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New Instability Results on Certain Fifth Order Nonlinear Vector Differential Equations

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Abstract
The purpose of this paper is to investigate instability of the trivial solution of non-linear vector differential equation of the fifth order by constructing a Lyapunov function to get two new instability results. Also, for each result an example is given to indicate the importance of the topic.Related to a scalar differential equations mentioned in the literature, the findings of this study are beneficial for results of instability of the trivial solution of five order non-linear vectoral diferential equations.

Key words: Nonlinear vector differential equation of fifth, Lyapunov function, instability.

1. Introduction

Instability of the trivial solution of scalar and vector differential equations of the fifth order were investigated by many authors. About the topic, we refer to the papers of Ezeilo ([1],[2]), Tiryaki [3], Li & Yu [4], Li & Duan [5], Sadek [6], Tunç ([7-9]), Tunç & Şevli [10], Tunç & Karta [11], Krasovskii [12]. In all of the papers mentioned above, authors used Krasovskii's criteria [12] and Lyapunov's second (or direct) method [13].

Now, we give these studies that were done on the instability of non-linear differential equations of the fifth order. According to observations in the literature, firstly, for the case of $n = 1$, Ezeilo ([1],[2]) investigated the instability of trivial solution of the fifth order scalar non-linear differential equations, respectively,

$$x^{(5)} + a_1 x^{(4)} + a_2 \dddot{x} + a_3 \ddot{x} + a_4 \dot{x} + f(x) = 0,$$

$$x^{(5)} + a_1 x^{(4)} + a_2 \dddot{x} + h(\dot{x})\ddot{x} + g(x)\dot{x} + f(x) = 0,$$

$$x^{(5)} + \psi(\ddot{x})\dddot{x} + \varphi(\ddot{x}) + \theta(\dot{x}) + f(x) = 0$$

and

$$x^{(5)} + a_1 x^{(4)} + a_2 \dddot{x} + g(\dot{x})\ddot{x} + h(x, \dot{x}, \ddot{x}, \dddot{x}, x^{(4)})\dot{x} + f(x) = 0,$$

in which a_1, a_2, a_3, a_4, are constants and f, g, h, ψ, ϕ and θ are continuous functions depending only on the arguments shown as $f(0) = \phi(0) = \theta(0) = 0$.

Tiryaki [3] studied the instability of trivial solution of the fifth order non-linear scalar differential equation of the form

$$x^{(5)} + a_1 x^{(4)} + k(x, \dot{x}, \ddot{x}, \dddot{x}, x^{(4)})\dddot{x} + g(\dot{x})\ddot{x} + h(x, \dot{x}, \ddot{x}, \dddot{x}, x^{(4)})\dot{x} + f(x) = 0.$$

Li & Yu [4] concerned the instability of trivial solution of the fifth order non-linear scalar differential equation

$$x^{(5)} + ax^{(4)} + b\dddot{x} + \psi(x, \dot{x}, \ddot{x}, \dddot{x}, x^{(4)})\ddot{x} + g(x)\dot{x} + f(x) = 0$$

By introducing a Lyapunov function, where a and b are some positive constant.

Li & Duan [5] showed the instability of trivial solution of the fifth order nonlinear scalar differential equation of the form

$$\dot{x}_i = x_{i+1} \qquad \dot{x}_i = x_{i+1} \qquad (i = 1,2,3,4)$$

$$\dot{x}_5 = -f_5(x_4)x_5 - f_4(x_3)x_4 - f_3(x_1, x_2, x_3, x_4, x_5)x_3 - f_2(x_2) - f_1(x_1)$$

$$(f_2(0) = f_1(0) = 0) \tag{1.1}$$

On the other hand, Sadek [6] examined the instability of trivial solutions of the fifth order vector differential equations described as

$$X^{(5)} + \Psi(\ddot{X})\dddot{X} + \Phi(\ddot{X}) + \Theta(\dot{X}) + F(X) = 0$$

and

$$X^{(5)} + AX^{(4)} + B\dddot{X} + H(\dot{X})\ddot{X} + G(X)\dot{X} + F(X) = 0.$$

In addition, respectively, Tunç ([7-9]) investigated the instability of trivial solution of the fifth order vector differential equations of the form

$$X^{(5)} + AX^{(4)} + \Psi(X, \dot{X}, \ddot{X}, \dddot{X}, X^{(4)})\dddot{X} + G(\dot{X})\ddot{X}$$

$$+ H(X, \dot{X}, \ddot{X}, \dddot{X}, X^{(4)})\dot{X} + F(X) = 0,$$

$$X^{(5)} + AX^{(4)} + B(t)\Psi(X, \dot{X}, \ddot{X}, \dddot{X}, X^{(4)})\dddot{X} + C(t)G(\dot{X})\ddot{X}$$

$$+ D(t)H(X, \dot{X}, \ddot{X}, \dddot{X}, X^{(4)})\dot{X} + E(t)F(X) = 0$$

and

$$X^{(5)} + \Psi(\dot{X}, \ddot{X})\dddot{X} + \Phi(\dot{X}, \ddot{X}, \dddot{X}, X^{(4)}) + \Theta(\dot{X}) + F(X) = 0.$$

Tunç & Şevli [10] showed a similar study for the instability of trivial solution of the fifth order vector differential equation

$$X^{(5)} + \Psi(\dot{X}, \ddot{X})\dddot{X} + \Phi(X, \dot{X}, \ddot{X})\ddot{X} + \theta(\dot{X}) + F(X) = 0.$$

Furthermore, Tunç & Karta [11] analyzed sufficient conditions which ensure the trivial solution of vector differential equation

$$X^{(5)} + AX^{(4)} + B\dddot{X} + \Psi(X, \dot{X}, \ddot{X}, \dddot{X}, X^{(4)})\ddot{X} + G(X)\dot{X} + F(X)X = 0$$

By introducing a Lyapunov function, in which A and B are constant nxn −symmetric matrices; Ψ, G and F are nxn − symmetric continuous matrix functions depending, in each case, on the arguments shown.

In this article, based on study produced by Li & Duan [5], we install two new results under different conditions for the instability of the solution $X = 0$ of the fifth order nonlinear vector differential equations of the form

$$X^{(5)} + \Psi(\dddot{X})X^{(4)} + \Phi(\ddot{X})\dddot{X} + \Theta(X, \dot{X}, \ddot{X}, \dddot{X}, X^{(4)})\ddot{X} + E(\dot{X}) + H(X) = 0 \tag{1.2}$$

in which $X \in \mathbb{R}^n$; Ψ,Φ and Θ are $n \times n$-symmetric continuous matrix functions depending, in each case ,on the arguments shown; $E, H: \mathbb{R}^n \to \mathbb{R}^n$ are continuous n-vector functions. It is supposed that $E(0) = H(0) = 0$. Let $J_H(X)$, $J_\Psi(W)$, $J_\Phi(Z)$ *and* $J_E(Y)$ display the Jacobian matrices corresponding to the functions $H(X)$, $\Psi(W)$, $\Phi(Z)$ *and* $E(Y)$ respectively,

$$J_H(X) = \left(\frac{\partial h_i}{\partial x_j}\right),\ J_\Psi(W) = \left(\frac{\partial w_i}{\partial w_j}\right)$$

$$J_{\Phi(Z)} = \left(\frac{\partial \Phi_i}{\partial z_j}\right),\ J_E(Y) = \left(\frac{\partial e_i}{\partial y_j}\right), \quad \text{(i,j=1,2,…,n)}$$

in which $(x_1, x_2, \dots, x_n)$, $(y_1, y_2, \dots, y_n)$, $(z_1, z_2, \dots, z_n)$, $(w_1, w_2, \dots, w_n)$, $(h_1, h_2, \dots, h_n)$, $(\psi_1, \psi_2, \dots, \psi_n)$,$(\varphi_1, \varphi_2, \dots, \varphi_n)$, $(e_1, e_2, \dots, e_n)$ are the components of $X, Y, Z, W, H, \Psi, \Phi, and\ E$ respectively.The symbol $\langle X, Y\rangle$ corresponding to any pair X,Y in $\mathfrak{R}^n$ stands for he usual scalar product $\sum_{i=1}^{n} x_i y_{i.}$

Throughout this paper,we consider the following differential systems which are equivalent to the equation (1.2) which was attained as usual by setting $\dot{X} = Y, \ddot{X} = Z, \dddot{X} = W, X^{(4)} = U$ from (1.2):

$$\dot{X} = Y, \dot{Y} = Z, \dot{Z} = W, \dot{W} = U$$

$$\dot{U} = -\Psi(W)U - \Phi(Z)W - \Theta(X, Y, Z, W, U)Z - E(Y) - H(X) \tag{1.3}$$

However, with respect to our observations in the literature, even though many papers have been reviewed, there are a few example about the subject. Therefore, we give an example to indicate the importance of the topic. Also, it should be expressed that the assumptions and Lyapunov [13] function used here are completely different than those mentioned in the literature.

2.Main Results

Our main result is the following two theorems.

Theorem 2.1. In addition to the basic conditions given above for coefficients Ψ, Φ, Θ, E and H of (1.2) equation, we suppose that following conditions hold as;

(i) $H(0) = 0, H(X) \neq 0$ if $X \neq 0, E(0) = 0, E(Y) \neq 0$ if $Y \neq 0$;

Ψ, Φ, Θ symetric and $\lambda_i(J_H(X)) < 0$, $\quad (i = 1,2, \dots, n)$,

(ii) $\lambda_i\big(\Theta(X, Y, Z, W, U)\big) \geq 0, \lambda_i(\Psi(W)) < 0$ for all $X, Y, Z, W, U \in \mathbb{R}^n$ $(i = 1,2, \dots, n)$,

Then trivial solution $X = 0$ of (1.2) is instability.

Proof. As basic tool for proof of Theorem (2.1), we will use Lyapunow function $V_1(X, Y, Z, W, U)$ given as

$$V_1 = -\langle Z, U\rangle - \left\langle Z, \int_0^1 \langle \Psi(\sigma W), W\rangle d\sigma \right\rangle + \frac{1}{2}\langle W, W\rangle - \int_0^1 \langle \Phi(\sigma Z)Z, Z\rangle d\sigma$$

$$-\int_0^1 \langle E(\sigma Y), Y\rangle d\sigma - \langle H(X), Y\rangle \qquad (2.1)$$

in which, under the conditions of Theorem (2.1), it will be indicated that Lyapunov function $V_1 = V_1(X, Y, Z, W, U)$ satisfies the entire Krasovskii [12] criteria:

(K_1) In every neighborhood of (0,0,0,0,0),there exists a point $(\xi, \eta, \varsigma, \mu, \rho)$ such as $V_1\ (\xi, \eta, \varsigma, \mu, \rho) > 0$.

(K_2) The time derivative $\dot{V}_1 = (\frac{d}{dt})V(X, Y, Z, W, U)$ along solution paths of system (1.3) is positive semidefinite.

(K_3) The only solution $V_1\ (X, Y, Z, W, U) = (X(t), Y(t), Z(t), W(t), U(t))$ of system (1.3) which satisfies $\dot{V}_1\ = 0 (t \geq 0)$ is the trivial solution (0,0,0,0,0).These properties guarantee that the trivial solution of (1.2) is unstable. It is clear that V_1(0,0,0,0,0)=0. Additionally,it is easy to see that

$$V_1(0,0,0,\varepsilon,0) = \frac{1}{2}\langle\varepsilon,\varepsilon\rangle = \frac{1}{2}\|\varepsilon\|^2 > 0$$

for all arbitrary $\varepsilon \neq 0, \varepsilon \in \mathbb{R}^n$. If this happens, it displays (K_1) feature of Krasoskii [12]. Let $(X, Y, Z, W, U) = (X(t), Y(t), Z(t), W(t), U(t))$ be an arbitrary solution of system (1.3). Differentiating (2.1) with respect to t, along this solution,calculations give that

$$\dot{V}_1 = \langle Z, \Psi(W)U\rangle + \langle Z, \Phi(Z)W\rangle + \langle Z, \Theta(X, Y, Z, W, U)Z\rangle$$

$$+\langle Z, E(Y)\rangle - \left\langle \frac{d}{dt} Z, \int_0^1 \langle \Psi(\sigma W), W\rangle d\sigma \right\rangle \qquad (2.2)$$

$$-\frac{d}{dt}\int_0^1 \langle \sigma\Phi(\sigma Z)Z, Z\rangle d\sigma - \frac{d}{dt}\int_0^1 \langle E(\sigma Y), Y\rangle d\sigma - \langle J_H(X)Y, Y\rangle$$

Now, recall that

$$\frac{d}{dt}\left\langle Z, \int_0^1 \langle \Psi(\sigma W), W\rangle d\sigma \right\rangle = \left\langle W, \int_0^1 \langle \Psi(\sigma W), W\rangle d\sigma \right\rangle$$

$$+\left\langle Z, \int_0^1 \langle \Psi(\sigma W), U\rangle d\sigma \right\rangle$$

$$+\left\langle Z, \int_0^1 \langle \sigma\, \boldsymbol{J}_{\Psi}(\sigma W)U, W\rangle d\sigma \right\rangle$$

$$= \left\langle W, \int_0^1 \langle \Psi(\sigma W), W\rangle d\sigma \right\rangle + \left\langle Z, \int_0^1 \langle \Psi(\sigma W), U\rangle d\sigma \right\rangle$$

$$+\left\langle Z, \int_0^1 \sigma \frac{\partial}{\partial\sigma}\langle \Psi(\sigma W), U\rangle d\sigma \right\rangle$$

$$= \left\langle W, \int_0^1 \langle \Psi(\sigma W), W \rangle d\sigma \right\rangle + \left\langle Z, \int_0^1 \langle \Psi(\sigma W), U \rangle d\sigma \right\rangle$$

$$+ \sigma \langle Z, \Psi(\sigma W) U \rangle \Big|_0^1$$

$$= \left\langle W, \int_0^1 \langle \Psi(\sigma W), W \rangle d\sigma \right\rangle + \langle Z, \Psi(W) U \rangle, \tag{2.3}$$

$$\frac{d}{dt} \int_0^1 \langle \sigma \Phi(\sigma Z) Z, Z \rangle d\sigma = \int_0^1 \langle \sigma \Phi(\sigma Z) W, Z \rangle d\sigma + \int_0^1 \sigma^2 \langle J_\Phi(Z) Z W, Z \rangle d\sigma$$

$$+ \int_0^1 \langle \sigma \Phi(Z) Z, W \rangle d\sigma$$

$$= \int_0^1 \langle \sigma \Phi(\sigma Z) W, Z \rangle d\sigma + \int_0^1 \sigma \frac{\partial}{\partial \sigma} \langle \sigma \Phi(\sigma Z) W, Z \rangle d\sigma$$

$$= \sigma^2 \langle \Phi(\sigma Z) W, Z \rangle \Big|_0^1 = \langle \Phi(Z) W, Z \rangle \tag{2.4}$$

$$\frac{d}{dt} \int_0^1 \langle E(\sigma Y), Y \rangle d\sigma = \int_0^1 \sigma \langle J_E(\sigma Y) Z, Y \rangle d\sigma + \int_0^1 \langle E(\sigma Y), Z \rangle d\sigma$$

$$= \int_0^1 \sigma \frac{\partial}{\partial \sigma} \langle E(\sigma Y), Z \rangle d\sigma + \int_0^1 \langle E(\sigma Y), Z \rangle d\sigma$$

$$= \sigma \langle E(\sigma Y), Z \rangle \Big|_0^1 = \langle E(Y), Z \rangle \tag{2.5}$$

Substituting (2.3), (2.4) and (2.5) into (2.2) and taking into account the conditions of the Theorem 2.1, we obtain

$\dot{V}_1 = -\langle W, \int_0^1 \langle \Psi(\sigma W), W \rangle d\sigma \rangle + \langle Z, \Theta(X, Y, Z, W, U) Z \rangle - \langle J_H(X) Y, Y \rangle > 0.$

If this happens, it shows (K_2) feature of Krasovskii [12]. Furthermore, $\dot{V}_1 = 0$ $(t \geq 0)$ necessarily implies that $Y = 0$ for all $t \geq 0$, and also $X = \xi$ (a constant vector) , $Z = \dot{Y} = 0, W = \ddot{Y} = 0, \dot{W} = \dddot{Y} = 0$, for all $t \geq 0$.If these statements are written in (1.3), we can clearly see that it is $H(\xi) = 0$, which necessarily shows that ξ=0 because $H(0) = 0$ and $H(X) \neq 0$ if $X \neq 0$.Thus, it follows as

$X = Y = Z = W = U = 0$ for all $t \geq 0$.

If this happens, it shows (K_3) feature of Krasovskii [12] is satisfied. As a result, taking into account the conditions of Theorem 2.1, the function V_1 ensures the entire criteria of Krasovskii [12]. Thus, the fundamental properties of the function $V_1(X, Y, Z, W, U)$, which were proved above, imply that the zero solution of system (1.3) is imply that the zero solution of system (1.3) is unstable. The system (1.3) is equivalent to the differential equation (1.2). Therefore, the proof of Theorem (2.1) is complete.

Theorem 2.2. In addition to the basic conditions given above for coefficients Ψ, Φ, Θ, E and H of (1.2) equation, we suppose that following conditions hold as;

(i) $H(0) = 0$, $H(X) \neq 0$ if $X \neq 0$ *and* $E(0)=0$, $E(Y) \neq 0$ if $Y \neq 0$;

Ψ, Φ ve Θ symmetric and $\lambda_i(J_H(X))>0$, $(i=1,2,...,n.)$,

(ii) $\lambda_i(\Theta(X,Y,Z,W,U)) \leq 0$, $\lambda_i(\Psi(W)) > 0$ for all $X,Y,Z,W,U \epsilon \mathbb{R}^n$,

$\lambda_i(\Phi(Z)) > k$, $k>-1$, $(i=1,2,\dots,n)$.

Then trivial solution $X = 0$ of (1.2) is instability.

Proof. As basic tool for proof of Theorem (2.1), we will use Lyapunow function $V_2(X,Y,Z,W,U)$ given as

$$V_2 = \langle Z,U\rangle + \left\langle Z, \int_0^1 \langle \Psi(\sigma W),W\rangle d\sigma \right\rangle - \frac{1}{2}\langle W,W\rangle + \int_0^1 \langle \Phi(\sigma Z)Z,Z\rangle d\sigma$$

$$+\int_0^1 \langle E(\sigma Y),Y\rangle d\sigma + \langle H(X),Y\rangle \tag{2.6}$$

In which, under the conditions of Theorem 2.1, it will be indicated that Lyapunov function $V_2 = V_2(X,Y,Z,W,U)$ satisfies the entire Krasovskii [12] criteria:

It is clear from (1.5) that $V_2(0,0,0,0,0)=0$. Additionally, It is easy to see that

$$V_2(0,0,\varepsilon,0,\varepsilon) = \langle \varepsilon,\varepsilon\rangle + \int_0^1 \langle \Phi(\sigma\varepsilon)\varepsilon,\varepsilon\rangle d\sigma$$

$$> \|\varepsilon\|^2 + k\|\varepsilon\|^2 = (1+k)\|\varepsilon\|^2 > 0,\ k > -1$$

for all arbitrary $\varepsilon \neq 0, \varepsilon \in \mathbb{R}^n$. If this happens, it displays (K_1) feature of Krasoskii [12]. Let $(X,Y,Z,W,U) = (X(t),Y(t),Z(t),w(t),U(t))$ be an arbitrary solution of system (1.3). Differentiating (2.6) with respect to t, along this solution,calculations give that

$$\begin{aligned}\dot{V}_2 &= -\langle Z,\Psi(W)U\rangle - \langle Z,\Phi(Z)W\rangle - \langle Z,\Theta(X,Y,Z,W,U)Z\rangle - \langle Z,E(Y)\rangle \\ &+ \frac{d}{dt}\left\langle Z, \int_0^1 \langle \Psi(\sigma W),W\rangle d\sigma \right\rangle + \frac{d}{dt}\int_0^1 \langle \sigma\Phi(\sigma Z)Z,Z\rangle d\sigma \\ &+ \frac{d}{dt}\int_0^1 \langle E(\sigma Y),Y\rangle d\sigma - \langle J_H(X)Y,Y\rangle\end{aligned}, \tag{2.7}$$

substituting (2.3), (2.4) and (2.5) into (2.7) and taking into account the conditions of the Theorem 2.1, we obtain

$$\dot{V}_2 = \left\langle W, \int_0^1 \langle \Psi(\sigma W),W\rangle d\sigma \right\rangle - \langle Z,\Theta(X,Y,Z,W,U)Z\rangle + \langle J_H(X)Y,Y\rangle > 0$$

If this happens, it shows (K_2) feature of Krasovskii [12]. Furthermore, $\dot{V}_2 = 0$ $(t \geq 0)$ necessarily implies that $Y = 0$ for all $t \geq 0$, and also $X = \xi$ (a constant vector) , $Z = \dot{Y} = 0, W = \ddot{Y} = 0, \dot{W} = \dddot{Y} = 0$, for all $t \geq 0$.If these statements are written in (1.3), we can clearly see that it is $H(\xi) = 0$, which necessarily shows that ξ=0 because $H(0) = 0$ and $H(X) \neq 0$ if $X \neq 0$.Thus, it follows as

$X = Y = Z = W = U = 0$ for all $t \geq 0$.

If this happens, it shows (K_3) feature of Krasovskii [12] is satisfied. As a result, taking into account the conditions of Theorem 2.1, the function V_2 ensures the entire criteria of Krasovskii [12]. Thus, the fundamental properties of the function $V_2(X, Y, Z, W, U)$, which were proved above, imply that the zero solution of system (1.3) is imply that the zero solution of system (1.3) is unstable. The system (1.3) is equivalent to the differential equation (1.2). Therefore, the proof of Theorem (2.1) is complete.

Now, we give an example for Theorem (2.1) and Theorem (2.2).

Example 2.1.

As special cases of system (1.3), if we take the matrixes below for $n = 2$

$$\Psi = \begin{bmatrix} -2 - z_1^2 - z_2^2 & 1 \\ 1 & -2 - z_1^2 - z_2^2 \end{bmatrix}, \ \Theta = \begin{bmatrix} \dfrac{1}{x_1^2 + 1} + 4 & 3 \\ 3 & \dfrac{1}{x_1^2 + 1} + 4 \end{bmatrix}$$

$$H = \begin{bmatrix} -\dfrac{x_1^3}{3} - 5x_1 \\ -\dfrac{x_2^2}{3} - 5x_2 \end{bmatrix}$$

Then, respectively, we have

$$\lambda_1(\Psi) = -z_1^2 - z_2^2 - 1, \ \lambda_2(\Psi) = -z_1^2 - z_2^2 - 3$$

$$\lambda_1(\Theta) = \frac{1}{x_1^2 + 1} + 1, \qquad \lambda_2(\Theta) = \frac{1}{x_1^2 + 1} + 7$$

In addition, $J_H(X)$ Jacobian matrix for H vector is obtained as below;

$$J_H(X) = \begin{bmatrix} -x_1^2 - 5 & 0 \\ 0 & -x_2^2 - 5 \end{bmatrix}$$

And we have

$$\lambda_1(J_H) = -x_1^2 - 5, \quad \lambda_2(J_H) = -x_2^2 - 5$$

Example 2.2.

As special cases of system (1.3), if we take the matrixes below for $n = 2$

$$\Psi = \begin{bmatrix} 4z_1^2 + 3z_2^2 + 3 & 2 \\ 2 & 4z_1^2 + 3z_2^2 + 3 \end{bmatrix}, \quad \Theta = \begin{bmatrix} -\dfrac{3}{2 + x_1^2 + x_2^2} & \dfrac{2}{2 + x_1^2 + x_2^2} \\ \dfrac{2}{2 + x_1^2 + x_2^2} & -\dfrac{3}{2 + x_1^2 + x_2^2} \end{bmatrix},$$

$$\Phi = \begin{bmatrix} 2 + z_1^2 + z_2^2 & 1 \\ 1 & 2 + z_1^2 + z_2^2 \end{bmatrix}, \quad H = \begin{bmatrix} x_1^3 + x_1 \\ x_2 + x_2^3 \end{bmatrix}$$

Then, respectively, we have

$$\lambda_1(\Psi) = 4z_1^2 + 3z_2^2 + 1 \,, \; \lambda_2(\Psi) = 4z_1^2 + 3z_2^2 + 5,$$

$$\lambda_1(\Theta) = -\frac{1}{1 + x_1^2 + x_2^2}, \qquad \lambda_2(\Theta) = -\frac{5}{1 + x_1^2 + x_2^2}$$

$$\lambda_1(\Phi) = z_1^2 + z_2^2 + 1, \quad \lambda_2(\Phi) = z_1^2 + z_2^2 + 3$$

In addition, $J_H(X)$ Jacobian matrix for H vector is obtained as below;

$$J_H(X) = \begin{bmatrix} 3x_1^2 + 1 & 0 \\ 0 & 3x_2^2 + 1 \end{bmatrix}$$

And we have

$$\lambda_1(H) = 3x_1^2 + 1, \quad \lambda_2(H) = 3x_2^2 + 1$$

It is obvious that all criteria of Theorem 2.1 and Theorem 2.2 are satisfied. That means, it becomes instability of trivial solution of vector differential equation corresponding for two dimentional arbitrary E vector, arbitrary symetric $2x2 -$ matrix Φ for Ψ, Θ *and* H chosen above.

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