A Simple Methodology for Quantum Mechanical Theory of Tardyons and Tachyons

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Abstract
The all moving substances in the universe can be analyzed as three parts in terms of speed. The first group is called tardyons. They have nonzero mass and the speed lower than the speed of light. The second group is called luxons which is moving with the speed of light. The third group of particles is called tachyon whose speed faster than the speed of light. The possibility of describing the tachyons have been discussed by using a simple methodology in tachyonic regime of the quantum fluctuations. A quantum field theory of noninteracting, spinless and faster-than-light particles have been described. The topics have been studied included mass, momentum and energy of the tardyons and tachyons.

Key Words: Tardyons, Tachyons, Special Relativity.

1. Introduction
Einstein’s special theory of relativity (SR) adumbrate that the mass of any object is infinite when access any object's speed to the speed of light [1]. The possibility of the existence of particles (superluminal or tardyons) that has greater velocity of light in a vacuum has been discussed by several researchers [2-14]. The meaning of tachyons are “swift, quick, fast, rapid”, and the tachyon was coined in 1967 by Feinberg [15]. Feinberg proposed that tachyonic particles (hypothetical particle) could be quanta of a quantum field with negative squared mass. Negative squared mass fields are commonly referred to as “tachyons” [16]. The possibility of particles moving faster than light was first suggested by Bilaniuk, Deshpande, and George Sudarshan [2].

The energy and momentum equations show that the mass would be imaginary as exceed the speed of light [2-14]. Wang and co-workers in their experiment show that the superluminal light pulse is faster 300 times than the normal speed of light (299,792,458 ms-1). In this experiment, a superluminal light pulse move with negative speed as -c/310 in a specially designed place. Wang and co-workers defend that the result of this experiment does not contradict with the causality or SR. It is observed that the superluminal light propagation is a result of the wave nature of light. To generate a superluminal light pulse, combine infinite number of waves at different frequencies is required [14].

Clay and Crouch showed on work about the cosmic rays the presence of rays faster than light [17]. The existence of hypothetical particles/tachyons was discovered in laboratory experiments. Several researchers have studied tachyons using both laboratory particle sources and cosmic rays. A brief description of such particles, i.e. tachyons, is the case of spinless, noninteracting particles. The tachyons cannot be quantized by Bose statistic, can be quantized Fermi statistic. There are two possible approaches that could detect tachyons. First one is the tachyons should appear as particles with (missing mass) 2 < 0. Second one is detect the Čerenkov radiation emitted by the tachyons in a vacuum. Charged tachyons can gain energy in an electromagnetic field [17-21].

In the scope of this work, we have calculated the mass, momentum and energy of tardyons and tachyons with using a simple methodology. For this work, quantum triangles as Figures 1a and 1b for c>v<sub>1</sub> (tardyons), for v> c Figures 1b and 1c have been used. As a result, tachyons which have real/imaginary mass, momentum and energy have been shown in this study.
2. Theoretical Model

The new definition quantum right-angled triangles for tardyons and tachyons have been demonstrated in Figures 1a, 1b and 1c as below.

![Fig. 1. Quantum right-angled triangles (a) $c>v_1$ (b) $m/m_0$ and (c) $v_2>c$.](image)

Let’s consider a simple wave in two-dimensional space for the studt tachyons (faster than the light particles). For this work, two type triangles for a new phenomenology of a scholar in two dimensional space have been used as Figures 1a and 1b for $c>v_1$ (tardyons), for $v_2>c$ (tachyons) Figures 1b and 1c as below. The trigonometric function that’s associated with the Pythagorean Theorem onto ordinary trigonometric functions shows the qualitatively properties of the tachyons. For tardyons ($c>v_1$), the biggest side is hypotenuse and it is shown as $c$ in figures 1a. that’s for tardyons $c$ is more bigger than the $v_1$.

For tachyons ($v_2>c$), the biggest side is hypotenuse and it is shown as $v_2$ in figure 1c. The Pythagorean theorem in mathematics is the square of the length of the hypotenuse (the side opposite the right-angled triangles) is equal to the sum of the squares of the lengths of the other two sides. The lengths of the hypotenuse are $c$ for tardyons and $v_2$ for tachyons, the lengths of the other two sides are $\sqrt{c^2 - v_1^2}$ and $v_1$ for tardyons and $\sqrt{v_2^2 - c^2}$ and $c$ for tachyons as shown on top in quantum right-angled triangles in Figures 1a and 1c, respectively. The strong link between the sides/angles and trigonometric ratios (sine, cosine) is valid also for these quantum right-angled triangles. In the quantum right-angled triangles, square of the trigonometric ratios (sine, cosine) is equal to the 1. This can be stated in equation form as

$$\sin^2(2\alpha) + \cos^2(2\alpha) = 1 \tag{1}$$

$$\sin(2\alpha) = 2 \sin \alpha \cdot \cos \alpha \tag{2}$$

$$\cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha \tag{3}$$

Here, $v_1$ and $v_2$ are speeds of particles, $c$ is speed of light, $m_0$ is rest mass and $m$ is the mass of particle (tardyon/bradyon, tachyon). $\hat{r}, \hat{k}$ and $\hat{n}$ are unit vectors. Sine and cosine of $\alpha$ have been calculated from the quantum right-angled triangles for tardyons ($c>v_1$) in Figures 1a and 1b as below.

$$\sin \alpha = \frac{v_1}{c} \text{ and } \cos \alpha = \frac{m_0 \hat{k}}{m \hat{s}} \tag{4}$$

Einstein’s theory of relativity has been found as below by using equation (1).

$$\frac{m_0^2}{m^2} + \frac{v_1^2}{c^2} = \pm 1 \tag{5}$$
We have firstly used positive (+1) part of the equation (5).

\[ \frac{m_0^2}{m^2} + \frac{v_1^2}{c^2} = +1 \]  

We have calculated real mass of these particles (tardyons, \( c>v_1 \)) by using equation (6).

\[ m = \frac{m_0}{\sqrt{1 - \frac{v_1^2}{c^2}}} \]  

In additional, equation (8) has been obtained by using the equation (6).

\[ (mv_1)^2 + (m_0c)^2 = (mc)^2 \]  

This expression (Eq.(8)) corresponds to equation (9) in below.

\[ p^2c^2 + E_0^2 = E^2 \]  

Here,

\[ p = mv_1 = \frac{m_0v_1}{\sqrt{1 - \frac{v_1^2}{c^2}}} \]  

\[ E_0 = m_0c^2 \]  

\[ E = mc^2 = \frac{m_0c^2}{\sqrt{1 - \frac{v_1^2}{c^2}}} \]  

On the other hand, equation (12) can be calculated as the same by using the differential form of the energy \((dE=dm.c^2)\) with equation (7). In equation (8), we chose

\[ m_0 = i\mu \]  

in order to write in relation between energy and momentum as below. The new form of equation (9) is found from equation (13):

\[ p^2c^2 - E^2 = \mu^2c^4. \]  

The imaginary mass of particle (tardyons) has been calculated in the case of \( c>v_1 \) by using the quantum right-angled triangles in Figures 1a and 1b. In this case, to obtain the imaginary mass we have used some mathematical equations as below by using negative (-1) part of the equation (5).

\[ \frac{m_0^2}{m^2} + \frac{v_1^2}{c^2} = -1 \]  

\[ m = \frac{i m_0}{\sqrt{1 + \frac{v_1^2}{c^2}}} \]  

Equation (17) is then given by using the equation (15).

\[ p^2c^2 + E_0^2 = -E^2 \]  

Here,
\[ p = mv_1 = \frac{im_0v_1}{\sqrt{1 + \frac{v_1^2}{c^2}}} = \frac{-\mu v_1}{\sqrt{1 + \frac{v_1^2}{c^2}}} \]  
(18)

\[ E = mc^2 = \frac{im_0c^2}{\sqrt{1 + \frac{v_1^2}{c^2}}} = \frac{-\mu c^2}{\sqrt{1 + \frac{v_1^2}{c^2}}} \]  
(19)

In addition, equations (18) and (19) can be calculated as the same by using the differential form of the momentum \((dp = m dv + v dm)\) and energy \((dE = dm c^2)\) with equation (16). Where, \(E_0\) is the same with equation (11). The new form of equation (17) by using equation (13) may therefore be written:

\[ p^2c^2 + E^2 = \mu^2c^4 \]  
(20)

The real mass of particle (tachyon/superluminal) has been calculated in the case of \(v_2 > c\) by using the quantum right-angled triangles in Figures 1b and 1c. In this case, to obtain the mass of tachyon we have used some mathematical equations as below,

\[ \sin \alpha = \frac{c}{v_2} \text{ and } \cos \alpha = \frac{m_0}{m_0^*} \]  
(21)

By using equations (1), (2) and (3) we have obtained equation (22) as below,

\[ \frac{m_0^2}{m^2} + \frac{c^2}{v_2^2} = 1 \]  
(22)

We have firstly used positive (+1) part of the equation (22).

\[ \frac{m_0^2}{m^2} + \frac{c^2}{v_2^2} = +1 \]  
(23)

We have calculated the real mass of tachyon by using equation (23).

\[ m = \frac{m_0}{\sqrt{1 - \frac{c^2}{v_2^2}}} \]  
(24)

Furthermore, the equation (25) may still be defined by using equation (23)

\[ (mc)^2 + (m_0v_2)^2 = (mv_2)^2 \]  
(25)

Equation (25) may therefore be written as equation (26).

\[ p_0^2c^2 + E^2 = p^2c^2 \]  
(26)

This can be seen directly from the equations for momentum and energy

\[ p = mv_2 = \frac{m_0v_2}{\sqrt{1 - \frac{c^2}{v_2^2}}} \]  
(27)

\[ p_0 = m_0v_2 \]  
(28)
\[ E = mc^2 = \frac{m_0c^2}{\sqrt{1 - \frac{c^2}{v^2}}} \] (29)

In addition, equations (27) and (29) can be calculated as the same by using the differential form of the momentum \((dp = mdv + vdm)\) and energy \((dE = dm \cdot c^2)\) with equation (24). It can be observed that the change of equation (26) occurs as equation (30) with using equation (13).

\[ E^2 - p^2c^2 = \mu^2v_2^2c^2 \] (30)

To obtain the imaginary mass of tachyon we have used some mathematical equations as below by using negative (-1) part of the equation (22).

\[ \frac{m_0^2}{m^2} + \frac{c^2}{v_2^2} = -1 \] (31)

\[ m = \frac{im_0}{\sqrt{1 + \frac{c^2}{v_2^2}}} \] (32)

Then, we shall observe equation (33) by using equation (31).

\[ p_0^2c^2 + E^2 = -p^2c^2 \] (33)

Here,

\[ p = mv_2 = \frac{im_0v_2}{\sqrt{1 + \frac{c^2}{v_2^2}}} = \frac{-\mu v_2}{\sqrt{1 + \frac{c^2}{v_2^2}}} \] (34)

\[ E = mc^2 = \frac{im_0c^2}{\sqrt{1 + \frac{c^2}{v_2^2}}} = \frac{-\mu c^2}{\sqrt{1 + \frac{c^2}{v_2^2}}} \] (35)

Where, \(p_0\) is the same with equation (28). In additional, equations (34) and (35) can be calculated as the same by using the differential form of the momentum \((dp = mdv + vdm)\) and energy \((dE = dm \cdot c^2)\) with equation (32). The new form of equation (33) by using equation (13) may therefore be written:

\[ p^2c^2 + E^2 = \mu^2v_2^2c^2 \] (36)

The real and imaginary masses of tardyons (slower than light) and tachyons (faster than light) have been summarized in Figure 2. On the left side, real and imaginary masses for \(c > \nu_1\) obtained by using Figures 1a, 1b, equations (1), (6) and (15). On the right side, real and imaginary masses for \(\nu_2 > c\) obtained by using Figures 1b, 1c, equations (1), (23) and (31).

**Fig. 2.** The masses of tardyons (slower than light) and tachyons (faster than light).
The results of these difference methods have been found as the same with previously obtained momentum and energy. Results of masses, momentum and energy for the tardyons (slower than light) are appropriate with previous works in literatures. It is important that the real and imaginary masses and their energy and momentum of the tachyons (faster than light) have been observed in this study. It is probability that the demonstration to the “rest mass” µ of the tachyons.

3. Conclusion
A new methodology has been developed which describes the treatment of tachyon mechanics and has been demonstrated that this part of the physics of tachyons. We have used the quantum right-angled triangles to calculate the real/imaginary masses, momentum and energy for tardyons and tachyons. The momentum and energy for tardyons and tachyons have been calculated by using the real/imaginary masses. The tachyons’ characteristic properties can be written as the tachyons have imaginary rest mass, always faster than speed of light(\nu^2>c). An important issue that was beyond our consideration is the quantization of tachyons, which have real and imaginary mass.

References