An Application Of Extreme Value Theory In Modelling Electricity Production In Kenya

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ABSTRACT

Extreme Value Theory provides a well-established statistical model for the computation of extreme risk measure which includes, Value at Risk and Expected Shortfall. In this paper we apply Univariate Extreme Value Theory to model extreme production for the Kenyan Electricity. We demonstrate that Extreme value theory can successfully be applied in predicting future Value at Risk to the electricity production. This will provide solutions to the problems faced by producers and consumers in the electricity market. In this paper Value at Risk is estimated using a Peak Over Threshold method. This technique models the distribution of exceedances over a high threshold rather than the individual observations. It concentrates on observation that exceeds central limits, focusing on the tail of the distribution. Extreme value theory is also applied to compute the tail risk measures at given confidence interval. An overview of the Extreme Value Theory and Peaks Over Threshold Method are also given. These methods are applied to electricity production in Kenya and the data exhibit some trend and modeled as a Gumbel distribution since the shape parameter is not significantly different from zero.

Keywords: Risk Modeling, Value at Risk, Extreme Value Theory.

1.0 INTRODUCTION

Over the previous years there has been significant uncertainty in electricity production in Kenya and Africa in general. This has led to several criticisms about the current risk management system and has inspired scholars to search for a more appropriate methodology to cope up with rare events that have got huge consequences. The problem is how to model the extreme phenomena that lies outside the range of available observation. These extreme observations no matter how large or small they are, depending on the data, they can cause serious challenges in the industry. For such anxieties one need to rely on a theory which can be used to build statistical models describing rare events. The question one should is “if things go wrong how wrong can they go” (Manfred and Elvis, 2006).

The main objective of this research is to apply extreme value theory in modeling the outliers in the Kenyan electricity production. Electricity being more like a service than a commodity by nature makes its total production very liquid compared to the production of many other commodities. This is due to its production, consumption, and delivery, which makes it an “instantaneous” product. The instantaneity is due to the fact that electricity cannot be stored, but its production and consumption need to be perfectly synchronized. As a result, the demand and supply of electricity have to be in perfect balance at all times, which in turn affects the electricity production considerably, as sudden large peaks and drops in demand and supply are reflected directly in the production. However, this has forced many industries in Kenya to operate in shifts due to fluctuation in electricity production. Another challenge that often arises is that there has been subsequent blackout in the country. Power disasters are particularly serious at sites where the environment and public safety are at risk.

Numerous studies have been done on electricity development. Mhilu (2007) applied the multiple regression technique for the development of regional equations for the estimation of low flow regimes of small catchments, On power transmission, Sebetoi A.B (2010) and Okou R, (2010) discusses the transmission of energy in Sub-Saharan African and the waste associate with transmission and came up with electricity tariff models on energy efficient. Mbugua et al. (2012) discuss on how quantile estimation and extreme value theory can be used to model extreme daily rate of change in electricity usage at macro level and to the daily rate of change in fuel prices. Another objective of this paper is to determine the statistical characteristics of Kenya daily electricity production and to model behavior of the tail distribution of electricity production.

The significant advantage of applying (EVT) on electricity production model residuals is that we can directly model the extreme production movements. (EVT) namely allows us to model the tails directly, instead of modeling the whole production. It is a well-known fact that distributions of many production exhibit tail fatness.

This distributional feature is not only observable in power production but also with the progress of Basel II regulation implementation, the prospect of (EVT) application is far from dismal. (EVT) has been used to quantify the probabilistic behavior of the unusually daily rate of change and it has arisen as a new methodology to analyze the tail behavior of electricity production, McNeil and Frey (2000). Another objective is to use extreme value theory to compute tail risk measures.

2.0 RISK MEASURE

Some of the most frequently used measure of risk in extreme quantile estimation includes value-at-risk (VaR) and Expected Shortfall (ES) and return level. This corresponds to the determination of the value at given variable exceed with a given probability. These risk measures will be discussed in detail.

2.1 Value-at-Risk

VaR is generally defined as the risk capital sufficient, in most instances to cover losses from a portfolio over a holding period of a fixed number of days. Suppose a random variable X, with a distribution function F describe negative returns on a certain financial instrument over a certain time horizon. Then VaR can be defined as the q-th quantile of the distribution F

$$VaR_q = F^{-1}(q) \quad (1)$$

Where $F^{-1}$is the inverse of the distribution F(q). The inverse of the distribution at a particular probability level is the called quantile. For risk management, q is usually taken to be greater than 0.95 and the quantile in this case is referred to us Value-at-Risk.

2.2 Expected shortfall

Another informative measure of risk is the expected shortfall (ES) or the tail conditional expectation which estimates the potential size of the loss exceeding VaR at q probability level. Mathematically we can define the (ES) as the expected size of a loss that exceed VaR_q

$$ES_q = E\left\lfloor X / X > VaR_q \right\rfloor \quad (2)$$

Artzner et al. (1999) argue that VaR is not a coherent risk measure, but proved that ES is a coherent risk measure.

3.0 EXTREME VALUE THEORY

Extreme value theory begins with the assumption that there is a sequence of independent and identical distribution observation $X_1 \ldots X_n$ from unknown distribution. The limit law for the block maxima denoted by M_n where n is the sizes of the sample (block) were introduced by Fisher and Tippet (1928) and later Gnedneko (1943) stated in their theorem. They stated that let X_n be a sequence of iid random variables. If there exist constants $c_n>0$, d_n and some non-degenerate distribution function H such that

$$\frac{M_n - d_n}{c_n} \to H \quad (3)$$

Then H belongs to one of the three standard extreme value distributions.

$$\text{Fretchet} \quad \phi_\alpha(x) = \left\{ \begin{matrix} 0, x \le 0 \\ \exp(-x^{-\alpha}), x > o \end{matrix}, \alpha > 0 \right\} \quad (4)$$

$$\text{Weibull} \quad \varphi_\alpha(x) = \left\{ \begin{matrix} \exp(-x^{-\alpha}), x \le 0 \\ 0, x > 0 \end{matrix}, \alpha > o \right\} \quad (5)$$

$$\text{Gumbel} \quad \lambda_\alpha(x) = \exp(\exp(-x)), x \in \Re \quad (6)$$

These distributions were generalized by Jenkinvon and Van Mises (1936) in the following one parameter representation. The GEV distribution function is

$$H_{\xi}(x) = \begin{cases} \exp\left[-(1+\xi x)^{\frac{-1}{\xi}}\right], \xi \neq 0 \\ \exp[-\exp(-x)], \xi = 0 \end{cases} \tag{7}$$

Where the iid random variable X is such that $1+\xi x>0$ ξ is the shape parameter. H(x) is called the standard Generalized Extreme Value (GEV) distribution.

The shape parameter ξ is crucial in determining the class of GEV distribution. When $\xi>0$ the distribution is known as Frechet which have got heavy tailed distribution, when $\xi=0$ it is a Gumbel distribution which is characterized by thin tailed distribution and when $\xi<0$ it is a Weibull distribution characterized by finite endpoint distribution. The classic extreme value theory can be divided into two groups; (Block maximum and Peak over threshold).

3.1 Method of Block minimum

The data is divided into consecutive blocks then the method focuses on the series of maxima or minima in these blocks (block maxima methods), then the generalized extreme value distributions to the minima are fitted and finally the GEV estimates are computed.

3.2 Peak over threshold

The peak over threshold (POT) is a more modern method used to model all large observations exceeding a high threshold u. The method is used to observe observations that exceed certain high threshold u. The excess distribution of the random variable X with distribution F over the threshold is defined

$$F_u(x) = \Pr\{X - u \leq x | x > u\} = \frac{F(x+u) - F(u)}{1 - F(u)} \tag{8}$$

In this case a fully parametric model based on the generalized Pareto distribution (GPD) can be used to estimate $F_u(x)$.

The generalized Pareto distribution use in modeling excess is defined by

$$G_{\xi,\beta(u)}(x) = \begin{cases} 1 - \left(1 - \frac{\xi x}{\beta(u)}\right)^{-\frac{1}{\xi}}, \xi \neq 0 \\ 1 - \exp\left\{\frac{-x}{\beta(u)}\right\}, \xi = 0 \end{cases} \tag{9}$$

With the scale parameter is $\beta(u)>0$ and $x\geq0$ with shape parameter $\xi>0$ or

The generalized Pareto distribution is a generalization in the same sense as the GEV; ξ can take positive negative or zero value. The shape parameter is the independent of **u** and is the same as for GEV distribution. The generalized Pareto distribution is supported by Balkema-de- Haan and Pickand (1975) theorem which is defined as

$$\lim_{u \to e_u} Sup\left|F_u(x) - G_{\xi,\beta(U)}(x)\right| = 0 \tag{10}$$

The result state that if F is in the maximum domain of attraction of the Fretchet distribution then as the threshold u approaches the endpoint of F the GPD asymptotically approximates excess distribution function $F_u(x)$. We can therefore write

$$G_{\xi,\beta(u)}(x)=\frac{F(x+u)-F(u)}{1-F(u)} \qquad x>u \tag{11}$$

Assuming a GPD function for the tail distribution, analytical expression for VaR_q and ES_q can be defined as a function of GPD parameters. Isolating F(u+x) from (11)

And replacing $F_u(x)$ by the GPD and F(u) by the estimate $(n-N_u)/n$ where n is the total number of observations and N_u the number of observation above the threshold u, we obtain

$$F(x)=\frac{N_u}{n}\left(1-\left(1+\frac{\xi}{\beta(u)}(x-u)\right)^{\frac{-1}{\xi}}\right)+\left(1-\frac{N_u}{n}\right),$$

this simplifies to

$$\hat{F}(x)=1-\frac{N_u}{n}\left(1+\frac{\xi}{\beta(u)}(x-u)\right)^{\frac{-1}{\xi}} \tag{12}$$

Inverting (12) for a given probability q we obtain

$$\hat{VaR}_q=u+\frac{\sigma}{\xi}\left(\left(\frac{n}{N_u}q\right)^{-\xi}-1\right), \tag{13}$$

Let as rewrite the expected shortfall as

$ES_q= VaRq + E(X- VaR_q / X > VaR_q)$

Where the second term on the right is the expected exceedances over the threshold VaR_q. It is known as the mean excess function for the GPD with parameter $\xi<1$ is

$$e(x)=E(X-Z/X>Z)=\frac{\sigma+\xi x}{1-\xi} \qquad \sigma+\xi x>0 \tag{14}$$

The function gives the average of the excesses of X over varying values of the threshold x

$$ES_p=\frac{VaR_p}{1-\xi}+\frac{\sigma-\xi u}{1-\xi} \tag{15}$$

The Peak Over Threshold method is used in electricity generation so that the threshold value is chosen and the generalized Pareto distribution is fitted accordingly to the sample data of extreme observation that exceeds the threshold. The parameters which will be estimated are the tail index ξ and the scale parameter β(u) of which the tail index is the most crucial one as it sets the rate how quickly the tail of $G_{\xi,\beta(u)}(x)$ decay away.

The Peak over threshold involves the following steps: we select the threshold u, then fit the GPD functions to the exceedances over u and then compute the point estimate for the expected shortfall and value-at-risk. Much theory tells us that u should be high as possible to satisfy equation (9) but the higher the threshold the less observation are left for the estimate of the parameters of the tail distribution function. The problem with POT is in the selection of correct threshold, if correct threshold is chosen then it is a better method that can be used in forecasting. The threshold value selected should be as high as possible with a sufficient data so that it can be used in fitting the GPD. So far no algorithm satisfactory performance for the selection of threshold u is available. A graphical diagnostic tool that is useful in selection of the threshold u in the mean excess plots (MEF) is defined by (McNeil, 1997);

$$\left(u, e_n(u)\right), \qquad x_1^n < u < x_n^n,$$

Where $e_n(u)$ is the sample excess function defined as

$$e_n(u) = \frac{\sum_{i=k}^{n}\left(x_i^n - u\right)}{n-k+1}, k = \min\left\{i / x_i^n > u\right\} \tag{16}$$

Where n-k+1 is the number of observations exceeding the threshold u. If the empirical Mean Excess plot is a positively sloped straight line above a certain threshold u, it is an indication that the data follows the GPD with a positive parameter ξ. On Exponential distributed data would show a horizontal Mean Excess plot while short tailed data would have a negatively sloped line. The value estimate which will be obtained will be used in comparing the values obtained from models routinely used in the electricity production.

The second method that is used in determining threshold or validation is Hill's plot. By ordering data with respect to their values $X_{1n}, X_{2n},..., X_{nn}$ where $X_{1n}>X_{2n}>\ldots>X_{nn}$

The Hill's estimator of the tail index, $\alpha=\frac{1}{\xi}>0$, is given by

$$\hat{\alpha} = \frac{1}{k}\sum_{j=1}^{k} \ln X_{j,n} - \ln X_{k,n}, \tag{17}$$

Where $k \to \infty$ is upper order statistics, n is the sample size and $\alpha=\frac{1}{\xi}$ is the tail index. A threshold is selected from the plot where the shape parameter ξ or tail index ά is fairly stable.

4.0 EMPERICAL RESULTS AND DISCUSSION

From Figure 1 it shows that the electricity production over the last two years has increased with time. This is due in particular to deterioration in the production-side outlook. There is also uncertainty over projected increase in demand. Although it is clear that the risks to electricity demand are increasing, it is very difficult to accurately estimate the level of security of production that will be provided by the producers. In particular, this is because of uncertainties regarding the level of demand, commercial decisions about generating plant.

Table 1: Summary of electricity production

1Q	MEDIAN	MEAN	3Q	STDEV	SKEWNESS	KURTOSIS
533.98	563.85	557.37	587.85	48.93	-0.560	0.26995

Table 1 Show basics statistical measures. There is a slight negative asymmetry and the kurtosis value obtained is a Leptokurtic

Figure 1: Depicts minimum electricity production

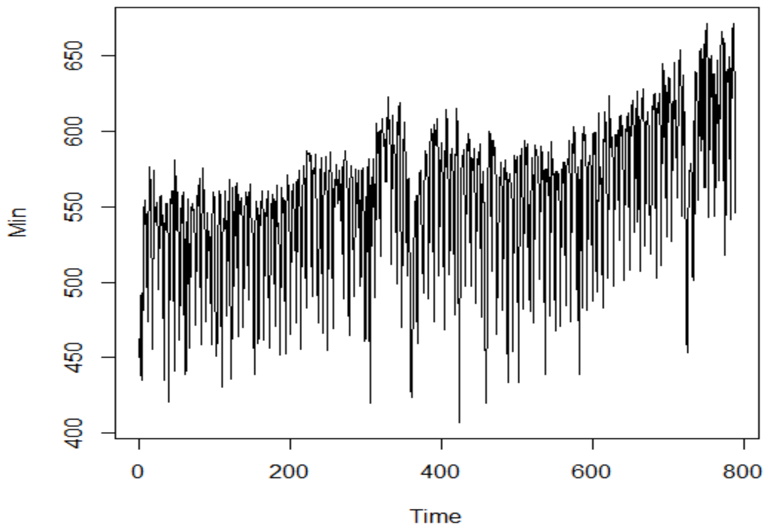
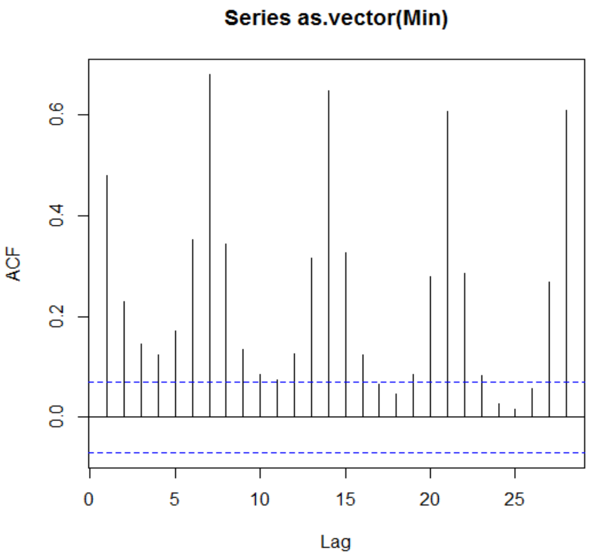


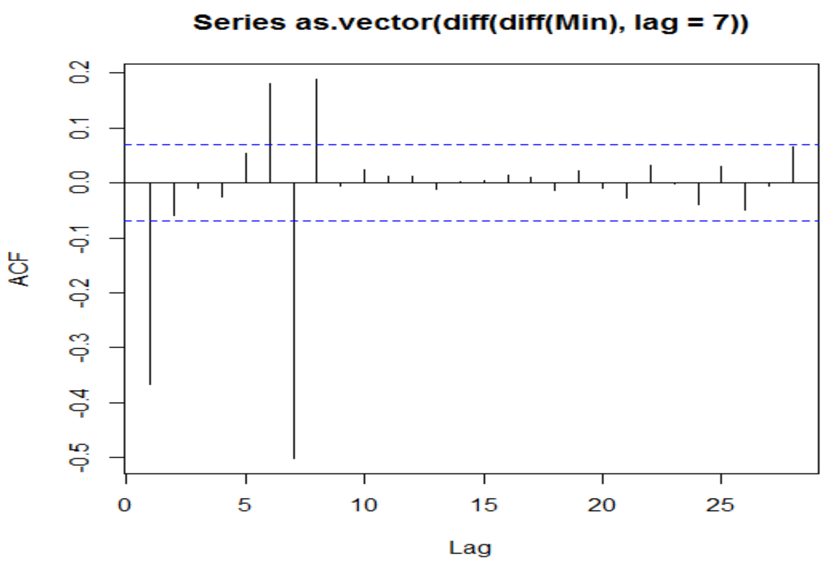
Figure 1 shows that the data is non-stationary over the period under observation. There is some trend meaning that there is an increase of production with time. Since data is of minimum daily production, at least weak dependence can be assumed that enable us to use the results presented. The serial autocorrelation function shows a trend that after every lag7 there is dependences and so on as depicted in Figure 2.

Figure 2: Autocorrelation function for electricity production



The data cannot be predicted therefore we transformed the data by differencing it at lag 7. After transformation the serial correlation is significant at lag 7 meaning that after every 7 days there is a minimum value that occurs.

Figure 3: Autocorrelation function for transformed electricity production



The correlation of production is evidently high; that is why we observe volatility clustering where there is a tendency of large values being followed by other large values and small being followed by the smaller ones. Exceedances of high or low threshold appear in clusters, indicating that there is dependence in the tails. The daily rate of change has another property that given their stationarity, there is statistical evidences that the fourth and fifth moment of the underlying marginal distribution might not exist.

Figure 4: Transformed data for electricity production

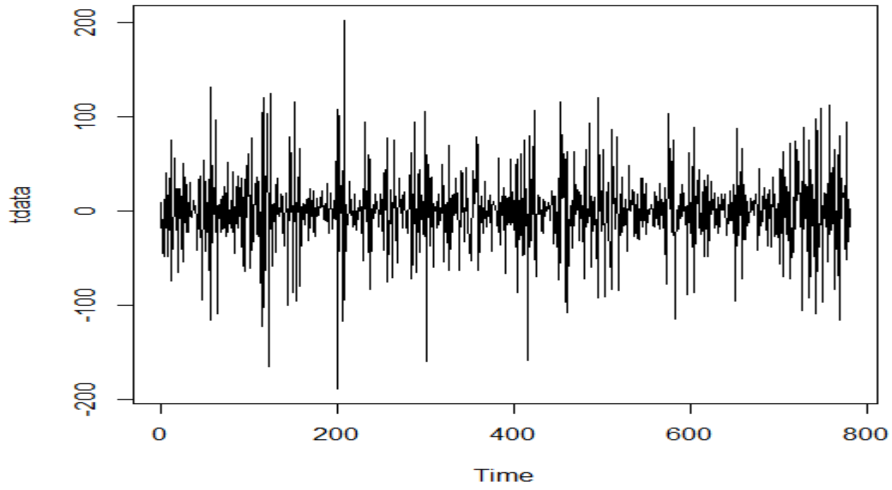


Table 2: Summary descriptive statistics of electricity production

1Q	MEDIAN	MEAN	3Q	STDEV	SKEWNESS	KURTOSIS
-19.48	0.13	-0.111	18.68	41.99	-0.097907	2.343

After twice differencing, Table 2 show basics statistical summary of electricity production. First we consider the distribution of the block maxima, which allows the determination of the production level. The only problem with this kind of method is the choice of the periods defining the blocks. The minimal productions in each of the block constitute the data points of the sample of the minimum M which was used to estimate the generalized extreme value distribution in equation (7). The estimates for the location, scale, and shape with the standard errors in parenthesis are

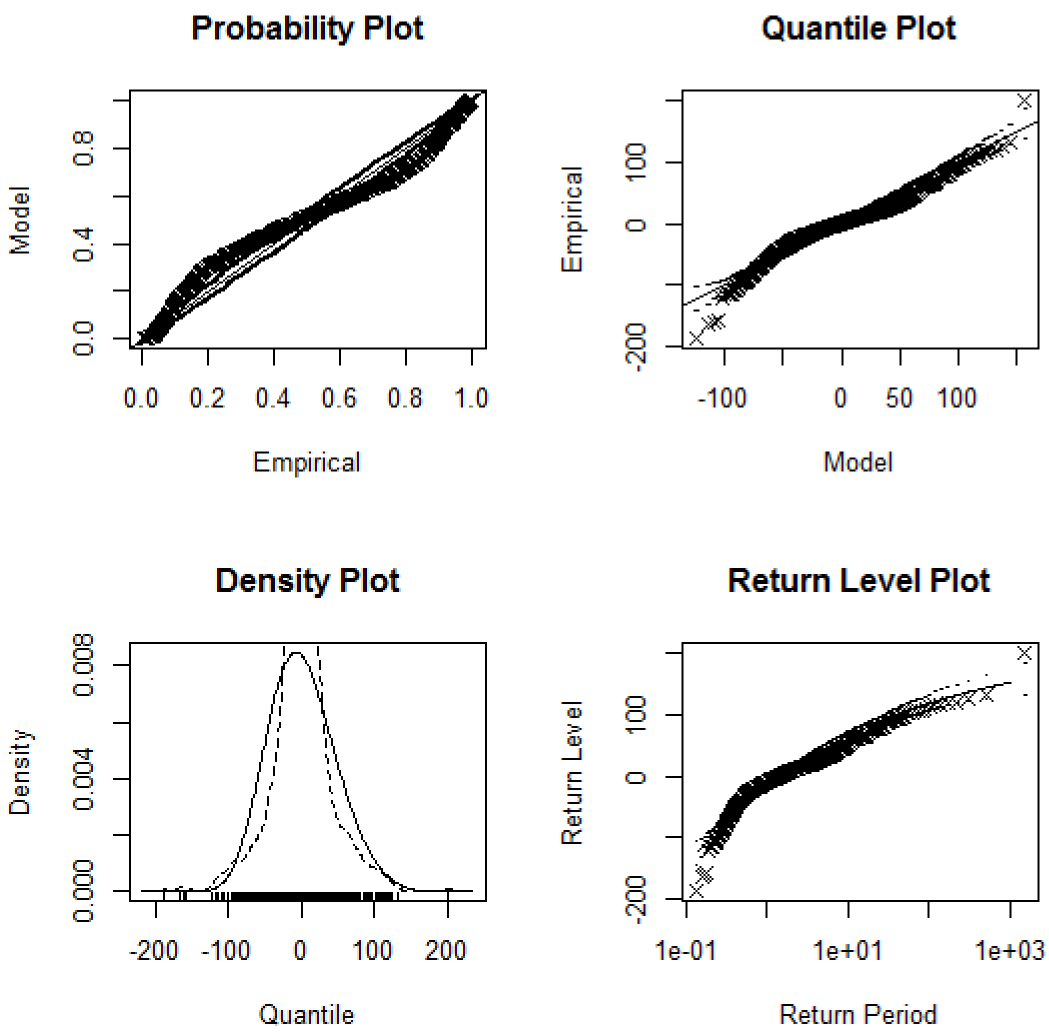
Table 3: Estimates for location, scale and shape parameter

Loc	Scale	Shape
534.17	26.60(2.5)	0.054(0.07)

Although the maximum like hood estimate for ξ is negative, what could respond to a bounded distribution, the confidences interval for shape include zero so lead us to reject the null hypothesis ξ =0. The Gumbel distribution is then a possible distribution to model the minimum electricity production therefore making us to conclude that the tail of the distribution is a medium tailed.

A diagnostic plot for accessing the accuracy of the extreme value model can be performed as shown in Figure 5. Both probability plot and quantile plot show a reasonable Extreme Value fit, the plots are not linear, so they cause a lot doubt on the adequacy of Extreme Value fitting using generalized extreme value in equation (7). The return level plot is not linear therefore it shows a slight convexity.

Figure 5: Shows probability plot, QQ-plot, Density plot and Return Plot



To confirm the suspicion we carried out an analysis of deviance between the two models

Table 4: Deviance analysis

Model	Df	Deviance	Df	Chisq	Pr(>chisq)
M1	3	8127.9			
M2	2	8317.9	1	190.02	2.2e-16***

As observed in the deviance analysis the results obtained, shows there is no differences between the two models, therefore M1 can be used in modeling since it has few parameters. BMM is not the best method in computing risk measures for electricity production.

Using POT method we get the maximum likehood fitting for the extreme value distribution. The estimates of shape and scale parenthesis are:

Table 5: Estimates for Shape and Scale parameters and standard errors using POT

Threshold	Scale	Shape
U=10	26.63680(2.5)	0.05449(0.07)
U=40	35.268(4.29)	-0.131(0.07)

Figure 6 shows the threshold at given points; we assume that where there is a tendency of straight line that is where we choose our threshold.

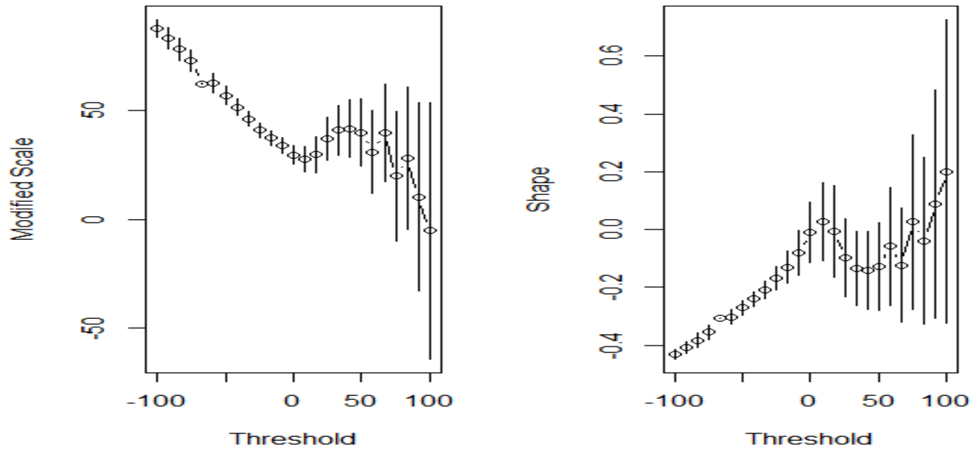


Figure 6: Show threshold determination for electricity production

Another informative method for determining the threshold is the mean residual life plot shown in Figure 7. The mean residual is a plot which is used to determine the threshold and normally we pick a threshold where the upward and downward movements tend to be a straight.

Figure 7: Mean Residual Life Plot

From Figure 8 a diagnostic plot for accessing the accuracy of the Extreme Value model is done, both the probability plot and quantile plot shows a reasonable Extreme Value fit, the plots are almost linear so they do not cause any doubts on the adequacy of EV fitting. The return plot is almost linear and shows a slight convexity (actually estimated to be negative). Even the extreme values in the return plot are within the confidence interval. We conclude that the daily demand for electricity is from the Gumbel distribution meaning that the distribution is a medium tailed. The diagnostic plot in Figure 8 obtained show that the Gumbel distribution seems more appropriate to model electricity production.

Figure 8: Show Probability plot, QQ-plot, Density plot and Return plot

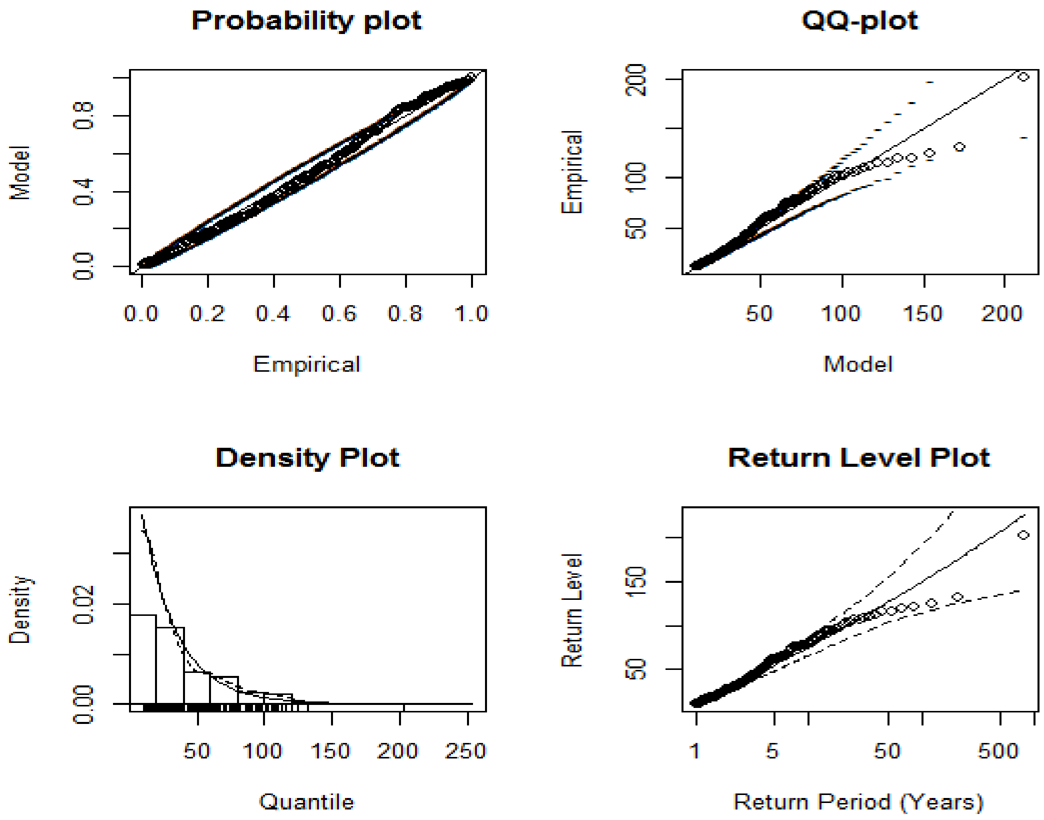


Table 6: Risk measures computed via peak over threshold measures at 5%, 1% and 0.1% probability

Probability	Quantile	Shortfall
0.95	66.43198	97.83132
0.99	116.37581	150.63300
0.999	195.83510	234.63906

From Table 5, with 5% probability the minimum production should be as low as 66.43198 and given that the production is less than 66.43198 the average production will be 97.83132. A drop in demand of this magnitude may compensate for the expected net production-side reductions, resulting in lower risks of electricity production.

5.0 SUMMARY AND CONCLUSION

Our assessment show that the data exhibit some trend and can be modeled as a Gumbel distribution since the shape parameter is not significantly different from zero. The average loss at a given level of electricity production, results may come from a large number of small events where demand exceeds supply in principle but it be can managed by Kenya Power through a set of mitigation actions available to their operating systems. For example, electricity production could be significantly lower than expected due to severe drought conditions. The results may also come from a small number of large events (e.g. the production deficit is more than expected) where controlled disconnections cannot be avoided. We have illustrated how extreme value theory can be used to model tail related risk measures such as value at risk and expected shortfall applying it to the Kenyan electricity production.

Our conclusion is that EVT can be useful for accessing the size of extreme events. From a practical point of view the problem can be approached in numerous ways depending on time horizon, the data availability, data frequency and the level of complexity one is willing to introduce in the model. One can choose to use conditional or unconditional approach, the POT or BMM method and compute risk measures.

In our scenario the POT approach proves to be the best method as it exploits the information in the sample data. It can be seen from the empirical results that for long time behavior we use unconditional approach. Finally we compute the risk measure as they provide better understanding on the restructuring of electricity production.

We therefore recommend future researchers to research on the maximum of minimum production to consider the magnitude of hydro-power production. Again one should also look at the minimum of the minimum power production under hydro electricity production.

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