Mathematical Theory and Modeling
ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)
Vol.2, No.3, 2012

Behavioural Analysis of a Complex Manufacturing System Having Queue in the Maintenance Section

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Abstract

In this paper reliability characteristic of a complex manufacturing system incorporating queue in service is studied. The considered system consists of three units namely A, B and C connected in series. Unit A consists of a main unit a_1 and an active redundant unit a_2, unit B consists of a main unit b_1 and a cold redundant unit b_2 and unit C consists of two units' c_1 and c_2 connected in parallel configuration. Considered system can completely fail due to failure of any of the subsystems. It is also assumed that the system can fail due to catastrophic failure. General repair facility is available for units c_1 and c_2 whereas there exits a maintenance section with one repairmen for repairing the units a_1, a_2, b_1 and b_2. Various reliability characteristics such as steady state behavior, availability, reliability, and MTTF and cost analysis have been obtained using supplementary variable technique and Gumble-Hougaard copula methodology.

Keywords: Supplementary variable technique, reliability, MTTF, asymptotic behavior, markov process, Gumble-Hougaard copula, profit function, queue.

1. Introduction

In modern engineering systems standby redundancy is used for improving the reliability and availability of components/units. Liebowitz (1966); Mine et al (1968) and Subba Rao (1970), while studying redundant system have assumed that a unit, immediately after failure, enters repair. Gupta et al (1983) and Pandey et al (2008) have assumed that the repair times of the failed units are independently distributed. This implies that there exist a fairly large number of independent repair facilities which would take up each unit as, and when, it fails. However, one can see in many practical situations, it is not feasible to have more than one repair facilities, in which the units that fail queue up for repair.

Queuing Theory plays a vital part in almost all investigations of service facilities. Queuing models provide a useful tool for predicting the performance of many service systems including computer systems, telecommunication systems, computer/communication networks, and flexible manufacturing systems. Traditional queuing models predict system performance under the assumption that all service facilities provide failure-free service. It must, however, be acknowledged that service facilities do experience failures and that they get repaired. Trivedi (1982) argues that failure/repair behavior of such systems' is commonly modeled separately using techniques classified under reliability/availability modeling. In recent years, it has been increasingly recognized that this separation of performance and reliability/ availability models is no longer adequate. Also Altiok's (1997) focused on the Performance Analysis of Manufacturing Systems. Mangey & Singh (2010) analyzed a complex system with common cause failures using Gumble-Hougaard copula methodology. Barlow & Proschan (1975) give the statistical theory of reliability and life testing.

Keeping above facts in view, the present paper deals with the reliability characteristics of a complex manufacturing system having 3-units A, B and C, connected in series, incorporating queue in service. Unit A consists of a main unit a_1 and an active redundant unit a_2, Unit B consists of a main unit b_1 and a cold redundant unit b_2 whereas unit C consists of two units' c_1 and c_2 connected in parallel configuration. The system can completely fail due to failure of any of the subsystems. Initially when the system starts functioning, the main units of subsystem A and B and both units of the subsystem C are operational. When the main units a_1 and b_1 of the subsystems A and B fail, The units in standby are switched on automatically and failed units are taken up for repair to maintenance section. An unusual situation is discussed here that when main units and standby units of A and B fail and they are taken up for repair to maintenance section, where repairmen is busy in repairing of other machine of the system. At this situation a queue is generated at the maintenance section. So here study is focused on the issue that all the four units after failure are in the queue waiting for repair. It is also assumed that the system can fail due to catastrophic failure. Once the system is failed due to catastrophic failure (CSF), two types of repairs i.e. constant and exponential are involved to repair the system. Hence the joint distribution is obtained with the help of Gumble-Hougaard copula. General repair facility is available for the repairing of units c_1 and c_2 whereas there exists a maintenance section with one repairmen for repairing the units a_1, a_2, b_1 and b_2 .For the units c_1 and c_2 failures follows exponential time distribution while repairs follow general time distribution and for the units a_1, a_2, b_1 and b_2 failure and repairs both follows exponential time distribution with Poisson arrivals. System specification and transition diagram is shown by figures 1 and 2 respectively. Table 1 shows the state specification of the system.

The following reliability characteristics of interest are obtained.

(i) Transition and steady state probabilities.

(ii) Mean operating time between successive failures for different failures.

(iii) Availability of the system.

(iv) Profit analysis.

2. Assumptions

(1) Initially the system is in good state.

(2) Subsystems A, B and C are connected in series.

(3) System has two states namely good and failed.

(4) For the units' c_1 and c_2 failures follow exponential time distribution while repairs follow general time distribution.

(5) For the units' a_1, a_2, b_1 and b_2 failure and repairs both follows exponential time distribution with Poisson arrivals.

(6) There are two types of repairs from state S_0 to S_{14} one is constant and other is exponential.

(7) Subsystem a_1, a_2, b_1 and b_2 can be repaired only at maintenance section.

(8) A special situation is discussed here that when all four machines a_1, a_2, b_1 and b_2 are in queue for repair at maintenance section because of repair of other machine.

(9) Once the system is failed due to catastrophic failure two types of repairs are involved to repair the system i.e. constant and other is exponential.

(10) Joint probability distribution of repair rates follows Gumbel-Hougaard copula.

3. Notations

Pr	Probability
$P_0(t)$	Pr (at time t system is in good state S_0)
$P_i(t)$	Pr {the system is in failed state due to the failure of the i^{th} subsystem at time t}, where i=2, 5, 7, 14.
K	Elapsed repair time, where k= x, y, z, u, q, g.
λ_i	Failure rates of subsystems, where i=a_1, a_2, b_1, b_2, c_1, c_2, CSF.
ψ	Arrival rate of unit's a_1, a_2, b_1, b_2 to the maintenance section.

μ Repair rate of unit's a_1, a_2, b_1, b_2.

$\phi_i(k)$ General repair rate of i^{th} system in the time interval (k, $k+\Delta$), where $i= c_1, c_2, CSF$ and $k=v, g, r, l$.

$P_3(t)$ Pr (at time t there is a queue (a_1, a_2, b_1, b_2) in the maintenance section due to servicing of some other unit and all four machines are waiting for repair.

$P_i(j,k,t)$ Pr (at time t system is in failed state due to the failure of j^{th} unit when k^{th} unit has been already failed, where i=9, 11. $j=g, v$. and $k=v, g$.

K_1, K_2 Revenue cost per unit time and service cost per unit time respectively.

Let $u_1 = e^l$ and $u_2 = \phi_{CSF}(l)$ then the expression for joint probability according to Gumbel-Hougaard family of copula is given as $\phi_{CSF}(l) = \exp[l^{\theta} + (\log\phi_{CSF}(l))^{\theta})^{1/\theta}]$.

4. Formulation of the Mathematical Model

Using elementary probability considerations and limiting procedure, we obtain the following set of difference-differential equations governing the behavior of considered system, continuous in time and discrete in space:

$$\left[\frac{d}{dt} + \lambda_{a_1} + \lambda_{a_2} + \lambda_{b_1} + \lambda_{b_2} + \lambda_{c_1} + \lambda_{c_2} + \lambda_{CSF}\right]P_0(t) = \int_0^{\infty}\mu\ (i)P_3(t)di + \phi_{c_1}P_8(t) + \phi_{c_2}P_{10}(t) + \int_0^{\infty}\phi_{CSF}(l)P_{14}(l,t)dl \quad \text{... (1)}$$

$$\left[\frac{\partial}{\partial t} + \lambda_{a_2} + \lambda_c\right]P_1(t) = \lambda_{a_1}P_0(t) + \int_0^{\infty}\phi_C(r)P_{12}(r,t)dr \quad \text{... (2)}$$

$$\left[\frac{\partial}{\partial t} + \psi\right]P_2(t) = \lambda_{a_2}P_1(t) \quad \text{... (3)}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial i} + (\mu + \psi)\right]P_3(t) = \psi[P_2(t) + P_5(t) + P_6(t) + P_7(t)] + \frac{(\psi t)^3 e^{-\psi t}}{6} \quad \text{... (4)}$$

$$\left[\frac{\partial}{\partial t} + \lambda_{a_1}\right]P_4(t) = \lambda_{a_2}P_0(t) \quad \text{... (5)}$$

$$\left[\frac{\partial}{\partial t} + \psi\right]P_5(t) = \lambda_{a_1}P_4(t) \quad \text{... (6)}$$

$$\left[\frac{\partial}{\partial t} + \lambda_{b_2} + \psi\right]P_6(t) = \lambda_{b_1}P_0(t) + \int_0^{\infty}\phi_C(r)P_{13}(r,t)dr \quad \text{... (7)}$$

$$\left[\frac{\partial}{\partial t}+\psi\right]P_7(t)=\lambda_{b_2}P_6(t)$$

… (8)

$$\left[\frac{\partial}{\partial t}+\phi_{c_1}(v)+\lambda_{c_2}\right]P_8(t)=\lambda_{c_1}P_0(t)+\int_0^\infty\phi_{c_2}(g)P_{11}(g,v,t)dg \quad \dots (9)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial g}+\phi_{c_2}(g)\right]P_9(g,v,t)=0 \quad \dots (10)$$

$$\left[\frac{\partial}{\partial t}+\phi_{c_2}(g)+\lambda_{c_1}\right]P_{10}(t)=\lambda_{c_2}P_0(t)+\int_0^\infty\phi_{c_1}(v)P_{11}(v,g,t)dv \quad \dots (11)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial v}+\phi_{c_1}(v)\right]P_{11}(v,g,t)=0 \quad \dots (12)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial r}+\phi_C(r)\right]P_{12}(r,t)=0 \quad \dots (13)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial r}+\phi_C(r)\right]P_{13}(r,t)=0 \quad \dots (14)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial l}+\phi_{CSF}(l)\right]P_{14}(l,t)=0 \quad \dots (15)$$

Boundary Conditions:

$$P_3(i=0,t)=\psi[P_2(t)+P_3(t)+P_6(t)+P_7(t)] \quad \dots (16)$$

$$P_8(0,t)=\lambda_{c_1}P_0(t) \quad \dots (17)$$

$$P_9(0,v,t)=\lambda_{c_2}P_8(t) \quad \dots (18)$$

$$P_{10}(0,t)=\lambda_{c_2}P_0(t) \quad \dots (19)$$

$$P_{11}(0,g,t)=\lambda_{c_1}P_{10}(t) \quad \dots (20)$$

$$P_{12}(0,t)=\lambda_C P_1(t) \quad \dots (21)$$

$$P_{13}(0,t)=\lambda_C P_6(t) \quad \dots (22)$$

$$P_{14}(0,t) = \lambda_{CSF} P_0(t)$$

… (23)

Initial condition:

$P_0(0) = 1$, otherwise zero. ... (24)

5. Solution of the Model

Taking Laplace transforms of equation (1) through (23) subject to initial and boundary conditions and then on solving them one by one; we obtain the following:

$$\overline{P}up = \overline{P}_0(s) + \overline{P}_1(S) + \overline{P}_4(s) + \overline{P}_6(s) + \overline{P}_8(s) + \overline{P}_{10}(s)$$

$$= \frac{1}{K(s)}[\,1 + \frac{\lambda_{a_1}}{[s+\lambda_{a_2}+\lambda_C-\lambda_C\overline{S}_{\phi_C}(s)]} + \frac{\lambda_{a_2}}{[s+\lambda_{a_1}]} + \frac{\lambda_{b_1}}{[s+\lambda_{b_2}+\psi-\lambda_C\overline{S}_{\phi_C}(s)]} +$$

$$\frac{\lambda_{c_1}}{[s+\lambda_{c_2}+\phi_{c_1}(v)-\lambda_{c_2}\overline{S}_{\phi_{C_2}}(s)]} + \frac{\lambda_{c_2}}{[s+\lambda_{c_1}+\phi_{c_2}(g)-\lambda_{c_1}\overline{S}_{\phi_{c1}}(s)]}] \quad \ldots (25)$$

$$\overline{P}_{Down} = \overline{P}_2(s) + \overline{P}_5(s) + \overline{P}_7(s) + \overline{P}_9(s) + \overline{P}_{11}(s) + \overline{P}_{12}(s) + \overline{P}_{13}(s) + \overline{P}_{14}(s)$$

$$= \frac{\lambda_{a_1}\lambda_{a_2}}{[s+\psi][s+\lambda_{a_2}+\lambda_C-\lambda_C\overline{S}_{\phi_C}(s)]}\frac{1}{K(s)} + \frac{\lambda_{a_1}\lambda_{a_2}}{[s+\psi][s+\lambda_{a_1}]}\frac{1}{K(s)} +$$

$$\frac{\lambda_{b_1}\lambda_{b_2}}{[s+\psi][s+\lambda_{b_2}+\psi-\lambda_C\overline{S}_{\phi_C}(s)]}\frac{1}{K(s)} + \frac{\lambda_{c_1}\lambda_{c_2}D_{\phi_{c_2}}(s)}{[s+\lambda_{c_2}+\phi_{c_1}(v)-\lambda_{c_2}\overline{S}_{\phi_{c_2}}(s)]}\frac{1}{K(s)} +$$

$$\frac{\lambda_{c_1}\lambda_{c_2}D_{\phi_{c_1}}(s)}{[s+\lambda_{c_1}+\phi_{c_2}(g)-\lambda_{c_1}\overline{S}_{\phi_{c1}}(s)]}\frac{1}{K(s)} + \frac{\lambda_C\lambda_{a_1}D_{\phi_c}(s)}{[s+\lambda_{a_2}+\lambda_C-\lambda_C\overline{S}_{\phi_C}(s)]}\frac{1}{K(s)} +$$

$$\frac{\lambda_C\lambda_{b_1}D_{\phi_c}(s)}{[s+\lambda_{b_2}+\psi-\lambda_C\overline{S}_{\phi_C}(s)]}\frac{1}{K(s)} + \frac{\lambda_{CSF}D_{\phi_{CSF}}(s)}{K(s)} \quad \ldots (26)$$

Where,

$$K(s) = s+\lambda_{a_1}+\lambda_{a_2}+\lambda_{b_1}+\lambda_{b_2}+\lambda_{c_1}+\lambda_{c_2}+\lambda_{CSF}-\psi\{[\frac{\lambda_{a_1}\lambda_{a_2}}{[s+\psi][s+\lambda_{a_2}+\lambda_C-\lambda_C\overline{S}_{\phi_C}(s)]}$$

$$+\frac{\lambda_{a_1}\lambda_{a_2}}{[s+\psi][s+\lambda_{a_1}]} + \frac{\lambda_{b_1}}{[s+\lambda_{b_2}+\psi-\lambda_C\overline{S}_{\phi_C}(s)]} + \frac{\lambda_{b_1}\lambda_{b_2}}{[s+\psi][s+\lambda_{b_2}+\psi-\lambda_C\overline{S}_{\phi_C}(s)]}]D_{\mu}(s)$$

$$+\frac{\psi^3}{(s+\psi)^4}\}-\frac{\lambda_{c_1}\phi_{c_1}(v)}{[s+\lambda_{c_2}+\phi_{c_1}(v)-\lambda_{c_2}\overline{S}_{\phi_{C_2}}(s)]}-\frac{\lambda_{c_2}\phi_{c_2}(g)}{[s+\lambda_{c_1}+\phi_{c_2}(g)-\lambda_{c_1}\overline{S}_{\phi_{c1}}(s)]}$$
$$-\lambda_{CSF}\overline{S}_{\phi_{CSF}}(s)$$
... (27)

$$M(s)=\psi\{[\frac{\lambda_{a_1}\lambda_{a_2}}{[s+\psi][s+\lambda_{a_2}+\lambda_C-\lambda_C\overline{S}_{\phi_C}(s)]}+\frac{\lambda_{a_1}\lambda_{a_2}}{[s+\psi][s+\lambda_{a_1}]}+\frac{\lambda_{b_1}}{[s+\lambda_{b_2}+\psi-\lambda_C\overline{S}_{\phi_C}(s)]}$$
$$+\frac{\lambda_{b_1}\lambda_{b_2}}{[s+\psi][s+\lambda_{b_2}+\psi-\lambda_C\overline{S}_{\phi_C}(s)]}]D_\mu(s)++\frac{\psi^3}{(s+\psi)^4}\} \quad \text{... (28)}$$

$$D_\mu(s)=\frac{1-\overline{S}_\mu(s)}{s+\psi} \quad \text{... (29)}$$

$$\phi_{CSF}(l)=\exp[l^\theta+(\log\phi_{CSF}(l))^\theta)^{1/\theta}] \quad \text{... (30)}$$

6. Asymptotic Behaviour of the System

Using Abel's lemma in Laplace transforms, viz;

$$\lim_{s\to 0} s\bar{f}(s)=\lim_{t\to\infty} f(t)=f(\text{say}) \quad \text{... (31)}$$

provided the limit on the right hand side exists, the time independent operational probabilities for up and down states are obtained as follows:

$$\overline{P}_{up}(s)=\frac{1}{K(0)}[1+\frac{\lambda_{a_1}}{\psi\lambda_{a_2}}+\frac{\lambda_{a_2}}{\lambda_{a_1}}+\frac{\lambda_{b_1}}{\lambda_{b_2}+\psi-\lambda_c}+\frac{\lambda_{c_1}}{\phi_{c_1}(v)}+\frac{\lambda_{c_2}}{\phi_{c_2}(g)}] \quad \text{... (32)}$$

$$\overline{P}_{down}=\frac{1}{K(0)}[\frac{\lambda_{a_1}}{\psi}+M(0)+\frac{\lambda_{a_2}}{\psi}+\frac{\lambda_{b_1}\lambda_{b_2}}{\psi(\lambda_{b_2}+\psi-\lambda_C)}+\frac{\lambda_{c_1}\lambda_{c_2}M_{\phi_{c_2}}}{\phi_{c_1}(v)}+\frac{\lambda_{c_1}\lambda_{c_2}M_{\phi_{c_1}}}{\phi_{c_2}(g)}+$$
$$\frac{\lambda_C\lambda_{a_1}M_{\phi_C}}{\lambda_{a_2}}+\frac{\lambda_C\lambda_{b_1}M_{\phi_C}}{\lambda_{b_2}+\psi-\lambda_C}+\lambda_{CSF}M_{\phi_{CSF}}] \quad \text{... (33)}$$

where,

$$M(0)=\lim_{s\to 0} M(s) \quad \text{... (34)}$$

$$M_{\phi_i}=\lim_{s\to 0}\frac{1-\overline{S}_{\phi_i}(s)}{s} \quad \text{... (35)}$$

$$S_{\phi_i}(s) = \frac{\phi_i}{s+\phi_i}$$

… (36)

6.1Particular Case

A particular case is also discussed as given below:

When all repairs follow exponential time distribution: In this case setting,

$$\overline{S}_{\phi_i}(s) = \frac{\phi_i}{s+\phi_i}, \forall i, \overline{S}_{\mu}(s) = \frac{\mu}{s+\mu} \text{ and } \overline{S}_{\phi_{CSF}}(s) = \frac{\exp\left[l^{\theta} + [\log \phi_{CSF}(l)]^{\theta}\right]^{1/\theta}}{s+\exp\left[l^{\theta} + [\log \phi_{CSF}(l)]^{\theta}\right]^{1/\theta}}$$ in equation (25) and (26), we can obtain the Laplace transforms of various state probabilities of the system.

7. Numerical Computation

7.1 Availability of the considered system

For the numerical computation let us consider the values:

$\lambda_{a_1} = 0.3, \lambda_{a_2} = .5, \lambda_{b_1} = .4, \lambda_{b_2} = .9, \lambda_{c_1} = .5, \lambda_{c_2} = .6, \lambda_C = .8, \lambda_{CSF} = .7, \psi = .5$ *and* $\phi_C = \phi_{c_1} = \phi_{c_2} = \mu = 1$ and x=*y*=*z*=*l*=*v*=*g*=*u*=*1*.

Putting all these values in equation (32) and taking inverse Laplace transform, we get

$$Pup=.1686680814e^{(-4.323965466t)}-.008768799073e^{(-2.117902295t)}-.04845342456e^{(-2.071912570t)}+.09122411851e^{(-2.062984318t)}+.4158882739e^{(-1.735709603t)}-.02422312588e^{(-1.114879943t)}+.005458900298e^{(-.8020535367t)}+.2063822881e^{(-.5004630522t)}\cos(.5063484832t)-.02881439167e^{(-.5004630522t)}\sin(.5063484832t)+.1082411055*10^{(-89)}I.1331028150*10^{(89)}e^{(-.5004630522t)}\cos(.5063484832t)-.9533452524*10^{(89)}e^{(-.5004630522t)}\sin(.5063484832t))+.1082411055*10^{(-89)}*I(.1331028150*10^{(89)}e^{(-.5004630522t)}\cos(.5063484832t)+.9533452524*10^{(89)}e^{(-.500463052t)}\sin(.5063484832t))-.1143200992*10^{(-5)}e^{(-.4693387571t)}-.001713357894e^{(-.3432408717t)}+.0005397902571e^{(-.2917232784t)}-.0002082937963e^{(-.2487154115t)}+.1949572589e^{(.08335215466t)}$$ …(37)

Now in equation (37) setting t=0, 1, 2….10 one can obtain the Figure 3.

7.2 Reliability Analysis

Let $\lambda_{a_1} = 0.3, \lambda_{a_2} = .5, \lambda_{b_1} = .4, \lambda_{b_2} = .9, \lambda_{c_1} = .5, \lambda_{c_2} = .6, \lambda_C = .8, \lambda_{CSF} = .7, \psi = .5$ and $\phi_C = \phi_{c_1} = \phi_{c_2} = \mu = 0$ and *x*=*y*=*z*=*l*=*v*=*g*=*u*=1.

Putting all these values in equation (32) and taking inverse Laplace transform, we get,

$$Pup=-.0003992289179e^{(-.6000000000t)}+.1301800390e^{(-1.400000000t)}+.2561750275e^{(-3.885958190t)}+.2579813141e^{(-1.229778949t)}-.0237396891e^{(-1.034271878t)}+.1939947786e^{(-.5178813385t)}\cos(.4067318666t)-.07626612243e^{(-.5178813385t)}\sin(.4067318666t)-.4458387683*10^{(-35)}I(.8553105723*10^{34})e^{(-.5178813385t)}\cos(.4067318666t)+.217561855*10^{35}e^{(-.5178813385t)}\sin(.4067318666t)-.4458387683*10^{(-35)}I(.8553105723*10^{34})e^{(-.5178813385t)}\cos(.4067318666t)-.217561855*10^{35}e^{(-.5178813385t)}\sin(.4067318666t)-.002547494817e^{(.3188919519t)}+.1883552536e^{(.004663645435t)}$$ …(38)

Now varying time in equation (38), one can obtain the Figure 4.

7.3 MTTF Analysis

MTTF of the system can be obtained as

$$MTTF = \lim_{s \to 0} \overline{P}\text{up}(s) \text{ (as } s \text{ tends to 0)}$$ ... (39)

7.3.1 With respect to λ_{a_1} :

Suppose $\lambda_{a_2} = .5, \lambda_{b_1} = .4, \lambda_{b_2} = .9, \lambda_{c_1} = .5, \lambda_{c_2} = .6, \lambda_C = .8, \lambda_{CSF} = .7, \psi = .5$ in equation (39) and putting $\lambda_{a_1} = 0.1, .5, 1, 1.5, 2...$ one can obtain the MTTF for different values of λ_{a_1} as shown in Figure 5.

7.3.2 With respect to λ_{c_1} :

Consider $\lambda_{a_1} = 0.3, \lambda_{a_2} = .5, \lambda_{b_1} = .4, \lambda_{b_2} = .9, \lambda_{c_2} = .6, \lambda_C = .8, \lambda_{CSF} = .7, \psi = .5$ in equation (39) and putting $\lambda_{c_1} = .1, .5, 1, 1.5, 2....$ one can obtain the MTTF for different values of λ_{c_1} as depicted in Figure 6.

7.3.3 With respect to λ_{c_2} :

Assume $\lambda_{a_1} = 0.3, \lambda_{a_2} = .5, \lambda_{b_1} = .4, \lambda_{b_2} = .9, \lambda_{c_1} = .5, \lambda_C = .8, \lambda_{CSF} = .7, \psi = .5,$ in equation (39) and taking $\lambda_{c_2} = .1, .5, 1, 1.5, 2....$ we have Figure 6 which shows the variation of MTTF for a range of values of λ_{c_2} .

7.3.4 With respect to λ_{CSF} :

Setting $\lambda_{a_1} = 0.3, \lambda_{a_2} = .5, \lambda_{b_1} = .4, \lambda_{b_2} = .9, \lambda_{c_1} = .5, \lambda_{c_2} = .6, \lambda_C = .8, \psi = .5$ in equation (39) and putting $\lambda_{CSF} = .1, .5, 1, 1.5,...$ one can get Figure 7 which exhibits the variation of MTTF for different values of λ_{CSF} .

7.4 Cost Analysis

Letting $\lambda_{a_1} = 0.3, \lambda_{a_2} = .5, \lambda_{b_1} = .4, \lambda_{b_2} = .9, \lambda_{c_1} = .5, \lambda_{c_2} = .6, \lambda_C = .8, \lambda_{CSF} = .7, \psi = .5$ and repair rates are $\phi_C = \phi_{c_1} = \phi_{c_2} = \mu = 1$ and x=y=z=l=v=g=u=1. Furthermore, if the repair follows exponential distribution then, on putting all these values and taking inverse Laplace transform one can obtain equations (37). If the service facility is always available, then expected profit during the interval (0, t] is given by

$$G(t) = K_1[.03900773092e^{(-4.323965466t)}+.00414032759e^{(-2.117902295 t)}-.02338584420 e^{(-2.071912570 t)}-.04421949198 e^{(-2.062984318t)}-.2396070594e^{(-1.735709603t)}+.02172711681e^{(-1.114879943t)}-.006806154350 e^{(-.8020535367 t)}-.1749950223 e^{(-.5004630522t)}\cos(.5063484832t)+.2346284213e^{(-.5004630522t)}\sin(.5063484832t)+.1082411055*10^{(-89)}I(.1083823101*10^{(90)}e^{(-.5004630522t)}\cos(.5063484832t)+.8083575158*10^{(89)}e^{(-.5004630522t)}\sin(.5063484832t))+.1082411055*10^{(-89)}I(-.1083823101*10^{(90)}e^{(-.5004630522t)}\cos(.5063484832t)-.8083575158*10^{(89)}e^{(-.5004630522t)}\sin(.5063484832t))+.2435764807*10^{(-5)}e^{(.4693387571t)}+.004991707093e^{(-.3432408717t)}-.001850350306e^{(-.2917232784t)}+.0008374784427e^{(-.2487154115t)}+2.338958839e^{(.083352154661t)}-1.840786251]-K_2t \quad \ldots (40)$$

Keeping $K_1 = 1$ and varying K_2 at 0.1, 0.2, 0.3 in equations (40), one can obtain Figure 8.

8. Results and Discussion

In this paper, we have evaluated availability, reliability, MTTF and cost function for the considered system by employing Supplementary variables technique and copula methodology. Also, we have computed asymptotic behavior and a particular case to improve practical utility of the system. M. Jain & Charu Bhargava (2008) have done the analysis of bulk arrival retrial queue. Indra & Sweety Bansal (2010) have given the concept of vacations to unreliable M/G/1 queue. But we have analysed a common phenomenon of queue, which usually occurs in the manufacturing system. Analysis of the Figure 3 gives us the idea of the availability of the system with respect to time *t*. Critical examination of Figure 3 yields that the values of the availability decreases approximately in a constant manner with the increment in time.

The Figure 4 shows the trends of reliability of the system with respect to time when all the failures and repair rates have some fixed values. From the graph we conclude that the reliability of the system decreases rapidly with passage of time when all failures follows exponential time distribution. The reason for the rapid decrement is queue in the maintenance section due to which the system is in down state for a large period of time.

Next, we study the effect of various parameters on the MTTF. A critical examination of Figures.5, 6, 7 shows that MTTF decreases with increment in λ_{a_1} , λ_{c_1} , λ_{c_2} and λ_{CSF} . An unusual phenomenon can be seen by observing the graph that for all the parameter, initially MTTF is negative due to queue in maintenance section and later it becomes positive.

Finally, Figure 8 represents the graph of the "Cost function vs. time. In this Figure we plotted a cost function G (t) for different values of cost K_1 and K_2. One can easily observe that increasing service cost leads decrement in expected profit.

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Table 1 State specification chart

States	Description	System State
S_0	When the system is in fully operational condition.	G
S_1	When the system is in operating state when unit a_1 is failed.	G
S_2	When the system is in failed state due to the failure of unit a_2.	F
S_3	When all four units a_1, a_2, b_1, b_2 are in queue at maintenance section due to repairing of some other unit of the manufacturing system. The system is in	F

	failed state at this time.	
S_4	When the system is in operable state when unit a_2 is failed.	G
S_5	When the system is in failed state due to the failure of unit a_1.	F
S_6	When the system is in operable condition when unit b_1 is failed.	G
S_7	When the system is in failed state due to failure of unit b_2.	F
S_8	When the system is in operable condition when unit c_1 is failed.	G
S_9	When the system is in failed state from the state S_8 due to failure of unit c_2.	F_R
S_{10}	When the system is in operable condition when unit c_2 is failed.	G
S_{11}	When the system is in failed state from the state S_{10} due to failure of unit c_1.	F_R
S_{12}	When the system is in failed state from the state S_1 due to failure of unit C.	F_R
S_{13}	When the system is in failed state from the state S_6 due to failure of unit C.	F_R
S_{14}	When the system is in failed state due to catastrophic failure.	F_R

G: Good state; F: Failed State; F_R= Failed state and under repair.

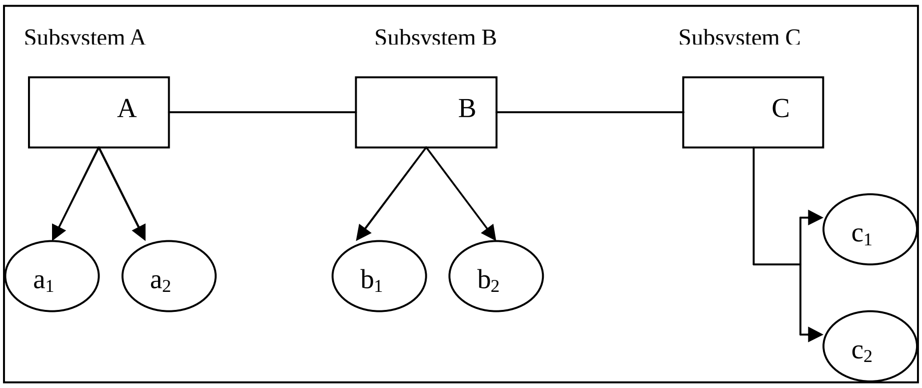

Figure 1: System Configuration

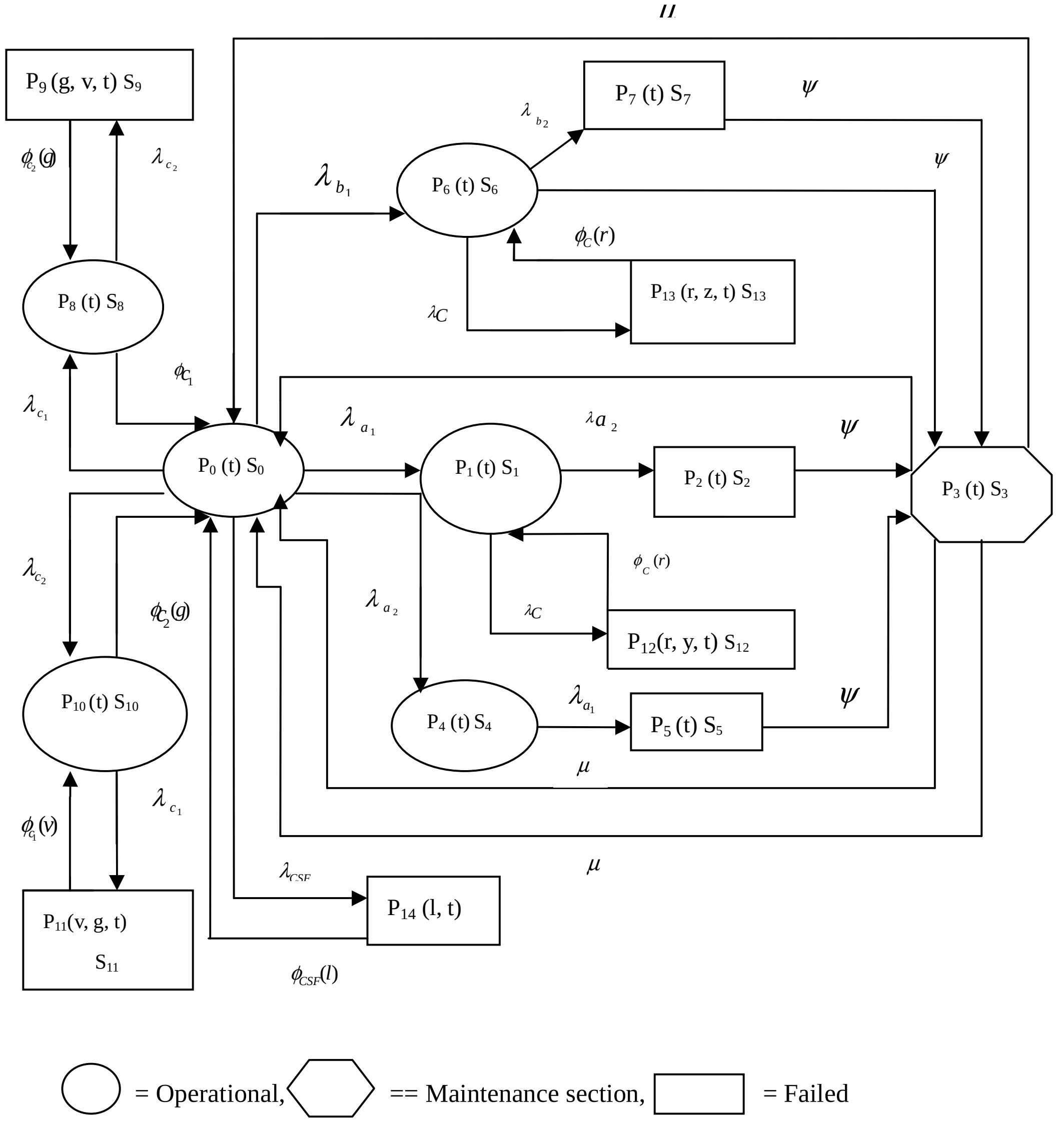

Figure 2: State transition diagram

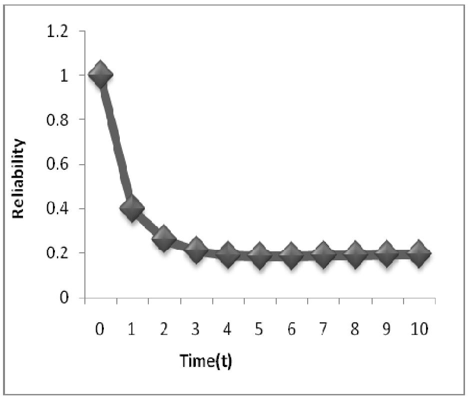

Figure 3. Availability Vs Time

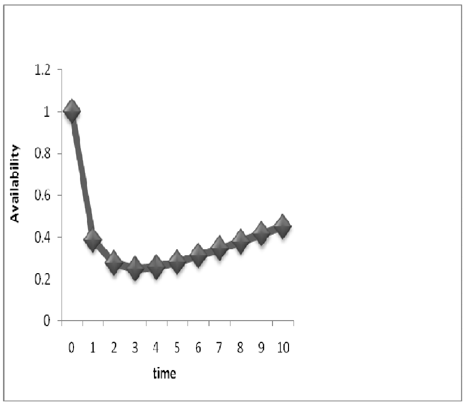

Figure 4. Reliability Vs Time

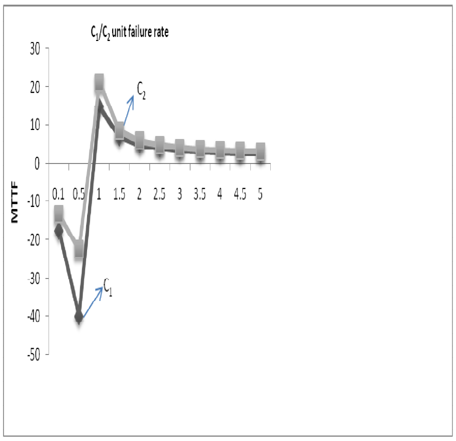

Figure 5. MTTF Vs λ_{a_1}

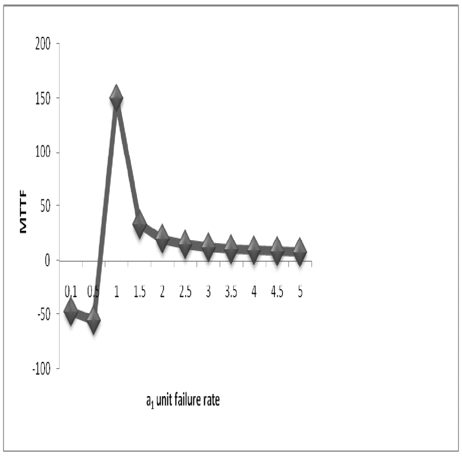

Figure 6. MTTF Vs λ_{c_1} and λ_{c_2}

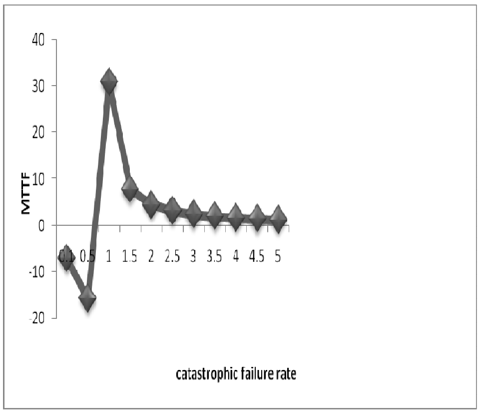

Figure 7. MTTF Vs λ_{CSF}

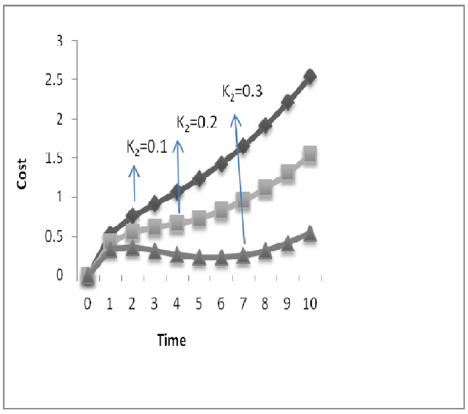

Figure 8. Cost Vs time