Journal of Natural Sciences Research
ISSN 2224-3186 (Paper) ISSN 2225-0921 (Online)
Vol.8, No.14, 2018

www.iiste.org

Spectral Energies and Radiative Lifetimes of Rydberg States in Neutral Hydrogen

Salman Raza*
Department of Physics, Government National College, University of Karachi
J-97, PIB Colony Karachi-5, Pakistan

Muhammad Noman Hameed
Department of Physics, University of Karachi
1-J-19/2. Muslim league Quarters Nazimabad#1 Karachi, Pakistan

Naveed Ali
Department of Physics, University of Karachi, Aga Khan Higher Secondary School, Karachi,
Saleem Nawaz Fazaia College, PAF Base Masroor Karachi
Plot# A 9/1 Muhammadi Colony Azizabad # 8, FB Area, Karachi, Pakistan

Abstract

Spectral Energies and radiative lifetimes under the influence of weakest bound electron (WBE) potential model theory for the following series: $ns^2S_{1/2}(n\geq2)$, $np^2P^o_{1/2,3/2}(n\geq2)$, $nd^2D_{3/2,5/2}(n\geq3)$ ***and,*** $nf^2F^o_{5/2,7/2}(n\geq4)$ of Rydberg States in Neutral Hydrogen (H I) have been calculated. The use of Martin's Expression and the Quantum Defects provided by Ritz's formula enable us to calculate the radiative lifetimes of mentioned series. This is the very first time that we have employed WBE potential model theory and calculated spectral Energies and radiative lifetimes of above mentioned series of Rydberg states in neutral hydrogen (H I). Quantum defects and radiative lifetimes of few high lying Rydberg states in neutral hydrogen were also obtained for the first time. In comparison with previously published data, a good agreement is achieved.

Introduction

In past decades recent advancements in optics, laser physics, plasma physics, thermo-nuclear fusion research, physical chemistry and in astronomy. The spectroscopic data like fine structure, Oscillator Strength, Transition probabilities, high lying Rydberg Levels, Quantum defects, Radiative lifetimes and etc. becomes the momentous research area in both theoretical and experimental fields. So, many measurement and theoretical computational techniques were developed for the exact measurement of spectroscopic parameters, like Many-body perturbation theory, Multi-channel Quantum defect theory, R-matrix method, Self-consistent field methods, Hartree-Fock approximation and semi-empirical method etc. these theoretical computational techniques always have some difficulties in exact calculation of spectroscopic parameters and sometimes calculations becomes complicated specially in many electron systems . Instead the recently newly developed unified (means for both relativistic and non-relativistic) WBE potential model theory is easy and efficient method in calculating the Rydberg states energies and radiative lifetimes. Recently more attention paid on this theory for theoretical computation of spectroscopic data.

Before initiating this work in literature review [1-16] author's studied that WBE potential model theory [17] works significantly very well with the many electrons system by considering them as hydrogenic atoms and always shows good agreement with the experimental and previously known theoretical values. In this work author's decided to show the validation of theory by theoretical computational of Rydberg States and radiative liftimes of simplest one-electron system (neutral Hydrogen) for the following series: $ns^2S_{1/2}(n\geq2)$, $np^2P^o_{1/2,3/2}(n\geq2)$, $nd^2D_{3/2,5/2}(n\geq3)$ ***and,*** $nf^2F^o_{5/2,7/2}(n\geq4)$ up to n=50, principal Quantum number(n), by exploiting experimental data obtained from Reference[23,24].

Theory:

The theory developed by Zheng et al. in 2004[17] is the simplest, easiest and at the same time very effective for calculating spectral energies of high lying Rydberg states. Unlike self-consistent field method it rely on a simple concept of weakest bound electron (WBE) and non-weakest bound electrons (NWBE) including nucleus forming an ionic-core of +(Z-1) charge. The WB electron moves under the influence of effective potential of an ion-core. During consecutive ionization step by step WBE separates from the ion-core, in each step only one weakest bound electron (WBE) ionize and rest non-weakest bound electrons (NWBE) forming a new ion-core with effective potential called central potential field. The formation of central potential is due to the effects of shielding, polarization and penetration. The potential function of WBE can be written as [17]:

$$V(x) = -\frac{Z}{x} + \frac{Y}{x^2} \tag{1}$$

where, $Y = \frac{m(m+1)+2ml}{2}$

In equation (1) the first and second terms at right hand side represent coulomb potential and dipole potential respectively. The dipole potential created by the polarization of ion-core WBE.

In first term, x is the separation distance between WBE's and nucleus, Z is the effective nucleus charge and in Y, *l* is the angular quantum number of WBE's and m is a measureable factor need to be determined but not necessarily an integer.

So, the energy formula for WBE's is:

$$E = -R\left(\frac{Z^*}{n^*}\right)^2 \tag{2}$$

In equation (2) R is the Rydberg constant, n* is the effective 'n' (principal quantum number).The Z* and n* are unknown values, but the problem is solved by the transformation between Eigen-values of Quantum defect theory (QDT) and WBE potential model theory, given by:

$$\frac{Z^*}{n^*} = \frac{Z_o}{n-\delta_n} \tag{3}$$

In equation (3) Z_o is the atomic kernel net charge number ($Z_o = 1, 2,$ and 3... for the ion-core charge in QDT) and δ_n is the quantum defect.

Now, The spectral energies of Rydberg states of an atomic system in the WBE potential model theory written as: [18]

$$R_L = I_o + E \tag{4}$$

In equation (4) the first term in right hand side is the ionization limit I_o and 2nd term is the energy E of WBE's.

Now, by combining equations (3) and (4), we can rewrite (4) as:

$$R_L = I_o - R\left(\frac{V_o}{n-\delta_n}\right)^2 \tag{5}$$

The quantum defect (δ_n) in equation (5) are computed by using Ritz's formula [19].

$$\delta_{n=}\ a + b(n-\delta)^{-2} + c(n-\delta)^{-4} + d(n-\delta)^{-6} \tag{6}$$

In equation (6) δ is the lowest Rydberg state quantum defect of the particular series , coefficients (a, b, c, d) in (6) are obtained by the method of least-square fitting, by using the first few given experimental values of the spectrum like energy levels series. [20].

Rykova's formula given below with the coefficients of (6) is used to determine the radiative lifetimes [21].

$$T = T_o(n-\delta_n)^{\alpha} \tag{7}$$

Similarly, in equation (7) the coefficients of Rykova's expression T_o and α are also measured with the method of least-square fitting of first few given published values of lifetimes of the spectral energies of Rydberg states by exploiting WBE potential model theory [22].

Result and discussion:

This theoretical study reports spectral energies and radiative lifetimes of Rydberg states in neutral hydrogen by utilizing WBE potential model theory and NIST probabilities for the following series: $ns^2S_{1/2}$, $np^2P^o_{1/2,3/2}$, $nd^2D_{3/2,5/2}$ ***and,*** $nf^2F^o_{5/2,7/2}$ up to n=50.

The coefficients for the calculations of spectral energies and radiative lifetimes of Rydberg states was determined by least square fitting from at least first three experimental values. Experimental Spectral energies of Rydberg levels directly obtained from National Institute of Standards and Technology (NIST) but lifetimes are calculated from NIST data of Einstein's coefficient A_{ik} by using expression of lifetime given by Verolaĭnen YF et al. [24] and used as a reference for computation. Exploiting Ritz's and Rykova's expression (see equation's: 6 and 7) through a program on macros spread sheet to obtain the coefficients for calculating energies and lifetimes. The coefficients shown in Table I and II, these coefficients were used to obtain the above mentioned series along with their radiative lifetimes. The computed values show good agreement with available published results (see figure 1 and 2).

Table I shows the values of 'δ' for every series that is the quantum defect of minimum spectral energy Rydberg state series, it also displays clearly that every series of quantum defects converges at the same ionization potential towards the value of 'a' which further tells that all series of hydrogen spectrum are core- polarization series. In table II the values of lifetime 'T_o' for quantum states are shown. It also shows the highest of 'T_o' is for $ns^2S_{1/2}(n\geq3)$ and smallest for $np^2P^o_{1/2,3/2}(n\geq2)$, and the value of 'α' which is less than three.

Table I: Coefficients of spectral energies of Rydberg states in neutral Hydrogen

H I (Z=1,isoelectronic sequence) $^2S_{1/2}$ 109 678.77174307 cm-1					
Spectral Series	**a**	**b**	**c**	**d**	**Δ**
$ns\,^2S_{1/2}$ **(n=2-50)**	-0.00213	0.026039	-0.16366	0.340976	-0.00053
$np\,^2P^o_{1/2,3/2}$ **(n=2-50)**	-0.00213	0.026039	-0.16366	0.340984	-0.00053
$nd\,^2D_{3/2,5/2}$ **(n=3-50)**	-0.00272	0.05475	-0.60542	2.410002	-0.00081
$nf\,^2F^o_{5/2,7/2}$ **(n=4-50)**	-0.00256	0.041871	-0.29156	3.4E-07	-0.00108

Table II: Coefficients of Radiative lifetimes of spectral Rydberg states in neutral Hydrogen

H I (Z=1, isoelectronic sequence)		
Spectral Series	**T_o**	**α**
ns $^2S_{1/2}$ (n=2-50)	2.81E-08	1.55
np $^2P^o_{1/2,3/2}$ (n=2-50)	2.06E-10	2.95
nd $^2D_{3/2,5/2}$ (n=3-50)	6.09E-10	2.9448
nf $^2F^o_{5/2,7/2}$ (n=4-50)	1.21E-09	2.9511

Figure 1 shows the quantum defects for the series: ***$ns^2S_{1/2}(n\geq2)$, $np^2P^o_{1/2,3/2}(n\geq2)$, $nd^2D_{3/2,5/2}(n\geq3)$*** and***, $nf^2F^o_{5/2,7/2}(n\geq4)$*** as the function of principal quantum number 'n' as it displays clearly that for: ***$np^2P^o_{1/2,3/2}(n\geq2)$, $nd^2D_{3/2,5/2}(n\geq3)$*** and***, $nf^2F^o_{5/2,7/2}(n\geq4)$*** quantum defects for j=l+1/2 and j=l-1/2 overlaps each other, and ***$ns^2S_{1/2}(n\geq2)$*** and, ***$np^2P^o_{1/2,3/2}(n\geq2)$*** have no fine splitting also for low lying quantum numbers all series converge towards the same value. Similarly, Figure 2 shows the radiative lifetimes as a function of principal quantum numbers for above mentioned series. All series display exponentially increasing behavior of lifetimes. Among all the steepest curve is of the series: ***$nf^2F^o_{5/2,7/2}(n\geq4)$*** shows the highest value of lifetime at n=50 (125139.8106nS). The ***$ns^2S_{1/2}(n\geq3)$*** and ***$np^2P^o_{1/2,3/2}(n\geq2)$*** are the closest.

Figure 1. Quantum defects of Rydberg states series vs Quantum numbers

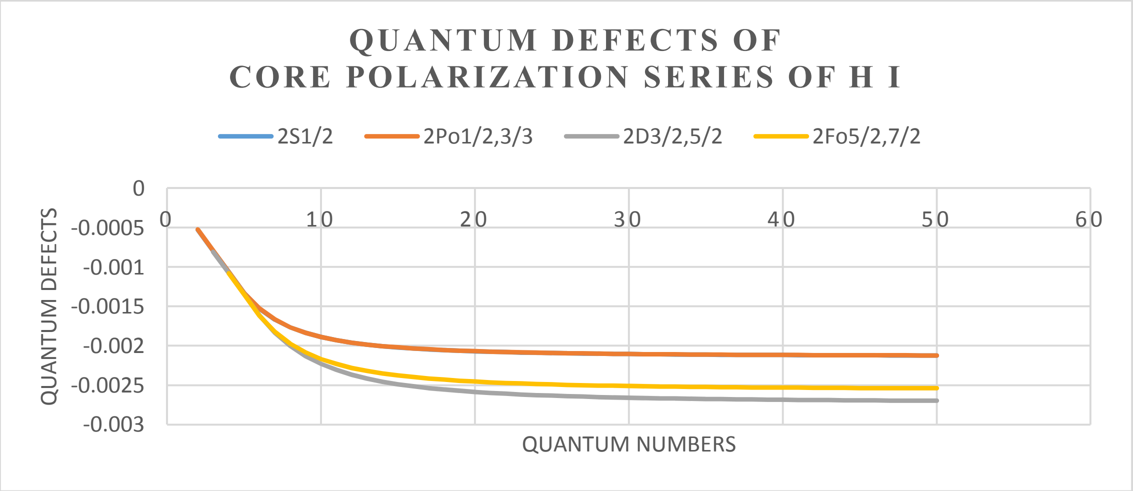
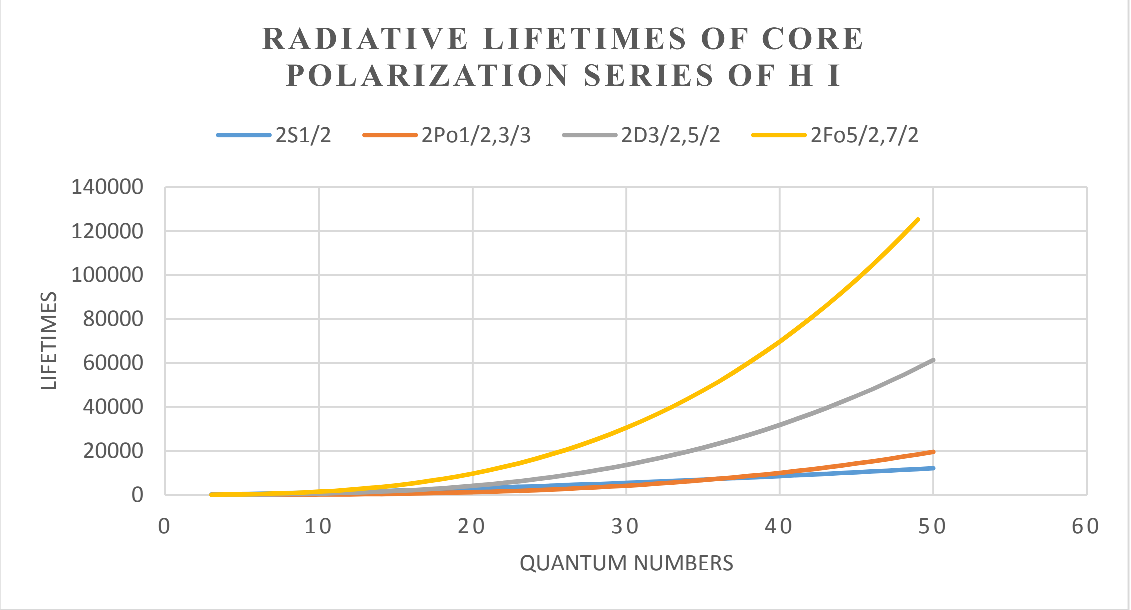


Figure 2. Radiative life times of Rydberg states series vs Quantum numbers



Now in tables III-VI presented the spectral energies of Rydberg states series: ***$ns^2S_{1/2}$, $np^2P^o_{1/2,3/2}$, $nd^2D_{3/2,5/2}$ and, $nf^2F^o_{5/2,7/2}$*** up to n=50 along with their lifetimes. The values from previously published literature were also listed, shows good agreement.

Table III. Experimental and Theoretical values of Rydberg levels and Radiative lifetimes of series: ns $^2S_{1/2}$ in neutral Hydrogen.

n	$R_{exp}cm^{-1}$	$R_{cal}cm^{-1}$	ΔRcm^{-1}	δ_n	$T_{exp}nS_{[23]}$	$T_{cal}nS$	$T_{ref}nS_{[24]}$
2	82259	82259	0.00	-0.0005			
3	97492.2	97492.2	0.00	-0.0008	158.371	154.242	160
4	102824	102824	0.00	-0.0011	226.55	240.913	227
5	105292	105292	0.00	-0.0013	352.28	340.463	352
6	106632	106632	-0.0741	-0.0015		451.641	534
7	107440	107440	-0.1312	-0.0017		573.522	782
8	107965	107965	-0.1618	-0.0018		705.384	1103
9	108325	108325	-0.1737	-0.0018		846.644	1511
10	108582	108582	-0.1744	-0.0019		996.815	2009
11	108772	108772	-0.169	-0.0019		1155.48	2610
12	108917	108917	-0.1606	-0.002		1322.3	3334
13		109030		-0.002		1496.93	
14		109119		-0.002		1679.12	
15		109191		-0.002		1868.61	
16		109250		-0.002		2065.19	
17		109299		-0.002		2268.63	
18		109340		-0.0021		2478.77	
19		109375		-0.0021		2695.43	
20		109404		-0.0021		2918.46	
21		109430		-0.0021		3147.7	
22		109452		-0.0021		3383.03	
23		109471		-0.0021		3624.32	
24		109488		-0.0021		3871.45	
25		109503		-0.0021		4124.31	
26		109516		-0.0021		4382.79	
27		109528		-0.0021		4646.8	
28		109539		-0.0021		4916.24	
29		109548		-0.0021		5191.03	
30		109557		-0.0021		5471.08	
31		109565		-0.0021		5756.31	
32		109572		-0.0021		6046.64	
33		109578		-0.0021		6342.01	
34		109584		-0.0021		6642.35	
35		109589		-0.0021		6947.58	
36		109594		-0.0021		7257.65	
37		109599		-0.0021		7572.49	
38		109603		-0.0021		7892.05	
39		109607		-0.0021		8216.26	
40		109610		-0.0021		8545.08	
41		109613		-0.0021		8878.45	
42		109617		-0.0021		9216.33	
43		109619		-0.0021		9558.66	
44		109622		-0.0021		9905.39	
45		109625		-0.0021		10256.5	
46		109627		-0.0021		10611.9	
47		109629		-0.0021		10971.6	
48		109631		-0.0021		11335.5	
49		109633		-0.0021		11703.6	
50		109635		-0.0021		12075.9	

Note: "δ_n" is the quantum defect", "n" is principal quantum number, and "ΔR" is the difference between experimental (R_{exp}, T_{exp}, T_{ref}) and calculated values (R_{cal}. T_{cal}).

Table IV. Experimental and Theoretical values of Rydberg levels and Radiative lifetimes of series: np $^2P^o_{1/2,3/2}$ in neutral Hydrogen.

n	R_{exp}cm^{-1}	R_{cal}cm^{-1}	ΔRcm^{-1}	δ_n	T_{exp}nS$_{[23]}$	T_{cal}nS	T_{ref}nS$_{[24]}$
2	82258.9	82258.9	0	-0.0005	1.59619	1.59492	1.59
3	97492.2	97492.2	0	-0.0008	5.27151	5.2448	5.4
4	102824	102824	0	-0.0011	12.3051	12.2363	12.4
5	105292	105292	0	-0.0013	23.7889	23.6065	23.8
6	106632	106632	-0.0741	-0.0015	40.8189	40.3822	40.7
7	107440	107440	-0.1312	-0.0017		63.579	64.4
8	107965	107965	-0.1618	-0.0018		94.2033	95.6
9	108325	108325	-0.1737	-0.0018		133.254	136
10	108582	108582	-0.1744	-0.0019		181.721	186
11	108772	108772	-0.169	-0.0019		240.592	247
12	108917	108917	-0.1606	-0.002		310.847	321
13		109030		-0.002		393.46	
14		109119		-0.002		489.402	
15		109191		-0.002		599.64	
16		109250		-0.002		725.136	
17		109299		-0.002		866.851	
18		109340		-0.0021		1025.74	
19		109375		-0.0021		1202.75	
20		109404		-0.0021		1398.85	
21		109430		-0.0021		1614.96	
22		109452		-0.0021		1852.05	
23		109471		-0.0021		2111.04	
24		109488		-0.0021		2392.88	
25		109503		-0.0021		2698.52	
26		109516		-0.0021		3028.87	
27		109528		-0.0021		3384.88	
28		109539		-0.0021		3767.48	
29		109548		-0.0021		4177.6	
30		109557		-0.0021		4616.16	
31		109565		-0.0021		5084.09	
32		109572		-0.0021		5582.32	
33		109578		-0.0021		6111.76	
34		109584		-0.0021		6673.34	
35		109589		-0.0021		7267.97	
36		109594		-0.0021		7896.58	
37		109599		-0.0021		8560.08	
38		109603		-0.0021		9259.39	
39		109607		-0.0021		9995.42	
40		109610		-0.0021		10769.1	
41		109613		-0.0021		11581.3	
42		109617		-0.0021		12432.9	
43		109619		-0.0021		13324.9	
44		109622		-0.0021		14258.2	
45		109625		-0.0021		15233.7	
46		109627		-0.0021		16252.2	
47		109629		-0.0021		17314.7	
48		109631		-0.0021		18422.1	
49		109633		-0.0021		19575.3	
50		109635		-0.0021		20775.2	

Note: "δ_n" is the quantum defect", "n" is principal quantum number, and "ΔR" is the difference between experimental (R_{exp}, T_{exp}, T_{ref}) and calculated values (R_{cal}. T_{cal}).

Table V. Experimental and Theoretical values of Rydberg levels and Radiative lifetimes of series: nd $^2D_{3/2,5/2}$ in neutral Hydrogen.

n	$R_{exp}cm^{-1}$	$R_{cal}cm^{-1}$	ΔRcm^{-1}	δ_n	$T_{exp}nS_{[23]}$	$T_{cal}nS$	$T_{ref}nS_{[24]}$
3	97492.3	97492.3	0	-0.0008	15.4673	15.4754	15.5
4	102824	102824	0	-0.0011	36.1492	36.1045	36
5	105292	105292	0	-0.0013	69.6696	69.6533	69.8
6	106632	106632	0	-0.0016	119.115	119.156	119
7	107440	107440	-0.0324	-0.0018		187.608	187
8	107965	107965	-0.0653	-0.002		277.978	277
9	108325	108325	-0.0885	-0.0021		393.213	394
10	108582	108582	-0.1022	-0.0022		536.236	537
11	108772	108772	-0.1087	-0.0023		709.958	712
12	108917	108917	-0.1104	-0.0024		917.268	927
13		109030		-0.0024		1161.05	
14		109119		-0.0025		1444.15	
15		109191		-0.0025		1769.44	
16		109250		-0.0025		2139.76	
17		109299		-0.0025		2557.93	
18		109340		-0.0026		3026.77	
19		109375		-0.0026		3549.1	
20		109405		-0.0026		4127.71	
21		109430		-0.0026		4765.42	
22		109452		-0.0026		5464.99	
23		109471		-0.0026		6229.21	
24		109488		-0.0026		7060.85	
25		109503		-0.0026		7962.69	
26		109516		-0.0026		8937.47	
27		109528		-0.0026		9987.95	
28		109539		-0.0027		11116.9	
29		109548		-0.0027		12327	
30		109557		-0.0027		13621.1	
31		109565		-0.0027		15001.8	
32		109572		-0.0027		16471.9	
33		109578		-0.0027		18034.1	
34		109584		-0.0027		19691.1	
35		109589		-0.0027		21445.7	
36		109594		-0.0027		23300.5	
37		109599		-0.0027		25258.3	
38		109603		-0.0027		27321.7	
39		109607		-0.0027		29493.5	
40		109610		-0.0027		31776.3	
41		109613		-0.0027		34172.8	
42		109617		-0.0027		36685.8	
43		109619		-0.0027		39317.8	
44		109622		-0.0027		42071.6	
45		109625		-0.0027		44949.8	
46		109627		-0.0027		47955.2	
47		109629		-0.0027		51090.3	
48		109631		-0.0027		54357.8	
49		109633		-0.0027		57760.5	
50		109635		-0.0027		61300.9	

Note: "δ_n" is the quantum defect", "n" is principal quantum number, and "ΔR" is the difference between experimental (R_{exp}, T_{exp}, T_{ref}) and calculated values (R_{cal}. T_{cal}).

Table VI. Experimental and Theoretical values of Rydberg levels and Radiative lifetimes of series: $nf\,^2F^o_{5/2,7/2}$ in neutral Hydrogen.

n	$R_{exp}cm^{-1}$	$R_{cal}cm^{-1}$	ΔRcm^{-1}	δ_n	$T_{exp}nS_{[23]}$	$T_{cal}nS$	$T_{ref}nS_{[24]}$
4	102824	102824	0.00	-0.0011	72.5258	72.5413	72.5
5	105292	105292	0.00	-0.0013	140.319	140.145	140
6	106632	106632	0.00	-0.0016	239.941	240.02	240
7		107440		-0.0018		378.271	378
8		107965		-0.002		560.951	559
9		108325		-0.0021		794.076	791
10		108582		-0.0022		1083.62	1079
11		108772		-0.0022		1435.54	1426
12		108917		-0.0023		1855.73	1849
13		109030		-0.0023		2350.1	
14		109119		-0.0024		2924.51	
15		109191		-0.0024		3584.81	
16		109250		-0.0024		4336.81	
17		109299		-0.0024		5186.33	
18		109340		-0.0024		6139.14	
19		109375		-0.0024		7201.03	
20		109404		-0.0025		8377.74	
21		109430		-0.0025		9675.02	
22		109452		-0.0025		11098.6	
23		109471		-0.0025		12654.2	
24		109488		-0.0025		14347.4	
25		109503		-0.0025		16184.1	
26		109516		-0.0025		18169.8	
27		109528		-0.0025		20310.3	
28		109539		-0.0025		22611.2	
29		109548		-0.0025		25078	
30		109557		-0.0025		27716.6	
31		109565		-0.0025		30532.4	
32		109572		-0.0025		33531.2	
33		109578		-0.0025		36718.4	
34		109584		-0.0025		40099.8	
35		109589		-0.0025		43680.9	
36		109594		-0.0025		47467.3	
37		109599		-0.0025		51464.5	
38		109603		-0.0025		55678.1	
39		109607		-0.0025		60113.8	
40		109610		-0.0025		64776.9	
41		109613		-0.0025		69673.2	
42		109617		-0.0025		74808	
43		109619		-0.0025		80187.1	
44		109622		-0.0025		85815.8	
45		109625		-0.0025		91699.7	
46		109627		-0.0025		97844.3	
47		109629		-0.0025		104255	
48		109631		-0.0025		110938	
49		109633		-0.0025		117897	
50		109635		-0.0025		125140	

Note: "δ_n" is the quantum defect", "n" is principal quantum number, and "ΔR" is the difference between experimental (R_{exp}, T_{exp}, T_{ref}) and calculated values (R_{cal}. T_{cal}).

Conclusion

In this work we presented spectral energies of Rydberg states and radiative lifetimes for the following series: $ns^2S_{1/2}$, $np^2P^o_{1/2,3/2}$, $nd^2D_{3/2,5/2}$ and, $nf^2F^o_{5/2,7/2}$ up to **n=50,** in neutral Hydrogen (HI). The spectral energies of Rydberg states series and radiative lifetimes compared with published literature, found in good agreement. It shows that Martin's and Rykova's expression provides simple and effective way for calculating Rydberg states and radiative lifetimes. It shows the semi-empirical method like WBE potential model theory in which one or two more parameters need to be adjusted by utilizing previously published data is a good computational method for the computation of spectroscopic data for complex and complicated system especially for many high lying Rydberg states.

Besides the effectiveness of WBE potential model theory. The following deductions made from neutral Hydrogen theoretical computation:

i. The NIST data for Rydberg levels for all seven series of neutral Hydrogen up to n=50 are well agreed.
ii. For lifetimes first we estimated lifetimes from the data of Einstein's coefficient A_{ik} available at NIST, and then this data was utilized for theoretical computation of lifetimes. Computed values shows good agreement with previously published data by Verolaĭnen YF et al. [24].
iii. All seven series of neutral hydrogen converges towards the Ritz's expression coefficient 'a' at the high principal quantum number, which clearly display that all seven series are core-polarization series. At low quantum numbers all series overlaps (see fig. I).
iv. In series: $np^2P^o_{1/2,3/2}$, $nd^2D_{3/2,5/2}$ and, $nf^2F^o_{5/2,7/2}$ quantum defects of J=l+1/2 and J=l-1/2 completely overlaps each other and there is no fine splitting between $ns^2S_{1/2}$ and, $np^2P^o_{1/2,3/2}$.
v. All seven series of radiative lifetimes shows exponentially increasing behavior. Among them the $nf^2F^o_{5/2,7/2}$ has the steepest curve but series: $ns^2S_{1/2}$ and, $np^2P^o_{1/2,3/2}$ are closest (see fig. II).
vi. The lifetimes measured ranges from 1.59nS (for $2p^2P^o_{1/2,3/2}$) to 125139.8106nS (for $50f^2F^o_{5/2,7/2}$) and there is a slight deviation in lifetimes of high Rydberg states of $ns^2S_{1/2}$ from the values published by Verolaĭnen YF et al. [24].

Acknowledgement

The authors greatly acknowledge Dr. Zaheeruddin from university of Karachi, Department of Physics for supporting in this project and imparting skills that have shaped our thinking in personal and professional lives. We can never really thank you enough.

References

[1] Wang J, Greene CH. Quantum-defect analysis of 3 p and 3 d H 3 Rydberg energy levels. Physical Review A. 2010 Aug 11;82(2):022506.
[2] Glukhov IL, Nikitina EA, Ovsiannikov VD. Lifetimes of Rydberg states in ions of the group II elements. Optics and Spectroscopy. 2013 Jul 1;115(1):9-17.
[3] Hua J, Shi-Wei Y, Chang-Jian D. Lifetimes of Rydberg states of Eu atoms. Chinese Physics B. 2015 Jan;24(1):013203.
[4] Ovsiannikov VD, Glukhov IL, Nekipelov EA. Radiative lifetime and photoionization cross-section for Rydberg states in alkali-metal atoms. Optics and Spectroscopy. 2011 Jul 1;111(1):25.
[5] Çelik G, Ateş Ş, Özarslan S, Taşer M. Transition probabilities, oscillator strengths and lifetimes for singly ionized magnesium. Journal of Quantitative Spectroscopy and Radiative Transfer. 2011 Sep 1;112(14):2330-4.
[6] Deller A, Alonso AM, Cooper BS, Hogan SD, Cassidy DB. Measurement of Rydberg positronium fluorescence lifetimes. Physical Review A. 2016 Jun 29;93(6):062513.
[7] Zhang W, Palmeri P, Quinet P, Biémont E, Du S, Dai Z. Radiative-lifetime measurements and calculations of odd-parity highly excited levels in Ba I. Physical Review A. 2010 Oct 14;82(4):042507.
[8] Zhang W, Feng Y, Dai Z. Radiative lifetime measurements of odd-parity moderately excited levels belonging to J= 0, 1, 2, 3 series in Sm I. JOSA B. 2010 Nov 1;27(11):2255-61.
[9] Çelik G, Erol E, Taşer M. Transition probabilities, oscillator strengths and radiative lifetimes for Zn II. Journal of Quantitative Spectroscopy and Radiative Transfer. 2013 Nov 1;129:263-71.
[10] Shizhong H, Qiufeng S. Calculation of the Rydberg Energy Levels for Francium Atom. Physics Research International. 2010 Dec 16;2010.
[11] Li S, Lei W, Hai-Feng Y, Xiao-Jun L, Hong-Ping L. Lifetime Measurement for 6snp Rydberg States of Barium. Chinese Physics Letters. 2011 Apr;28(4):043101.
[12] Ateş Ş, Uğurtan HH. Lifetimes of excited levels for atomic silicon. Indian Journal of Physics. 2013 Jan 1;87(1):9-17.
[13] Çelik G, Doğan D, Ateş Ş, Taşer M. Transition probabilities and radiative lifetimes of levels in FI. Atomic Data and Nuclear Data Tables. 2012 Jul 1;98(4):566-88.
[14] Çelik G, Ateş Ş, Erol E. Oscillator strengths and lifetimes for Cu I. Canadian Journal of Physics. 2015 Feb 11;93(10):1015-23.
[15] Çelik G, Atalay B, Ateş Ş. Radiative Lifetimes for Singly Ionized Beryllium. detail. 2016 Sep 1;15:20.
[16] Zhou C, Wang ZM, He HY. Energy levels of the 5d3/2 nf, 6p3/2 nd autoionizing series of Hg I. C anadian

Journal of Physics. 2015 Jul 28;93(12):1541-3.
[17] Zheng NW, Wang T, Ma DX, Zhou T, Fan J. Weakest bound electron potential model theory International journal of quantum chemistry. 2004 Jan 1;98(3):281-90.
[18] Zheng N, Ma D, Yang R, Zhou T, Wang T, Han S. An efficient calculation of the energy levels of the carbon group. The Journal of Chemical Physics. 2000 Aug 1;113(5):1681-7.
[19] Martin WC. Series formulas for the spectrum of atomic sodium (Na I). JOSA. 1980 Jul 1;70(7):7848.
[20] Shizhong H, Qiufeng S. Calculation of the Rydberg Energy Levels for Francium Atom. Physics Research International. 2010 Dec 16;2010.
[21] Zhou C, Liang L, Zhang L. Theoretical calculation of energy levels and radiative lifetimes of Tl I. Chinese Optics Letters. 2008 Mar 1;6(3):161-4.
[22] Jönsson G, Lundberg H. Natural radiative lifetimes in the 2 S 1/2 and 2 D 5/2, 3/2 sequences of aluminum. Zeitschrift für Physik A Atoms and Nuclei. 1983 Sep 1;313(3):151-4.
[23] https://physics.nist.gov/PhysRefData/ASD/lines_form.html
[24] Verolaĭnen YF, Nikolaich AY. Radiative lifetimes of excited states of atoms. Soviet Physics Uspekhi. 1982;25(6):431.