Study On Feebly Lambda – Functions

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Abstract

in this research we study a several characterization of feeby λ-functions and investigate the relationship between such functions .

Keywords: feeby open set , λ-open set , feeby continuous function , feeby open mapping , feeby closed mapping , feeby λ- continuous function , feeby λ-open mapping , feeby λ-closed mapping , perfectly continuous function , feeby λ-perfectly continuous function .

1. Introduction

The notion of feeby open set introduced by S. N. Maheshwari and P. C. Jain [1982] , after that some mathematician uses this definition in a topological space (X,T) , Dalal ibraheem in [2012] study feeby continuous and proved several results in her paper . Some Results of Feebly Open and Feebly Closed Mappings in introduced by dalal [2009] . Dalal ibraheem[2007] define the feeby generalized closed set also sina greenwood and ivan L. reilly [ 1986 ] introduced the feeby closed mappings with some result on It .S . Pious Missier and E. Sucila [2013 ] introduced the perfectly continuous function. Yiezi Al-talkany [2007] in this previous research we are define the λ-open set in bitopological space after that , H. shaheed and S. Kadham [2006] introduced the λ- continuous function in bitopological space. Now in this paper we define some feeby function by using the λ-open set and study some theorems.

2. Preliminaries:

A subset A of a topological space X is said to be feeby open [S. N. Maheshwari and P.C. Jain 1982] if there exists an open set U such that U ⊆ A ⊆ cl(U) , Jankovic D. S. , Reidly I. L. [1985] proved that the complement of feeby open set is feeby closed set

For a subset A of a space X the closure and interior of A with respect to a topological space T are denoted by cl(A) and int(A). Some basic theorems and definitions we needed in this paper we give it now:

2-1 Definition [Dalal Ibraheem 2007]

A function F:X→ Y is called feeby closed function if the image of each closed set in X is feeby closed set in Y.

2-2. Definition [Dalal Ibraheem 2009]

A function F: X → Y is called feeby open function if the image of each open set in X is feeby open set in Y.

2-3. Theorem [Dalal Ibraheem 2007]

Every feeby open mapping is feeby open mapping.

2-4 Definition [H. Shaheed and S. Kadham2006]: a function f:(X,T,Tα)→(Y,V,V α) is called λ-continuous function if the inverse image of each open set in X is λ-open set in Y.

2-5 Definition [S. N. Maheshwari and P.C.Jain1982]: a function f:(X,T)→(Y,V) is said to be feeby continuous if the inverse image of each open set in Y is feeby open set in X.

2-6 Definition [yiezi AL-talkany 2007]: let (X,T,T α) be a bitopological space a subset A in X is said to be λ-open set if there exist U∈ T α such that A⊆ U and A⊆ int T(U)

2-6 Remark: [Dalal Ibraheem 2009]

1- Every open set is feeby open set

2- Every closed set is feeby closed set
2-7 **Theorem:** [Yiezi AL-Talkany2007] every open set is $\lambda$-open set

2-8 **Theorem** [H. Shaheed and S. Kadham 2006] every continuous function is $\lambda$-continuous

2-9 **Theorem** [S. N.Maheshwari and P.C.Jain 1982] every continuous mapping is feebly continuous mapping.

Dalal Ibraheem in her research proof the following theorems:

2-10 **Theorem:**[Dalal Ibraheem 2009] every closed mapping is feebly closed mapping.

2-11 **Theorem:**[Dalal Ibraheem 2009]: every open mapping is feebly open mapping.

2-12 **Theorem:** [Dalal Ibraheem 2009] the composition of two closed function is feebly closed function.

all the above theorems are not exist in our research.

3 **Feebly $\lambda$-continuous function**

**Definition:** a function $f:(X,T,T^a) \rightarrow (Y,V,V^a)$ is said to be feebly $\lambda$-continuous iff the inverse image for every $\lambda$-open set in $Y$ is feebly open set in $X$.

3-1 **Theorem:** [Saad Naji Al-Azawi, Jamhour Mahmoud AL-obaidi, Aco Saied2008]

Every continuous function is feebly continuous function

3-2 **Theorem:** if the function $f:(X,T,T^a) \rightarrow (Y,V,V^a)$ is feebly $\lambda$-continuous then

$f:(X,T) \rightarrow (Y,V)$ is feebly continuous.

Proof: let $H$ be open set in $Y$, by remark (1-5) $H$ is $\lambda$-open set, since $f$ is feebly $\lambda$-continuous then $f^{-1}(H)$ is feebly open set and then $f$ is feebly continuous.

3-3 **Theorem:**

Let $f:(X,T,T^a) \rightarrow (Y,V,V^a)$ is feebly $\lambda$-continuous and $g:(Y,V,V^a) \rightarrow (Z,W,W^a)$ is $\lambda$-continuous then gof is feebly $\lambda$-continuous

Proof: let $A$ be open set in $Z$, since $g$ is $\lambda$-continuous then $g^{-1}(A)$ is $\lambda$-open set in $Y$ and since $f$ is feebly $\lambda$-continuous then

$f^{-1}(g^{-1}(A))=(gof)^{-1}(A)$ is feebly open set in $X$.

3-4 **Theorem:** Let $f:(X,T,T^a) \rightarrow (Y,V,V^a)$ is feebly $\lambda$-continuous and $g:(Y,V,V^a) \rightarrow (Z,W,W^a)$ is $\lambda$-continuous then gof is feebly $\lambda$-continuous

Proof: exist by definitions

3-6 **Theorem:** Let $f:(X,T,T^a) \rightarrow (Y,V,V^a)$ be a map then the following are equivalent:

1- $f$ is feebly $\lambda$-continuous
2- The inverse image of each $\lambda$-closed set in $Y$ is feebly closed set in $X$
3- $Cl(f^{-1}(A)) \subseteq f^{-1}(cl(A))$ for each $A$ in $Y$
4- $f(cl(A)) \subseteq cl(f(A))$ for each $A$ in $X$
5- $f^{-1}(int(B)) \subseteq int(f^{-1}(B))$ for each $B$ in $Y$

Proof: (1)$\Rightarrow$ (2) obvious by definition
(2)$\Rightarrow$(3) let $A$ is subset of $Y$, then $cl(A)$ is closed set in $Y$ and then it is $\lambda$-closed set in $Y$, by (2) $f^{-1}(cl(A))$ is feebly closed set in $X$.

Since $f^{-1}(A) \subseteq f^{-1}(cl(A))$ then $cl(f^{-1}(A)) \subseteq cl(f^{-1}(cl(A))= f^{-1}(cl(A))$.

(3)$\Rightarrow$(4) let $A$ is closed set in $X$, then by (3) we get $cl(A) \subseteq cl(f^{-1}(f(A))) \subseteq f^{-1}(cl(f(A)))$ then $f(cl(A)) \subseteq cl(f(A))$.

(4)$\Rightarrow$(5) let $B$ is any subset of $Y$, by (4) $f(cl(X-f^{-1}(B))) \subseteq cl(f(X-f^{-1}(B)))$ and then $f(X-int(f^{-1}(B)) \subseteq cl(f(X-f^{-1}(B)))$ then we get that $X-int(f^{-1}(B)) \subseteq f^{-1}(Y-int(B))$ and then $f^{-1}(int(B)) \subseteq int(f^{-1}(B))$.

(5)$\Rightarrow$(1) let $A$ is $\lambda$-open set in $Y$, then by (5) $f^{-1}(int(A)) \subseteq f^{-1}(A)$ and then $f^{-1}(A) \subseteq int(f^{-1}(A))$, from that we get $f^{-1}(A)$ is feebly open set in $X$.

3-7 **Example:**
4 -feitly \( \lambda \)-open function and feitly \( \lambda \)-closed function

4-1. Definition: A function \( f:(X,T^a) \rightarrow (Y,V^a) \) is said to be feitly \( \lambda \)-open function if \( f(G) \) is feitly open set in \( Y \) foe every \( \lambda \)-open set \( G \) in \( X \).

4-2. Definition: A function \( f:(X,T^a) \rightarrow (Y,V^a) \) is said to be feitly \( \lambda \)-closed if \( f(G) \) is feitly closed set in \( Y \) for every \( \lambda \)-closed set \( G \) in \( X \).

4-3. Theorem: Let \( f:(X,T^a) \rightarrow (Y,V^a) \) is feitly \( \lambda \)-open and bijective function then \( f \) is feitly \( \lambda \)-closed function.

Proof: Let \( H \) is \( \lambda \)-closed set in \( X \) then \( X-H \) is \( \lambda \)-open set, since \( f \) is bijective then \( f(X-H)=Y-f(H) \) is feitly open set in \( Y \) and then \( f(H) \) is feitly closed set in \( Y \).

4-4. Theorem: Let \( f:(X,T^a) \rightarrow (Y,V^a) \) and \( g:(Y,V^a) \rightarrow (Z,W^a) \) are two function such that \( Gof \) is \( \lambda \)-open function and \( g \) is feitly \( \lambda \)-continuous injective function then \( f \) is feitly \( \lambda \)-open function.

Proof: Let \( A \) is \( \lambda \)-open set in \( X \), then \( (gof)(A) \) is feitly open function, since \( g \) is feitly \( \lambda \)-continuous then \( g^{-1}(gof)(A)=f(A) \) is feitly open set in \( Y \), and then \( f \) is feitly \( \lambda \)-open function.

4-5. Theorem: Let \( f:(X,T^a) \rightarrow (Y,V^a) \) and \( g:(Y,V^a) \rightarrow (Z,W^a) \) are two function such that \( Gof \) is feitly \( \lambda \)-open function and \( f \) is feitly \( \lambda \)-continuous surjective function then \( g \) is feitly \( \lambda \)-open function.

Proof: Let \( B \) is \( \lambda \)-open set in \( Y \), since \( f \) is feitly \( \lambda \)-continuous then \( f^{-1}(B) \) is feitly open set in \( X \), and since \( g \) is feitly open function then \( (gof)(f^{-1}(B))=g(B) \) is feitly open set in \( Z \).

4-6. Theorem: Let \( f:(X,T^a) \rightarrow (Y,V^a) \) and \( g:(Y,V^a) \rightarrow (Z,W^a) \) are two function such that \( Gof \) is \( \lambda \)-closed function and \( g \) is feitly \( \lambda \)-continuous injective function then \( f \) is feitly \( \lambda \)-closed function.

Proof: Let \( H \) is \( \lambda \)-closed set in \( X \), then \( (gof)(H) \) is \( \lambda \)-closed set in \( Z \), since \( g \) is feitly \( \lambda \)-continuous then \( g^{-1}(gof)(H)=f(H) \) is feitly closed set in \( Y \), \( f \) is feitly \( \lambda \)-closed map.

4-7. Theorem: Let \( f:(X,T^a) \rightarrow (Y,V^a) \) and \( g:(Y,V^a) \rightarrow (Z,W^a) \) are two function such that \( f \) is \( \lambda \)-closed function and \( g \) is feitly \( \lambda \)-closed function then \( gof \) is feitly \( \lambda \)-closed function.

Proof: Let \( H \) is \( \lambda \)-closed set in \( X \), then \( f(H) \) is \( \lambda \)-closed set in \( Y \) and then \( g(f(H))=gof(H) \) is feitly closed set in \( Z \), then \( gof \) is feitly \( \lambda \)-closed function.

5- Feitly \( \lambda \)-perfectly continuous function

5-1. Definition [S.Pious Missier and E. Sucila, 2013]:
A mapping \( f:(X,T) \rightarrow (Y,V) \) is said to be perfectly continuous if the inverse image of each open set in \( Y \) is both open and closed in \( X \).

5-2. Definition: A function \( f:(X,T^a) \rightarrow (Y,V^a) \) is said to be feitly \( \lambda \)-perfectly continuous function if the inverse image of each \( \lambda \)-open set in \( Y \) is feitly open set and feitly closed set in \( X \).

5-3. Theorem: Every feitly \( \lambda \)-perfectly continuous function is feitly continuous function.

Proof: Let \( A \) is open set in \( Y \), and then it is \( \lambda \)-open set since \( f \) is feitly \( \lambda \)-perfectly continuous, then \( f^{-1}(A) \) is feitly open set in \( X \) and then \( f \) is feitly continuous.

5-4. Theorem: Every feitly \( \lambda \)-perfectly continuous function is feitly \( \lambda \)-continuous function.

Proof: Exist by definitions.

References


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Perfectly continuous $\xrightarrow{\text{feebl}}$ feebly continuous $\xleftarrow{\text{feebl}}$ feebly $\lambda$-perfectly continuous

\begin{align*}
\lambda\text{-continuous} & \quad \rightarrow \quad \text{continuous} \\
\text{feebl }\lambda\text{- continuous} & \quad \leftarrow \quad \text{feebly continuous}
\end{align*}
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