$\delta\beta-$Rough Probability In Topological Spaces

Mostafa A. Elgendy [1] Galal. Elkobrosy [1] Amr M. Amin [1*]
1. Department of Engineering Mathematics and Physics, Faculty of Engineering,
Alexandria University, Alexandria 21544, Egypt
* E-mail of the corresponding author: amr.mahmoud.amin@gmail.com

Abstract

The main aim of the rough set is reducing the boundary region by increasing the lower approximation and decreasing the upper approximation, since $\delta\beta-$open sets is stronger than any topological open or near open sets. In this paper, the topological generalization using $\delta\beta-$open sets method is applied for constructing $\delta\beta-$rough probability in topological spaces which we can consider them as results from the general relations on the approximation spaces. The basic concepts of $\delta\beta-$rough probability were introduced and sufficiently illustrated. Moreover, proved results and examples are provided.

Keywords: $\delta\beta-$topologized Stochastic approximation space, $\delta\beta-$rough expectation, $\delta\beta-$rough variance, $\delta\beta-$rough probability generating function, $\delta\beta-$rough characteristic function.

1. Introduction

Rough Sets were introduced by Pawlak (1982) in the early eighties and developed further as a mathematical method of knowledge representation and processing under uncertain data and incomplete information. Nowadays, rough set theory is widely recognized to have a great importance in several fields, which is witnessed by the increasing number of papers concerned with applications and theoretical foundations of rough set. Partitioning of a set with an equivalence relation is the core concept behind Pawlak's rough set theory. But this is too restrictive to deal with different real life situations. Topology is an important branch of mathematics, topology has its own theory and own significance. Rough set theory combined with topology is expected to provide us with new area to study.

Many existing approaches, for obtaining probability in topological spaces which we can consider them as results from the general relations on the approximation spaces , such as, M (2010) and Luay *et al.* (2013) proposed the concept of rough probability , M. E *et al.* (2011) defined the near rough probability . The topological generalization using $\delta\beta-$open sets was introduced by A.S. (2015). He shows that $\delta\beta-$open sets is stronger than any topological open or near open sets. In this paper, the basic concepts of $\delta\beta-$rough probability were introduced and sufficiently illustrated. Moreover, proved results and examples are provided.

2. Preliminaries

A topological space (Kelley 1955) is a pair (U,τ) consisting of a set U and family τ of subsets of U satisfying the following conditions:

1) $\phi\in\tau$, $U\in\tau$.
2) τ is closed under arbitrary union.
3) τ is closed under finite intersection.

Throughout this paper (U,τ) denotes a topological space, the elements of U are called points of the space, The subsets of U belonging to τ are called open sets in the space, the complement of the subsets of U belonging to τ are called closed sets in the space, and the family of all τ - closed subsets of U is denoted by τ^*, the family τ of open subsets of U is also called a topology for U . A family $\mathrm{B}\subseteq\tau$ is called a base for (U,τ) iff every nonempty open subset of U can be represented as a union of subfamily of B . A family $\mathrm{S}\subseteq\tau$ is called a subbase iff the family of all finite intersections is a base for (U,τ). The τ -closure of a subset A in U is denoted by $cl(A)$ and is given by $cl(A)=\bigcap\{F\subseteq U: A\subseteq F \text{ and } F\in\tau^*\}$. The τ -interior of a subset A is denoted by $\text{int}(A)$ and is defined by $\text{int}(A)=\bigcup\{G\subseteq U: G\subseteq A \text{ and } G\in\tau\}$. The boundary of a subset $A\subseteq U$ is denoted by $Bnd(A)$ and defined by $Bnd(A)=cl(A)-\text{int}(A)$.

Some forms of near open sets are introduced in the following definition.

Definition 2.1. Let (U,τ) be a topological space, if $A\subseteq U$, then A is called

1) Regular –open (M. E *el al*. 1982) (briefly $r-$open) if $A=\text{int}(cl(A))$.
2) Semi – open (N. 1963) (briefly $s-$open) if $A\subseteq cl(\text{int}(A))$.
3) Pre –open (O. 1965) (briefly $p-$open) if $A\subseteq \text{int}(cl(A))$.
4) α -open (briefly $\alpha-$open) if $A=\text{int}(cl(\text{int}(A)))$.
5) Semi pre-open –open (M. E *el al*. 1983) (briefly $\beta-$open) if $A\subseteq cl(\text{int}(cl(A)))$.

The complement of $r-$open (resp. $s-$open, $p-$open, $\alpha-$open, $\beta-$open) set is called $r-$closed (resp. $s-$closed, $p-$ closed, $\alpha-$ closed, $\beta-$ closed) set. The family of all $r-$open (resp. $s-$open, $p-$open, $\alpha-$open, $\beta-$open) sets in (U,τ) is denoted by $RO(U)$ (resp. $SO(U)$, $PO(U)$, $\alpha O(U)$, $\beta O(U)$). The family of all $r-$closed (resp. $s-$ closed, $p-$ closed, $\alpha-$ closed, $\beta-$ closed) sets in (U,τ) is denoted by $RC(U)$ (resp. $SC(U)$, $PC(U)$, $\alpha C(U)$, $\beta C(U)$). The regular-closure (resp. $s-$ closure, $p-$ closure, $\alpha-$ closure, $\beta-$ closure) of A in (U,τ) is denoted by ${}_rcl(A)$ (resp. ${}_scl(A)$, ${}_pcl(A)$, ${}_\alpha cl(A)$ ${}_\beta cl(A)$) and defined to be the intersection of all regular –closed (resp. semi -closed, $p-$ closed, $\alpha-$ closed, $\beta-$ closed) sets containing A. The regular-interior (resp. $s-$ interior, $p-$ interior, $\alpha-$ interior, $\beta-$ interior) of A in (U,τ) is denoted by ${}_r\text{int}(A)$ (resp. ${}_s\text{int}(A)$, ${}_p\text{int}(A)$, ${}_\alpha\text{int}(A)$ ${}_\beta\text{int}(A)$) and defined to be the union of all regular –open (resp. semi -open, $p-$ open, $\alpha-$open, $\beta-$ open) sets contained in A. The regular-boundary (resp. $s-$ boundary, $p-$ boundary, $\alpha-$ boundary, $\beta-$ boundary) of A in (U,τ) is denoted by ${}_rBnd(A)$ (resp. ${}_sBnd(A)$, ${}_pBnd(A)$, ${}_\alpha Bnd(A)$, ${}_\beta Bnd(A)$) and is defined by ${}_rBnd(A)={}_rcl(A)-{}_r\text{int}(A)$.

We shall recall some concepts about $\delta\beta-$open sets which are essential for our present study.

Definition 2.2. (A. S. 2015). Let (U,τ) be a topological space, for $A\subseteq U$, then the $\delta-$closure of A is denoted by $cl_\delta(A)$ and is defined by:

$$cl_\delta(A)=\{a\in U: A\cap \text{int}(cl(G))\neq\phi, G\in\tau \text{ and } a\in G\}$$

Hence, A is called $\delta-$closed if $A=cl_\delta(A)$, the complement of $\delta-$closed set is called $\delta-$open set.

Definition 2.3. (A. S. 2015). Let (U,τ) be a topological space, for $A\subseteq U$, then A is called $\delta\beta-$open set if $A\subseteq cl(\text{int}(cl_\delta(A)))$, the family of all $\delta\beta-$open sets in U denoted by $\delta\beta O(U)$. The complement of $\delta\beta-$open set is called $\delta\beta-$closed set, the family of all $\delta\beta-$closed sets are denoted by $\delta\beta C(U)$.

Definition 2.4. (A. S. 2015). Let (U,τ) be a topological space, if $A\subseteq U$, then

1) The union of all $\delta\beta-$open sets contained in inside X is called $\delta\beta-$interior of A in U and denoted by $\beta\,\text{int}_\delta(A)$ and is defined by:
$$\beta\,\text{int}_\delta(A)=\bigcup\{G\subseteq U: G\subseteq A, \text{and } G\in\delta\beta O(U)\}.$$
2) The intersection of all $\delta\beta-$closed sets containing A is called $\delta\beta-$closure of A in U and denoted by $\beta\,\text{cl}_\delta(A)$ and is defined by:
$$\beta\,cl_\delta(A)=\bigcap\{F\subseteq U: F\supseteq A, \text{and } F\in\delta\beta C(U)\}.$$
3) The $\delta\beta-$boundary of A in U is denoted by $\beta Bnd_\delta(A)$ and is defined by

$$\beta Bnd_\delta(A) = \beta cl_\delta(A) - \beta \operatorname{int}_\delta(A).$$

Definition 2.5. (A. S. 2015). Let (U,τ) be a topological space, for $A \subseteq U$, then

1) $\beta \operatorname{int}_\delta(A) = A \cap cl(\operatorname{int}(cl(A)))$.
2) $\beta \, cl_\delta(A) = A \cup \operatorname{int}(cl(\operatorname{int}(A)))$.

The $\delta\beta-$open sets are stronger than any topological near open sets such that regular open, semi open, $\beta-$open, $\alpha-$open and $\delta-$ open. The arbitrary union on $\delta\beta-$open sets is again $\delta\beta-$open set, but the intersection of two $\delta\beta-$open may not be $\delta\beta-$open set. Thus the $\delta\beta-$open sets do not form a topology in U.

Theorem 2.6. Let (U,τ) be a topological space, if $A \subseteq U$, then

$$\beta \operatorname{int}_\delta(A^c) = (\beta \, cl_\delta(A))^c .$$

Proof.

$$(\beta cl_\delta(A))^c = U - \beta cl_\delta(A) = U - \bigcap\{F \subseteq U : A \subseteq F \text{ and } F \in \delta\beta C(U)\}.$$
$$= \bigcup\{U - F \subseteq U : U - F \in \delta\beta O(U) \text{ and } U - F \subseteq U - A\} = \beta \operatorname{int}_\delta(A^c).$$

Definition 2.7. The pair $K = (U, R_{\delta\beta})$ is called $\delta\beta-$approximation space where U a finite non empty universe and $R_{\delta\beta}$ is the general relation on U used to get a sub-base for topology τ_K on U which generates the class $\delta\beta O(X)$ of all $\delta\beta-$open sets. Then the triple $\kappa = (U, R_{\delta\beta}, \tau_K)$ is called $\delta\beta-$ topologized approximation space.

Definition 2.8. Let $\kappa = (U, R_{\delta\beta}, \tau_K)$, be a $\delta\beta-$topologized approximation space, for $A \subseteq U$. Then $\delta\beta-$lower approximation (resp. $\delta\beta-$upper approximation) of A in U is denoted by $\underline{R}_{\delta\beta}(A)$ (resp. $\overline{R}_{\delta\beta}(A)$) and is defined by:

$$\underline{R}_{\delta\beta}(A) = \bigcup\{G \in \delta\beta O(U) : G \subseteq A\}.$$
$$\left(\text{resp. } \overline{R}_{\delta\beta}(A) = \bigcap\{F \in \delta\beta C(U) : F \supseteq A\}\right).$$

Lemma 2.9. (A. S. 2015). Let $\kappa = (U, R_{\delta\beta}, \tau_K)$, be the $\delta\beta-$topologized approximation space, for $A \subseteq U$ .Then from relation

$$\operatorname{int}(A) \subseteq_\beta \operatorname{int}(A) \subseteq \beta \operatorname{int}_\delta(A) \subseteq A \subseteq \beta cl_\delta(A) \subseteq_\beta cl(A) \subseteq cl(A)$$

we have

$$\underline{R}(A) \subseteq \underline{R}_\beta(A) \subseteq \underline{R}_{\delta\beta}(A) \subseteq X \subseteq \overline{R}_{\delta\beta}(A) \subseteq \overline{R}_s(A) \subseteq \overline{R}(A) .$$

3. $\delta\beta-$Rough Probability in Topological Spaces

Definition 3.1. The pair $K = (U, R_{\delta\beta})$ is called $\delta\beta-$approximation space where U a finite non empty universe and $R_{\delta\beta}$ is the general relation on U used to get a subbase for topology τ_K on U which generates the class $\delta\beta O(X)$ of all $\delta\beta-$open sets. Then the order 4-tuples $S = (U, R_{\delta\beta}, P, \tau_K)$ is called the $\delta\beta-$topologized stochastic approximation space, where P is the probability.

3.1 $\delta\beta-$Rough Probability

Definition 3.2. Let A be an event in the $\delta\beta-$topologized stochastic approximation space $S = (U, R_{\delta\beta}, P, \tau_K)$ then, the $\delta\beta-$ lower (resp. $\delta\beta-$upper) probability of A is denoted by ${}_{\delta\beta}\underline{P}(A)$ (resp. ${}_{\delta\beta}\overline{P}(A)$) and is defined by:

$$_{\delta\beta}\underline{P}(A) = P(\underline{R}_{\delta\beta}(A)).$$
$$\left(\text{resp. } _{\delta\beta}\overline{P}(A) = P(\overline{R}_{\delta\beta}(A))\right).$$

where $\underline{R}_{\delta\beta}(A) = \beta\operatorname{int}_{\delta}(A)$ and $\overline{R}_{\delta\beta}(A) = \beta cl_{\delta}(A)$. Cleary, $0 \le {}_{\delta\beta}\underline{P}(A) \le 1$ and $0 \le {}_{\delta\beta}\overline{P}(A) \le 1$.

Definition 3.3. Let A be an event in the $\delta\beta-$topologized stochastic approximation space $S = (U, R_{\delta\beta}, P, \tau_K)$. The $\delta\beta-$ rough probability of A, denoted by $_{\delta\beta}P^*(A)$ is given by:

$$_{\delta\beta}P^*(A) = ({}_{\delta\beta}\underline{P}(A),\ {}_{\delta\beta}\overline{P}(A)).$$

Proposition 3.4 . Let A, B be two events in the $\delta\beta-$topologized stochastic approximation space $S = (U, R_{\delta\beta}, P, \tau_K)$.

1) $_{\delta\beta}\underline{P}(A) \le P(A) \le {}_{\delta\beta}\overline{P}(A)$.
2) $_{\delta\beta}\underline{P}(\phi) = {}_{\delta\beta}\overline{P}(\phi) = 0$.
3) $_{\delta\beta}\underline{P}(U) = {}_{\delta\beta}\overline{P}(U) = 1$.
4) $_{\delta\beta}\underline{P}(A^c) = 1 - {}_{\delta\beta}\overline{P}(A)$.
5) $_{\delta\beta}\overline{P}(A^c) = 1 - {}_{\delta\beta}\underline{P}(A)$.
6) $_{\delta\beta}\overline{P}(A \cup B) \le {}_{\delta\beta}\overline{P}(A) + {}_{\delta\beta}\overline{P}(B) - {}_{\delta\beta}\overline{P}(A \cap B)$.
7) $_{\delta\beta}\underline{P}(A \cup B) \ge {}_{\delta\beta}\underline{P}(A) + {}_{\delta\beta}\underline{P}(B) - {}_{\delta\beta}\underline{P}(A \cap B)$.

Definition 3.5. Let A be a event in the $\delta\beta-$topologized stochastic approximation space $S = (U, R_{\delta\beta}, P, \tau_K)$. Then from Lemma 2.9. We have

$$\underline{P}(A) \subseteq {}_{\beta}\underline{P}(A) \subseteq {}_{\delta\beta}\underline{P}(A) \subseteq P(A) \subseteq {}_{\delta\beta}\overline{P}(A) \subseteq {}_{\beta}\overline{P}(A) \subseteq \overline{P}(A).$$

Example 3.6. Consider the experiment of choosing one card from four cards numbered from one to four. The collection of the four elements forms the outcome space. Hence

$$U = \{1,2,3,4\}$$

Let R be a binary relation defined on U such that $R = \{(1,1), (1,2), (1,3), (2,3), (3,3), (3,4), (4,2)\}$. Thus $U/R = \{\{1,2,3\}, \{3\}, \{3,4\}, \{2\}\}$. Let $K = (U, R_{\delta\beta})$ be a $\delta\beta-$ approximation space and τ_K is the topology associated to R . Thus

$$\tau_K = \{U, \phi, \{2\}, \{3\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{2,3,4\}\} \text{ and } \tau_K^* = \{U, \phi, \{1\}, \{4\}, \{1,2\}, \{1,4\}, \{1,2,4\}, \{1,3,4\}\}.$$

Then the class $\delta\beta O(X)$ and $\delta\beta C(X)$ associated with this topology

$$\delta\beta O(U) = P(U) - \{1\} \text{ and } \delta\beta C(U) = P(U) - \{2,3,4\}.$$

where $P(U)$ the family of all subsets of U.

Define the random variable X to be the number on the chosen card. We can construct Table 3.1. Which contains the $\delta\beta-$lower and the $\delta\beta-$upper probabilities of a random variable $X = x$ as the following:

Table 3.1. represent $\delta\beta-$Lower and $\delta\beta-$upper probabilities of a random variable X .

X	1	2	3	4
${}_{\delta\beta}\underline{P}(X=x)$	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
${}_{\delta\beta}\overline{P}(X=x)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

3.2 Rough Distribution Function

The distribution function of a random variable X gives the probability that X does not exceed X. We define the $\delta\beta-$lower and $\delta\beta-$upper distribution functions of a random variable X .

Definition 3.7. Let X be a random variable in the $\delta\beta-$topologized stochastic approximation space $S=(U,R_{\delta\beta},P,\tau_K)$. The $\delta\beta-$lower distribution (resp. $\delta\beta-$upper distribution) function of X is denoted by ${}_{\delta\beta}\underline{F}(x)$ (resp. ${}_{\delta\beta}\overline{F}(x)$) and is defined by:

$$ {}_{\delta\beta}\underline{F}(x)={}_{\delta\beta}\underline{P}(X\le x) . $$
$$ \left(\text{resp. } {}_{\delta\beta}\overline{F}(x)={}_{\delta\beta}\overline{P}(X\le x)\right). $$

Definition 3.8. Let X be a random variable in the $\delta\beta-$topologized stochastic approximation space $S=(U,R_{\delta\beta},P,\tau_K)$. The $\delta\beta-$rough distribution function of X is given by:

$$ {}_{\delta\beta}F^{*}(x)=({}_{\delta\beta}\underline{F}(x),{}_{\delta\beta}\overline{F}(x)) . $$

Example 3.9. Consider the same experiment in Example 3.6. Then the $\delta\beta-$lower and $\delta\beta-$upper distributions of X are

$$ {}_{\delta\beta}\underline{F}(x)=\begin{cases} 0 & -\infty<x<2 \\ \frac{1}{4} & 2\le x<3 \\ \frac{2}{4} & 3\le x<4 \\ \frac{3}{4} & 4\le x<\infty \end{cases} \text{, and } {}_{\delta\beta}\overline{F}(x)=\begin{cases} 0 & -\infty<x<1 \\ \frac{1}{4} & 1\le x<2 \\ \frac{2}{4} & 2\le x<3 \\ \frac{3}{4} & 3\le x<4 \\ \frac{4}{4} & 4\le x<\infty \end{cases} . $$

3.3 Rough Expectation

Definition 3.10. Let X be a random variable in the $\delta\beta-$topologized stochastic approximation space $S=(U,R_{\delta\beta},P,\tau_K)$. The $\delta\beta-$lower (resp. $\delta\beta-$upper) expectation of X is denoted by ${}_{\delta\beta}\underline{E}(X)$ (resp. ${}_{\delta\beta}\overline{E}(X)$) and is defined by:

$$ {}_{\delta\beta}\underline{E}=\sum_{k=1}^{n}x_k \ {}_{\delta\beta}\underline{P}(X=x_k) $$

$$ \left(\text{resp. } {}_{\delta\beta}\overline{E}(X)=\sum_{k=1}^{n}x_k \ {}_{\delta\beta}\overline{P}(X=x_k)\right). $$

Definition 3.11. Let X be a random variable in the $\delta\beta-$topologized stochastic approximation space $S=(U,R_{\delta\beta},P,\tau_K)$. The $\delta\beta-$rough expectation of X is given by:

$$ {}_{\delta\beta}E^{*}(X)=({}_{\delta\beta}\underline{E}(X),{}_{\delta\beta}\overline{E}(X)) . $$

Example 3.12. consider the same Experiment in example 3.6. Then $\delta\beta-$lower and $\delta\beta-$upper expectations of X are

$$_{\delta\beta}\underline{E}(X)=2.25, \qquad _{\delta\beta}\overline{E}(X)=2.5.$$

Hence, the $\delta\beta-$rough expectation

$$_{\delta\beta}E^{*}(X)=(2.25\,,\,2.5).$$

Theorem 3.13. Let X be a random variable in the $\delta\beta-$topologized stochastic approximation space $S=(U,R_{\delta\beta},P,\tau_K)$. For any a , b , we have,

$$_{\delta\beta}\underline{E}(a\,X+b)=a\ _{\delta\beta}\underline{E}(X)+b\,c\,,\quad 0\le c\le 1.$$

Proof.

$$_{\delta\beta}\underline{E}(a\,X+b)=\sum_{k=1}^{n}(a\,x_k+b)\ _{\delta\beta}\underline{P}(X=x_k)=\sum_{k=1}^{n}a\,x_k\ _{\delta\beta}\underline{P}(X=x_k)+\sum_{k=1}^{n}b\ _{\delta\beta}\underline{P}(X=x_k)$$

$$=a\sum_{k=1}^{n}x_k\ _{\delta\beta}\underline{P}(X=x_k)+b\sum_{k=1}^{n}{}_{\delta\beta}\underline{P}(X=x_k)$$

$$=a\ _{\delta\beta}\underline{E}(X)+b\,c\,,\quad c=\sum_{k=1}^{n}{}_{\delta\beta}\underline{P}(X=x_k).\ \text{i.e.}\ 0\le c\le 1.$$

Theorem 3.14. Let X be a random variable in the $\delta\beta-$topologized stochastic approximation space $S=(U,R_{\delta\beta},P,\tau_K)$. For any a , b , we have,

$$_{\delta\beta}\overline{E}(a\,X+b)=a\ _{\delta\beta}\overline{E}(X)+b\,d\,,\quad 0\le d\le 1,\ d\in N^{+}\ .$$

$$\text{where }\ d=\textstyle\sum_{k=1}^{n}{}_{\delta\beta}\overline{P}(x_k)\,.$$

Proof. The proof is similar to Theorem 3.13.

3.4 Rough Variance

Definition 3.15. Let X be a random variable in the $\delta\beta-$topologized stochastic approximation space $S=(U,R_{\delta\beta},P,\tau_K)$. The $\delta\beta-$ lower (resp. $\delta\beta-$ upper) variance of X is denoted by $_{\delta\beta}\underline{V}(X)$ (resp. $_{\delta\beta}\overline{V}(X)$) and is defined by:

$$_{\delta\beta}\underline{V}(X)={}_{\delta\beta}\underline{E}(X-{}_{\delta\beta}\underline{E}(X))^2.$$

$$\left(\text{resp. } _{\delta\beta}\overline{V}(X)={}_{\delta\beta}\overline{E}(X-{}_{\delta\beta}\overline{E}(X))^2\right).$$

Definition 3.16. Let X be a random variable in the $\delta\beta-$topologized stochastic approximation space $S=(U,R_{\delta\beta},P,\tau_K)$. The $\delta\beta-$rough variation of X is given by:

$$_{\delta\beta}V^{*}(X)=(\ _{\delta\beta}\underline{V}(X),{}_{\delta\beta}\overline{V}(X)).$$

Example 3.17. consider the same Experiment in Example 3.6. Then $\delta\beta-$lower and $\delta\beta-$upper variance of X are

$$_{\delta\beta}\underline{V}(X)=0.921875\,, \qquad _{\delta\beta}\overline{V}(X)=1.25\,.$$

Hence the $\delta\beta-$rough variance

$$_{\delta\beta}V^{*}(X)=(0.921875\,,1.25)\,.$$

Theorem 3.18. Let X be a random variable in a $\delta\beta-$topologized stochastic approximation space $S=(U,R_{\delta\beta},P,\tau_K)$. For any a , b we have,

$$_{\delta\beta}\underline{V}(X){=}_{\delta\beta}\underline{E}(X)^2-(2-c)\,(_{\delta\beta}\underline{E}(X))^2\,.$$

$$\text{where } c=\sum_{k=1}^{n}{}_{\delta\beta}\underline{P}(x_k)\,.$$

Proof.

$$_{\delta\beta}\underline{V}(X){=}_{\delta\beta}\underline{E}\left(X-_{\delta\beta}\underline{E}(X)\right)^2{=}_{\delta\beta}\underline{E}\left(X^2-2X\,_{\delta\beta}\underline{E}(X)+(_{\delta\beta}\underline{E}(X))^2\right)$$

$$={}_{\delta\beta}\underline{E}\left(X^2\right)-2_{\delta\beta}\underline{E}(X)_{\delta\beta}\underline{E}(X)+_{\delta\beta}\underline{E}\left(_{\delta\beta}\underline{E}(X)\right)^2$$

$$={}_{\delta\beta}\underline{E}\left(X^2\right)-2\left(_{\delta\beta}\underline{E}(X)\right)^2+c\left(_{\delta\beta}\underline{E}(X)\right)^2 \text{ where } c=\sum_{k=1}^{n}{}_{\delta\beta}\underline{P}(x_k)$$

$$={}_{\delta\beta}\underline{E}\left(X^2\right)-(2-c)\left(_{\delta\beta}\underline{E}(X)\right)^2.$$

Theorem 3.19. Let X be a random variable in a $\delta\beta-$topologized stochastic approximation space $S=(U,R_{\delta\beta},P,\tau_K)$. For any a , b we have,

$$_{\delta\beta}\overline{V}(X){=}_{\delta\beta}\overline{E}(X)^2-(2-d)\left(_{\delta\beta}\overline{E}(X)\right)^2 \quad \text{where } d=\sum_{k=1}^{n}{}_{\delta\beta}\overline{P}(x_k)\,.$$

Proof. The proof is similar to Theorem 3.18.

Theorem 3.20. Let X be a random variable in the $\delta\beta-$topologized stochastic approximation space $S=(U,R_{\delta\beta},P,\tau_K)$. For any a , b we have,

$$_{\delta\beta}\underline{V}(aX+b)=a^2\,_{\delta\beta}\underline{E}(X)^2-(2a-c)\left(_{\delta\beta}\underline{E}(X)\right)^2+2b(a-c)_{\delta\beta}\underline{E}(X)+b^2c$$

$$\text{where } c=\sum_{k=1}^{n}{}_{\delta\beta}\underline{P}(x_k)\,.$$

Proof.

$$_{\delta\beta}\underline{V}(aX+b){=}_{\delta\beta}\underline{E}\left((aX+b)-_{\delta\beta}\underline{E}(X)\right)^2$$

$$={}_{\delta\beta}\underline{E}(aX+b)^2-2_{\delta\beta}\underline{E}(aX+b)_{\delta\beta}\underline{E}(X)+_{\delta\beta}\underline{E}\left(_{\delta\beta}\underline{E}(X)\right)^2$$

$$={}_{\delta\beta}\underline{E}\left(a^2X^2+2abX+b^2-2aX\,\underline{E}(X)-2b\underline{E}(X)+_{\delta\beta}\underline{E}\left(_{\delta\beta}\underline{E}(X)\right)^2\right)$$

$$={}_{\delta\beta}\underline{E}\left(a^2X^2\right)+2ab\,{}_{\delta\beta}\underline{E}(X)+b^2c-2a\left({}_{\delta\beta}\underline{E}(X)\right)^2-2bc\,{}_{\delta\beta}\underline{E}(X)+c\left({}_{\delta\beta}\underline{E}(X)\right)^2$$

$$\text{where } c=\sum_{k=1}^{n}{}_{\delta\beta}\underline{P}(x_k)$$

$$=a^2\,{}_{\delta\beta}\underline{E}(X)^2-(2a-c)\left({}_{\delta\beta}\underline{E}(X)\right)^2+2b\,(a-c)\,{}_{\delta\beta}\underline{E}(X)+b^2c$$

Theorem 3.21. Let X be a random variable in the $\delta\beta-$topologized stochastic approximation space $S=(U,R_{\delta\beta},P,\tau_K)$. For any a, b we have,

$${}_{\delta\beta}\overline{V}(aX+b)=a^2\,{}_{\delta\beta}\overline{E}(X)^2-(2a-d)\left({}_{\delta\beta}\overline{E}(X)\right)^2+2b(a-d)\,{}_{\delta\beta}\overline{E}(X)+b^2d$$

$$\text{where } d=\sum_{k=1}^{n}{}_{\delta\beta}\overline{P}(x_k).$$

Proof. The proof is similar to Theorem 3.20.

3.5 Rough Moment Generating Function and Rough Characteristic Function

Definition 3.22. Let X be a random variable in the $\delta\beta-$topologized stochastic approximation space $S=(U,R_{\delta\beta},P,\tau_K)$. Then the $\delta\beta-$ lower (resp. $\delta\beta-$ upper) moment generating function of X is denoted by ${}_{\delta\beta}\underline{M_X}(t)$ (resp. ${}_{\delta\beta}\overline{M_X}(t)$) and is defined by:

$${}_{\delta\beta}\underline{M_X}(t)={}_{\delta\beta}\underline{E}\left(e^{tX}\right).$$

$$\left(\text{resp. } {}_{\delta\beta}\overline{M_X}(t)={}_{\delta\beta}\overline{E}\left(e^{tX}\right)\right).$$

Definition 3.23. Let X be a random variable in the $\delta\beta-$topologized stochastic approximation space $S=(U,R_{\delta\beta},P,\tau_K)$. Then the $\delta\beta-$rough moment generating function of X is defined by:

$${}_{\delta\beta}M^*(t)=\left({}_{\delta\beta}\underline{M_X}(t),{}_{\delta\beta}\overline{M_X}(t)\right).$$

Example 3.24. consider the same Experiment in Example 3.6. Then $\delta\beta-$ lower and $\delta\beta-$ upper moment generating function of X are

$${}_{\delta\beta}\underline{M_X}(t)=\frac{1}{4}\left(e^{2t}+e^{3t}+e^{4t}\right) \text{ and } {}_{\delta\beta}\overline{M_X}(t)=\frac{1}{4}\left(e^{t}+e^{2t}+e^{3t}+e^{4t}\right).$$

Definition 3.25. Let X be a random variable in the $\delta\beta-$topologized stochastic approximation space $S=(U,R_{\delta\beta},P,\tau_K)$. If $Z=\dfrac{X+a}{b}$,for any two constants a,b we have

$${}_{\delta\beta}\underline{M_Z}(t)=e^{\left(\frac{a}{b}\right)t}\,{}_{\delta\beta}\underline{M_X}\left(\frac{t}{b}\right)$$

Proof.

$$_{\delta\beta}\underline{M}_Z(t) = {}_{\delta\beta}\underline{E}\left(e^{t\left(\frac{X+a}{b}\right)}\right) = {}_{\delta\beta}\underline{E}\left(e^{\left(\frac{a}{b}\right)t} .e^{t\left(\frac{X}{b}\right)}\right).$$

$$= e^{\left(\frac{a}{b}\right)t} \; {}_{\delta\beta}\underline{E}\left(e^{t\left(\frac{X}{b}\right)}\right) = e^{\left(\frac{a}{b}\right)t} \; {}_{\delta\beta}\underline{M}_X\left(\frac{t}{b}\right).$$

Definition 3.26. Let X be a random variable in the $\delta\beta-$topologized stochastic approximation space $S=(U,R_{\delta\beta},P,\tau_K)$. If $Z=\dfrac{X+a}{b}$,for any two constants a,b we have

$$_{\delta\beta}\overline{M}_Z(t) = e^{\left(\frac{a}{b}\right)t} \; {}_{\delta\beta}\overline{M}_X\left(\frac{t}{b}\right)$$

Proof. The proof is similar to Theorem 3.25.

Definition 3.27. Let X be a random variable in the $\delta\beta-$topologized stochastic approximation space $S=(U,R_{\delta\beta},P,\tau_K)$. Then the $\delta\beta-$ lower (resp. $\delta\beta-$upper) characteristic function of X is denoted by $_{\delta\beta}\underline{\phi_X}(t)$ (resp. $_{\delta\beta}\overline{\phi_X}(t)$) and is defined by:

$$_{\delta\beta}\underline{\phi_X}(t) = {}_{\delta\beta}\underline{E}\left(e^{itX}\right) = \sum_X e^{itX} \; {}_{\delta\beta}\underline{P}(X).$$

$$\left(\text{resp. } {}_{\delta\beta}\overline{\phi_X}(t) = {}_{\delta\beta}\overline{E}\left(e^{itX}\right) = \sum_X e^{itX} \; {}_{\delta\beta}\overline{P}(X)\right).$$

Example 3.28. consider the same Experiment in Example 3.6. Then $\delta\beta-$lower and $\delta\beta-$upper characteristic function of X are

$$_{\delta\beta}\underline{M}_X(t) = \frac{1}{4}\left(e^{2it}+e^{3it}+e^{4it}\right) \text{ and } {}_{\delta\beta}\overline{M}_X(t) = \frac{1}{4}\left(e^{it}+e^{2it}+e^{3it}+e^{4it}\right).$$

Definition 3.29. Let X be a random variable in the $\delta\beta-$topologized stochastic approximation space $S=(U,R_{\delta\beta},P,\tau_K)$. If $Z=aX+b$,for any two constants a,b we have

$$_{\delta\beta}\underline{\phi_Z}(t) = e^{ibt} \; {}_{\delta\beta}\underline{\phi_X}(at).$$

Proof.

$$_{\delta\beta}\underline{\phi_Z}(t) = {}_{\delta\beta}\underline{E}\left(e^{it(aX+b)}\right) = {}_{\delta\beta}\underline{E}\left(e^{ibt} .e^{i(ta)X}\right).$$

$$= e^{ibt} \; {}_{\delta\beta}\underline{E}\left(e^{i(ta)X}\right) = e^{ibt} \; {}_{\delta\beta}\underline{\phi_X}(at).$$

Definition 3.30. Let X be a random variable in a $\delta\beta-$topologized stochastic approximation space

$S = (U, R_{\delta\beta}, P, \tau_K)$. If $Z = a\,X + b$, for any two constants a, b we have

$$_{\delta\beta}\overline{\phi_Z}(t) = e^{ibt}\ _{\delta\beta}\overline{\phi_X}(at).$$

Proof. The proof is similar to Theorem 3.29.

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