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Theoretical Investigation of Radiative Lifetimes and Rydberg Levels Sequence in Indium I:

Naveed Ali [1, 2,3] Muhammad Noman Hameed [1] Salman Raza [1, 4]
Arif Akhter Azam[1, 5]
1.Department of Physics, University of Karachi
2.Aga Khan Higher secondary School
3.Saleem Nawaz Fazaia College P.A.F Base Masroor
4.Government National College #1
5.Government Degree College, Gulshan-e-Iqbal, Karachi

Abstract
The weakest bound electron potential model theory (WBEPMT) is an effective semi-empirical method developed by Zheng et al. in 2004 for commutating atomic spectroscopic parameters, particularly for high lying Rydberg levels. Employing this theory paper, we theoretically computed the series of Rydberg levels and Radiative lifetimes with their quantum defects in neutral Indium (In I) for the states having term values: $^2D_{3/2}$, $^2D_{5/2}$, $^2F°_{5/2}$, $^2F°_{7/2}$, $^2P°_{1/2}$, $^2P°_{3/2}$ *and* $^2S_{1/2}$. The result of lifetime is compared with Yildiz's work and found to be in good agreement. This work is also an extension of Yildiz's work. The Rydberg energies and quantum defects of $^2F°_{5/2}$, $^2F°_{7/2}$ are presented first time.
Keywords: Quantum defects, Radiative lifetime, Weakest bound electric potential model, Rydberg levels, Indium

Introduction

Neutral Indium (In^{49}) is a post-transition metal lying at the intersection of group 13 and period 5 of the periodic table of chemical elements. It has an electronic configuration of [Kr]$4d^{10}5s^25p^1$ with ground state configuration [Cd]$5p^2P°_{1}2$ and ionized level $5s2^1S_0$ with ionization potential of 5.7863556eV. Being a member of post transition metals, the inert pair effect allows neutral Indium to have multiple oxidation states among which +3 oxidation state is most stable followed by the +1-oxidation state. This inert pair effect increases the range of applications of Indium from the semiconductor industry, where it used as a high-speed transistor, LCD screens, photo-voltaic cells to ultra-high vacuum applications and further as control rods in nuclear reactors.

G. Jönsson et al. experimentally obtained radiative lifetimes of Indium of $n \leq 20$ of $5s^2ns^2S_{1/2}$ and $5s^2nd^2D_{3/2,5/2}$. They used a UV pulsed laser. They observed normal behavior of S state lifetime and D sequence to be strongly perturbed [1]. Yildiz M calculated atomic lifetimes and energy levels of some Rydberg series by using weakest bound electron potential model theory for neutral Indium. To calculate lifetimes, he used quantum defect method and Martin's expression. The results are in excellent agreement with experimental data available. The method also provided some unknown lifetimes and energy levels too [2]. Browaeys et al. have summarized works done over the last decade by several researchers to study interacting Rydberg atoms in a skillful way [3]. Civiš S et al. used time resolved Fourier transform and LIBS technique to obtain spectra of neutral Indium. They analyzed that the method is suitable for Rydberg atomic levels. Time delay in pulse of laser gave advantage to filter low intensity spectral lines. They recorded spectra at different wave numbers with a resolution of 0.017 cm^{-1}. Through this technique they found five new levels of neutral Indium [4]. Burnham used Argon-Fluoride laser (ArF) at 193nm found a 451 nm line in neutral Indium. The used pulses had energies of 0.5 mJ which resulted from the absorption of 30 mJ [5]. Ewiss et al. used a pulse dye laser in their experimental setup. They found radiative lifetimes of $6p^2P$ state of In I [6]. Norton M and Gallagher A found experimentally $(7.0\pm0.3) \times 10^{-9}$sec lifetime of neutral Indium in 6^2S_{12} state. They used the Hanle effect technique [7]. Ewiss MZ and Snoek C. used a Nitrogen laser-pumped dye laser for the transition of neutral Indium. They used photomultiplier to observe the decay of Indium atoms. They experimentally found the lifetime of $5s^2ns^2S_{1/2}$ and $5s^2nd^2D_{5/2}$ ($n<11$) levels in In I. The results are in good agreement with literature [8]. Hong FL et al. studied 2F Rydberg states of neutral Indium. They used microwave resonance spectroscopy. Experimentally through microwave transition from 2F5/2 and 2F7/2 Rydberg series they observed inversion in the fine structure. They observed that the inner core is closely related with the valance electrons in a long-range interaction in the fine structure of 2F states [9]. Andersen T and Sørensen G. also studied atomic lifetimes of Indium among other elements by using beam-foil technique. They observed the atomic lifetime of lowest S level is 7.5±0.7 ns. They compared their result with the results observed experimentally by Hanle-effect technique. The results are in very good agreement. They also compared their result with theoretical data based upon a single-configuration coulomb approximation [10]. Havey MD et al. found lifetimes of some group three elements from N_2 laser-pumped dye laser. Lifetime of neutral indium of $^2S_{1/2}$ state was found to be (7.4 ± 0.3) ns [11]. Neijzen JH and Dönszelmann A. have studied $5p^2P_{3/2}$---$np^2P_{1/2,3/2}$ states for < 24 $n < 44$ by using pulsed dye laser. They verified ionization limit (46670.107 cm^{-1}) by accurate values of quantum defects. They also discussed the differences in their results from those of previous works. They further experimentally determined

the relation between the real and the virtual intermediate levels, which was previously known only theoretically. They also studied $np^2P_{1/2,3/2}$ Rydberg series of neutral Gallium and Indium. They excited the atoms using pulsed dye laser. Their results showed that these series are unperturbed. They found very accurate level values by using Rydberg-Ritz formulae. The result ionization limit (46 670.106±0.006 cm^{-1}) agreed well with previous values of neutral Indium. They also studied the ionization of neutral Indium by using external fields of force. They first excited an atom in UV from ground state to some higher state or by stepwise excitation in visible region. After excitation, they applied a force field on highly excited atoms that triggered ionization in. They compared their experimental results for n values with models [12, 13, 14]. De Vlieger GJ et al. used dye laser to study $np^2P_{1/2,3/2}$ and $nd^2D_{3/2,5/2}$ Rydberg states for 15 n < 32. They observed Hydrogenic behavior of fine structure. They observed the perturbation of nd^2D series due to $5s5p^{22}D$ term, and a value for corresponding term is also provided [15]. Gallagher TF studied the properties of Rydberg atoms by considering the Coulomb radial functions. Using these he described the energy level structure. He discussed the atoms (Rydberg) in electric fields and analyzed vacuum fluctuations. He surveyed collisions of Rydberg atoms with un-excited atoms, molecules, charge particles and similar atoms (Rydberg) [16].

Theory

In this section the analytical treatment of weakest bound electron potential model theory (WBEPMT) will be discussed. According to WBEPMT the Rydberg levels energy of weakest bound electrons (WBE's) can be written as: [17]

$$T = T_{limit} + E, \tag{1}$$

In equation (1) the first term on right hand side is the ionization limit and second term represents the energy of weakest bound electron (WBE).

The WBE is assumed to move in the effective potential of an ion core produced due to core penetration, core polarization and shielding effect. Analytical expression of effective potential can be written as [18]

$$V(r) = -\frac{Z'}{r} + \frac{k(k+1)+2kl}{2r^2} \tag{2}$$

In equation (2) the first term on right hand side represents the Coulomb potential and the second term represents the dipole potential. Z' is difference of charge of nucleus and non-weakest bound electrons (NWBE's), and in second term k is a parameter that is adjusted accordingly.

Put the expression of effective potential into radial equation from equation (2) in equation (3)

$$\frac{1}{R}\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} + \left[E + \frac{Z'}{r} - \frac{l'(l'+1)}{2r^2}\right]R = 0 \tag{3}$$

With

$$l' = l + k \tag{4}$$

We get

$$R_{nl}(r) = Aexp\left(-\frac{Z'r}{n'}\right)r^{l'}L_{n-l-1}^{2l'+1}\left(\frac{2Z'r}{n'}\right), \tag{5}$$

$$E = -R\left(\frac{Z'}{n'}\right)^2$$

In the above equations on right hand side n' = n + k, A is the normalization constant, and R the Rydberg constant. Let

$$\frac{Z'}{n'} = \frac{Z_{net}}{n-\delta_n} \tag{7}$$

In the above equation on right hand side Z_{net} is the net charge number of the atomic kernel (Z_{net}=1 for neutral atoms) and δ_n is the quantum defect. Now we can re-write equation (1) as

$$T = T_{limit} + E = T_{limit} - R\left(\frac{Z_{net}}{n-\delta_n}\right)^2 \tag{8}$$

The δ_n in equation (8) can be easily determined by exploiting Ritz's expression [19]. We can write

$$\delta_n = \sum_{i=0}^{3}\frac{a_i}{(n-\delta_0)^{2i}} \tag{9}$$

In above equation on right hand side δ_0 is representing to quantum defect of lowest energy level in a given series, a_i are coefficients that can be obtained through regression of equation (9).

The Rykova's expression for atomic lifetimes [20] and [24].

$$\tau = \tau_0(n-\delta_n)^{\alpha} \quad \ldots(10)$$

In above equation T_o and α are the coefficients in Rykova's expression. These are obtained using the method of least-square fitting of first few values of atomic lifetimes taken from previously published data [2].

Procedure

The theoretical computation of spectral energies of Rydberg levels in In I by using semi-empirical method WBEPMT is carried out. The experimental data of In I, used to calculate quantum defects, was obtained from

NIST web site [22]. The data was used in a simple program on macros of Excel that calculated quantum defects and co-efficient of corresponding states. To calculate radiative lifetimes, initial data from [2] was utilized.

Results and Discussions

We have calculated quantum defects up to quantum number = 50. To calculate quantum defects, spectral data was obtained from NIST. The coefficients in Martin's formula were calculated using few values of spectral energies form NIST. The quantum defects along with principle quantum numbers are used to calculate effective principal quantum numbers needed in the calculation of lifetimes of In I. The lifetimes of neutral Indium up to n = 50 for term values $^2S_{1/2}$, $^2D_{5/2}$, $^2D_{3/2}$, $^2P°_{3/2}$, $^2P°_{1/2}$, $^2F°_{5/2}$, $^2F°_{7/2}$ have been calculated.

Figs. I-VII show graphs between QD (quantum defects) and n (principle quantum numbers). Figs. VIII-XII show the graphs between the τ (lifetimes calculated) and τ (lifetimes references) and n (principle quantum numbers) along the x-axis. The red curve shows the work of Yildiz, the blue curve an extension of red curve shows the lifetime calculated in this work. The two curves in all cases ($^2S_{1/2}$, $^2D_{5/2}$, $^2D_{3/2}$, $^2P°_{3/2}$, $^2P°_{1/2}$) perfectly overlap.

In Table III-IX: first column shows principal quantum numbers (n), second column shows experimental Rydberg energy levels (E_{exp}), third column shows calculated Rydberg energy levels (E_{cal}), fourth column shows the difference between E_{exp} and E_{cal} (ΔE), fifth column shows quantum defects (δ_n). In column six, seven and eight different experimental values are shown (wherever available, in cm^{-1}). Similarly, in table X-XIV: the first column shows principal quantum numbers (n), second column shows reference lifetimes ($\tau_{(REF)}$), third column shows calculated lifetimes ($\tau_{(CAL)}$), fourth and fifth column shows reference lifetimes ($\tau_{(REF)}$) (where ever available); all in nanoseconds.

Conclusions

The updated data of neutral Indium In I (version 5.5.6, 6th April 2018) from NIST was utilized to compute various spectral properties.

- The energy sequence of Rydberg levels with their quantum defects of the series ($^2D_{3/2}$, $^2D_{5/2}$, $^2F°_{5/2}$, $^2F°_{7/2}$, $^2P°_{1/2}$, $^2P°_{3/2}$ and $^2S_{1/2}$) up to n = 50 have been calculated using WBEPMT.
- The Rydberg energies have been compared with the NIST values when available. The agreement is good except from some values where the differences in energies are up to 4 cm^{-1}. The difference indicates the levels are perturbed. The Martin's formula works well for unperturbed energy levels.
- The Rydberg energies and quantum defects of the series are presented for the first time.
- The lifetime of the series ($^2D_{3/2}$, $^2D_{5/2}$, $^2F°_{5/2}$, $^2F°_{7/2}$, $^2P°_{1/2}$, $^2P°_{3/2}$ and $^2S_{1/2}$) have also been calculated and compared with the work of Yildiz. This work is an extension of Yildiz's work. The agreement in the overlap area of the two works is excellent (as can be seen in fig. VIII-XII).

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Table I: The coefficient's a_0, a_1, a_2, a_3 and δ_o of spectral series $^2D_{3/2}, ^2D_{5/2}, ^2F°_{5/2}, ^2F°_{7/2}, ^2P°_{1/2}, ^2P°_{3/2}$ and $^2S_{1/2}$.:

In I (Z=49,isoelectronic sequence) $5s^{21}S_o$ 46670.106 cm-1					
Spectral Series	**a_0**	**a_1**	**a_2**	**a_3**	**δ_0**
$5s^2ns\ ^2S_{1/2}$ (n=6-50)	3.720717	0.237949	0.342326	-0.19786	3.781539
$5s^2nd\ ^2D_{3/2}$ (n=4-50)	2.365481	-4.6356	43.84257	-149.945	2.177815
$5s^2nd\ ^2D_{5/2}$ (n=5-50)	2.322831	-3.22122	22.98154	-53.1715	2.175425
$5s^2np\ ^2P^o_{1/2}$ (n=6-50)	3.223083	0.353411	0.376211	1.643687	3.281886
$5s^2np\ ^2P^o_{3/2}$ (n=6-50)	3.197117	0.31517	0.95521	-0.6629	3.254177
$5s^2nf^2F^o_{5/2}$ (n=4-50)	0.041222079	-0.170178116	-0.302195262	2.995047616	0.0299732
$5s^2nf^2F^o_{7/2}$ (n=4-50)	0.041222079	-0.170178116	-0.302195262	2.995047616	0.0299732

Note: "δ_o" is the quantum defect of the lowest Rydberg State of the series and "n" is principal quantum number.

Table II: The coefficient's α and T_o of spectral series $^2S_{1/2}, ^2D_{5/2}, ^2D_{3/2}, ^2P°_{1/2}, ^2P^o_{3/2}$:

In I (Z=49, isoelectronic sequence)		
Spectral Series	**T_o**	**α**
$5s^2ns\ ^2S_{1/2}$ (n=6-50)	9.69E-10	2.7319
$5s^2nd\ ^2D_{3/2}$ (n=5-50)	5.95E-10	2.7137
$5s^2nd\ ^2D_{5/2}$ (n=5-50)	1.94E-10	3.4787
$5s^2np\ ^2P^o_{1/2}$ (n=6-50)	1.62E-09	3.7075
$5s^2np\ ^2P^o_{3/2}$ (n=6-50)	1.53E-09	3.6364

Note: "T_o" is the minimum value of lifetime for n=1 Rydberg state and "n" is principal quantum number.

Table III. The calculated spectral energies compared with experimental energies [22] of Rydberg series: $5s^2ns\ ^2S_{1/2}$ (n=6-50) of In I.

n	$E_{exp}cm^{-1}$	E_{cal} cm^{-1}	ΔE cm^{-1}	δ_n	$E_{exp}cm^{-1}$ Ref[2]
6	24372.96	24372.96	0.00	3.781539	22297
7	36301.86	36301.86	0.00	3.746701	10368
8	40636.98	40636.98	0.00	3.735134	6033
9	42719.02	42719.02	0.00	3.729907	3951
10	43881.26	43881.32	0.06	3.727096	2788
11	44595.86	44596.45	0.59	3.725408	2070
12	45067.19	45067.8	0.61	3.724315	1600
13	45394.13	45394.87	0.74	3.723564	1274
14	45630.44	45631.09	0.65	3.723027	1038
15	45806.88	45807.25	0.37	3.722629	862
16		45942.12		3.722326	727
17		46047.67		3.72209	622
18		46131.82		3.721903	538
19		46199.99		3.721751	470
20		46255.98		3.721627	
21		46302.53		3.721524	
22		46341.66		3.721437	
23		46374.85		3.721364	
24		46403.25		3.721301	
25		46427.75		3.721247	
26		46449.02		3.721201	
27		46467.6		3.72116	
28		46483.94		3.721124	
29		46498.38		3.721092	
30		46511.2		3.721064	
31		46522.64		3.721039	
32		46532.88		3.721017	
33		46542.1		3.720996	
34		46550.41		3.720978	
35		46557.94		3.720962	
36		46564.79		3.720947	
37		46571.02		3.720933	
38		46576.72		3.720921	
39		46581.94		3.720909	
40		46586.73		3.720899	
41		46591.14		3.720889	
42		46595.22		3.72088	
43		46598.98		3.720872	
44		46602.47		3.720864	
45		46605.71		3.720857	
46		46608.72		3.720851	
47		46611.52		3.720845	
48		46614.14		3.720839	
49		46616.58		3.720834	
50		46618.87		3.720829	78

Table IV. The calculated spectral energies compared with experimental energies [22] of Rydberg series: $5s^2ns\ 5s^2nd\ ^2D_{3/2}$ (n=5-50) of In I.

n	$E_{exp}cm^{-1}$	$E_{cal}\ cm^{-1}$	$\Delta E\ cm^{-1}$	δ_n	$E_{exp}cm^{-1}$ Ref[2]
5	32892.23	32892.23	0.00	2.177815	13778
6	39048.53	39048.53	0.00	2.205504	7809
7	41836.41	41836.41	0.00	2.235286	4834
8	43335.93	43335.93	0.00	2.263034	3334
9	44234.7	44236.71	2.01	2.284633	2594
10	44815.06	44818.88	3.82	2.300775	1949
11	45211.51	45216	4.49	2.31284	1517
12	45493.98	45498.51	4.53	2.321975	1215
13	45702.08	45706.4	4.32	2.329004	994
14	45859.3	45863.71	4.41	2.334503	829
15		45985.55		2.338873	701
16		46081.8		2.342397	601
17		46159.13		2.345275	521
18		46222.19		2.347654	456
19		46274.28		2.349641	402
20		46317.79		2.351316	358
21		46354.52		2.352742	
22		46385.79		2.353964	
23		46412.64		2.35502	
24		46435.86		2.355938	
25		46456.08		2.356741	
26		46473.79		2.357447	
27		46489.39		2.358072	
28		46503.2		2.358627	
29		46515.49		2.359122	
30		46526.47		2.359565	145
31		46536.32		2.359964	
32		46545.19		2.360324	
33		46553.21		2.36065	
34		46560.48		2.360946	
35		46567.09		2.361215	
36		46573.13		2.361462	
37		46578.64		2.361688	
38		46583.7		2.361895	
39		46588.36		2.362086	
40		46592.64		2.362262	78
41		46596.6		2.362424	
42		46600.26		2.362575	
43		46603.65		2.362715	
44		46606.81		2.362845	
45		46609.74		2.362966	
46		46612.48		2.363079	
47		46615.03		2.363184	
48		46617.42		2.363283	
49		46619.65		2.363375	
50		46621.75		2.363462	48

Table V. The calculated spectral energies compared with experimental energies [22] of Rydberg series: $5s^2nd\ ^2D_{5/2}$ (n=5-50) of In I.

n	$E_{exp}cm^{-1}$	E_{cal} cm^{-1}	ΔE cm^{-1}	δ_n	$E_{exp}cm^{-1}$ Ref[2]
5	32915.54	32915.54	0.00	2.175425	13755
6	39098.38	39098.38	0.00	2.193033	7697
7	41861.96	41861.96	0.00	2.222643	4808
8	43355.08	43355.08	0.00	2.246487	3315
9	44249.77	44251.78	2.01	2.263737	2530
10	44827.02	44830.68	3.66	2.276117	1907
11	45221.08	45225.22	4.14	2.285143	1489
12	45502.04	45505.76	3.72	2.291866	1194
13	45709.02	45712.16	3.14	2.29698	979
14	45982.73	45868.33	-114.39	2.300949	817
15		45989.3		2.304083	692
16		46084.87		2.306598	594
17		46161.68		2.308645	515
18		46224.32		2.310331	451
19		46276.07		2.311736	398
20		46319.32		2.312918	354
21		46355.83		2.313923	
22		46386.92		2.314783	
23		46413.62		2.315525	
24		46436.72		2.316169	
25		46456.83		2.316732	
26		46474.45		2.317227	
27		46489.98		2.317664	
28		46503.73		2.318053	
29		46515.96		2.318399	
30		46526.89		2.318709	144
31		46536.7		2.318987	
32		46545.54		2.319239	
33		46553.53		2.319466	
34		46560.77		2.319673	
35		46567.36		2.319861	
36		46573.37		2.320033	
37		46578.86		2.320191	
38		46583.91		2.320335	
39		46588.54		2.320468	
40		46592.81		2.320591	77
41		46596.76		2.320704	
42		46600.41		2.320809	
43		46603.79		2.320907	
44		46606.93		2.320997	
45		46609.86		2.321082	
46		46612.59		2.32116	
47		46615.13		2.321234	
48		46617.51		2.321302	
49		46619.74		2.321367	
50		46621.83		2.321427	48

Table VI. The calculated spectral energies compared with experimental energies [22] of Rydberg series: $5s^2np\ ^2P^o_{1/2}$ (n=6-50) of In I.

n	$E_{exp}cm^{-1}$	E_{cal} cm^{-1}	ΔE cm^{-1}	δ_n	$E_{exp}cm^{-1}$ Ref[2]
6	31816.98	31816.98	0.00	3.281886	14853
7	38861.43	38861.43	0.00	3.251238	7809
8	41827.1	41827.1	0.00	3.239868	4843
9	43369.09	43369.09	0.00	3.234291	3288
10		44275.03		3.231116	2389
11		44852.86		3.22913	1814
12		45244.05		3.227802	1424
13		45521.2		3.226869	1148
14		45724.71		3.226189	1148
15		45878.55		3.225678	791
16		45997.67		3.225283	672
17		46091.79		3.224972	578
18		46167.44		3.224723	578
19		46229.16		3.22452	440
20		46280.17		3.224353	389
21		46322.81		3.224213	
22		46358.83		3.224095	
23		46389.51		3.223995	
24		46415.88		3.223909	
25		46438.69		3.223834	
26		46458.57		3.223769	
27		46475.99		3.223713	
28		46491.34		3.223663	
29		46504.94		3.223618	
30		46517.05		3.223579	153
31		46527.87		3.223544	
32		46537.59		3.223512	
33		46546.34		3.223484	
34		46554.25		3.223458	
35		46561.43		3.223435	
36		46567.96		3.223414	
37		46573.92		3.223394	
38		46579.37		3.223377	
39		46584.37		3.223361	
40		46588.97		3.223346	81
41		46593.21		3.223332	
42		46597.12		3.223319	
43		46600.75		3.223307	
44		46604.11		3.223297	
45		46607.23		3.223286	
46		46610.14		3.223277	
47		46612.84		3.223268	
48		46615.37		3.22326	
49		46617.74		3.223252	
50		46619.95		3.223245	50

Table VII. The calculated spectral energies compared with experimental energies [22] of Rydberg series: $5s^2np\ ^2P^o_{3/2}$ (n=6-50) of In I.

n	$E_{exp}cm^{-1}$	E_{cal} cm^{-1}	ΔE cm^{-1}	δ_n	$E_{exp}cm^{-1}$ Ref[2]
6	32115.25	32115.25	0.00	3.254177	14555
7	38972.9	38972.9	0.00	3.224191	7697
8	41881.44	41881.44	0.00	3.212936	4789
9	43399.53	43399.53	0.00	3.207522	3259
10		44293.76		3.204497	2371
11	44865.79	44865.19	0.60	3.202633	1802
12		45252.6		3.2014	1416
13		45527.36		3.200541	1142
14		45729.3		3.199918	940
15		45882.07		3.199452	787
16		46000.42		3.199093	669
17		46093.98		3.198812	576
18		46169.21		3.198587	500
19		46230.61		3.198404	439
20		46281.38		3.198253	388
21		46323.83		3.198128	
22		46359.69		3.198022	
23		46390.25		3.197932	
24		46416.51		3.197855	
25		46439.24		3.197788	
26		46459.05		3.19773	
27		46476.41		3.197679	
28		46491.72		3.197635	
29		46505.28		3.197595	
30		46517.35		3.19756	152
31		46528.14		3.197528	
32		46537.83		3.1975	
33		46546.55		3.197475	
34		46554.45		3.197452	
35		46561.61		3.197431	
36		46568.12		3.197412	
37		46574.07		3.197395	
38		46579.51		3.197379	
39		46584.5		3.197365	
40		46589.09		3.197351	81
41		46593.32		3.197339	
42		46597.22		3.197328	
43		46600.84		3.197317	
44		46604.19		3.197307	
45		46607.31		3.197298	
46		46610.21		3.19729	
47		46612.91		3.197282	
48		46615.44		3.197275	
49		46617.8		3.197268	
50		46620.01		3.197262	50

Table VIII. The calculated spectral energies compared with experimental energies [22] of Rydberg series: $5s^2nf\ ^2F^o_{7/2}$ (n=4-50) of In I.

n	E_{exp} cm^{-1}	E_{cal} cm^{-1}	ΔE cm^{-1}	δ_n
4	39707.59	39707.59	0.00	0.029973
5	42220.25	42220.25	0.00	0.034036
6	43584.66	43584.66	0.00	0.036276
7	44406.31	44406.31	0.00	0.037617
8	44938.81	44938.85	0.04	0.03848
9		45303.49		0.039066
10		45564.02		0.039483
11		45756.59		0.039789
12		45902.93		0.040021
13		46016.74		0.0402
14		46106.98		0.040343
15		46179.74		0.040457
16		46239.26		0.04055
17		46288.57		0.040628
18		46329.87		0.040692
19		46364.82		0.040747
20		46394.64		0.040794
21		46420.3		0.040834
22		46442.53		0.040868
23		46461.92		0.040898
24		46478.94		0.040925
25		46493.95		0.040948
26		46507.26		0.040969
27		46519.12		0.040988
28		46529.72		0.041004
29		46539.25		0.041019
30		46547.84		0.041032
31		46555.61		0.041044
32		46562.67		0.041055
33		46569.09		0.041065
34		46574.95		0.041074
35		46580.31		0.041083
36		46585.24		0.04109
37		46589.77		0.041097
38		46593.95		0.041104
39		46597.81		0.04111
40		46601.38		0.041115
41		46604.69		0.041121
42		46607.77		0.041125
43		46610.64		0.04113
44		46613.32		0.041134
45		46615.82		0.041138
46		46618.15		0.041141
47		46620.34		0.041145
48		46622.4		0.041148
49		46624.32		0.041151
50		46626.14		0.041154

Table IX. The calculated spectral energies compared with experimental energies [22] of Rydberg series: $5s^2nf\ ^2F^o_{5/2}$ (n=4-50) of In I.

n	E_{exp} cm^{-1}	E_{cal} cm^{-1}	ΔE cm^{-1}	δ_n
4	39707.59	39707.59	0.00	0.029973
5	42220.25	42220.25	0.00	0.034036
6	43584.66	43584.66	0.00	0.036276
7	44406.31	44406.31	0.00	0.037617
8		44938.85		0.03848
9		45303.49		0.039066
10		45564.02		0.039483
11		45756.59		0.039789
12		45902.93		0.040021
13		46016.74		0.0402
14		46106.98		0.040343
15		46179.74		0.040457
16		46239.26		0.04055
17		46288.57		0.040628
18		46329.87		0.040692
19		46364.82		0.040747
20		46394.64		0.040794
21		46420.3		0.040834
22		46442.53		0.040868
23		46461.92		0.040898
24		46478.94		0.040925
25		46493.95		0.040948
26		46507.26		0.040969
27		46519.12		0.040988
28		46529.72		0.041004
29		46539.25		0.041019
30		46547.84		0.041032
31		46555.61		0.041044
32		46562.67		0.041055
33		46569.09		0.041065
34		46574.95		0.041074
35		46580.31		0.041083
36		46585.24		0.04109
37		46589.77		0.041097
38		46593.95		0.041104
39		46597.81		0.04111
40		46601.38		0.041115
41		46604.69		0.041121
42		46607.77		0.041125
43		46610.64		0.04113
44		46613.32		0.041134
45		46615.82		0.041138
46		46618.15		0.041141
47		46620.34		0.041145
48		46622.4		0.041148
49		46624.32		0.041151
50		46626.14		0.041154

Table X. Calculated and compared Lifetimes of Rydberg states series: $5s^2ns\ ^2S_{1/2}$ (n=6-50) for In I.

n	$\tau_{(REF)}$ns Ref[2]	$\tau_{(CAL)}$nS	$\tau_{(EXP)}$ns Ref[1]	$\tau_{(EXP)}$ns Ref[10]	$\tau_{(SD)}$ns Ref[23]	$\tau_{(EXP)}$ns Ref[6,8]
6	7	8.542739073		7.5±0.7	7.04	
7	29	24.31285199	27±6		21.5	19.53±1.5
8	56	50.93970634	55±6		47.4	53±5
9	94	90.81413331	104±12		89.4	118±10
10	147	146.1579964	163±13			
11	217	219.0750768	244±19			
12	306	311.5779489	330±21			
13	417	425.6048388	490±35			
14	551	563.0313557	625±60			
15	712	725.6791792	785±785			
16	902	915.32276	1025±70			
17	1122	1133.694644	1170±95			
18	1377	1382.489798	1360±135			
19	1666	1663.369196	1690±200			
20	1995	1977.962829	2000±300			
21		2327.872267				
22		2714.672865				
23		3139.915682				
24		3605.129158				
25		4111.820596				
26		4661.477479				
27		5255.568645				
28		5895.545347				
29		6582.842203				
30	7898	9882.55				
31		10981.95				
32		12157.31				
33		13410.94				
34		14745.09				
35		16162.01				
36		17663.95				
37		19253.12				
38		20931.75				
39		22702.04				
40	20242	24566.17				
41		26526.33				
42		28584.69				
43		30743.39				
44		33004.59				
45		35370.43				
46		37843.04				
47		40424.53				
48		43117.02				
49		45922.61				
50	41425	48843.38				

Table XI. Calculated and compared Lifetimes of Rydberg states series: $5s^2nd\ ^2D_{3/2}$ (n=5-50) for In I.

n	$\tau_{(REF)}$ns Ref[2]	$\tau_{(CAL)}$nS	$\tau_{(EXP)}$ns Ref[1]	$\tau_{(EXP)}$ns Ref[10]	$\tau_{(SD)}$ns Ref[23]
5	9	9.93198229		6.3±0.5	6.45
6	22	22.1785391		21±3	19.2
7	42	41.1406348	200±4	50±5	42
8	71	68.0955022	317±22		75.5
9	109	104.400242	550±50		
10	158	151.298132	455±40		
11	218	209.949585	490±50		
12	289	281.456869	485±40		
13	374	366.878419	500±30		
14	471	467.237483	570±40		
15	583	583.5278	635±40		
16	710	716.717604	735±60		
17	853	867.75263	820±65		
18	1011	1037.55846	895±60		
19	1186	1227.04244	1075±70		
20	1379	1437.09523	1275±115		
21		1668.59216			
22		1922.3944			
23		2199.34989			
24		2500.29428			
25		2826.05165			
26		3177.43527			
27		3555.24811	4375		
28		3960.28353			
29		4393.32565			
30		4855.14993			
31		5346.5235			
32		5868.2056			
33		6420.94791			
34		7005.49487			
35		7622.584			
36		8272.94619			
37		8957.30593			
38		9676.38157			
39		10430.8856			
40	9663	11221.5246			
41		12049			
42		12914.0076			
43		13817.2382			
44		14759.3777			
45		15741.107			
46		16763.1025			
47		17826.0359			
48		18930.5747			
49		20077.382			
50	17675	21267.1168			

Table XII. Calculated and compared Lifetimes of Rydberg states series: $5s^2nd\ ^2D_{5/2}$ (n=5-50) for In I.

n	$\tau_{(REF)}$ns Ref[2]	$\tau_{(CAL)}$nS	$\tau_{(EXP)}$ns Ref[1]	$\tau_{(EXP)}$ns Ref[10]	$\tau_{(SD)}$ns Ref[23]	$\tau_{(EXP)}$ns Ref[6,8]
5	7	7.173554329		7.6±0.5	6.78	
6	20	20.26116514		22±3	20.1	18.6±1.5
7	45	44.6369208	147±10	50±5	44.0	154±10
8	87	85.22815564	238±20		77.2	300±60
9	151	147.5112806	640±50			
10	242	237.4203652	780±80			
11	365	361.3160582	760±80			
12	527	525.9604717	780±65			
13	733	738.4949728	845±60			
14	990	1006.42063	1005±70			
15	1304	1337.581023	1185±110			
16	1681	1740.147087	1335±125			
17	2129	2222.60366	1540±200			
18	2653	2793.737476	1810±210			
19	3263	3462.626376	1985±250			
20	3963	4238.629541	2215±250			
21		5131.378625				
22		6150.769637				
23		7306.955489				
24		8610.339126				
25		10071.56715				
26		11701.52392				
27		13511.32601				
28		15512.31706				
29	17693	17716.06294				
30		20134.3472				
31		22779.16676				
32		25662.72788				
33		28797.44229				
34		32195.9236				
35		35870.98379				
36		39835.62998				
37		44103.06123				
38		48686.66558				
39		53600.01721				
40	49452	58856.87367				
41		64471.17324				
42		70457.03247				
43		76828.74371				
44		83600.77283				
45		90787.75692				
46		98404.50222				
47		106465.982				
48		114987.3344				
49		123983.8609				
50	108296	133471.0241				

Table XIII. Calculated and compared Lifetimes of Rydberg states series: $5s^2np\ ^2P^o_{1/2}$ (n=6-50) for In I.

n	$\tau_{(REF)}$ns Ref[2]	$\tau_{(CAL)}$nS	$T_{(SD)}$ns Ref[23]	N	$\tau_{(REF)}$ns Ref[2]	$\tau_{(CAL)}$nS	$\tau_{(SD)}$ns Ref[23]
6	69	66.19048	69.7	29		277238.6	
7	218	217.9904	219	30	321185	319257.2	
8	525	528.4706	473	31		365746.5	
9	1065	1075.498		32		416995	
10	1929	1949.352		33		473299.1	
11	3221	3252.105		34		534962.4	
12	5053	5097.096		35		602295.7	
13	7551	7608.485		36		675617.3	
14	10850	10920.88		37		755252.4	
15	15096	15179		38		841533.4	
16	20447	20537.41		39		934799.8	
17	27068	27160.24		40	1046525	1035398	
18	35134	35221		41		1143681	
19	44832	44902.31		42		1260010	
20	56356	56395.8		43		1384751	
21		69901.86		44		1518278	
22		85629.52		45		1660971	
23		103796.3		46		1813218	
24		124628.2		47		1975413	
25		148359.2		48		2147956	
26		175231.5		49		2331253	
27		205495.5		50	2560953	2525718	
28		239409.2					

Table XIV. Calculated and compared Lifetimes of Rydberg states series: $5s^2np\ ^2P^o_{3/2}$ (n=6-50) for In I.

n	$\tau_{(REF)}$ns Ref[2]	$\tau_{(CAL)}$nS	$\tau_{(EXP)}$ns Ref[6,8]	$\tau_{(SD)}$ns Ref[23]
6	63	60.17905	55±4	63.7
7	192	191.6441		192
8	450	454.2137		414
9	898	908.5242		
10	1606	1623.854		
11	2650	2677.54		
12	4116	4154.53		
13	6098	6147.007		
14	8695	8754.079		
15	12014	12081.52		
16	16168	16241.51		
17	21279	21352.49		
18	27472	27538.91		
19	34880	34931.12		
20	43640	43665.17		
21		53882.75		
22		65730.97		
23		79362.36		
24		94934.64		
25		112610.7		
26		132558.6		
27		154951.1		
28		179966.2		
29		207786.5		
30	240164	238599.5		
31		272597.1		
32		309976.1		

33		350937.8		
34		395687.9		
35		444436.7		
36		497398.6		
37		554792.7		
38		616842.1		
39		683774.3		
40	764514	755821.1		
41		833218		
42		916205.2		
43		1005027		
44		1099930		
45		1201168		
46		1308996		
47		1423675		
48		1545467		
49		1674641		
50	1839146	1811468		

Figure I. Graph between principle quantum numbers and quantum defects of Rydberg series: $5s^2ns\ ^2S_{1/2}$ (cm^{-1})

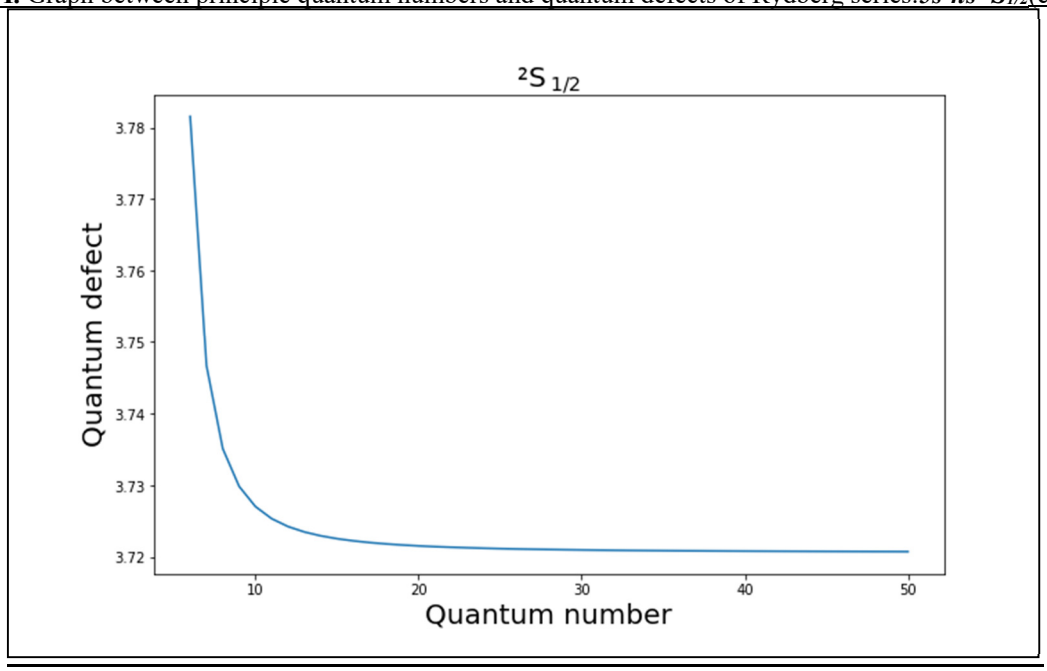

Figure II. Graph between principle quantum numbers and quantum defects of Rydberg series:$5s^2nd\ ^2D_{3/2}$(cm^{-1})

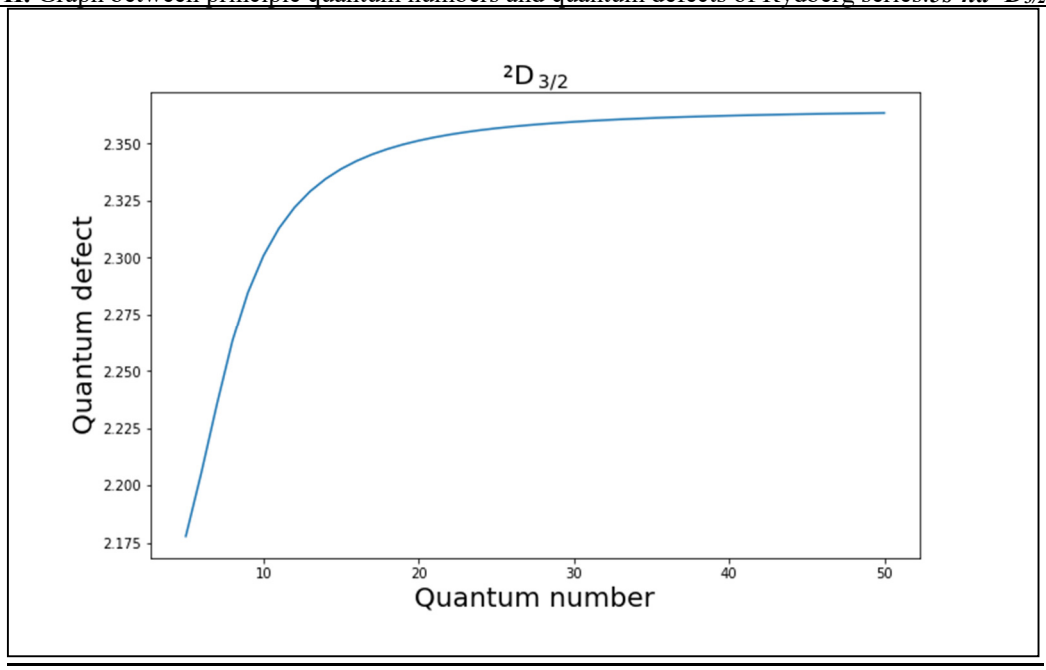

Figure III. Graph between principle quantum numbers and quantum defects of Rydberg series:$5s^2nd\ ^2D_{5/2}$(cm^{-1})

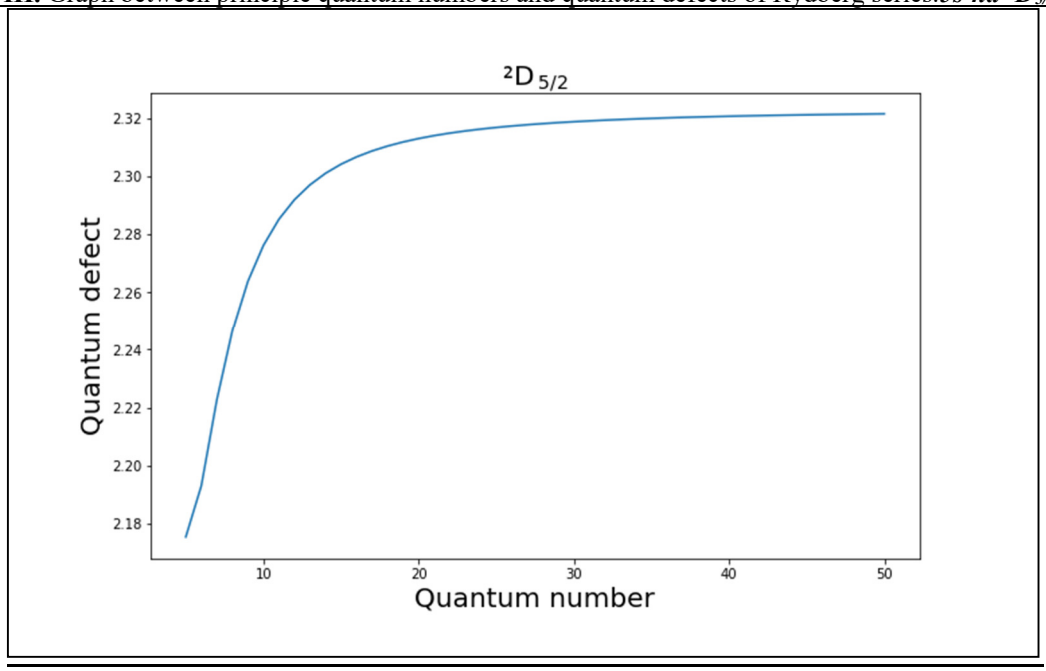

Figure IV. Graph between principle quantum numbers and quantum defects of Rydberg series:$5s^2np\ ^2P^o_{1/2}$(cm^{-1})

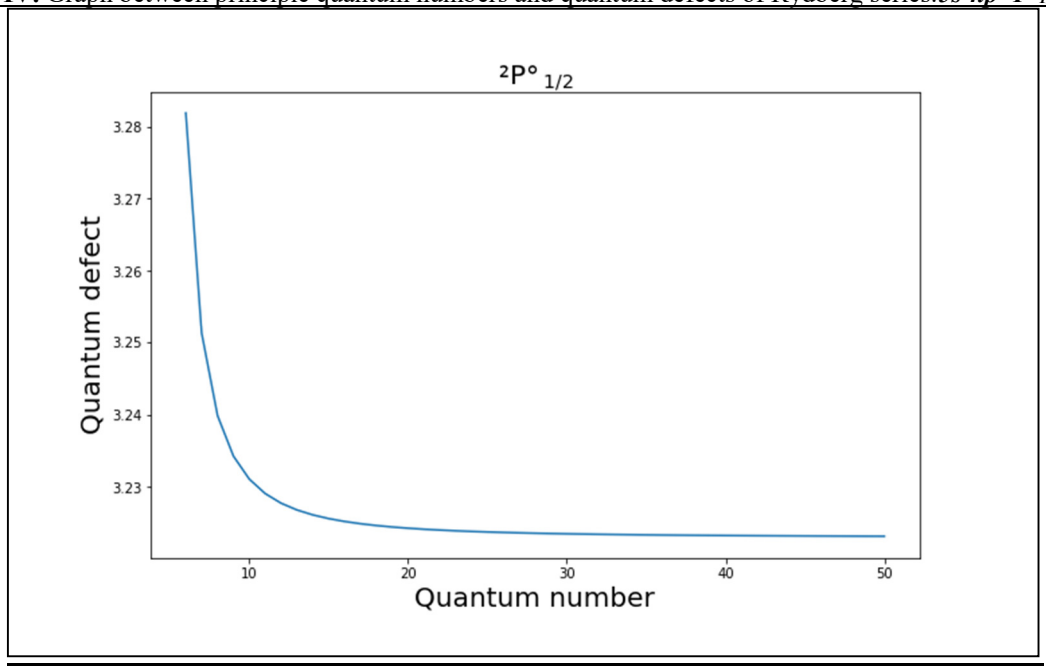

Figure V. Graph between principle quantum numbers and quantum defects of Rydberg series:$5s^2np\ ^2P^o_{3/2}$(cm^{-1})

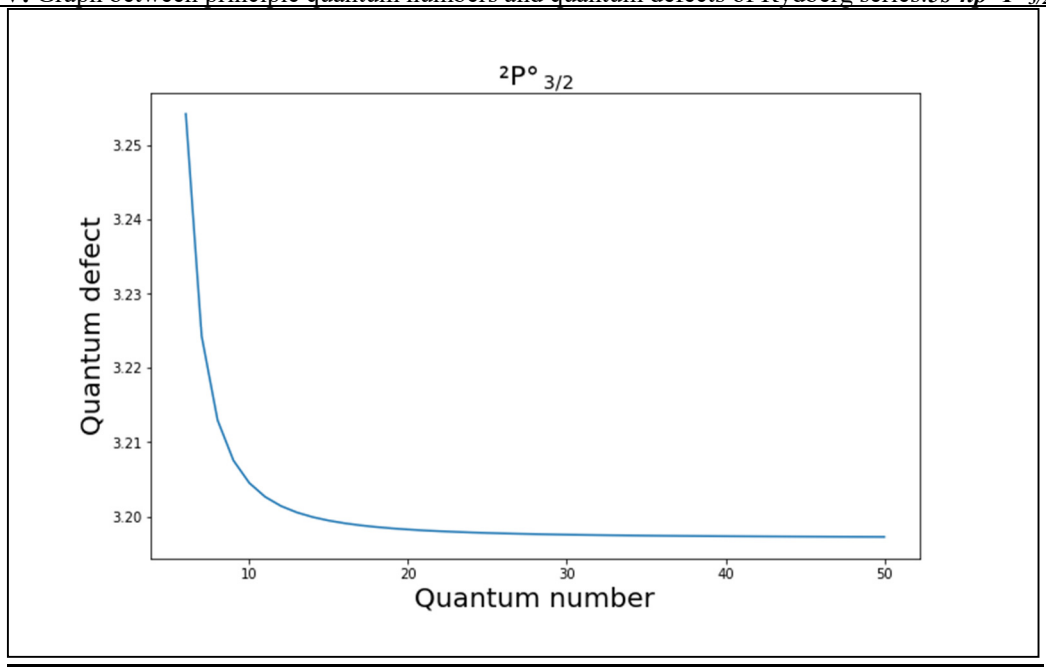

Figure VI. Graph between principle quantum numbers and quantum defects of Rydberg series:$5s^2nf\,^2F^o_{5/2}$(cm^{-1})

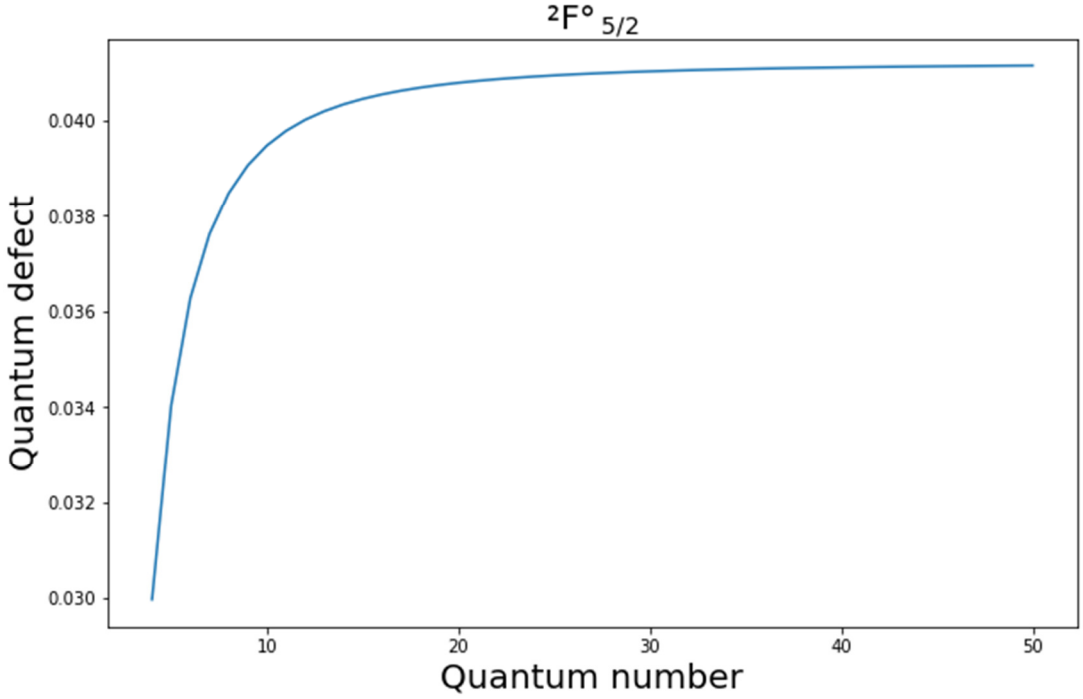

Figure VII. Graph between principle quantum numbers and quantum defects of Rydberg series:$5s^2nf\,^2F^o_{7/2}$(cm^{-1})

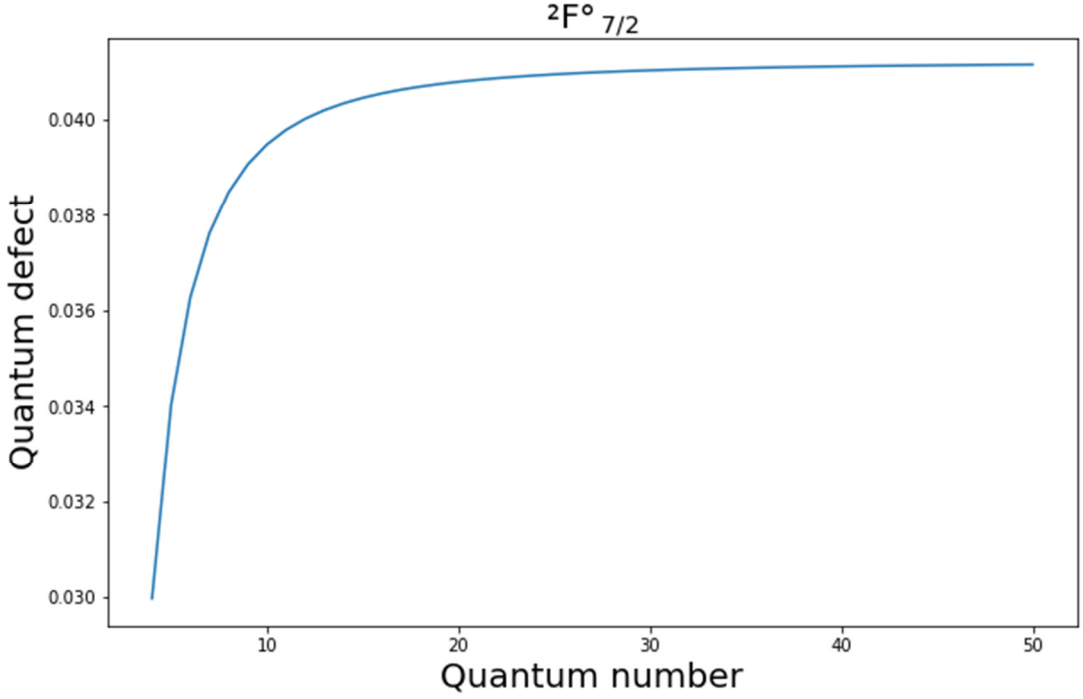

Figure VIII. Graph between lifetimes and principle quantum numbers of Rydberg series:$5s^2ns\ ^2S_{1/2}$(cm^{-1})

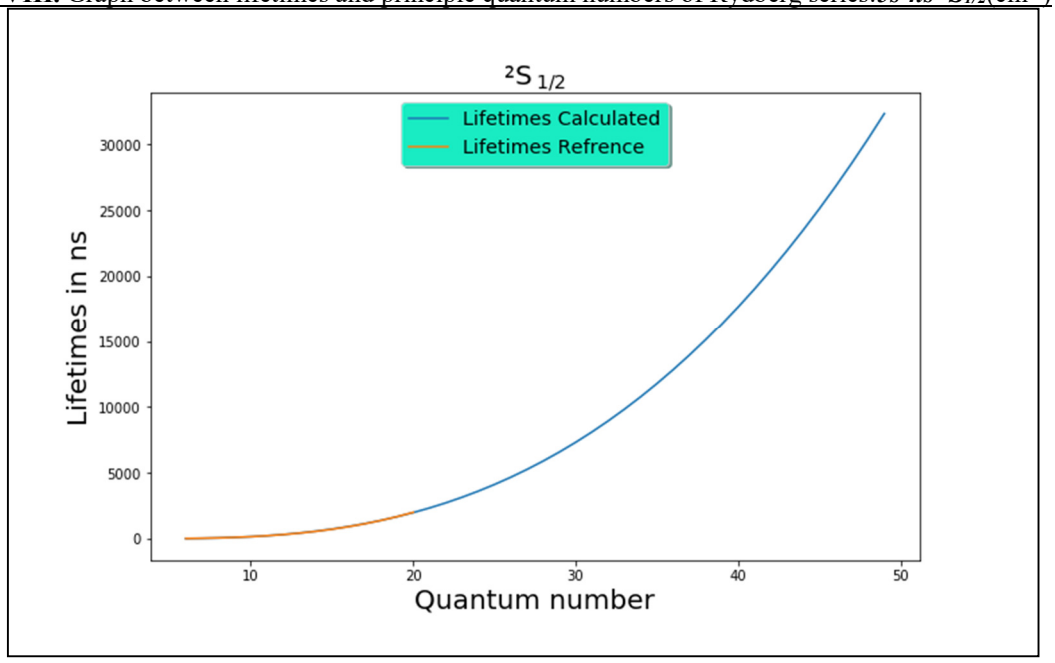

Figure IX. Graph between lifetimes and principle quantum numbers of Rydberg series:$5s^2nd\ ^2D_{3/2}$(cm^{-1})

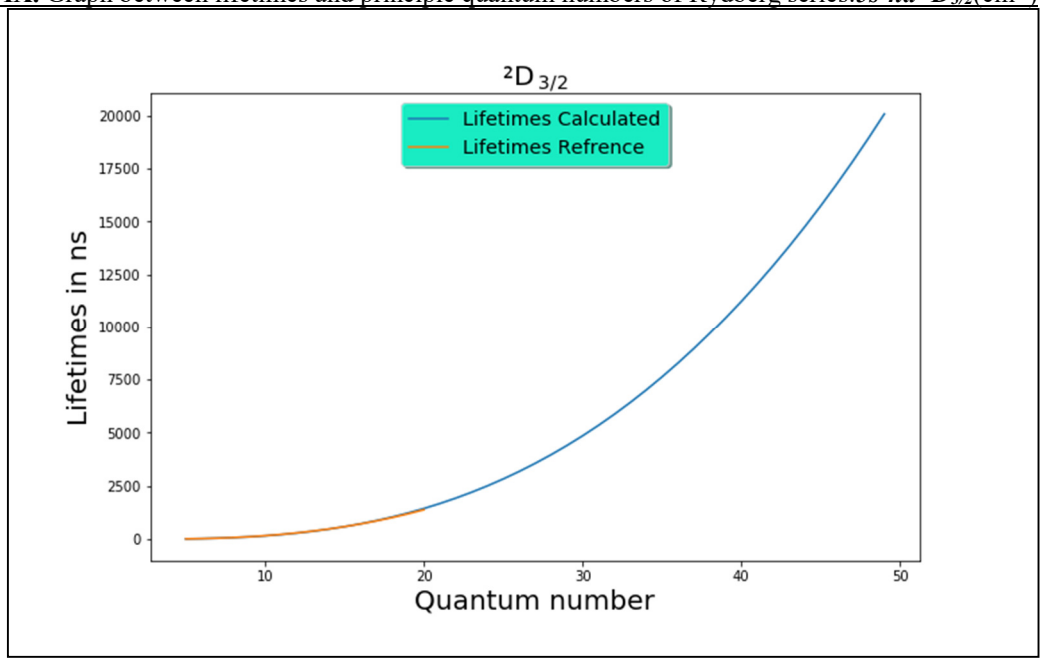

Figure X. Graph between lifetimes and principle quantum numbers of Rydberg series:$5s^2nd\ ^2D_{5/2}$(cm^{-1})

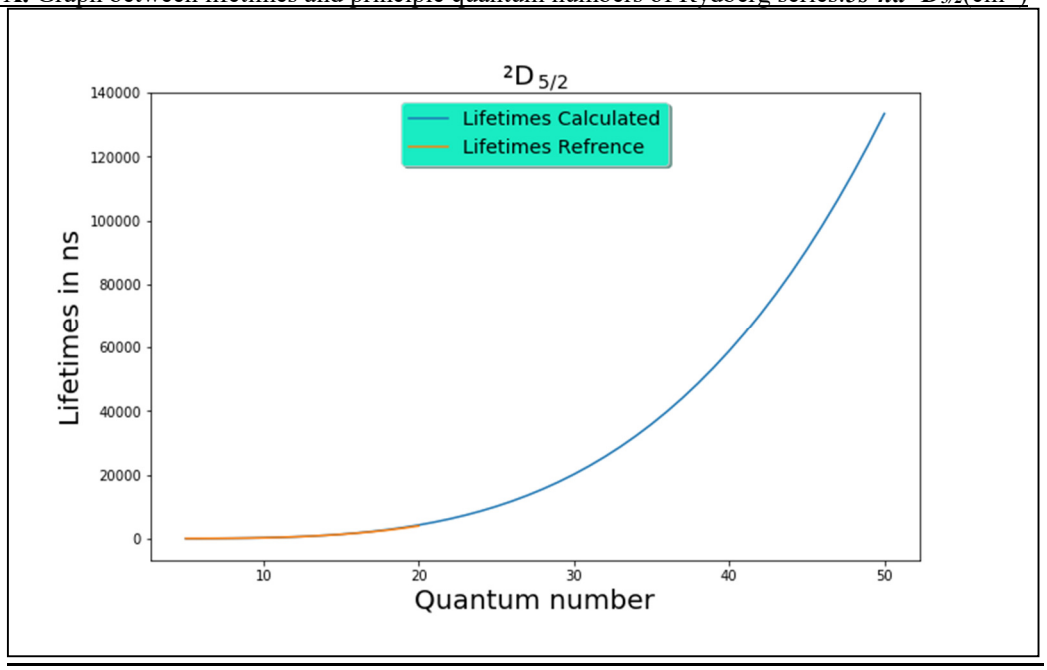

Figure XI. Graph between lifetimes and principle quantum numbers of Rydberg series:$5s^2np\ ^2P^o_{1/2}$(cm^{-1})

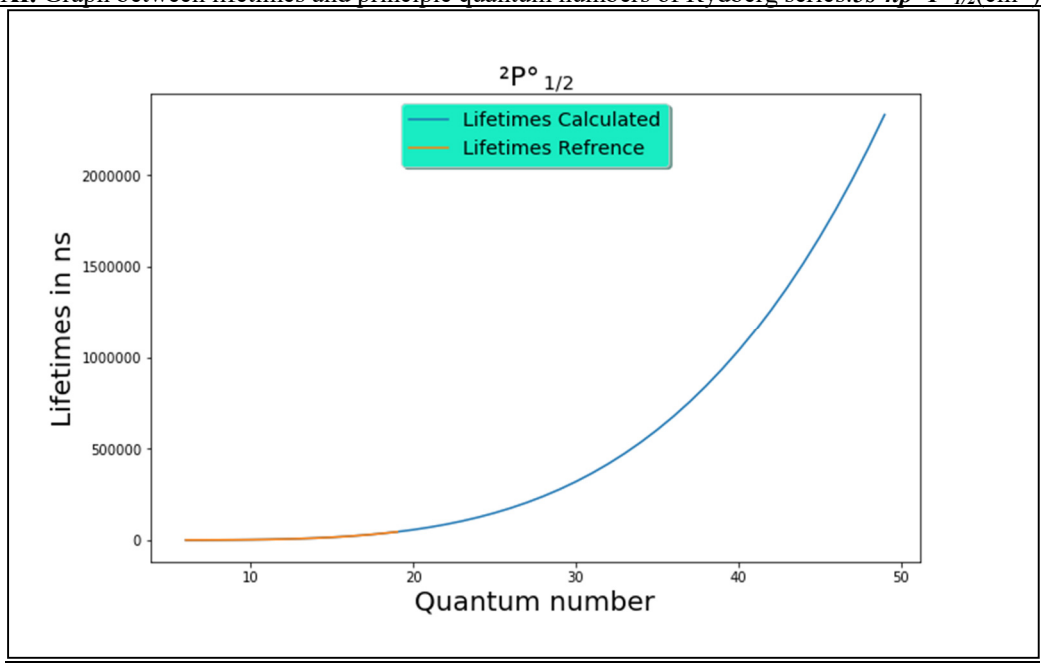

Figure XII. Graph between lifetimes and principle quantum numbers of Rydberg series:$5s^2np\ ^2P^o_{3/2}$(cm^{-1})