Mathematical Theory and Modeling www.iiste.org
ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)
Vol.2, No.6, 2012

Correlation between Averages Times of Random Walks On an Irregularly Shaped Objects and the Fractal Dimensions

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Abstract

This study is strongly motivated by fractal dimensions concept and dearth of relevant literatures aimed at correlating the average time of Random Walks on irregularly shaped planar objects and their corresponding fractal dimensions. Twenty selected countries maps from around the World were investigated to determine their random walk parameters and their fractal dimensions. The images (maps) were scanned, burned and saved as black and white Jpeg files. A program – visual basic 6.0 was developed for fractal dimensions and random parameters estimation. The program calculated: for each image number of boxes around the boundary of an image for 20x20 grids, and the distance covered by 100 random walkers on an image for the same number of grids. Thereafter, the program displayed a log-log graph of boundary count versus grids and log-log average time versus distance. The corresponding slope of line of best fit represents respectively the fractal dimension and random walks parameters. Preliminary investigation focusing grid size suggests 20x20 grids as non-compromising in terms of results reliability and computation expenses. The random parameters estimated based on 20x20 ranges from 1.976 to 2.995 while the estimated fractal dimensions range from 1.116 to1.212. The correlation of fractal dimensions and random walks parameter had regression value $R^2 = 0.014$. The estimated fractal dimension showed that maps (images) boundaries are statistically fractal. However, there is no correlation between their fractal dimensions and the corresponding random parameters.

KEY WORDS: Random walk, Shapes, Grids, Parameters, Dimensions

1. Introduction

Dimensions can be defined as the number of parameters or coordinates that are used for describing mathematical objects. For example, lines are defined as one-dimensional, squares as two-dimensional and cones as three-dimensional still, there are some objects such as clouds, trees veins, lungs, oceans, mountains, coastline and so on that could not be described adequately with a whole number dimensions Thus, measuring or drawing many irregularly shaped objects with Euclidean dimensions would have posed problems to science world if not for the timely discoveries of fractals and chaos.

Fractals proffer a different way of observing and modelling the complex in nature than Euclidean geometry. Consequently, blending disciplinary science of complexity coupled with the power of computers bring new tools and techniques for studying real world systems (Arakawa & Krotkov 1994).

Chaos theory provides us with a new perspective to view the World. For centuries, we thought of all things around us as made up of Cartesians such as lines, rectangles, polygons, etc as building blocks to understand objects around us. The emerging science of complex systems brought about remarkable insights into nature of universe and life. It cuts across all disciplinary boundaries as complex systems; abound in physical, biological, mathematical, meteorological and many other fields (Gleick, 1998).

Meanwhile, chaos science uses a different geometry called fractal geometry – a new language to describe models and analyze complex found in nature (Kellert 1993).

There has been considerable interest recently in chaos theory and fractal geometry as we find that many processes in the world can be accurately described using the theory. In fact the computer graphics industry is rapidly incorporating these techniques in dedicated graphic rendering CADS to generate stunningly; images as well as realistic natural looking structures (http://www.faemalia.net/fractals:visited: December, 2006.)

However, as a complete surprise, rules of random growth which are as simple as seem trivial are in fact capable of generating an unimaginably rich variety of fractal shapes. The finding has immediately fired many skilful and ardent investigators worldwide and soon proved to involve a fascinating interface between the roughness associated with fractals and the smoothness associated with the Laplace equation (Mandelbrot 1991)

The concept 'fractal' was known for more than two decades ago. The term fractal comes from "fractus" which is a Latin word for broken or fragmented object. Mandelbrot proposed the idea of fractal (fractional dimension) as a way to cope with problems of scale in the real World. He then defined a fractal to be any curve or surface that is independent of scale. This property, referred to as SELF SIMILARITY, means that any portion of the curve, if blown up in scale, would appear identical to the whole curve (Mandelbrot 1991).

Another key characteristic is a mathematical parameter called its fractal dimensions. Unlike Euclidean dimension, fractal is generally expressed by non integer (fraction) rather than by a whole number The concept of "fractal dimension" provides a way to measure how rough fractal curves are. We normally consider lines to have a dimension of 1, surfaces a dimension of 2 and solids a dimension of 3.

However, a rough curve (say) wanders around on a surface; in extreme may be so rough that it effectively fills the surface on which it lies. Twisted surfaces, such as a tree's foliage or internal surfaces of lungs may effectively be described as three- dimensional structures. We can therefore think of roughness as an increase in dimension: a rough curve has a dimension somewhere between 2 and 3. The dimension of a fractal curve is a number that characterizes the way in which the measured length between given points increase and decrease.

Whilst the topological dimension of a line is always 1 and that of surface is always 2, hence the fractal dimension may be any real number between 1 and 2.

The significance of random walk and Brownian motion are felt in the sciences, such as mathematics, chemistry and physics. In random walk each steps is made without thought and also, with no direction. Brownian motion is the fractional Brownian motion model. This model regards naturally occurring rough surfaces (like mountains, clouds and trees as the end result of "random walks" and utilizes a random iteration algorithm to produce fractal patterns. As a result, many scientific theories and applications related to it have been developed and they subsequently play major roles in the world of physics. After series of experiments made, a conclusion that the origin of this motion was physical, not biological but was caused by collisions with invisible molecules (Thomas Spencer, 2006).

From the foregoing it became obvious that there is dearth of literature that focuses issue bothering on concepts of fractal dimension and random walks parameter which is the object of present investigation.

2. Methodology

Image is defined as a representation of an object. Image is the technical name for a picture when it is stored in a computer, production of permanent images by means of the action of light on sensitized surfaces, giving rise to a new form of visual arts, historical documents and scientific tool (Microsoft Encarta 2004).

For this work, twenty different types of maps were scanned, burned on compact disc and saved as black and white Jpeg files. A program-visual basic 6.0 version – was written to perform the analysis. Fractal dimension was then used to analyse each of the images (maps).

Consequently, the fractal images used were all loaded one after the other and subdivided into grids for the purpose of analysis. The area to be traversed by the random walkers was coloured black while the space outside the boundary was coloured white for each map investigated. This does not mean that other colours should not be used but at a time, two colours must be used for the program to recognize the image as well as the boundary.

To save the time and the resources of the computer, the algorithm was written to favour fig 1b or c i.e. 2, 1 is equivalent to 1, 2; 2, 3 is also equivalent to 3, 2 and so on.

11	12	13
21	22	23
31	32	33

a

11	12	13
	22	23
		33

b

OR

11		
21	22	
31	32	33

c

Fig. 1

For random walk

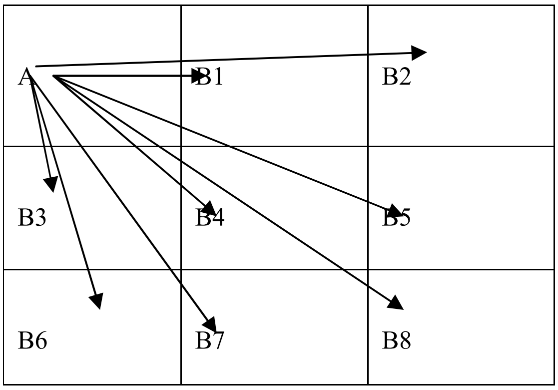

Fig.: 2

The governing equation for random walks parameters is given as

$$T_{ave} \propto D_r^{\beta}$$

For boundary box counts

This shows the number of boxes covered by the image to the number of grids at a time, for example when the number of grid was 1 the box covered by the image was 1, when the number of the grid increased to 2, the number of boxes covered by the image was 2 and soon.

Here, the program also calculates the number of boxes on the boundary of an image investigated and the number of dimensions of the grids at a time. The log-log boundary box counts versus grids were then plotted for each of the maps investigated. Fractal dimension then found the slope of the best fit straight line which is the fractal dimensions of the image.

Governing equation for fractal Dimension

$$Box\,Count(B) \propto (Grid\ Size)^{D} \qquad (1.2)$$

The results got for each map for both the random walk parameters and fractal dimensions were then correlated.

3. Results and Discussions

To select the numbers of the grids and the random walkers to be used for the analysis some pertinent questions came up to mind as to any essential features that characterized each image and their significance, viza- viz:

- What effect does the numbers of random walkers has on the results?
- Does the roughness and size of the image edge have any significant effect on the results?

- What effect does the number of divisions of the grids have on the results?

Preliminary investigation experimented different numbers of grid size. Thereafter 20x20 grids was identified as minimum computational expenses without compromising results reliability.

3.1 Analysis

The various existing maps of the world were fed as input into the program codes. The program was divided into two modules. The first module calculated the log-log average time versus distance while the second module calculated the log-log boundary box counts versus grids.

For each fractal image, two charts were obtained from the data generated. In essence, this work suppose to present a total number of forty one charts including that used for correlation of fractal dimensions and random walk parameters but nine charts and a table were presented because of the space.

4.0 Conclusion

Based on the investigation on fractal images sampled for this work, the following conclusions could be drawn.

- ✓ The outlines (boundary) of the images had fractal dimension ranging from 1.116 to 1.212 and for random walk the parameters ranging from 1.976 to 2.299.
- ✓ The value for regression R^2 (0.0134) was lesser than 0.5 for graph of random walk parameters versus fractal dimension for all the Maps investigated suggests no correlation. This means fractal dimension of the irregular edge of a planar object does not strongly depend on the parameter of a random walker on the same designated region.
- ✓ The fractal dimensions for all the images were greater than unity (>1) which suggests that the maps boundaries were all rough.

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Microsoft Encarta Premium suit 2004

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Engr. Olaiya, Kamorudeen Adesina is a graduate of University of Ibadan, Nigeria. He had his Bachelors Degree (B.Sc) and Masters of Science (M.Sc.) in Mechanical Engineering in the years 2000 and 2007 respectively.

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Engr. Olaiya K.A has many publications in the learned journals and international conference proceedings especially in area of design and related disciplines.

Engr. Olaiya K.A became a corporate member, Nigerian Society of Engineers (NSE) in 2009. He is a registered Engineer by the council for Regulations of engineering in Nigeria.(COREN) in 2010. He is married and blessed with children.

Dr. SALAU Tajudeen Abiola Ogunniyi is a senior Lecturer in the department of Mechanical Engineering, University of Ibadan, Nigeria. He joined the services of the University of Ibadan in February 1993 as Lecturer II in the department of Mechanical Engineering. By virtue of hard work, he was promoted to Lecturer 1 in 2002 and senior Lecturer in 2008.He had served the department in various capacities. He was the coordinator of the department for 2004/2005 and 2005/2006 Academic sessions. He was the recipient of M.K.O Abiola postgraduate scholarship in 1993/1994 session while on his Ph.D research programme in the University of Ibadan.

Dr. Salau has many publications in learned journals and international conference proceedings especially in the area of nonlinear dynamics. He had served as external examiner in departments of Mechanical Engineering of some institutions of higher learning in the country and a reviewer/rapporteur in some reputable international conference proceedings. His area of specialization is solid mechanics with bias in nonlinear dynamics: Fractal and chaos.

Dr. salau is a corporate member, Nigerian Society of Engineers (NSE). He is a registered Engineer by the council for Regulations of engineering in Nigeria.(COREN). He is happily married and blessed with children.

Appendices

Figure 1. Map of Argentina

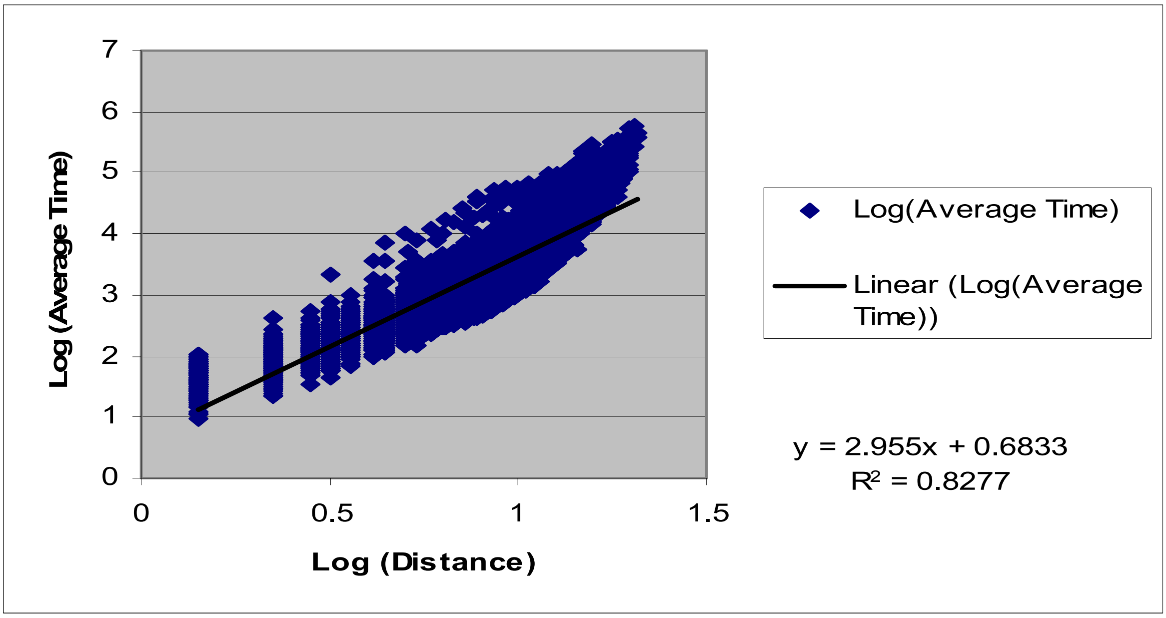

Figure 2. Map of Argentina for random walk

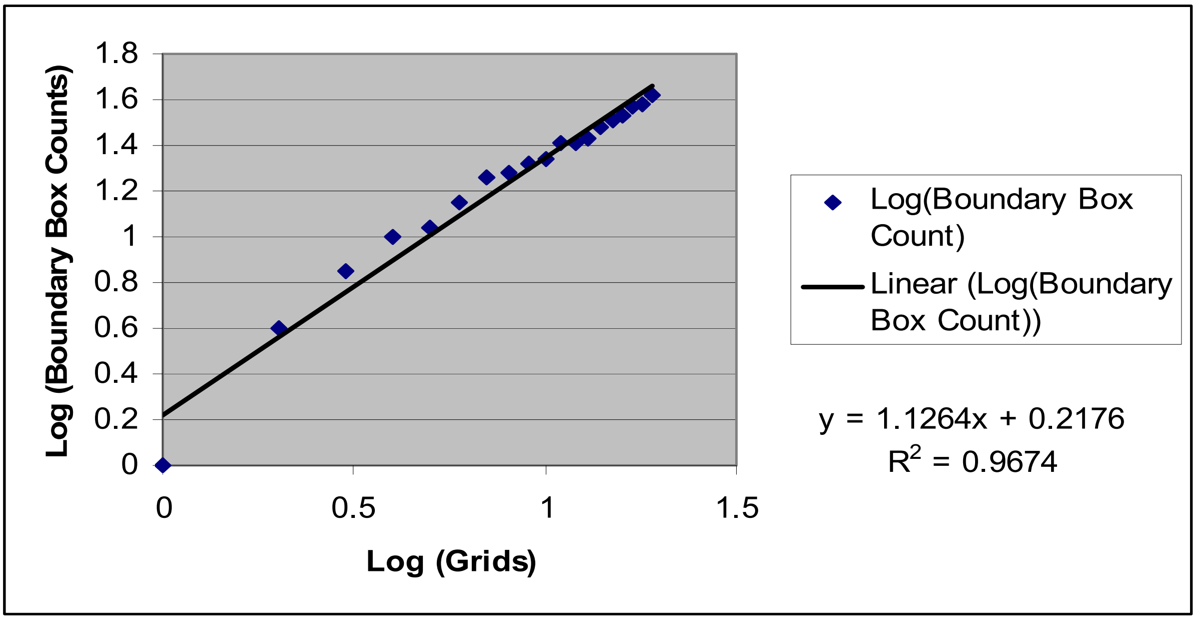

Figure 3. Map of Argentina for Boundary Box Counts

Figure 4. Map of Brazil

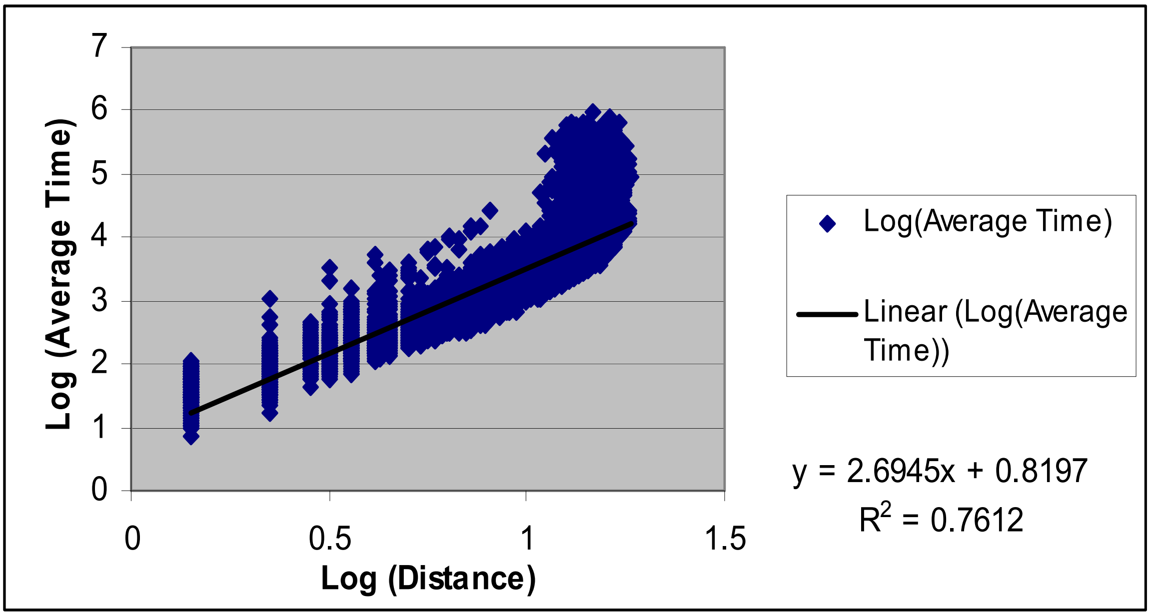

Figure 5. Map of Brazil for random walks

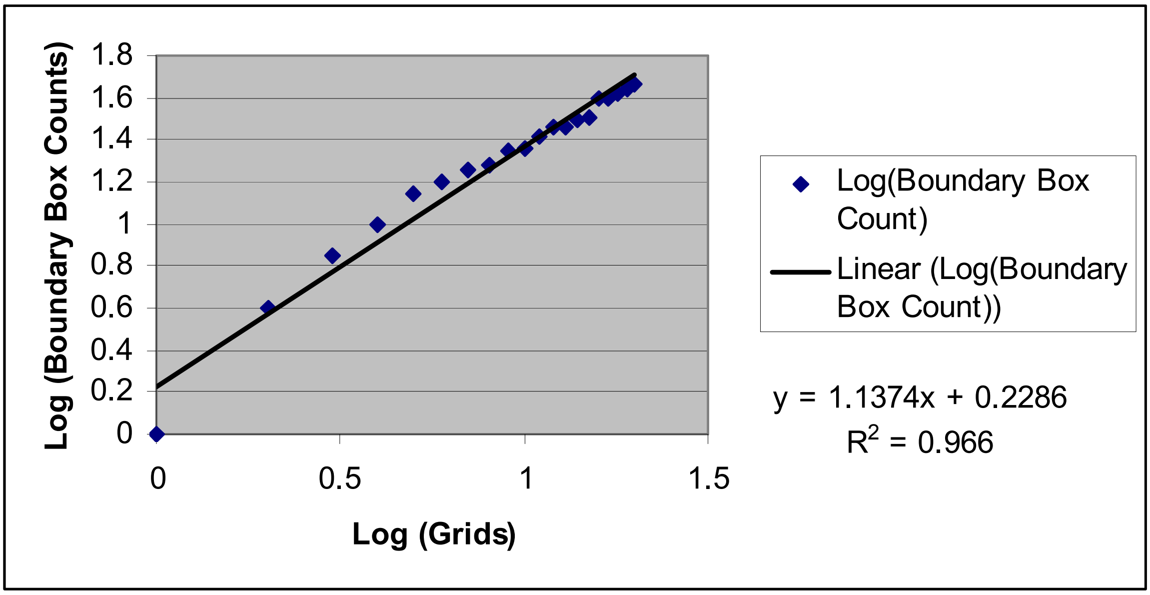

Figure 6. Map of Brazil for Boundary Box Counts

Figure 7. Map of Cameroon

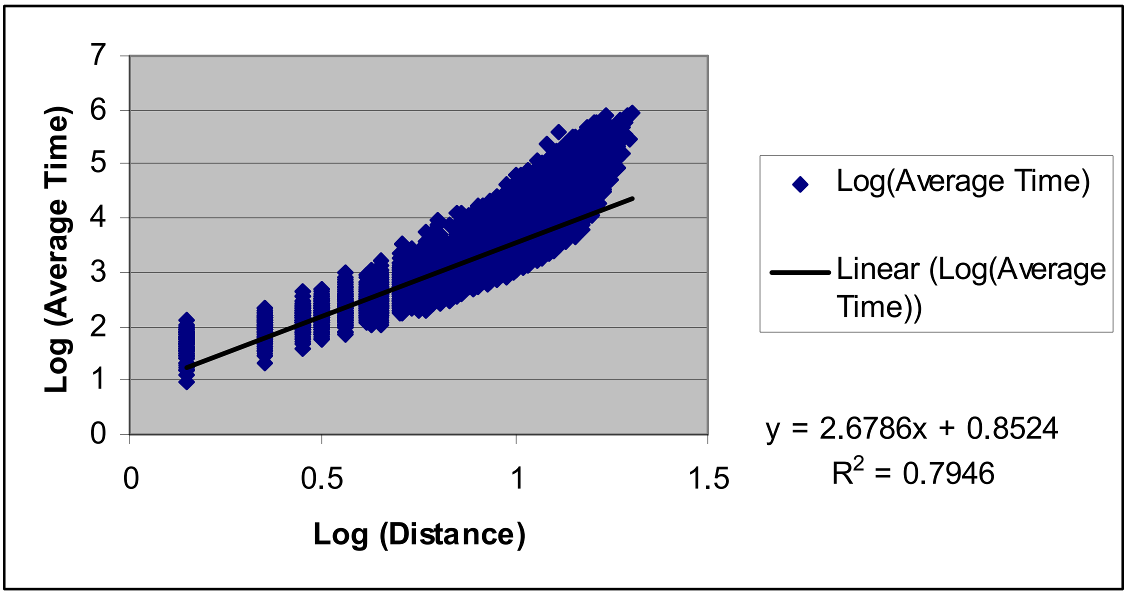

Figure 8. Map of Cameroon for random walks

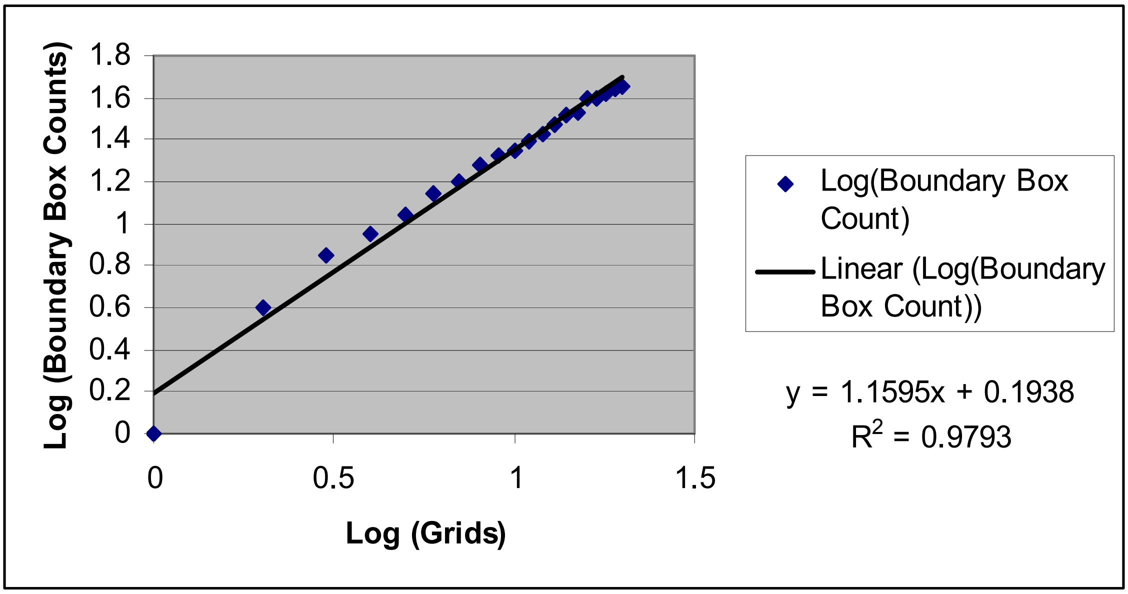

Figure 9. Map of Cameroon for Boundary Box Counts

Figure 10. Map of Chad

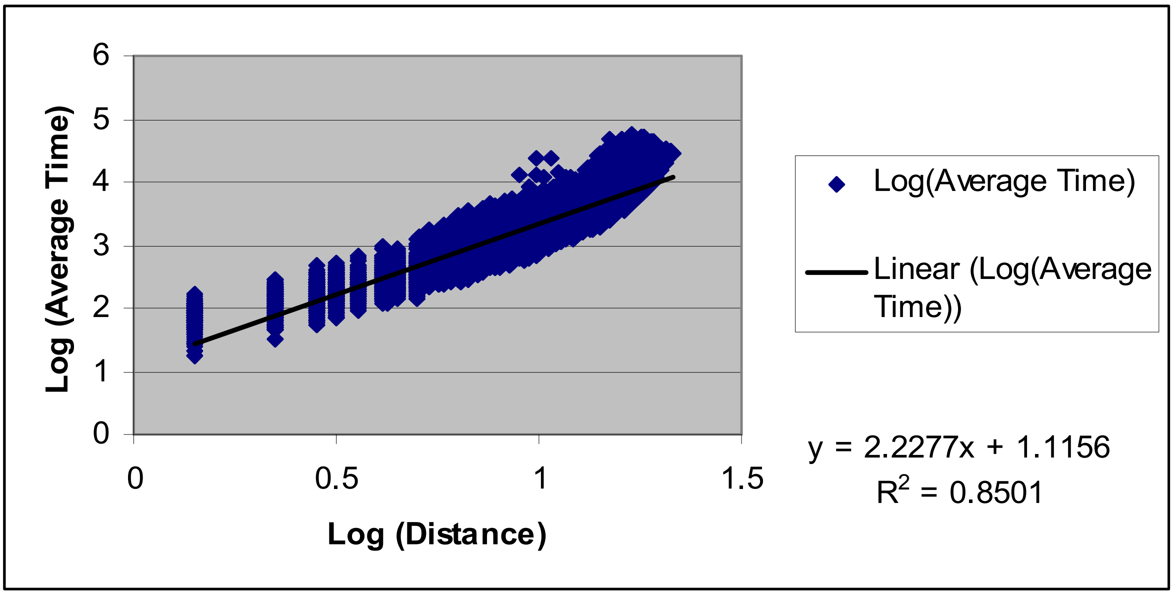

Figure 11. Map of Chad for random walks

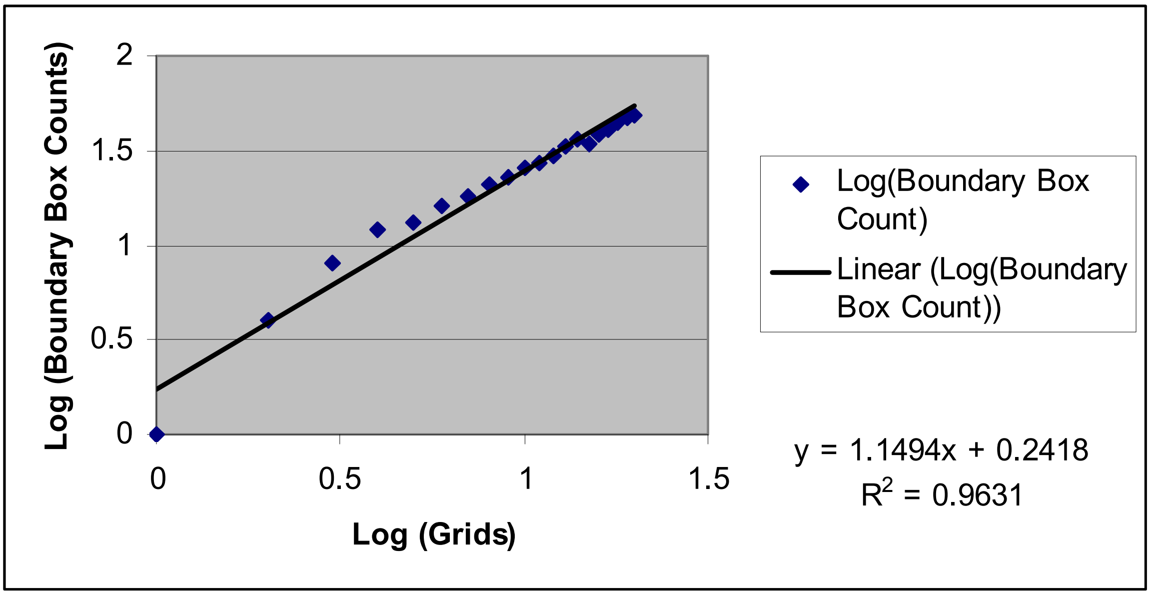

Figure 12. Map of Chad for Boundary Box Counts

Table 1. Table Used For The Correlation

S/N	FRACTAL IMAGES (MAP)	RANDOMWALK PARAMETERS	BOUNDARY BOXCOUNT DIMENSIONS
1	Argentina	2.9550	1.1231
2	Brazil	2.6945	1.1374
3	Cameroon	2.6786	1.1595
4	Chad	2.2277	1.1494
5	Egypt	2.2775	1.1684
6	Ethiopia	2.1409	1.1781
7	Finland	2.1661	1.1161
8	Ghana	1.9760	1.1543
9	Iraq	2.3724	1.2115
10	Libya	2.1153	1.2008
11	Mali	2.7974	1.2022
12	Mauritania	2.2146	1.1705
13	Morocco	2.6370	1.1897
14	Niger	2.3429	1.1675
15	Nigeria	2.0592	1.1865
16	Portugal	2.7373	1.1222
17	Romania	2.0547	1.1443
18	Saudi-Arabia	2.4464	1.1483
19	Spain	2.7033	1.1828
20	Venezuela	2.8659	1.1664

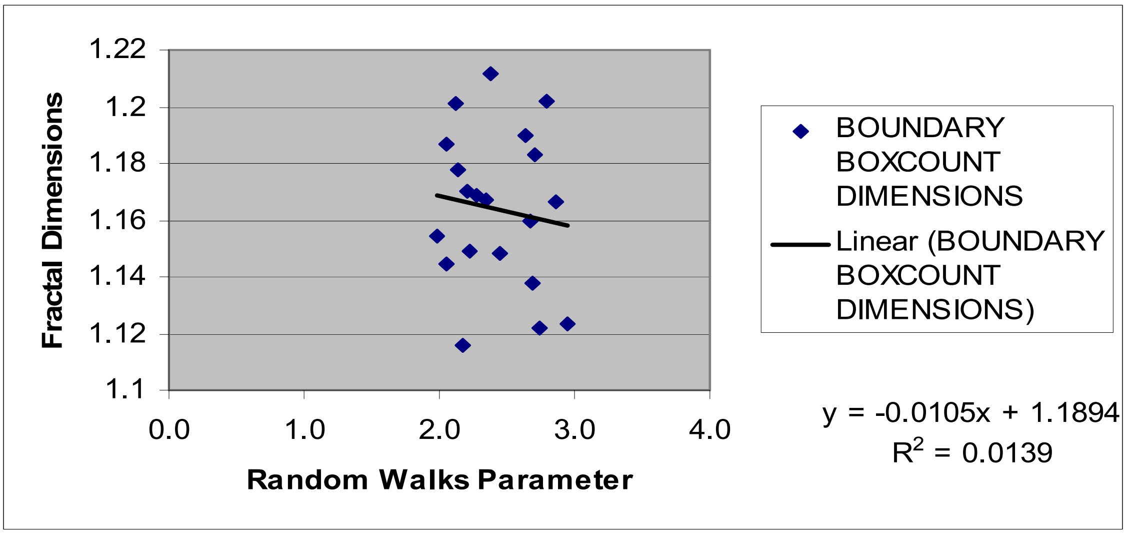

Figure 13. Correlated Values of Fractal Dimensions against Random Parameters for all the Images investigated