dt} = -((a'_{36})^{(7)} + (p_{36})^{(7)})G_{36} + (a_{36})^{(7)}G_{37} - (q_{36})^{(7)}G_{36}^{*}T_{37} \]

\[ \frac{dG_{37}}{dt} = -((a'_{37})^{(7)} + (p_{37})^{(7)})G_{37} + (a_{37})^{(7)}G_{36} - (q_{37})^{(7)}G_{37}^{*}T_{37} \]
2. The characteristic equation of this system is

\[
\begin{align*}
\frac{dG_38}{dt} &= - \left( (a'_{38})^{(7)} + (p_{38})^{(7)} \right) G_{38} + (a_{38})^{(7)} \mathcal{E}_{37} - (q_{38})^{(7)} \mathcal{T}_{37} \\
\frac{dT_{36}}{dt} &= - \left( (b'_{36})^{(7)} - (r_{36})^{(7)} \right) T_{36} + (b_{36})^{(7)} \mathcal{T}_{37} + \sum_{j=36}^{38} (s_{36}(j) \mathcal{T}_{36}) \\
\frac{dT_{37}}{dt} &= - \left( (b'_{37})^{(7)} - (r_{37})^{(7)} \right) T_{37} + (b_{37})^{(7)} \mathcal{T}_{36} + \sum_{j=36}^{38} (s_{37}(j) \mathcal{T}_{36}) \\
\frac{dT_{38}}{dt} &= - \left( (b'_{38})^{(7)} - (r_{38})^{(7)} \right) T_{38} + (b_{38})^{(7)} \mathcal{T}_{37} + \sum_{j=36}^{38} (s_{38}(j) \mathcal{T}_{36})
\end{align*}
\]

2. The characteristic equation of this system is

\[
\begin{align*}
\left( (\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)} \right) \left( (\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{15})^{(1)} \right) \\
\left[ \left( (\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)} \right) (q_{14})^{(1)} G_{14} + (a_{14})^{(1)} (q_{13})^{(1)} G_{13} \right] \\
\left( (\lambda)^{(1)} + (b'_{14})^{(1)} - (r_{14})^{(1)} \right) s_{(14),(13)} T_{14} + (b_{14})^{(1)} s_{(13),(13)} T_{13} \\
+ \left( (\lambda)^{(1)} + (a_{14})^{(1)} + (p_{14})^{(1)} \right) (q_{14})^{(1)} G_{14} + (a_{13})^{(1)} (q_{14})^{(1)} G_{14} \\
\left( (\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)} \right) s_{(14),(13)} T_{14} + (b_{14})^{(1)} s_{(13),(13)} T_{13} \\
\left( (\lambda)^{(1)} \right)^2 + \left( (a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)} \right) (\lambda)^{(1)} \\
\left( (\lambda)^{(1)} \right)^2 + \left( (b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)} \right) (\lambda)^{(1)} \\
+ \left( (\lambda)^{(1)} \right)^2 + \left( (a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)} \right) \left( (\lambda)^{(1)} \right)^2 G_{15} \\
+ \left( (\lambda)^{(1)} + (a_{13})^{(1)} + (p_{13})^{(1)} \right) \left( (a_{13})^{(1)} (q_{14})^{(1)} G_{14} + (a_{14})^{(1)} (a_{15})^{(1)} (q_{13})^{(1)} G_{13} \right) \\
\left( (\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)} \right) s_{(14),(15)} T_{14} + (b_{14})^{(1)} s_{(13),(15)} T_{13} \right] = 0 \\
+ \left( (\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)} \right) \left( (\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)} \right)
\end{align*}
\]
\[
\begin{align*}
\left[ \left( \lambda \right)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)} \right] & \left( q_{17} \right)^{(2)} G_{17} + (a_{17})^{(2)} (q_{16})^{(2)} G_{16} \\
+ \left( \left( \lambda \right)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)} \right) s_{(17),(17)} T_{17}^* + (b_{17})^{(2)} s_{(16),(17)} T_{17}^* \\
+ \left( \left( \lambda \right)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)} \right) (q_{16})^{(2)} G_{16} + (a_{16})^{(2)} (q_{17})^{(2)} G_{17} \\
+ \left( \left( \lambda \right)^{(2)} + (b'_{17})^{(2)} - (r_{17})^{(2)} \right) s_{(17),(16)} T_{17}^* + (b_{16})^{(2)} s_{(16),(16)} T_{16}^* \\
\left( \left( \lambda \right)^{(2)} + (a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)} \right) (q_{18})^{(2)} G_{18} \\
+ \left( \left( \lambda \right)^{(2)} + (a_{16})^{(2)} + (p_{16})^{(2)} \right) (a_{16})^{(2)} (q_{17})^{(2)} G_{17} + (a_{17})^{(2)} (a_{18})^{(2)} (q_{16})^{(2)} G_{16} \\
\left( \left( \lambda \right)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)} \right) s_{(17),(16)} T_{17}^* + (b_{17})^{(2)} s_{(16),(16)} T_{16}^* \} = 0
\end{align*}
\]

+ 

\[
\begin{align*}
\left( \lambda \right)^{(3)} + (b'_{22})^{(3)} - (r_{22})^{(3)} \right) & \left( \left( \lambda \right)^{(3)} + (a'_{22})^{(3)} + (p_{22})^{(3)} \right) \\
\left[ \left( \left( \lambda \right)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)} \right) \left( q_{21} \right)^{(3)} G_{21} + (a_{21})^{(3)} (q_{20})^{(3)} G_{20} \right] \\
\left( \left( \lambda \right)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)} \right) s_{(21),(21)} T_{21}^* + (b_{21})^{(3)} s_{(20),(21)} T_{21}^* \\
+ \left( \left( \lambda \right)^{(3)} + (a'_{21})^{(3)} + (p_{21})^{(3)} \right) (q_{20})^{(3)} G_{20} + (a_{20})^{(3)} (q_{21})^{(3)} G_{21} \\
+ \left( \left( \lambda \right)^{(3)} + (b'_{21})^{(3)} - (r_{21})^{(3)} \right) s_{(21),(20)} T_{21}^* + (b_{21})^{(3)} s_{(20),(20)} T_{20}^* \\
\left( \left( \lambda \right)^{(3)} + (a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)} \right) (q_{22})^{(3)} G_{22} \\
+ \left( \left( \lambda \right)^{(3)} + (a_{20})^{(3)} + (p_{20})^{(3)} \right) (a_{22})^{(3)} (q_{21})^{(3)} G_{21} + (a_{21})^{(3)} (a_{22})^{(3)} (q_{20})^{(3)} G_{20}
\end{align*}
\]
\[
\left( (\lambda)^{(3)} + (b_{20}^{3})^{(3)} - (r_{20})^{(3)} \right) s_{(21),(22)}^* T_{21}^{*} + (b_{21})^{(3)} s_{(20),(22)}^* T_{20}^{*} \right) = 0
\]

+ \[
\left( (\lambda)^{(4)} + (b_{20}^{4})^{(4)} - (r_{20})^{(4)} \right) \left( (\lambda)^{(4)} + (a_{20}^{4})^{(4)} + (p_{20})^{(4)} \right) \]
\[
\left[ \left( (\lambda)^{(4)} + (a_{24}^{4})^{(4)} + (p_{24})^{(4)} \right) g_{25}^{4} + (a_{25})^{(4)} (q_{24})^{(4)} g_{24}^{4} \right] \]
\[
\left( (\lambda)^{(4)} + (b_{24}^{4})^{(4)} - (r_{24})^{(4)} \right) s_{(25),(25)}^* T_{25}^{*} + (b_{25})^{(4)} s_{(24),(25)}^* T_{24}^{*} \right) + \left( (\lambda)^{(4)} + (a_{25}^{4})^{(4)} + (p_{25})^{(4)} \right) (q_{24})^{(4)} g_{24}^{4} + (a_{24})^{(4)} (q_{25})^{(4)} g_{24}^{4} \right)
\]
\[
\left( (\lambda)^{(4)} + (b_{24}^{4})^{(4)} - (r_{24})^{(4)} \right) s_{(25),(24)}^* T_{25}^{*} + (b_{25})^{(4)} s_{(24),(24)}^* T_{24}^{*} \right) = 0
\]

+ \[
\left( (\lambda)^{(5)} + (b_{30}^{5})^{(5)} - (r_{30})^{(5)} \right) \left( (\lambda)^{(5)} + (a_{30}^{5})^{(5)} + (p_{30})^{(5)} \right) \]
\[
\left[ \left( (\lambda)^{(5)} + (a_{28}^{5})^{(5)} + (p_{28})^{(5)} \right) (q_{28})^{(5)} g_{29}^{5} + (a_{29})^{(5)} (q_{28})^{(5)} g_{28}^{5} \right] \]
\[
\left( (\lambda)^{(5)} + (b_{28}^{5})^{(5)} - (r_{28})^{(5)} \right) s_{(29),(29)}^* T_{29}^{*} + (b_{29})^{(5)} s_{(28),(29)}^* T_{29}^{*} \right) + \left( (\lambda)^{(5)} + (a_{29})^{(5)} + (p_{29})^{(5)} \right) (q_{28})^{(5)} g_{28}^{5} + (a_{28})^{(5)} (q_{29})^{(5)} g_{28}^{5} \right)
\]
\[
\left( (\lambda)^{(5)} + (b_{28}^{5})^{(5)} - (r_{28})^{(5)} \right) s_{(29),(28)}^* T_{29}^{*} + (b_{29})^{(5)} s_{(28),(28)}^* T_{28}^{*} \right) = 0
\]
\[
\left(\left(\lambda\right)^{(5)}\right)^2 + \left(\left(a_{28}\right)^{(5)} + \left(a_{29}\right)^{(5)} + \left(p_{28}\right)^{(5)} + \left(p_{29}\right)^{(5)}\right)\left(\lambda\right)^{(5)}
\]
\[
\left(\left(\lambda\right)^{(5)}\right)^2 + \left(\left(b_{28}\right)^{(5)} + \left(b_{29}\right)^{(5)} - \left(r_{28}\right)^{(5)} + \left(r_{29}\right)^{(5)}\right)\left(\lambda\right)^{(5)}
\]
\[
+ \left(\left(\lambda\right)^{(5)}\right)^2 + \left(\left(a_{28}\right)^{(5)} + \left(a_{29}\right)^{(5)} + \left(p_{28}\right)^{(5)} + \left(p_{29}\right)^{(5)}\right)\left(\lambda\right)^{(5)}\left(q_{30}\right)^{(5)}G_{30}
\]
\[
+ \left(\left(\lambda\right)^{(5)} + \left(a_{28}\right)^{(5)} + \left(p_{28}\right)^{(5)}\right)\left(\left(a_{30}\right)^{(5)}\left(q_{29}\right)^{(5)}G_{29}^* + \left(a_{30}\right)^{(5)}\left(a_{30}\right)^{(5)}\left(q_{28}\right)^{(5)}G_{28}^*
\]
\[
\left(\left(\lambda\right)^{(5)} + \left(b_{28}\right)^{(5)} - \left(r_{28}\right)^{(5)}\right)S_{(29),(30)}T_{29}^* + \left(b_{29}\right)^{(5)}S_{(28),(30)}T_{28}^*
\right) = 0
\]
\[
+
\]
\[
\left(\left(\lambda\right)^{(6)} + \left(b_{34}\right)^{(6)} - \left(r_{34}\right)^{(6)}\right)\left(\left(\lambda\right)^{(6)} + \left(a_{44}\right)^{(6)} + \left(p_{44}\right)^{(6)}\right)
\]
\[
\left[\left(\left(\lambda\right)^{(6)} + \left(a_{32}\right)^{(6)} + \left(p_{32}\right)^{(6)}\right)\left(q_{33}\right)^{(6)}G_{33}^* + \left(a_{33}\right)^{(6)}\left(q_{32}\right)^{(6)}G_{32}^*
\right] + \left(\left(\lambda\right)^{(6)} + \left(b_{32}\right)^{(6)} - \left(r_{32}\right)^{(6)}\right)S_{(33),(33)}T_{33}^* + \left(b_{33}\right)^{(6)}S_{(32),(33)}T_{33}^*
\right)
\[
+ \left(\left(\lambda\right)^{(6)} + \left(a_{33}\right)^{(6)} + \left(p_{33}\right)^{(6)}\right)\left(q_{32}\right)^{(6)}G_{32}^* + \left(a_{32}\right)^{(6)}\left(q_{33}\right)^{(6)}G_{33}^*
\right)
\[
\left(\left(\lambda\right)^{(6)} + \left(b_{32}\right)^{(6)} - \left(r_{32}\right)^{(6)}\right)S_{(33),(32)}T_{33}^* + \left(b_{33}\right)^{(6)}S_{(32),(32)}T_{32}^*
\right)
\[
\left(\left(\lambda\right)^{(6)}\right)^2 + \left(\left(a_{32}\right)^{(6)} + \left(a_{33}\right)^{(6)} + \left(p_{32}\right)^{(6)} + \left(p_{33}\right)^{(6)}\right)\left(\lambda\right)^{(6)}
\]
\[
\left(\left(\lambda\right)^{(6)}\right)^2 + \left(\left(b_{32}\right)^{(6)} + \left(b_{33}\right)^{(6)} - \left(r_{32}\right)^{(6)} + \left(r_{33}\right)^{(6)}\right)\left(\lambda\right)^{(6)}
\]
\[
+ \left(\left(\lambda\right)^{(6)}\right)^2 + \left(\left(a_{32}\right)^{(6)} + \left(a_{33}\right)^{(6)} + \left(p_{32}\right)^{(6)} + \left(p_{33}\right)^{(6)}\right)\left(\lambda\right)^{(6)}\left(q_{34}\right)^{(6)}G_{34}
\]
\[
+ \left(\left(\lambda\right)^{(6)} + \left(a_{32}\right)^{(6)} + \left(p_{32}\right)^{(6)}\right)\left(\left(a_{34}\right)^{(6)}\left(q_{33}\right)^{(6)}G_{33}^* + \left(a_{33}\right)^{(6)}\left(a_{34}\right)^{(6)}\left(q_{32}\right)^{(6)}G_{32}^*
\right)
\[
\left(\left(\lambda\right)^{(6)} + \left(b_{32}\right)^{(6)} - \left(r_{32}\right)^{(6)}\right)S_{(33),(34)}T_{33}^* + \left(b_{33}\right)^{(6)}S_{(32),(34)}T_{32}^*
\right) = 0
\]
\[
+
\]
\[
\left(\left(\lambda\right)^{(7)} + \left(b_{38}\right)^{(7)} - \left(r_{38}\right)^{(7)}\right)\left(\left(\lambda\right)^{(7)} + \left(a_{38}\right)^{(7)} + \left(p_{38}\right)^{(7)}\right)
\]
\[
\left[\left(\left(\lambda\right)^{(7)} + \left(a_{36}\right)^{(7)} + \left(p_{36}\right)^{(7)}\right)\left(q_{37}\right)^{(7)}G_{37}^* + \left(a_{37}\right)^{(7)}\left(q_{36}\right)^{(7)}G_{36}^*
\right]
\]
\[
\left( (\lambda)^{(7)} + (b_{36}^{(7)}) - (r_{36}^{(7)} \right) s_{(37),(37)}T_{37} + (b_{37}^{(7)} s_{(36),(37)}T_{37}^2
\]
\[
+ \left( (\lambda)^{(7)} + (a_{37}^{(7)}) + (p_{37}^{(7)})(q_{36}^{(7)}g_{36} + (a_{36}^{(7)})(q_{37}^{(7)}g_{37}^2
\]
\[
\left( ((\lambda)^{(7)} + (b_{36}^{(7)} - (r_{36}^{(7)})) s_{(37),(36)}T_{37} + (b_{37}^{(7)} s_{(36),(36)}T_{36}^2
\]
\[
\left( ((\lambda)^{(7)} + (a_{36}^{(7)} + (a_{37}^{(7)} + (p_{36}^{(7)} + (p_{37}^{(7)})) (\lambda)^{(7)}
\]
\[
\left( (\lambda)^{(7)} + (b_{36}^{(7)})(r_{36}^{(7)} + (r_{36}^{(7)} + (r_{36}^{(7)})) (\lambda)^{(7)}
\]
\[
+ (\lambda)^{(7)} + (a_{36}^{(7)} + (a_{37}^{(7)} + (p_{36}^{(7)}) + (p_{37}^{(7)}) (\lambda)^{(7)}) (a_{36}^{(7)})(q_{36}^{(7)}g_{36}^2
\]
\[
\left( ((\lambda)^{(7)} + (b_{36}^{(7)} - (r_{36}^{(7)})\right) s_{(37),(36)}T_{37} + (b_{37}^{(7)} s_{(36),(36)}T_{36}^2) = 0
\]

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In F. Fernflores. The Equivalence of Mass and Energy. Stanford Encyclopedia of Philosophy

Note that the relativistic mass, in contrast to the rest mass \( m_0 \), is **not a relativistic invariant, and that the velocity is not a Minkowski four-vector, in contrast to the quantity \( \frac{d\tau}{dt} \), where \( \tau \) is the differential of the proper time. However, the energy-momentum four-vector is a genuine Minkowski four-vector, and the intrinsic origin of the square-root in the definition of the relativistic mass is the distinction between \( dt \) and \( d\tau \).


Conversions used: 1956 International (Steam) Table (IT) values where one calorie
≡ 4.1868 J and one BTU ≡ 1055.05585262 J. Weapons designers' conversion value of one gram TNT
≡ 1000 calories used.

(22) Assuming the dam is generating at its peak capacity of 6,809 MW.

(23) Assuming a 90/10 alloy of Pt/Ir by weight, a $C_p$ of 25.9 for Pt and 25.1 for Ir, a Pt-dominated average $C_p$ of 25.8, 5.134 moles of metal, and 132 J.K$^{-1}$ for the prototype. A variation of ±1.5 picograms is of course, much smaller than the actual uncertainty in the mass of the international prototype, which are ±2 micrograms.

(24) Article on Earth rotation energy. Divided by $c^2$.

(25) Earth's gravitational self-energy is $4.6 \times 10^{-10}$ that of Earth's total mass, or 2.7 trillion metric tons. Citation: The Apache Point Observatory Lunar Laser-Ranging Operation (APOLLO), T. W. Murphy, Jr. et al. University of Washington, Dept. of Physics (132 kB PDF, here.).

(26) There is usually more than one possible way to define a field energy, because any field can be made to couple to gravity in many different ways. By general scaling arguments, the correct answer at everyday distances, which are long compared to the quantum gravity scale, should be minimal coupling, which means that no powers of the curvature tensor appear. Any non-minimal couplings, along with other higher order terms, are presumably only determined by a theory of quantum gravity, and within string theory, they only start to contribute to experiments at the string scale.


(41)^ Ives, Herbert E. (1952), "Derivation of the mass-energy relation", Journal of the Optical Society of America 42 (8): 540–543, DOI:10.1364/JOSA.42.000540


Acknowledgments:

The introduction is a collection of information from various articles, Books, News Paper reports, Home Pages Of authors, Journal Reviews, Nature ‘s L:etters, Article Abstracts, Research papers, Abstracts Of Research Papers, Stanford Encyclopedia, Web Pages, Ask a Physician Column, Deliberations with Professors, the internet including Wikipedia. We acknowledge all authors who have contributed to the same. In the eventuality of the fact that there has been any act of omission on the part of the authors, we regret with great deal of compunction, contrition, regret, trepidation and remorse. As Newton said, it is only because erudite and eminent people allowed one to piggy ride on their backs; probably an attempt has been made to look slightly further. Once again, it is stated that the references are only illustrative and not comprehensive.
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