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Generalized Auto-regressive Conditional Heteroscedastic Modeling of the Volatility of Stock Returns in the Nairobi Securities Exchange

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Abstract

The most important characteristic of a stock or bond is its return or profit. This return is volatile and tomorrow's price is uncertain and must be described by a probability distribution. The purpose of this study was to develop a model of stock returns in the Nairobi Securities Exchange (NSE) using the Generalized Autoregressive Conditional Heteroscedastic (GARCH) model. Closing prices of Safaricom and Kenya Commercial Bank (KCB) were obtained from the NSE for the period January 2011 to October 2014 which formed 1000 observations excluding weekends and holidays. Test for normality and stationarity was done using the Shapiro–Wilk test and Augmented Dickey Fuller (ADF) respectively. All the return series exhibited, leptokurtosis, volatility clustering and negative skewness. The estimation results reveal that GARCH (1, 1) best fits both return series over the period of study.

Keywords: Heteroscedasticity. Stock Returns, Volatility

1. Introduction

Modeling stock return volatility has become one of the most aspects of financial markets providing an important input for portfolio management, option pricing and market regulation. Volatility is unobservable in financial market and it is measured by standard deviations or variance of return which can be directly considered as a measure of risk of assets. The choice an investor makes of a portfolio is intended to maximize the expected return subject to a risk constraint, or to minimize his risk subject to a return constraint. An efficient model for forecasting of an asset's price volatility provides a starting point for the assessment of investment risk. To price an option, one needs to know the volatility of the underlying asset. This can only be achieved through modeling the volatility. Volatility also has a great effect on the macro-economy. High volatility beyond a certain threshold will increase the risk of investor loses and raise concerns about the stability of the market and the wider economy (Hongyu, 2006).

Investing in the NSE has attracted many individuals. This can be evidenced by the number of people who showed interest in buying the Safaricom IPO's during its inception in 2008. Both academicians and practitioners recognize that volatility is not directly observable and that financial returns show certain characteristics that are specific to financial time series such as volatility clustering and leverage effect (Bollerslev, 1986).

Financial econometricians have developed many time-varying volatility models among them ,the Autoregressive Conditional Heteroscedastic (ARCH) model proposed by (Engle, 1982) and its extension, the Generalized Autoregressive Conditional Heteroscedastic (GARCH) developed by (Bollerslev, 1986), and (Taylor, 1986) which have been applied widely. This study seeks to develop a model of stock returns volatility in the NSE.

1.1 Objectives of the Study

The objective of this study is to develop a model for stock returns and apply to selected companies in the NSE.

2. Literature Review

Various linear and non-linear models have been developed in the literature and extensively applied in practice to describe stock return volatility. (Poterba, 1986) take into account the linear model and specify a stationary AR (1) process for volatility of the Standard and Poor, S&P 500 index. Another study by (French, 1987) uses a non-linear stationary ARIMA (0, 1, 3) model to describe the volatility of the S&P 500 index. Similarly, Schwert (1990) and Schwert and Seguin (1990) use a linear AR (12) as an approximation for monthly stock return volatility.

Linear time series models however are not robust to describe certain features of a volatility series. For instance there are well-defined empirical evidences that stock returns have a tendency to exhibit clusters of outliers (Mandelbrot (1963) and Fama (1965), implying that large variances tend to be followed by another large variance. They are unable to explain a number of important features common to much financial data, including leptokurtosis, volatility clustering, long memory, volatility smile and leverage effects. That is, because the assumption of homoscedasticity (or constant variance) is not appropriate when using financial data, and in such instances it is preferable to examine patterns that allow the variance to depend upon its history.

Thus such limitations of linear models have motivated many researchers to consider non-linear alternatives. The Autoregressive Conditional Heteroscedastic (ARCH) model of (Engle, 1982), the generalized ARCH (GARCH) model of (Bollerslev, 1986) and exponential GARCH (EGARCH) model of (Nelson, 1991)are the common non-linear models used in finance literature. These ARCH class models have been found to be useful in capturing certain non-linear features of financial time series such as heavy tailed distributions and clusters of outliers.

A study by (Akgiray, 1989) uses a GARCH(1,1) model to investigate the time series properties of the stock returns and reports GARCH to be the best of several models in describing and forecasting stock market volatility. (Anil & Higgins, 1993)investigated the volatility of the conventional ordinary least squares to estimate optimal hedge ratio estimates using future contracts. Similarly, (Najand, 1991) examines the relative ability of linear and non-linear models to forecast daily S&P 500 futures index volatility. The study finds that non-linear GARCH models perform best. (Benoit, 1963) utilized the infinite variance distributions, when considering the models for stock market price changes. (Fama, 1965) when modeling stock market prices attributed their discrepancies to the possibility of the process having stable innovations and thus fitted an adequate model on this basis.

Markov-Switching models have also been used to capture the volatility dynamics of financial time series. This is because they give rise to a plausible interpretation of nonlinearities. Markov switching model of stock returns was originally proposed by (Startz, & Nelson, 1989). (Bhar, 2004), among others employ markov switching models for the modeling of stock returns.

There is a significant amount of research on volatility of stock markets of developed countries. For instance, (Gary, 2004) applied the GARCH model to the Shanghai Stock Exchange while (Bertram, 2004) modeled Australian Stock Exchange using ARCH models. Other studies on these stock markets include (Baudouhat, 2004)who utilized the GARCH model in analyzing the Nordic financial market integration. (Walter, 2005) applied the structural GARCH model to portfolio risk management for the South African equity market as well (Hongyu, 2006) who forecasted the volatility of the Chinese stock market using the GARCH-type models. (Elie, 2012) compared the GARCH model and the EGARCH under three distribution assumptions: the Gaussian, the t-student and the general error distributions. He showed that the distribution of returns is far from being normally distributed with fat tails and volatility clustering being persistent. (Al-Jafari, 2012) utilized a non-linear symmetric GARCH(1,1) model and two non-linear asymmetric models, TARCH(1,1) and EGARCH(1,1) to Muscat Securities Market and the empirical findings provide no presence of day-of –the –week effect

The Sub-Saharan Africa has been under-researched as far as volatility modeling is concerned. Studies carried out in the African stock markets include, (Frimpong, 2006)who applied GARCH models to the Ghana Stock Exchange. (Brooks, 1997)examined the effect of political change in the South African Stock Market; (Appiah-Kusi, 1998) investigated the volatility and volatility spillovers in the emerging markets in Africa. More recently, (Emenike, 2010) applied the EGARCH model to the Kenyan and Nigerian Stock Market returns. From the available literature, the NSE just like other Sub Saharan Africa Equity Markets has been under-researched as far as market volatility is concerned and therefore this study contributes to the small literature available on the Nairobi Stock Exchange.

These developments in financial econometrics suggest the use of nonlinear time series structures to model the stock market prices and the expected returns. The focus of financial time series modeling has been on the ARCH model and its various extensions. However, the ARCH has limitations in that it treats negative and positive returns in the same way. It is also very restrictive in parameters and often over predicts the volatility because it responds slowly to large shocks. GARCH models have proved adequate in modeling and forecasting volatility. GARCH for instance takes into account excess kurtosis i.e. fat tail behavior and volatility clustering which are two important characteristics if time series. It also provides accurate forecast of variances and covariance of asset return through its ability to model time varying conditional variances.

3. Methodology

3.1 Data

The data used in this study comprised Safaricom's and KCB's daily closing price over the period January, 2011 to October, 2014 excluding weekends and public holidays forming a sample of 1000. The daily closing prices were obtained NSE. Since the return of an asset is a complete and scale free summary of an investment with attractive statistical features, use return series rather than the price series (Campell, Lo, & MacKinlay, 1997).

3.2 Method

ARCH Model

ARCH models based on the variance of the error term at time t depends on the realized values of the squared error terms in previous time periods. The model is specified as

$$y_t = \mu_t \quad (3.1)$$

$$\mu_t \backsim N(0, h_t) \quad (3.2)$$

$$h_t = \alpha_0 + \sum \alpha_j \mu_{t-i}^2 \quad (3.3)$$

This model is referred to as (q), where q refers to the order of the lagged squared returns included in the model. If we use $ARCH(1)$ model it becomes

$$h_t = \alpha_0 + \alpha_1 + \alpha_{t-1}^2 \quad (3.3)$$

Since h_t is a conditional variance, its value must always be strictly positive; a negative variance at any point in time would be meaningless. To have positive conditional variance estimates, all of the coefficients in the conditional variance are usually required to be non-negative. Thus coefficients must be satisfy $\alpha_0 \geq 0$ and $\alpha_1 \geq 0$

GARCH Model

Bollerslev (1986) and Taylor (1986) developed the $GARCH(p, q)$ model. The model allows the conditional variance of variable to be dependent upon previous lags; first lag of the squared residual from the mean equation and present news about the volatility from the previous period which is as follows

$$h_t = \alpha_0 + \sum \alpha_i \mu_{t-i}^2 + \sum \beta_i h_{t-i} \tag{3.4}$$

In the literature most used and simple model is the $GARCH(1,1)$ process, for which the conditional variance can be written as follows

$$h_t = \alpha_0 + \alpha_1 \mu_{t-1}^2 + \beta_1 h_{t-1} \tag{3.5}$$

Under the hypothesis of covariance stationarity, the unconditional variance h_t can be found by taking the unconditional expectation of equation 3.5.

We find that

$$h = \alpha_0 + \alpha_1 h + \beta_1 h \tag{3.6}$$

Solving the equation 3.5 we have

$$h = \frac{\alpha_0}{1-\alpha_1-\beta_1} \tag{3.7}$$

For this unconditional variance to exist, it must be the case that $\alpha_1 + \beta_1 < 1$and for it to be positive, we require that $\alpha_0 > 0$

4. Data Analysis and Results

4.1 Statistical Analysis

The daily closing price series are first converted to return series. Let Y_t denote the daily closing price of a stock at the end of the day t, the daily stock return series was generated by

$$r_t = ln\frac{Y_t}{Y_{t-1}} \tag{4.1}$$

where r_t is the return, Y_t is today's price and Y_{t-1} is yesterday's price.

Stationarity of the return series was checked using unit root test. Lagrange Multiplier (LM) and Ljung-Box statistics were used to test for ARCH effects on the squared residuals of the regressed AR (p) process. Under the null hypothesis that there is no ARCH effects the LM test statistic equal to TR^2 has asymptotic chi -squared distribution with p degree of freedom.

4.2 Results and Discussion

4.2.1 Data Exploration

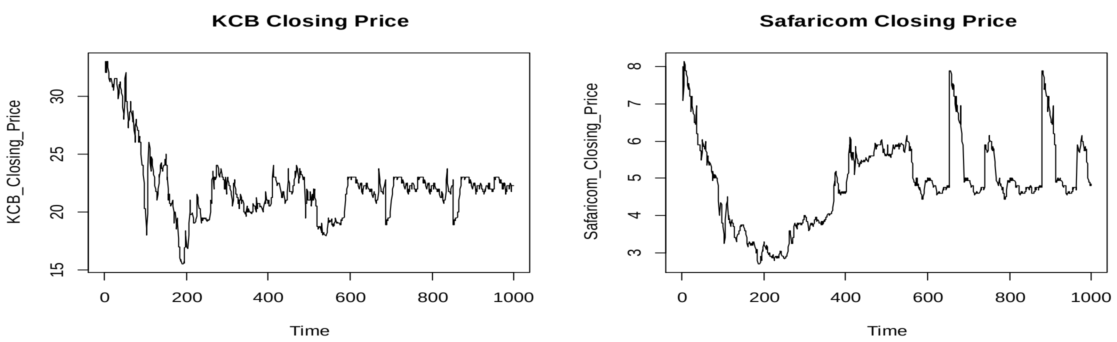

Figure 4.1: Time series plot of the closing prices

From **Figure 4.1** the closing prices are very irregular with varied degree of fluctuations. The time plots clearly show that the mean and variance are not constant, showing non-stationarity of the data. Series such as these cannot be used for further statistical inferences because of their implications (Gujarati, 2004), thus theneed to transform them to returns. The plots of daily returns of Safaricom and KCB are presented **[Insert Figure 4.2**].

The plots for returns are stationary and exhibit no trend and the amplitude vary with time a phenomenon called ARCH effects. Volatility clustering is also evident.

4.2.2 Descriptive Statistics for closing prices and returns

Table 4.2: Descriptive statistics for prices and returns

A. Prices	**KCB Closing price**	**Safaricom Closing price**
Mean	22.166	4.8966
Median	22.000	4.8000
Minimum	15.500	2.7000
Maximum	33.000	8.1500
Standard deviation	2.9001	1.1534
C.V.	0.13083	0.23555
Skewness	1.4555	0.32651
Ex. kurtosis	3.2942	-0.0026444
Observations	1000	1000
	1000	1000
B. Returns	**KCB returns**	**Safaricom returns**
Mean	-0.00039499	-0.00050147
Median	0.00000	0.00000
Minimum	-0.18540	-0.17556
Maximum	0.093930	0.50872
Standard deviation	0.021025	0.033079
C.V.	53.230	65.964
Skewness	-1.0826	7.5401
Ex. kurtosis	14.321	116.23
Observations	999	999

Table 4.2 above shows summary statistics for the two companies' return series. The results indicate high volatility and the risky nature of the market since the standard deviation of the market returns is high in comparison with the mean. Also the standard deviations are very close for both Safaricom and KCB with Safaricom being slightly volatile. Both price series have positive skewness implying that the distribution has a long right tail. On the other hand, the return series for Safaricom have negative skewness implying that the distribution has a long left tail and positive for KCB implying that the distribution has long right tail. The values for kurtosis are high (above three) for both return series implying they are leptokurtic. The Shapiro-Wilk test rejects normality at the 5% level for all series. So, the samples have all financial characteristics: volatility clustering and leptokurtosis.

4.2.3 Test for Unit Root

A stationary check for both closing prices and returns using Augmented Dickey Fuller (ADF) test shows that under the null hypothesis, unit root is not detected in both returns.

Table 4.3: ADF and PP test for prices and returns for Safaricom and KCB

Safaricom	**Closing Prices**	**Return series**
ADF Value	-3.5	-9.13
P-value	0.04	0.01
KCB		
ADF Value	-2.71	-9.38
P-value	0.27	0.01

4.2.4 Testing for ARCH effects in Returns

Before fitting the autoregressive models, the presence of ARCH effects in the residuals is tested. If there does not exist a significant ARCH effect in the residuals then the ARCH model is mis-specified. Testing the hypothesis of no significant ARCH effects is based on the Lagragian Multiplier (LM) approach as stated earlier on the methodology.

Table 4.4: Lagragian Multiplier test for Arch effects

Returns	**Chi-square**	**df**	**p-value**
KCB	117.15	6	<0.001
Safaricom	74.5019	4	<0.001

From **Table 4.4**the p-values for both series are less than 0.05 hence we reject the null hypothesis of no significant arch effect in the daily returns of Safaricom and KCB and conclude there are significant arch effects.

4.2.5 Parameter Estimation

From R output

	Estimate	Std. Error	t value	Pr (>|t|)
a0	4.101e-05	8.491e-06	4.829	1.37e-06 ***
a1	1.866e-01	2.617e-02	7.131	9.98e-13 ***
b1	7.209e-01	3.363e-02	21.438	< 2e-16 ***

For Safaricom the fitted GARCH (1, 1) model is

$$r_t = 5.76 + \varepsilon_t$$

$$\sigma_t^2 = 0.00004 + 0.186\, Z^2_{t-1} + 0.7209\, \sigma^2_{t-1}$$

From the following output for KCB

	Estimate	Std. Error	t value	Pr (>|t|)
a0	2.895e-05	5.833e-06	4.963	6.95e-07 ***
a1	1.912e-01	2.592e-02	7.376	1.63e-13 ***
b1	7.597e-01	2.561e-02	29.662	< 2e-16 ***

The fitted $GARCH$ (1, 1) model is

$$r_t = 20.18 + \varepsilon_t$$

$$\sigma_t^{\ 2} = 0.000028 + 0.19\, Z^{\ 2}_{t-1} + 0.7597\, \sigma^2_{t-1}$$

To assess the accuracy of the estimates, the standard errors are used the smaller the better. From the standard errors the estimates are precise. Based on 95% confidence level, the coefficients of the fitted $GARCH\ (1,1)$ model are significantly different from zero.

4.25 Goodness-of-Fit

Here the adequacy of the selected models is done. This is done by using standardized residuals which are assumed to follow either normal or standardized t distribution. It must satisfy the requirement of a white noise. The plots include normal plots, ACF plot time series plot and histogram. If the model fits the data well the histogram of the residuals should be symmetric. The normal probability plot should be a straight line while the time plot should exhibit random variation. For ACF plots all the correlation should be within the boundary line meaning the data is stationary. [**Insert Figure 4.3**]

It is clear that all the correlations are within the test bounds implying the fitted model is adequate. Further the $Q - Q$ plot shows that the models were adequate.

Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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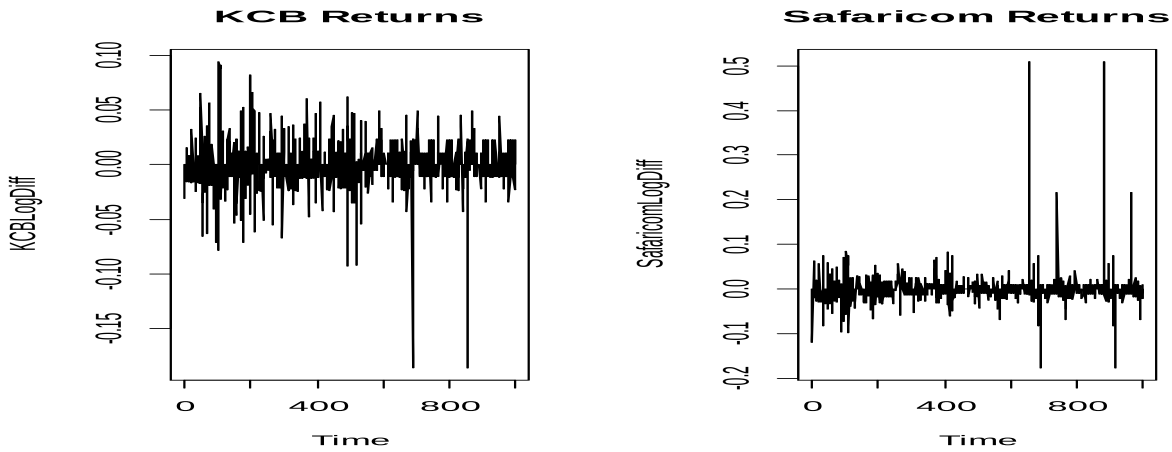

Figure 4.2: Time series plot of the return series

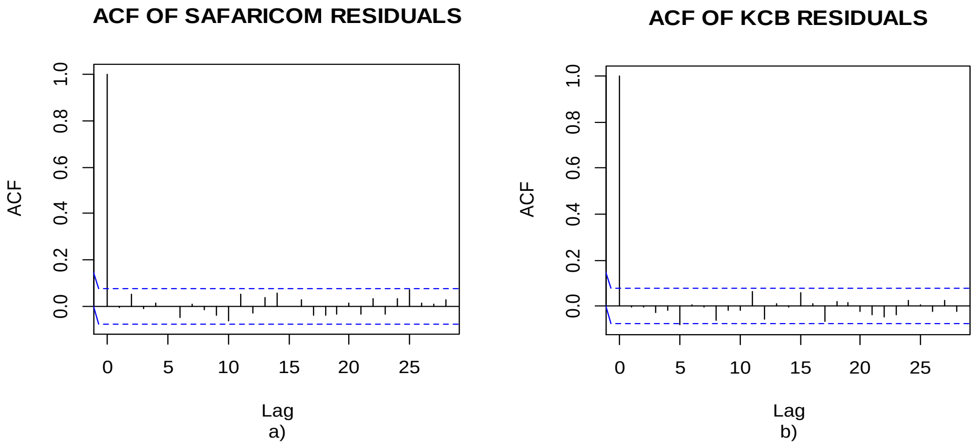

Figure 4.3: ACF plots of residuals for Safaricom and KCB