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Common Fixed Point of Weakly Compatible Maps on Intuitionistic Fuzzy Metric Spaces

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Abstract
In this paper, we introduce the concept of ϵ- chainable intuitionistic fuzzy metric space askin to the notion of ϵ-chainable fuzzy metric space and prove a common fixed point theorem for weakly compatible mappings in this newly defined space.

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Key words and phrases: ϵ- chainable intuitionistic fuzzy metric space, weakly Compatible mappings, Common fixed point.

1. Introduction

Alaca et al.[1] using the idea of intuitionistic fuzzy sets, they defined the notion of intuitionistic fuzzy metric space as Park[17] with the help of continuous t-norms and continuous t- co-norms as a generalization of fuzzy metric space due to Kramosil and Michalek[11].Further, they introduced the notion of Cauchy sequences in intuitionistic fuzzy metric spaces and proved the well known fixed point theorems of Banach[5] and Edelstein[7] extended to intuitionistic fuzzy metric spaces with help of Grabiec[8].

Turkoglu et al.[31] introduced the concept of compatible maps and compatible maps of types (α) and (β) in intuitionistic fuzzy metric spaces and gave some relations between the concepts of compatible maps and compatible maps of types (α) and (β).

Gregory et al.[9], Saadati et al.[25], Singaloti et al.[24],Ciric et al.[6], Jesic [10], Kutukcu [13] and many others studied the concept of intuitionistic fuzzy metric space and its applications. Sharma and Deshapande [27], [28] proved common fixed point theorems for finite number of mapping without continuity and compatibility on intuitionistic fuzzy metric spaces. In the study of fixed points of metric spaces Pant[18],[19],[21] has initiated work using the concept of non compatible maps in metric spaces. The common fixed point theorem of compatible maps we often require assumptions on the completeness of the space or the continuity of the mappings involved besides some contractive conditions but the study of non compatible maps can be extended to the class of non expansive or Lipschitz type mapping pairs even without assuming continuity of the mappings. Aamri and Moutawakil [4] introduced the property (E.A) and thus generalized the concept of non compatible maps.

Sharma, Kutukcu and Pathak [30] introduced the property(S-B) in intuitionistic fuzzy metric spaces and proved fixed point theorems on intuitionistic fuzzy metric spaces.

In 2006, cho, jungck introduced the notion of ϵ- chainable intuitionistic fuzzy metric space and proved common fixed point theorems for four weakly compatible mappings. In a similar mode, we introduced the concept of ϵ- chainable intuitionistic fuzzy metric space and proved common fixed point theorems for four weakly compatible mappings of ϵ- chainable intuitionistic fuzzy metric space.

2. Preliminaries

Definition 2.1[26]. A binary operation *: [0, 1] × [0, 1] → [0, 1] is a continuous t-norm if it satisfies the following conditions:

(1) * is associative and commutative,

(2) * is continuous,

(3) a * 1 = a for all a ∈ [0, 1],

(4) a * b ≤ c * d whenever a ≤ c and b ≤ d for all a, b, c, d ∈ [0, 1],

Two typical examples of continuous t-norm are a * b = ab and a * b = min (a, b).

Definition 2.2[26]. A binary operation ✧: [0, 1] × [0, 1] → [0, 1] is a continuous t-norm if it satisfies the following conditions:

(1) ✧ is associative and commutative,

(2) ✧ is continuous,

(3) a ✧1 = a for all a ∈ [0, 1],

(4) a ✧ b ≤ c ✧ d whenever a ≤ c and b ≤ d for all a, b, c, d ∈ [0, 1],

Two typical examples of continuous t-norm are a ✧ b = ab and a ✧ b = max (a, b).

Definition 2.3[1]. A 5-tuple (X, M, N, $*$, ✧) is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, ✧ is a continuous t-co norm and M,N are fuzzy sets on $X^2 \times [0,\infty)$ satisfying the following conditions:

(i) $M(x,y,t) + N(x,y,t) \leq 1 for\ all\ x,y \in X\ and\ t > 0;$

(ii) $M(x,y,0) = 0\ for\ all\ x,y \in X;$

(iii) $M(x,y,t) = 1\ for\ all\ x,y \in X\ and\ t > 0\ if\ and\ only\ if\ x = y;$

(iv) $M(x,y,t) = M(y,x,t)\ for\ all\ x,y \in X\ and\ t > 0;$

(v) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s) for\ all\ x,y \in X\ and\ s,t > 0;$

(vi) $for\ all\ x,y \in X, M(x,y,\ .) : [0,\infty) \rightarrow [0,1]\ is\ left\ continuous;$

(vii) $\lim_{t\rightarrow\infty} M(x,y,t) = 1\ for\ all\ x,y \in X\ and\ t > 0;$

$(viii)$ $N(x,y,0) = 1\ for\ all\ x,y \in X;$

(ix) $M(x,y,t) = 1\ for\ all\ x,y \in X\ and\ t > 0\ if\ and\ only\ if\ x = y;$

(x) $N(x,y,t) = N(y,x,t)\ for\ all\ x,y \in X\ and\ t > 0;$

(xi) $N(x,y,t) \diamond N(y,z,s) \geq N(x,z,t+s) for\ all\ x,y \in X\ and\ s,t > 0;$

(xii) $for\ all\ x,y \in X, N(x,y,\ .) : [0,\infty) \rightarrow [0,1]\ is\ right\ continuous;$

(xiii) $\lim_{t\to\infty} N(x\,,y\,,t) = 0\; for\; all\; x,y\; \in X\; and\; t > 0.$

Then (M, N) is called an intuitionistic fuzzy metric space on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of non-nearness between x and y w.r.t. t respectively.

Remark 2.1. The concept of triangular norms (t-norms) and triangular co norms (t-co norms) are known as the axiomatic skeletons that we use for characterizing fuzzy intersections and unions, respectively. These concepts were originally introduced by Menger [15] in his study of stastical metric spaces. Several examples
For these concepts by many authors [12], [34].

Remark 2.2.[2],[3]. Every fuzzy metric space $(X\,,M,\,*)$ is an intuitionistic fuzzy metric space of the form $(X\,,M,1-M,\,*,\diamondsuit)$ such that t-norm $*$ and t-co norm $\diamondsuit$ are associated as $x\diamondsuit y = 1-\big((1-x)*(1-y)\big) for\; all\; x\,,y \in X.$

Example 2.1.[14].Let (x ,d) be a metric space. Define a * b=min {a, b} and t-co norm a $\diamondsuit$ b = max (a, b) for all $x\,,y \in X, t > 0$, $M_d(x,y,t) = \frac{t}{t+d(x,y)}\; and\; N_d(x,y,t) = \frac{d(x\,,y)}{t+d(x,y)}$. Then (X, M, N, $*$, $\diamondsuit$) is an intuitionistic fuzzy metric space.

Definition 2.4.[20] Two maps A and S are called R-weakly commuting at a point x if $d(ASx,SAx) \le Rd(Ax,Sx)$ for some $R > 0$. A and S are called point wise R-weakly commuting on X if given x in X, there exists $R > 0$ such that $(ASx,SAx) \le Rd(Ax,Sx)$.

Definition 2.5.[33] Two maps A and S of a fuzzy metric space (X, M, $*$) are called R-weakly commuting at a point x if $M(ASx,SAx,t) \le M(Ax,Sx,\frac{t}{R})$ for some $R,t > 0$.

Definition 2.6.[32] A pair of self mappings (f,g) intuitionistic fuzzy metric space is said to be R-weakly compatible if there exists a positive real number R such that $M(fgx,gfx,t) \ge M(fx,gx,\frac{t}{R})$, $N(fgx,gfx,t) \le N(fx,gx,\frac{t}{R})$ for all x in X.

Definition 2.7.[23] Two maps A and S are called R-weakly commuting of type (Ag) if there exists a positive real number R such that $d(AAx,SAx,t) \le Rd(Ax,Sx,\frac{t}{R})$ for some $R,t > 0$

Definition 2.8.[22] Two maps A and S of a fuzzy metric space (X, M, $*$) are called R-weakly commuting of type (Ag) if there exists a positive real number R such that $M(AAx,SAx,t) \ge M(Ax,Sx,\frac{t}{R})$ for some $R,t > 0$, x in X.

Lemma 2.1[1]. Let (X, M, N, $*$, $\diamondsuit$) be an intuitionistic fuzzy metric space and $for\; all\; x,y \in X, t > 0$ and if for a number $k \in (0\,,1)$ such that $M(x\,,y\,,kt) \ge M(y\,,x\,,t)\; and\; N(x\,,y\,,kt) \le N(y\,,x\,,t)$then $x = y$.

Definition 2.9. Let (X, M, N, $*$, $\diamondsuit$) be intuitionistic fuzzy metric space. A finite sequence
$x = x_o, x_1, x_2, x_3 \ldots\ldots x_n = y$ is called ϵ- chain from x to y if there exists a positive number $\epsilon > 0$ such that $M(x_i, x_{i-1\,,}t) > 1-\in$ and $N\big(x_i, x_{i-1\,,}t\big) < 1-\in$ for all t > 0 & i = 1,.. n. An intuitionistic fuzzy metric space (X, M, N, $*$, $\diamondsuit$) is called ϵ- chainable if for any $x,y \in X$, there exists a ϵ- chain from x to y.

3. Main results

Theorem 3.1. Let A, B, S and T be self maps of a complete ϵ- chainable intuitionistic fuzzy metric spaces (X, M, N, $*$, $\diamond$) with continuous t-norm * and continuous t-co norm $\diamond$ defined by $t * t \geq t$ and $(1-t) \diamond (1-t) \leq (1-t)$ for all $t \in [0,1]$ satisfying the following condition:

(3.1) A(X) ⊆ T(X) and B(X) ⊆ S(X),

(3.2) A and S are continuous,

(3.3) The pairs (A, S) and (B, T) are weakly compatible,

(3.4) There exist $q \in (0,1)$ such that

$$M(Ax, By, qt) \geq \left\{ \begin{matrix} M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) \\ M(By, Sx, t) * M(Ax, Ty, t) * \left(\frac{M(Sx, Ty, t)}{M(Ax, Ty, t)}\right) * \\ \left(\frac{M(By, Ty, t)}{M(Ax, Sx, t)}\right) \end{matrix} \right\}$$

And

$$N(Ax, By, qt) \leq \left\{ \begin{matrix} N(Sx, Ty, t) \diamond N(Ax, Sx, t) \diamond N(By, Ty, t) \\ N(By, Sx, t) \diamond N(Ax, Ty, t) \diamond \left(\frac{N(Sx, Ty, t)}{N(Ax, Ty, t)}\right) \diamond \\ \left(\frac{N(By, Ty, t)}{N(Ax, Sx, t)}\right) \end{matrix} \right\}$$

$\forall x, y \in X$ and $t > 0$.Then A, B, S and T have a unique common fixed point in X.

Proof : Since A(X) ⊆ T(X), therefore for any $x_0 \in X$, there exists a point $x_1 \in X$ such that $Ax_0 = Tx_1$ and for the point x_1 , we can choose a point $x_2 \in X$ such that $Bx_1 = Sx_2$ as B(X) ⊆ S(X). Inductively, we can find a sequence $\{y_n\}$ in x as follows $y_{2n-1} = Tx_{2n-1} = Ax_{2n-2}$ and $y_{2n} = Sx_{2n} = Bx_{2n-1}$ for n = 0,1,2.. By theorem of Alaca et al. [1], we can conclude that $\{y_n\}$ is Cauchy sequence in X. Since X is complete, therefore sequence $\{y_n\}$ in X converges to z for some z in X and also the sequences $\{Tx_{2n-1}\}, \{Ax_{2n-2}\}$ and $\{Sx_{2n}\}, \{Bx_{2n-1}\}$ also converges to z. Since X is ϵ- chainable, there exists ϵ- chain from x_n to x_{n+1}, that is , there exists a finite sequence $x_n = y_1, y_2 \dots . y_l = x_{n+1}$ such that $M(x_i, x_{i-1}, t) > 1 - \epsilon$ and $N(x_i, x_{i-1}, t) < 1 - \epsilon$ for all t > 0 and i = 1,2….l. Thus we have

$$M(x_n, x_{n+1}, t) \geq M\left(y_1, y_2, {}^t/_l\right) * M\left(y_2, y_3, {}^t/_l\right) *..* M\left(y_{l-1}, y_l, {}^t/_l\right)$$
$$> (1-\epsilon) * (1-\epsilon) *..* (1-\epsilon) \geq (1-\epsilon) \text{ and}$$
$$N(x_n, x_{n+1}, t) \leq N\left(y_1, y_2, {}^t/_l\right) \diamond N\left(y_2, y_3, {}^t/_l\right) \diamond..\diamond N\left(y_2, y_3, {}^t/_l\right)$$
$$< (1-\epsilon) \diamond (1-\epsilon) \diamond..\diamond (1-\epsilon) \leq (1-\epsilon)$$

for m > n

$$M(x_n, x_m, t) \geq M(x_n, x_{n+1}, {}^t/m-n) * M(x_{n+1}, x_{n+2}, {}^t/m-n) *..* M(x_{m-1}, x_m, {}^t/m-n)$$
$$> (1-\epsilon) * (1-\epsilon) *..* (1-\epsilon) \geq (1-\epsilon) \text{ and}$$
$$N(x_n, x_m, t) \leq N(x_n, x_{n+1}, {}^t/m-n) \diamond N(x_{n+1}, x_{n+2}, {}^t/m-n) \diamond..\diamond N(x_{m-1}, x_m, {}^t/m-n)$$
$$< (1-\epsilon) \diamond (1-\epsilon) \diamond..\diamond (1-\epsilon) \leq (1-\epsilon)$$

Therefore, $\{x_n\}$ is a Cauchy sequence in X and hence there exists x in X such that $x_n \to x$. From (3.2),

$Ax_{2n-2} \to Ax, Sx_{2n} \to Sx$ as limit n → ∞. By uniqueness of limits, we have $Ax = z = Sx$. since pair (A, S) is weakly compatible, therefore, $ASx = SAx$ and so $Az = Sz$. From (3.2), we have $ASx_{2n} \to Sz$. Also, form Continuity of S, we have, $SSx_{2n} \to Sz$. from (3.4), we get

$$M(ASx_{2n}, Bx_{2n-1}, qt) \geq \left\{ \begin{array}{c} M(SSx_{2n}, Tx_{2n-1}, t) * M(ASx_{2n}, SSx_{2n}, t) * \\ M(Bx_{2n-1}, Tx_{2n-1}, t) * M(Bx_{2n-1}, SSx_{2n}, t) * \\ M(ASx_{2n}, Tx_{2n-1}, t) * \left(\frac{M(SSx_{2n}, Tx_{2n-1}, t)}{M(ASx_{2n}, Tx_{2n-1}, t)} \right) * \\ \left(\frac{M(Bx_{2n-1}, Tx_{2n-1}, t)}{M(ASx_{2n}, SSx_{2n}, t)} \right) \end{array} \right\}$$

And

$$N(ASx_{2n}, Bx_{2n-1}, qt) \leq \left\{ \begin{array}{c} N(SSx_{2n}, Tx_{2n-1}, t) \diamondsuit N(ASx_{2n}, SSx_{2n}, t) \diamondsuit \\ N(Bx_{2n-1}, Tx_{2n-1}, t) \diamondsuit N(Bx_{2n-1}, SSx_{2n}, t) \diamondsuit \\ N(ASx_{2n}, Tx_{2n-1}, t) \diamondsuit \left(\frac{N(SSx_{2n}, Tx_{2n-1}, t)}{N(ASx_{2n}, Tx_{2n-1}, t)} \right) \diamondsuit \\ \left(\frac{N(Bx_{2n-1}, Tx_{2n-1}, t)}{N(ASx_{2n}, SSx_{2n}, t)} \right) \end{array} \right\}$$

Proceeding limit as n → ∞ , we have

$$M(Sz, z, qt) \geq \left\{ \begin{array}{c} M(Sz, z, t) * M(Sz, Sz, t) * \\ M(z, z, t) * M(z, Sz, t) * \\ M(Sz, z, t) * \left(\frac{M(Sz, z, t)}{M(Sz, z, t)} \right) * \\ \left(\frac{M(z, z, t)}{M(Sz, Sz, t)} \right) \end{array} \right\}$$

And

$$N(Sz, z, qt) \leq \left\{ \begin{array}{c} N(Sz, z, t) \diamondsuit N(Sz, Sz, t) \diamondsuit \\ N(z, z, t) \diamondsuit N(z, Sz, t) \diamondsuit \\ N(Sz, z, t) \diamondsuit \left(\frac{N(Sz, z, t)}{N(Sz, z, t)} \right) \diamondsuit \\ \left(\frac{N(z, z, t)}{N(Sz, Sz, t)} \right) \end{array} \right\}$$

From lemma (2.1), we get

$Sz = z$, and hence $Az = Sz = z$. Since A(X) ⊆ T(X), there exists v in X such that $Tv = Az = z$.

From (3.4), we have

$$M(Ax_{2n}, Bv, qt) \geq \left\{ \begin{array}{c} M(Sx_{2n}, Tv, t) * M(Ax_{2n}, Sx_{2n}, t) * \\ M(Bv, Tv, t) * M(Bv, Sx_{2n}, t) * \\ M(Ax_{2n}, Tv, t) * \left(\frac{M(Sx_{2n}, Tv, t)}{M(Ax_{2n}, Tv, t)} \right) * \\ \left(\frac{M(Bv, Tv, t)}{M(Ax_{2n}, Sx_{2n}, t)} \right) \end{array} \right\}$$

And

$$N(Ax_{2n}\,, Bv, qt) \;\le \begin{Bmatrix} N(Sx_{2n}\,, Tv, t) \diamond N(Ax_{2n}\,, Sx_{2n}, t) \diamond \\ N(Bv\,, Tv, t) \diamond N(Bv\,, Sx_{2n}, t) \diamond \\ N(Ax_{2n}\,, Tv, t) \diamond \left(\dfrac{N(Sx_{2n}\,, Tv, t)}{N(Ax_{2n}\,, Tv, t)}\right) \diamond \\ \left(\dfrac{N(Bv\,, Tv, t)}{N(Ax_{2n}\,, Sx_{2n}, t)}\right) \end{Bmatrix}$$

Letting as n → ∞ , we have

$$M(z\,, Bv, qt) \;\ge \begin{Bmatrix} M(z\,, Tv, t) * M(z\,, z, t) * \\ M(Bv\,, Tv, t) * M(Bv\,, z, t) * \\ M(z\,, Tv, t) * \left(\dfrac{M(z\,, Tv, t)}{M(z\,, Tv, t)}\right) * \\ \left(\dfrac{M(Bv\,, Tv, t)}{M(z\,, z, t)}\right) \end{Bmatrix}$$

$$= \begin{Bmatrix} M(z\,, z, t) * M(z\,, z, t) * \\ M(Bv\,, z, t) * M(Bv\,, z, t) * \\ M(z\,, z, t) * \left(\dfrac{M(z\,, z, t)}{M(z\,, z, t)}\right) * \\ \left(\dfrac{M(Bv\,, z, t)}{M(z\,, z, t)}\right) \end{Bmatrix}$$

$M(z\,, Bv, qt) \;\ge \{M(Bv\,, z, t\}$

And

$$N(z\,, Bv, qt) \;\le \begin{Bmatrix} N(z\,, Tv, t) \diamond N(z\,, z, t) \diamond \\ N(Bv\,, Tv, t) \diamond N(Bv\,, z, t) \diamond \\ N(z\,, Tv, t) \diamond \left(\dfrac{N(z\,, Tv, t)}{N(z\,, Tv, t)}\right) \diamond \\ \left(\dfrac{N(Bv\,, Tv, t)}{N(z\,, z, t)}\right) \end{Bmatrix}$$

$$= \begin{Bmatrix} N(z\,, z, t) \diamond N(z\,, z, t) \diamond \\ N(Bv\,, z, t) \diamond N(Bv\,, z, t) \diamond \\ N(z\,, z, t) \diamond \left(\dfrac{N(z\,, z, t)}{N(z\,, z, t)}\right) \diamond \\ \left(\dfrac{N(Bv\,, z, t)}{N(z\,, z, t)}\right) \end{Bmatrix}$$

$N(z\,, Bv, qt) \;\le \{N(Bv\,, z, t\}$

By Lemma (2.1) , we have $Bv = z$ and therefore, we have $Tv = Bv = z$. Since (B ,T) is weakly compatible,

Therefore $TBv = BTv$ and hence $Tz = Bz$. From (3.4)

$$(Ax_{2n}\,, Bz, qt) \ge \begin{Bmatrix} M(Sx_{2n}\,, Tz, t) * M(Ax_{2n}\,, Sx_{2n}, t) * M(Bz\,, Tz, t) \\ M(Bz\,, Sx_{2n}, t) * M(Ax_{2n}\,, Tz, t) * \left(\dfrac{M(Sx_{2n}\,, Tz, t)}{M(Ax_{2n}\,, Tz, t)}\right) * \\ \left(\dfrac{M(Bz\,, Tz, t)}{M(Ax_{2n}\,, Sx_{2n}, t)}\right) \end{Bmatrix}$$

And

$$N(Ax_{2n}, Bz, qt) \leq \left\{ \begin{array}{c} N(Sx_{2n}, Tz, t) \diamond N(Ax_{2n}, Sx_{2n}, t) \diamond N(Bz, Tz, t) \\ N(Bz, Sx_{2n}, t) \diamond N(Ax_{2n}, Tz, t) \diamond \left(\frac{N(Sx_{2n}, Tz, t)}{N(Ax_{2n}, Tz, t)}\right) \diamond \\ \left(\frac{N(Bz, Tz, t)}{N(Ax_{2n}, Sx_{2n}, t)}\right) \end{array} \right\}$$

Letting as n → ∞ , we have

$$M(z, Bz, qt) \geq \left\{ \begin{array}{c} M(z, Tz, t) * M(z, z, t) * M(Bz, Tz, t) \\ M(Bz, z, t) * M(z, Tz, t) * \left(\frac{M(z, Tz, t)}{M(z, Tz, t)}\right) * \\ \left(\frac{M(Bz, Tz, t)}{M(z, z, t)}\right) \end{array} \right\}$$

$$= \left\{ \begin{array}{c} M(z, Bz, t) * M(z, z, t) * M(Bz, Bz, t) \\ M(Bz, z, t) * M(z, Bz, t) * \left(\frac{M(z, Bz, t)}{M(z, Bz, t)}\right) * \\ \left(\frac{M(Bz, Bz, t)}{M(z, z, t)}\right) \end{array} \right\}$$

$i.e\ M(z, Bz, qt) \geq \{M(z, Bz, t\}$ and

$$N(z, Bz, qt) \leq \left\{ \begin{array}{c} N(z, Tz, t) \diamond N(z, z, t) \diamond N(Bz, Tz, t) \\ N(Bz, z, t) \diamond N(z, Tz, t) \diamond \left(\frac{N(z, Tz, t)}{N(z, Tz, t)}\right) \diamond \\ \left(\frac{N(Bz, Tz, t)}{N(z, z, t)}\right) \end{array} \right\}$$

$$= \left\{ \begin{array}{c} N(z, Bz, t) \diamond N(z, z, t) \diamond N(Bz, Bz, t) \\ N(Bz, z, t) \diamond N(z, Bz, t) \diamond \left(\frac{N(z, Bz, t)}{N(z, Bz, t)}\right) \diamond \\ \left(\frac{N(Bz, Bz, t)}{N(z, z, t)}\right) \end{array} \right\}$$

$$i.e\ N(z, Bz, qt) \leq \{N(z, Bz, t\}$$

Which implies that $Bz = z$.

Therefore, $Az = Sz = Bz = z$. Hence A , B ,S and T have common fixed point z in X.

Uniqueness: Let w be another common fixed point of A , B ,S and T. Then from (3.4),

$$M(z, w, qt) = M(Az, Bw, qt) \geq \left\{ \begin{array}{c} M(Sz, Tw, t) * M(Az, Sz, t) * M(Bw, Tw, t) \\ M(Bw, Sz, t) * M(Az, Tw, t) * \left(\frac{M(Sz, Tw, t)}{M(Az, Tw, t)}\right) * \\ \left(\frac{M(Bw, Tw, t)}{M(Az, Sz, t)}\right) \end{array} \right\}$$

$$= \left\{ \begin{matrix} M(z,w,t) * M(z,z,t) * M(w,w,t) \\ M(z,z,t) * M(z,w,t) * \left(\frac{M(z,w,t)}{M(z,w,t)}\right) * \\ \left(\frac{M(w,w,t)}{M(z,z,t)}\right) \end{matrix} \right\}$$

$i.e\ M(z,w,qt) \geq \{M(z,w,t\}$ and

$$N(z,w,qt) = N(Az,Bw,qt) \leq \left\{ \begin{matrix} N(Sz,Tw,t) \diamond N(Az,Sz,t) \diamond N(Bw,Tw,t) \\ N(Bw,Sz,t) \diamond N(Az,Tw,t) \diamond \left(\frac{N(Sz,Tw,t)}{N(Az,Tw,t)}\right) \diamond \\ \left(\frac{N(Bw,Tw,t)}{N(Az,Sz,t)}\right) \end{matrix} \right\}$$

$$= \left\{ \begin{matrix} N(z,w,t) \diamond N(z,z,t) \diamond N(w,w,t) \\ N(z,z,t) \diamond N(z,w,t) \diamond \left(\frac{N(z,w,t)}{N(z,w,t)}\right) \diamond \\ \left(\frac{N(w,w,t)}{N(z,z,t)}\right) \end{matrix} \right\}$$

$$i.e\ N(z,w,qt) \leq \{N(z,w,t\}$$

Therefore, $Az = Sz = Bz = z$. Hence A , B ,S and T have common fixed point z in X.

Uniqueness: Let w be another common fixed point of A , B ,S and T. Then from (3.4),

By lemma (1.1), z = w. Hence A, B, S and T have unique common fixed point z in X.

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