

Fuzzy Fault Tree Analysis using Level (λ, ρ) Interval-Valued Fuzzy Numbers

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Abstract

In this study we have discussed a fuzzy fault tree using level (λ, ρ) interval-valued fuzzy numbers. Level (λ, ρ) interval-valued fuzzy numbers are one of the extensions of fuzzy numbers and they have been applied to fault-tree analysis in many studies. The arithmetic operations of (λ, ρ) interval-valued fuzzy numbers have also been addressed in the study. In fuzzy fault tree, we have computed failure rate of fire protection system taking failure rate of each component of the system as (λ, ρ) interval-valued fuzzy numbers.

Keywords: Fuzzy fault tree, interval-valued fuzzy number, Signed distance, Failure rate

1. Introduction

Ever since the fuzzy set was proposed by Zadeh in 1965, fuzzy numbers (triangular fuzzy numbers and trapezoidal fuzzy numbers) have been widely studied, developed, and applied to various fields, such as multi-criteria decision-making, logic programming, pattern recognition and many more. In fuzzy set, the degree of membership functions of the element in the universe is having a single value: either zero or one. Many times specialists are uncertain about the values of the membership of an element in a set. Hence, it is better to represent the values of the membership of an element in a set by intervals of possible real numbers instead of real numbers. An interval-valued fuzzy set on a universe X is a mapping from X to fall closed sub-intervals of the real interval [0, 1]. This type of fuzzy sets has been intensively investigated, not only its theoretical aspects, but also its numerous applications. Level (λ, ρ) interval-valued fuzzy numbers are one of the extensions of fuzzy numbers (triangular fuzzy numbers and trapezoidal fuzzy numbers).

The concept of fault tree analysis (FTA) was developed in 1962 at Bell Telephone Laboratories. FTA is now widely used in many fields, such as in nuclear reactor, chemical industry, aviation industry, etc. FTA is a logical and diagrammatic method for evaluating system reliability. It is a logical approach for systematically quantifying the possibility of abnormal system event. For such systems, it is, therefore, unrealistic to assume a crisp number (classical) for different basic events. Zadeh (1965) suggested a paradigm shift from a theory of total denial and affirmation to a theory of grading to give new concept of fuzzy set. Tanaka and Singer (1983, 1990) used fuzzy set theory to replace crisp numbers by fuzzy numbers for better estimation of possibility of top event in FTA. Suresh et al. (1996) used a method based on α-cuts to deal with FTA, treating the failure possibility as triangular and trapezoidal fuzzy numbers. Huey-Ming Lee (2012) provided fuzzy parallel system reliability analysis based on level (λ, ρ) interval-valued fuzzy numbers. Ching-Fen Fuh (2014) used level (λ, 1) interval-valued fuzzy numbers to evaluate fuzzy reliability of systems. G. S. Mahapatra et al. (2010) proposed fuzzy fault tree analysis using intuitionistic fuzzy numbers. Sanjay Kumar Tyagi et al. (2010) used fuzzy set to analysis fuzzy fault tree. Mahapatra and Roy (2009) presented triangular intuitionistic fuzzy number and used it for reliability evaluation. M. Chen (2003) evaluated fuzzy system reliability using vague set theory. Singer (1990) proposed a method using fuzzy numbers to represent the relative frequencies of the basic events. He used possibilistic 'AND' and 'OR' operators to construct possible fault tree. Wang Y.C. et al. (2000) developed fuzzy fault tree based on improved fuzzy arithmetic operator. Wang J. Q. et al. (2013) provided new operators on triangular intuitionistic fuzzy numbers and their applications in system fault analysis. Neeraj Lata (2013) presented the fuzzy fault tree analysis using intuitionistic fuzzy numbers. In above research articles, the authors used fuzzy and intuitionistic fuzzy number to analyze the fault tree. Level (λ, ρ) interval-valued fuzzy numbers are yet not used for fault tree analysis.

Keeping above fact in view, this study used level (λ, ρ) interval-valued fuzzy numbers to analyze the fuzzy fault tree. The failure rate of each component of fault tree represented by level (λ, ρ) interval-valued fuzzy numbers. Furthermore, to ensure easy defuzzification of failure of fault tree in the fuzzy sense we have used signed distance method. In this paper, section-2 defines level (λ, ρ) Interval-valued fuzzy sets. In section-3, we defined signed distance method. In section-4, the failures of components of fire protection system have been fuzzified. The last section concludes the work.

2. Preliminaries. In order to consider the fuzzy fault tree analysis based on level (λ, ρ) interval-valued fuzzy numbers, we provide following definitions:

Definition 2.1. A fuzzy set $\tilde{A}$ defined on R is called the level λ-triangular fuzzy number if its membership function is

$$\mu_{\tilde{A}}(x)=\begin{cases}\lambda\dfrac{(x-a)}{(b-a)}, & a\le x\le b\\ \lambda\dfrac{(c-x)}{(c-b)}, & b\le x\le c\\ 0, & \text{otherwise}\end{cases}$$

When $a<b<c$, $0<\lambda\le 1$, then $\tilde{A}$ is called the level λ fuzzy number and denoted by $\tilde{A}=(a,b,c;\lambda)$. When $\lambda=1$ it is known as triangular fuzzy number and denoted by $\tilde{A}=(a,b,c)$.

Definition 2.2. An interval-valued fuzzy set $\tilde{A}$ on R is given by

$$\tilde{A}=\{(x,[\mu_{\tilde{A}^L}(x),\mu_{\tilde{A}^U}(x)])\},\quad \forall x\in R,$$

where $0\le\mu_{\tilde{A}^L}(x)\le\mu_{\tilde{A}^U}(x)\le 1$ and $\mu_{\tilde{A}^L}(x),\mu_{\tilde{A}^U}(x)\in[0,1]$

and denoted by $\mu_{\tilde{A}}(x)=[\mu_{\tilde{A}^L}(x),\mu_{\tilde{A}^U}(x)],\ x\in R$ or $\tilde{A}=[\tilde{A}^L,\tilde{A}^U]$.

Let $\tilde{A}^L=(a,b,c;\lambda)$ and $\tilde{A}^U=(e,b,h;\rho)$, where $0<\lambda\le\rho\le 1$ and $e<a<b<c<h$.

Then the interval-valued fuzzy set is expressed as

$$\tilde{A}=[(a,b,c;\lambda),(e,b,h;\rho)].$$

The family of all level (λ, ρ) interval-valued fuzzy numbers is denoted by

$$F_{IV}(\lambda,\rho)=\{[(a,b,c;\lambda),(e,b,h;\rho)]\mid e<a<b<c<h\},$$

where $0<\lambda\le\rho\le 1$, $a,b,c,e,h\in R$.

The interval-valued fuzzy set $\tilde{A}$ indicates that, when the membership grade of X belongs to the interval $[\mu_{\tilde{A}^L}(x),\mu_{\tilde{A}^U}(x)]$, the largest grade is $\mu_{\tilde{A}^U}(x)$ and the smallest grade is $\mu_{\tilde{A}^L}(x)$.

Let

$$\mu_{\tilde{A}^L}(x)=\begin{cases}\lambda\dfrac{(x-a)}{(b-a)}, & a\le x\le b\\ \lambda\dfrac{(c-x)}{(c-b)}, & b\le x\le c\\ 0, & \text{otherwise}\end{cases}\tag{1}$$

Then, $\tilde{A}^L=(a,b,c;\lambda),\ a<b<c.$

Let

$$\mu_{\tilde{A}^U}(x)=\begin{cases}\rho\dfrac{(x-p)}{(b-p)}, & p\le x\le b\\ \rho\dfrac{(r-x)}{(r-b)}, & b\le x\le c\\ 0, & \text{otherwise}\end{cases}\tag{2}$$

Then, $\tilde{A}^L=(p,b,r;\rho),\ p<b<r.$

Consider the case in which $0<\lambda\le\rho\le 1$ and $p<a<b<c<r$. From (1) and (2), we

obtain $\tilde{A}=[\tilde{A}^L,\tilde{A}^U]=[(a,b,c:\lambda),(p,b,r;\rho)]$, which is called the Level (λ, ρ) interval-valued fuzzy number. Fig. 1 shows that, when $a = p$, $c = r$, $\lambda = 0$, the level (λ, ρ) interval-valued fuzzy number reduces to the level ρ triangular fuzzy number. The α-cut of interval-valued fuzzy set $\tilde{A}$ is shown in Fig. 2:

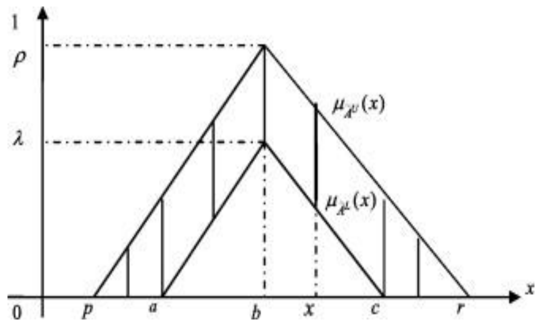

Figure.1 Level (λ, ρ) interval-valued fuzzy numbers

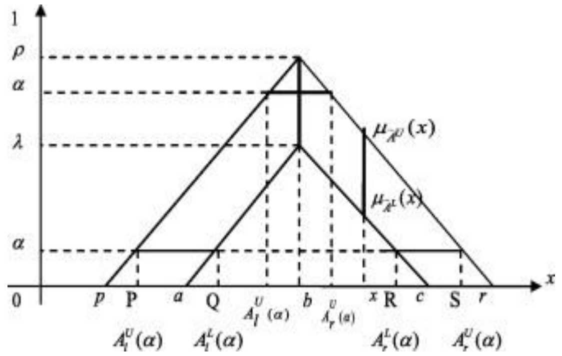

Figure.2 α-cut of (λ, ρ) interval-valued fuzzy numbers

If $0\le\alpha<\lambda$, then from equations (1) & (2), we can obtain

$$A_l{}^L(\alpha)=a+(b-a)\frac{\alpha}{\lambda},\qquad A_r{}^L(\alpha)=c-(c-b)\frac{\alpha}{\lambda}$$

$$A_l{}^U(\alpha)=e+(b-e)\frac{\alpha}{\lambda},\qquad A_r{}^U(\alpha)=h-(h-b)\frac{\alpha}{\lambda}$$

and if $\lambda\le\alpha\le\rho$,then

$$A_l{}^U(\alpha)=e+(b-e)\frac{\alpha}{\lambda},\qquad A_r{}^U(\alpha)=h-(h-b)\frac{\alpha}{\lambda}$$

where $\{A_l{}^L(\alpha),A_l{}^U(\alpha)\}$ and $\{A_r{}^L(\alpha),A_r{}^U(\alpha)\}$ are α-cuts of left and right side of (λ, ρ) interval-valued fuzzy numbers.

Let $\tilde{A}=[\tilde{A}^L,\tilde{A}^U]=[(a_1,b_1,c_1;\lambda),(a_2,b_1,c_2;\rho)]$, $\tilde{B}=[\tilde{B}^L,\tilde{B}^U]=[(p_1,q_1,r_1;\lambda),(p_2,q_1,r_2;\rho)]\in$ $F_{IV}(\lambda,\rho)$. Through operations level λ fuzzy numbers and level ρ fuzzy numbers, we can get the following:

$$\tilde{A}^L+\tilde{B}^L=(a_1+p_1,b_1+q_1,c_1+r_1;\lambda)$$
$$\tilde{A}^U+\tilde{B}^U=(a_2+p_2,b_1+q_1,c_2+r_2;\rho)$$

Definition 2.3. If $\tilde{A}=[\tilde{A}^L,\tilde{A}^U]=[(a_1,b_1,c_1;\lambda),(a_2,b_1,c_2;\rho)]$,
$\tilde{B}=[\tilde{B}^L,\tilde{B}^U]=[(p_1,q_1,r_1;\lambda),(p_2,q_1,r_2;\rho)]\in F_{IV}(\lambda,\rho)$ and $k\in R$, then

i. $\tilde{A}\oplus\tilde{B}=[\tilde{A}^L\oplus\tilde{B}^L,\tilde{A}^U\oplus\tilde{B}^U]$
ii. $\tilde{A}-\tilde{B}=[\tilde{A}^L-\tilde{B}^L,\tilde{A}^U-\tilde{B}^U]$
iii. $\tilde{A}\otimes\tilde{B}=[\tilde{A}^L\otimes\tilde{B}^L,\tilde{A}^U\otimes\tilde{B}^U]$
iv. $1-\tilde{A}=[1-\tilde{A}^L,1-\tilde{A}^U]$
v. $k\tilde{A}=[k\tilde{A}^L,k\tilde{A}^U]$

(a) when $k > 0$ then $k\tilde{A}=[(ka_1,kb_1,kc_1;\lambda),(ka_2,kb_1,kc_2;\rho)]$

(b) when $k < 0$ then $k\tilde{A}=[(kc_1,kb_1,ka_1;\lambda),(kc_2,kb_1,ka_2;\rho)]$

(c) when $k = 0$ then $k\tilde{A}=[(0,0,0;\lambda),(0,0,0;\rho)]$

3. Signed distance:

Definition 3.1. $d_0(a,0)$, $a\in R$ is defined as $d_0(a,0)=a$ and is called the signed distance from a to 0.

The meaning of d_0 is that when $a > 0$, $d_0(a,0)=a>0$ i.e. a is the right of 0 and the distance from 0 is a, when $a < 0$, $d_0(a,0)=a<0$ i.e. a is at the left of 0 and the distance from 0 is $-a$. That's why $d_0(a,0)$ is known as the signed distance of a from 0.

The signed on $F_{IV}(\lambda,\rho)$, by definition 3.1 can be defined by the following: if $\tilde{A}=[(a,b,c;\lambda).(p,b,r;\rho)]=[\tilde{A}^L,\tilde{A}^U]\in F_{IV}(\lambda,\rho)$ then the α-level set of $\tilde{A}=[\tilde{A}^L,\tilde{A}^U]$ is defined as $\{x\,|\,\mu_{\tilde{A}^U}(x)\ge\alpha\}-\{x\,|\,\mu_{\tilde{A}^L}(x)>\alpha\}$.

We have following one-one onto mapping for each α.

When $0 \le \alpha < \lambda$,

$$[A_l^U(\alpha), A_l^L(\alpha); \alpha] \text{ (corresspon ding } PQ) \leftrightarrow [A_l^U(\alpha), A_l^L(\alpha)]$$
$$[A_r^L(\alpha), A_r^U(\alpha); \alpha] \text{ (corresspon ding } RS) \leftrightarrow [A_r^L(\alpha), A_r^U(\alpha)]$$

and $[A_l^U(\alpha), A_l^L(\alpha)] \cap [A_r^L(\alpha), A_r^U(\alpha)] = \phi$

and when $\lambda \le \alpha \le \rho$,

$$[A_l^U(\alpha), A_r^U(\alpha); \alpha] \leftrightarrow [A_l^U(\alpha), A_r^U(\alpha)]$$

Using definition 3.1, we obtain the signed distance of $\tilde{A} = [(a, \underset{\sim}{b}, c; \lambda), (p, q, \underset{\sim}{r}; \rho)]$ from 0. For $0 \le \alpha < \lambda$, the signed distances of P, Q, R, S (see Fig. 2) from 0 are $d_0(\tilde{A}_l^U(\alpha), 0) = \tilde{A}_l^U(\alpha)$, $d_0(\tilde{A}_l^L(\alpha), 0) = \tilde{A}_l^L(\alpha)$, $d_0(\tilde{A}_r^L(\alpha), 0) = \tilde{A}_r^L(\alpha)$ and $d_0(\tilde{A}_r^U(\alpha), 0) = \tilde{A}_r^U(\alpha)$ respectively. Therefore, the signed distance of the interval $[\tilde{A}^L(\alpha).\tilde{A}^U(\alpha)]$ from 0 is given by:

$$d_0\left(\left[\tilde{A}_l^U(\alpha), \tilde{A}_l^L(\alpha)\right], 0\right) = \frac{1}{2}\left(d_0\left(\tilde{A}_l^U(\alpha), 0\right) + d_0\left(\tilde{A}_l^L(\alpha), 0\right)\right) = \frac{1}{2}\left(\tilde{A}_l^U(\alpha) + \tilde{A}_l^L(\alpha)\right)$$

$$= \frac{1}{2}\left[a + p + (b-a)\frac{\alpha}{\lambda} + (b-p)\frac{\alpha}{\rho}\right]$$

Similarly,

$$d_0\left(\left[\tilde{A}_r^U(\alpha), \tilde{A}_r^L(\alpha)\right], 0\right) = \frac{1}{2}\left(\tilde{A}_r^U(\alpha) + \tilde{A}_r^L(\alpha)\right) = \frac{1}{2}\left[c + r - (c-b)\frac{\alpha}{\lambda} - (r-b)\frac{\alpha}{\rho}\right] \quad (3)$$

When $\left[\tilde{A}_l^U(\alpha), \tilde{A}_l^L(\alpha)\right] \cap \left[\tilde{A}_r^U(\alpha), \tilde{A}_r^L(\alpha)\right] = \phi$, the signed distance of $\left[\tilde{A}_l^U(\alpha), \tilde{A}_l^L(\alpha)\right] \cup \left[\tilde{A}_r^U(\alpha), \tilde{A}_r^L(\alpha)\right]$ from 0 is obtained as

$$d_0\left(\left[\tilde{A}_l^U(\alpha), \tilde{A}_l^L(\alpha)\right] \cup \left[\tilde{A}_r^U(\alpha), \tilde{A}_r^L(\alpha)\right], 0\right)$$
$$= \frac{1}{2}\left[d_0\left(\left[\tilde{A}_l^U(\alpha), \tilde{A}_l^L(\alpha)\right], 0\right) + d_0\left(\left[\tilde{A}_r^U(\alpha), \tilde{A}_r^L(\alpha)\right], 0\right)\right]$$

$$= \frac{1}{4}\left[a + p + c + r + (2b - a - c)\frac{\alpha}{\lambda} + (2b - p - r)\frac{\alpha}{\rho}\right] \quad (4)$$

The function given in equation (4) is continuous on $0 \le \alpha < \lambda$ with respect to *α*. It follows that, by integration, we can find the average value

$$\frac{1}{\lambda}\int_0^\lambda d_0\left(\left[\tilde{A}_l^U(\alpha), \tilde{A}_l^L(\alpha)\right] \cup \left[\tilde{A}_r^U(\alpha), \tilde{A}_r^L(\alpha)\right], 0\right) d\alpha$$

$$= \frac{1}{8}[a + c + 2b + 2p + 2r + (2b - p - r)\frac{\lambda}{\rho}] \quad (5)$$

Similarly, for $\lambda \le \alpha \le \rho$, we obtain

$$d_0\left(\left[\tilde{A}_l^U(\alpha), \tilde{A}_r^U(\alpha)\right], 0\right) = \frac{1}{2}\left[p + r + (2b - p - r)\frac{\alpha}{\rho}\right]$$

This function is also continuous on $\lambda \le \alpha \le \rho$ with respect to *α*. Now through integration, find the average value,

$$\frac{1}{\rho-\lambda}\int_{\lambda}^{\rho} d_0\left(\left[\tilde{A}_l^U(\alpha),\tilde{A}_r^U(\alpha)\right],0\right)d\alpha=\frac{1}{4}[2b+p+r+(2b-p-r)\frac{\lambda}{\rho}] \tag{6}$$

Definition 3.2. Let $\tilde{A}=[(a,b,c;\lambda),(p,b,r;\rho)]$ then the signed distance from $\tilde{A}$ to 0 is defined as follows:

i. When $0<\lambda<\rho\le 1$,

$$d(\tilde{A},0)=\frac{1}{\lambda}\int_0^{\lambda}\frac{1}{4}[a+p+c+r+(2b-a-c)\frac{\alpha}{\lambda}+(2b-p-r)\frac{\alpha}{\rho}]d\alpha+$$

$$\frac{1}{\rho-\lambda}\int_{\lambda}^{\rho}\frac{1}{2}[p+r+(2b-p-r)\frac{\alpha}{\rho}]d\alpha$$

$$=\frac{1}{8}\left[6b+a+c+4e+4h+\frac{3\lambda}{\rho}(2b-h-e)\right]$$

ii. When $0<\lambda=\rho\le 1$,

$$d(\tilde{A},0)=\frac{1}{8}\left[4b+a+c+e+h\right]$$

Definition 3.3. Let $\tilde{A}=(p,b,r;\rho)\in F_N(\rho)$ then the signed distance of $\tilde{A}$ from 0 is given by

$$d_0(\tilde{A},0)=\frac{1}{2\rho}\int_0^{\rho}\left[\tilde{A}_l^U(\alpha)+\tilde{A}_r^U(\alpha)\right]d\alpha=\frac{1}{4}\left[2b+p+r\right]$$

4. Fuzzy Fault Tree:

Fault tree analysis is one of the most widely used methods in the industrial sector to evaluate reliability of engineering systems. In conventional fault tree analysis, the failure probabilities of system components are treated as exact values. It is often very difficult to estimate precise value of failure rates or probabilities of system components in dynamically changing environments or in systems where data is insufficient for statistical inferences. Fuzzy set has the capacity of dealing with such situations. Many times specialists are uncertain about the value of membership of an element in a set. Hence, it is better to represent the values of the membership of the element in a set by intervals of possible real numbers instead of real numbers.

4.1 *Fuzzy fault tree analysis of fire protection system*

The fault tree structure of fire protection system as shown in Fig.3 is taken for analysis. Failure of fire protection system depends on different facts like heat detection failure, pump failure, nozzle blocked and smoke detection failure. There are two major factors viz. fire detection system failure and water deluge system failure, each of which has two sub-factors. Following notations have been used to illustrate the system failure rate.

$\underset{\sim}{\tilde{f}_1}=\underset{\sim}{\tilde{f}_S}$ failure of smoke detection,

$\underset{\sim}{f_2}=\underset{\sim}{f_H}$ failure of heat detection,

$\underset{\sim}{f_3}=\underset{\sim}{f_P}$ failure of pump,

$\underset{\sim}{f_4}=\underset{\sim}{f_N}$ failure of nozzles blocked.

$\underset{\sim}{f_5}=\underset{\sim}{f_F}$ failure of fire detection system,

$\underset{\sim}{f_6}=\underset{\sim}{f_W}$ failure of water deluge system,

$f_7=f_{FP}$ failure of fire protection system,

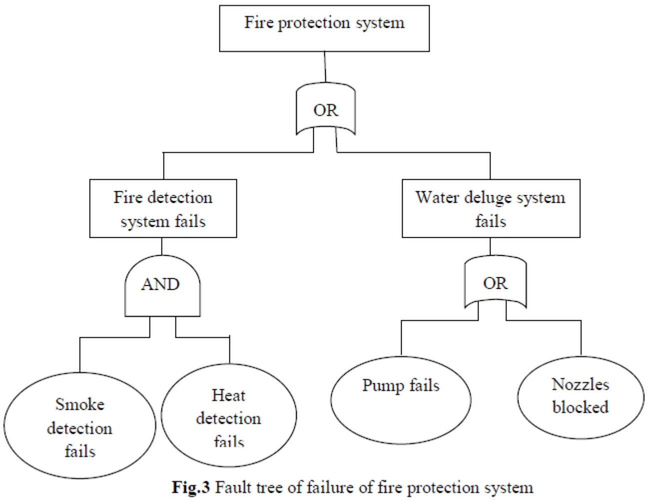

Fig.3 Fault tree of failure of fire protection system

Failure of fire protection system can be evaluated by using the following steps:

I. $\tilde{f}_5 = \tilde{f}_1 \otimes \tilde{f}_2$ and $\tilde{f}_6 = 1-(1-\tilde{f}_3)(1-\tilde{f}_4)$

II. $\tilde{f}_7 = 1-(1-\tilde{f}_5)(1-\tilde{f}_6)$

Let failure rate of all components of system represented by (λ, ρ) interval-valued fuzzy number. For any event

$$\tilde{f}_J = [\tilde{f}_J{}^L, \tilde{f}_J{}^U] = [(f_J - \delta_{2J}, f_J, f_J + \delta_{3J}; \lambda), (f_J - \delta_{1J}, f_J, f_J + \delta_{4J}; \rho)]$$

where $f_J - 1 \leq 0 < \delta_{2J} < \delta_{1J} < f_J$ and $0 < \delta_{3J} < \delta_{4J} < 1 - f_J$, $j = 1, 2, 3, 4, 5, 6.$

Failure rate of fire detection system $\tilde{f}_5$, fire water deluge system $\tilde{f}_6$, fire protection system $\tilde{f}_7$ given by equations (7), (8) & (9) respectively,

$$\tilde{f}_5 = [\{\underset{J=1}{\overset{2}{\otimes}}(\tilde{f}_J - \delta_{2J}), \underset{J=1}{\overset{2}{\otimes}}\tilde{f}_J, \underset{J=1}{\overset{2}{\otimes}}(\tilde{f}_J + \delta_{3J}); \lambda\}, \{\underset{J=1}{\overset{2}{\otimes}}(\tilde{f}_J - \delta_{1J}), \underset{J=1}{\overset{2}{\otimes}}\tilde{f}_J, \underset{J=1}{\overset{2}{\otimes}}(\tilde{f}_J + \delta_{4J}); \rho\}] \tag{7}$$

$$\begin{aligned}\tilde{f}_6 = [&\{1-\underset{J=3}{\overset{4}{\otimes}}(1-\tilde{f}_J + \delta_{2J}), 1-\underset{J=3}{\overset{4}{\otimes}}\tilde{f}_J, 1-\underset{J=3}{\overset{4}{\otimes}}(1-\tilde{f}_J - \delta_{3J}); \lambda\},\\ &\{1-\underset{J=3}{\overset{4}{\otimes}}(1-\tilde{f}_J + \delta_{1J}), 1-\underset{J=3}{\overset{4}{\otimes}}\tilde{f}_J, 1-\underset{J=3}{\overset{4}{\otimes}}(1-\tilde{f}_J - \delta_{4J}); \rho\}]\end{aligned} \tag{8}$$

$$\begin{aligned}\tilde{f}_7 = [&\{1-\underset{J=5}{\overset{6}{\otimes}}(1-\tilde{f}_J + \delta_{2J}), 1-\underset{J=5}{\overset{6}{\otimes}}\tilde{f}_J, 1-\underset{J=5}{\overset{6}{\otimes}}(1-\tilde{f}_J - \delta_{3J}); \lambda\},\\ &\{1-\underset{J=5}{\overset{6}{\otimes}}(1-\tilde{f}_J + \delta_{1J}), 1-\underset{J=1}{\overset{2}{\otimes}}\tilde{f}_J, 1-\underset{J=1}{\overset{2}{\otimes}}(1-\tilde{f}_J - \delta_{4J}); \rho\}]\end{aligned} \tag{9}$$

4.2 *Numerical solution of failure of fire protection system using (λ, ρ) interval-valued fuzzy number*

Consider an example of fire protection system in which failure rates of events are taken as

$\tilde{f}_1$ = failure of smoke detection = [(0.2, 0.4, 0.6; 0.8); (0.1, 0.4, 0.7; 0.98)]

$\tilde{f}_2$ = failure of heat detection = [(0.3, 0.5, 0.8; 0.8); (0.2, 0.5, 0.9; 0.98)]

$\tilde{f}_3$ = failure of pump = [(0.2, 0.3, 0.5; 0.8); (0.1, 0.3, 0.6; 0.98)]

$\tilde{f}_4$ = failure of nozzles blocked = [(0.2, 0.5, 0.7; 0.8); (0.1, 0.5, 0.8; 0.98)]

Using equations (7), (8) & (9), we get the failure rate of fire detection system, water deluge system, fire protection system respectively as

$\tilde{f}_5$ = [(0.06, 0.2, 0.48; 0.8); (0.02, 0.2, 0.63; 0.98)]

$\tilde{f}_6$ = [(0.36, 0.65, 0.15; 0.8); (0.19, 0.65, 0.92; 0.98)]

$\tilde{f}_7$ = [(0.3984, 0.72, 0.558; 0.8); (0.2062, 0.72, 0.9704; 0.98)] (10)

By definition 3.2, we can defuzzify, $\tilde{f}_7$ and get the failure rate of the system in the fuzzy sense as follows:

$$\frac{1}{2}d(\tilde{f}_7,0)=\frac{1}{16}[6b+a+c+4p+4r+\frac{3\lambda}{\rho}(2b-r-p)]$$

From equation (10) we have, $a=0.3984$, $b=0.72$, $c=0.558$, $p=0.2062$, $r=0.9704$, $\lambda=0.8$ and $\rho=0.98$. Then

$$\frac{1}{2}d(\tilde{f}_7,0)=0.66424$$

If $0<\lambda=\rho\leq 1$, then the signed distance of $\tilde{f}_7$ from 0 will be:

$$\frac{1}{2}d(\tilde{f}_7,0)=\frac{1}{16}[4b+a+c+p+r]$$

$$=.3133125$$

5. Conclusion

In this study we have proposed a new method to analyze the fuzzy fault tree. A fault tree of fire protection system is used to analyze the fuzzy failure rate. The failure rate of the components of the system is considered as level (λ, ρ) interval-valued fuzzy numbers. The definition of level (λ, ρ) interval-valued fuzzy numbers and the arithmetic operators of level (λ, ρ) interval-valued fuzzy numbers are also discussed. We have defined the definition of signed distance. We have also discussed defuzzification of fuzzy failure rate of components of the system using signed distance method.

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