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Flow Shop Scheduling Problem to minimize the Rental Cost under Fuzzy Environment

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Abstract

In this paper, we propose a new algorithm to fuzzy flow – shop scheduling problem in which processing time of jobs are uncertain. The uncertain parameters are represented by triangular fuzzy numbers. By using a new type of fuzzy arithmetic and a fuzzy ranking method, we propose a method to minimize the rental cost of machines under the specified rental policy; without converting the fuzzy processing times to classical numbers. A numerical example is provided to illustrate the proposed method.

Keywords: Triangular fuzzy number, fuzzy ranking, Rental policy, fuzzy processing time, fuzzy flow shop scheduling.

1. Introduction

Scheduling consists of planning and arranging jobs in an orderly sequence of operations in order to "satisfy the customer's requirements". Scheduling involves sequencing of activities under time and resource constrains to meet a specific objective. The schedule of jobs and the control of their flows through a production process are the most significant role in any modern manufacturing systems. It is a complex decision making problem because of conflicting goals, limited resources and the difficulty in accurately modeling real world problems. In an industrial job assignment problem context, scheduling activities are mapped to operations, and resources to machines. The purpose of the scheduler is to determine starting time for each operation to achieve desired performance, while satisfying capacity and technological constraints. A simpler version of this problem is "flow shop scheduling" in which all jobs pass through all machines in the same order. Flow shop scheduling is one of the most important problems in the area of production management. It can be briefly described as follows: There are a set of *m* machines and a set of *n* jobs. All jobs have the same processing operation order when passing through the machines. Operations cannot be interrupted and each machine can process only one operation at a time. The problem is to find the job sequence on the machines which minimize the make span. However, it is often difficult to apply those conventional approaches to real-world flow shop scheduling problems. For example, the production due dates of jobs are usually not as rigid as they were supposed to be in those algorithms. Also, the processing times of jobs could be imprecise due to either the incomplete knowledge or uncertain environment.

Fuzzy sets are often used to describe the processing time when the knowledge about the processing time is incomplete. A number of fuzzy approaches to flow shop scheduling problems have been developed and reported in literature. An algorithm given by Johnson's [05] for scheduling jobs in a two, three machine flow shop problems to minimize the time at which all jobs are completed is one of the earliest results in flow shop scheduling. The job sequencing with fuzzy processing time was addressed by MacCahon and Lee [07]. Hong and Chuang [03] proposed a new triangular fuzzy Johnson algorithm. Seyed Reza Hejari [13] etc introduced an improved version of Mc Cahon and Lee algorithm. Yager [14], Shukla and Chen [12], Marin and Roberto [06] have contributed remarkably in the field of flow shop scheduling problems. Ishibuchi and Lee [04] formulated the fuzzy flow shop scheduling problem with fuzzy processing time. Deepak Gupta etc [02] studied specially structured three stage flow shop scheduling to minimize the rental cost of machines.

In this paper, we consider a three-machine flow shop scheduling problem with triangular fuzzy processing time. Applying a new type of fuzzy arithmetic [08] and a fuzzy ranking method [01] we propose a method for finding the rental cost of the machines without converting the fuzzy processing times to classical numbers. A numerical example is provided to illustrate the proposed method. The rest of this paper is organized as follows. In section 2 we recall the basic concepts of fuzzy numbers, triangular fuzzy numbers, fuzzy arithmetic and their rankings. In section 3, we introduce the concept of fuzzy flow shop scheduling and an algorithm to minimize the rental cost of machines in a flow shop scheduling problem under a specified rental policy. In section 4, a numerical example is given to illustrate the method proposed in this paper.

2. Preliminaries

The aim of this section is to present some notations, notions and results which are useful in our further study.

Definition 2.1

A fuzzy set $\tilde{a}$ defined on the set of real numbers R is said to be a fuzzy number if its membership function $\tilde{a}: R \to [0,1]$ has the following study.

(a) $\tilde{a}$ is convex, $\tilde{a}\{\lambda x_1+(1\text{-}t)x_2\} = \min\{\tilde{a}(x_1), \tilde{a}(x_2)\}$, for all $x_1, x_2 \in R$ and $\lambda \in [0,1]$

(b) $\tilde{a}$ is normal i.e. there exists an $x \in R$ such that $\tilde{a}(x) = 1$

(c) $\tilde{a}$ is piecewise continuous.

Definition 2.2

A fuzzy number $\tilde{a}$ on R is said to be a triangular fuzzy number (TFN) or linear fuzzy number if its membership function $\tilde{a}: R \to [0,1]$ has the following characteristics:

$$\tilde{a}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & \text{for } a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & \text{for } a_2 \le x \le a_3 \\ 0, & \text{elsewhere} \end{cases}$$

We denote this triangular fuzzy number by $\tilde{a} = (a_1, a_2, a_3)$. We use F(R) to denote the set of all triangular fuzzy numbers.

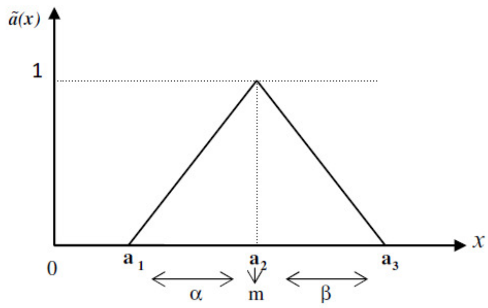

Fig 1. Triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3) = (\alpha, m, \beta)$.

Also if $m = a_2$ represents the modal value or midpoint, $\alpha = (a_2 - a_1)$ represents the left spread and $\beta = (a_3 - a_2)$ represents the right spread of the triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3)$, then the triangular fuzzy number $\tilde{a}$ can be represented by the triplet $\tilde{a} = (a_1, a_2, a_3) = (\alpha, m, \beta)$.

Definition 2.3

A triangular fuzzy number $\tilde{a} \in F(R)$ can also be represented as a pair $\tilde{a} = (\underline{a}, \overline{a})$ of functions $\underline{a}(r)$ and $\overline{a}(r)$ for $0 \le r \le 1$ which satisfies the following requirements:

(i). $\underline{a}(r)$ is a bounded monotonic increasing left continuous function.

(ii). $\overline{a}(r)$ is a bounded monotonic decreasing left continuous function.

(iii). $\underline{a}(r) \le \overline{a}(r)$, $0 \le r \le 1$

Definition 2.4

For an arbitrary triangular fuzzy number $\tilde{a} = (\underline{a}, \overline{a})$, the number $a_0 = \left(\frac{\underline{a}(1)+\overline{a}(1)}{2}\right)$ is said to be a location index number of $\tilde{a}$. The two non-decreasing left continuous functions $a_* = (a_0 - \underline{a})$, $a^* = (\overline{a} - a_0)$ are called the left fuzziness index function and the right fuzziness index function respectively. Hence every triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3)$ can also be represented by $\tilde{a} = (a_0, a_*, a^*)$

2.1 Ranking of triangular Fuzzy Numbers

Many different approaches for the ranking of fuzzy numbers have been proposed in the literature. Abbasbandy and Hajjari [01] proposed a new ranking method based on the left and the right spreads at some α -levels of fuzzy numbers.

For an arbitrary triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3) = (a_0, a_*, a^*)$ with parametric form $\tilde{a} = (\underline{a}(r), \overline{a}(r))$, we define the magnitude of the triangular fuzzy number $\tilde{a}$ by

$$\mathrm{Mag}(\tilde{a}) = \frac{1}{2}\left(\int_0^1 (\underline{a} + \overline{a} + a_0)\ f(r)\ dr\right)$$
$$= \frac{1}{2}\left(\int_0^1 (a^* + 4a_0 - a_*)\ f(r)\ dr\right).$$

where the function $f(r)$ is a non-negative and increasing function on [0,1] with $f(0) = 0$, $f(1) = 1$ and $\int_0^1 f(r)\ dr = \frac{1}{2}$. The function $f(r)$ can be considered as a weighting function. In real life applications, $f(r)$ can be chosen by the decision maker according to the situation. In this paper, for convenience we use $f(r) = r$. Then the magnitude of a triangular fuzzy number $\tilde{a}$ is given by

$$\mathrm{Mag}(\tilde{a}) = \left(\frac{a^* + 4a_0 - a_*}{4}\right).$$

The magnitude of a triangular fuzzy number $\tilde{a}$ synthetically reflects the information on every membership degree, and meaning of this magnitude is visual and natural. Mag($\tilde{a}$) is used to rank fuzzy numbers. Larger the Mag($\tilde{a}$), larger the fuzzy number. For any two triangular fuzzy numbers $\tilde{a} = (a_0, a_*, a^*)$ $\tilde{b} = (b_0, b_*, b^*)$ in F(R), we define the ranking of $\tilde{a}$ and $\tilde{b}$ by comparing the Mag ($\tilde{a}$) and Mag ($\tilde{b}$) on R as follows:

(i). $\tilde{a} \succeq \tilde{b}$ if and only if Mag($\tilde{a}$) $\geq$ Mag($\tilde{b}$)

(ii). $\tilde{a} \preceq \tilde{b}$ if and only if Mag($\tilde{a}$) $\leq$ Mag($\tilde{b}$)

(iii). $\tilde{a} \approx \tilde{b}$ if and only if Mag($\tilde{a}$) = Mag($\tilde{b}$)

Definition 2.5

A triangular fuzzy number $\tilde{a} = (a_0, a_*, a^*)$ is said to be symmetric if and only if $a_* = a^*$.

Definition 2.6

A triangular fuzzy number $\tilde{a} = (a_0, a_*, a^*)$ is said to be non-negative if and only if Mag($\tilde{a}$) $\geq$ 0 and is denoted by $\tilde{a} \succeq \tilde{0}$. Further if Mag($\tilde{a}$) > 0, then $\tilde{a} = (a_0, a_*, a^*)$ is said to be a positive fuzzy number and is denoted by $\tilde{a} \succ \tilde{0}$

.

Definition 2.7

Two triangular fuzzy numbers $\tilde{a} = (a_0, a_*, a^*)$ and $\tilde{b} = (b_0, b_*, b^*)$ in F(R) are said to be equivalent if and only if Mag($\tilde{a}$) = Mag($\tilde{b}$).That is $\tilde{a} \approx \tilde{b}$ if and only if Mag($\tilde{a}$) = Mag($\tilde{b}$).Two triangular fuzzy numbers $\tilde{a} = (a_0, a_*, a^*)$ and $\tilde{b} = (b_0, b_*, b^*)$ in F(R) are said to be equal if and only if $a_0 = b_0, a_* = b_*, a^* = b^*$. That is $\tilde{a} = \tilde{b}$ if and only if $a_0 = b_0, a_* = b_*, a^* = b^*$.

2.2 *Arithmetic operations on triangular Fuzzy Numbers*

Ming Ma et al. [08] have proposed a new fuzzy arithmetic based upon both location index and fuzziness index functions. The location index number is taken in the ordinary arithmetic, whereas the fuzziness index functions are considered to follow the lattice rule which is least upper bound in the lattice L. That is for $a, b \in L$ we define $a \vee b = \max\{a, b\}$ and $a \wedge b = \min\{a, b\}$.

For arbitrary triangular fuzzy numbers $\tilde{a} = (a_0, a_*, a^*)$ and $\tilde{b} = (b_0, b_*, b^*)$ and $* = \{+, -, \times, \div\}$, the arithmetic operations on the triangular fuzzy numbers are defined by $\tilde{a} * \tilde{b} = (a_0 * b_0, a_* \vee b_*, a^* \vee b^*)$.

In particular for any two triangular fuzzy numbers $\tilde{a} = (a_0, a_*, a^*)$ and $\tilde{b} = (b_0, b_*, b^*)$, we define

(i) Addition: $\tilde{a} + \tilde{b} = (a_0, a_*, a^*) + (b_0, b_*, b^*) = (a_0 + b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\})$

(ii) Subtraction: $\tilde{a} - \tilde{b} = (a_0, a_*, a^*) - (b_0, b_*, b^*) = (a_0 - b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\})$.

(iii) Multiplication: $\tilde{a} \times \tilde{b} = (a_0, a_*, a^*) \times (b_0, b_*, b^*) = (a_0 \times b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\})$.

(iv) Division: $\tilde{a} \div \tilde{b} = (a_0, a_*, a^*) \div (b_0, b_*, b^*) = (a_0 \div b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\})$.

3. Fuzzy Flow Shop scheduling

One kind of scheduling problem that frequently occurs in real world applications is the flow shop problem. Consider *n* different jobs that must pass through *m* processing machines. A machine can process one job at a time and we assume that the order of the jobs cannot be changed once the processing has begun. Given the time required for processing of each job on each machine, the question is in what order the jobs should be fed into the first machine in order to minimize the make span (the total time required to process all the jobs).

Property –1

When scheduling to optimize any regular measure of performance in a static deterministic flow shop, it is sufficient to consider only those schedules in which the same job sequence exists on all machines.

Property- 2

When scheduling to optimize make span in the static deterministic flow shop, it is sufficient to consider only those schedules in which same job sequence exists on machine 1 and 2 and the same job sequence on machines(m – 1) and m.

3.1 Assumptions:

(a) No job pre-emption is allowed.
(b) The machine can only process one job at a time.
(c) All jobs are available at the beginning of the scheduling time horizon.
(d) The machines set-up times are negligible.
(e) All jobs have deterministic processing times.
(f) Due dates are fuzzy.
(g) Machines may be idle.
(h) Processing times are independent of the schedule.
(i) To feed a job on a second machine, it must be completed on the first machine
(j) Each job has m operations.
(k) Each job must be completed once it is started.

3.2 Rental Policy

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required i.e. the first machine will be taken on rent in the starting of the processing the jobs, 2nd machine will be taken on rent at the time when 1st job is completed on 1st machine and transported to 2nd machine, 3rd machine will be taken on rent at the time when 1st job is completed on the 2nd machine and transported to 3rd machine and so on.

3.3 Notations

S_k : Sequence obtained by applying Johnson's procedure, k = 1, 2, 3,…,m.

M_j : Machine j, j= 1, 2, 3,…,m.

$\tilde{M}$: Minimum makes span

$\tilde{A}_{ij}$: Fuzzy processing time of i^{th} job on machine M_j i=1, 2, 3,....,n; j=1,2,3…,m

$\tilde{t}_{ij}$ (S_k): Completion time of i^{th} job of sequence S_k on machine M_j

$\tilde{I}_{ij}$ (S_k): Idle time of machine M_j for job i in the sequence (S_k)

$\tilde{U}_j(S_k)$: Utilization time for which machine M_j is required

$\tilde{R}(S_k)$: Total rental cost for the sequence (S_k) of all machine

$\tilde{C}_i$: Rental cost of i^{th} machine.

$\tilde{CT}$ (S_i): Total completion time of the jobs for sequence S_i

3.4 Problem Formulation

Let some job *i* (*i* =1, 2, 3, ..., n) is to be processed on j machines j=(1,2,…m) under the specified rental policy P. Let $\tilde{A}_{ij}$ be the processing time of i^{th} job on j^{th} machine described by triangular fuzzy numbers. Our aim is to find the sequence $\{S_k\}$ of the jobs which minimize the rental cost of the machines.

Mathematically, the problem can be stated as follows:

Minimize $R(S_k) = \sum_{i=1}^{n}\left[\tilde{A}_{i1} \times \tilde{C}_1 + \tilde{U}_2(S_K) \times \tilde{C}_2 + \tilde{U}_3(S_K) \times \tilde{C}_3\right]$

Subject to constraint: Rental Policy P.

Our objective is to minimize the rental cost of machines while minimizing their utilization time.

3.5 Algorithm

The following algorithm provides a heuristic approach to minimize the utilization time and hence rental cost of flow shop scheduling with processing time in fuzzy environment.

Let there be *m* machines M_1, M_2, …… M_m. This problem can be converted to a two machine problem, if one of the following conditions is satisfied.

Let $\tilde{A}_{i1}$, $\tilde{A}_{i2}$,…, $\tilde{A}_{im}$ be the processing times on machines M_1, M_2, …… M_m respectively.

If $$\min_i \tilde{A}_{i1} \succeq \max_i \tilde{A}_{ij}, j = 2,3,......,m-1$$

(or) $$\min_i \tilde{A}_{im} \succeq \max_i \tilde{A}_{ij}, j = 2,3,......,m-1.$$

Then this problem can be converted to a two machine problem. Introduce two fictitious machines H and K such that

$$\tilde{H}_i = \tilde{A}_{i1} + \tilde{A}_{i2} + \tilde{A}_{i3} + ... + \tilde{A}_{i(m-1)}, \quad i = 1,2,3 \dots , n.$$

$$\tilde{K}_i = \tilde{A}_{i2} + \tilde{A}_{i3} + \tilde{A}_{i4} + ... + \tilde{A}_{im}, \quad i = 1,2,3 \dots , n,$$

where $\tilde{H}_i$ and $\tilde{K}_i$ are the processing times for job *i* on machines H and K respectively.

Obtain the sequence S_k (say) by applying Johnson's [05] algorithm on machines H & K.

4. Numerical Illustration

Consider 5 jobs, 3 machines flow shop problem with processing times described by triangular fuzzy numbers as given in table 1. The rental cost per unit time for machines M_1, M_2 and M_3 are 4 units, 2 units and 3 units respectively, under the rental policy P. Our objective is to obtain an optimal schedule to minimize the total rental cost of the machines. [02].

Table 1- Fuzzy processing times $\tilde{a} = (a_1, a_2, a_3)$

Jobs	M_1	M_2	M_3
1	(7,8,9)	(6,7,8)	(3,4,5)
2	(12,13,14)	(5,6,7)	(4,5,6)
3	(8,10,12)	(4,5,6)	(6,7,8)
4	(10,11,12)	(5,6,7)	(11,12,13)
5	(9,10,11)	(5,6,8)	(8,9,10)

Here all the decision parameters are triangular fuzzy numbers represented in the form of $\tilde{a} = (a_1, a_2, a_3)$. According to Definition 2.4 these triangular fuzzy numbers can also be represented in a convenient form $\tilde{a} = (a_1, a_2, a_3) = \left(a_0, a_*, a^*\right)$, where a_0 - location index number, $a_* = (a_0 - \underline{a})$ - left fuzziness index function and $a^* = (\overline{a} - a_0)$ - right fuzziness index function of $\tilde{a}$ respectively. Hence the given data in the form $\tilde{a} = \left(a_0, a_*, a^*\right)$ is

Table 2- Fuzzy processing time $\tilde{a} = \left(a_0, a_*, a^*\right)$

Jobs	M_1	M_2	M_3
1	(8,1,1)	(7,1,1)	(4,1,1)
2	(13,1,1)	(6,1,1)	(5,1,1)
3	(10,2,2)	(5,1,1)	(7,1,1)
4	(11,1,1)	(6,1,1)	(12,1,1)
5	(10,1,1)	(6,1,1)	(9,1,1)

Min processing time on M_1 = (8, 1, 1)
Max Processing time on M_2 = (7, 1, 1)
Min processing time on M_3 = (4, 1, 1)
Min time of $M_1 \succeq$ Max time of M_2

Let H and K be 2 fictions machines such that $\tilde{H}_i = \tilde{M}_{i1} + \tilde{M}_{i2}$,叟$\tilde{}_i$ =熪$_{i2}$ + $\tilde{M}_{i3}$

Table-3

Jobs	H	K
1	(15,1,1)	(11,1,1)
2	(19,1,1)	(11,1,1)
3	(15,2,2)	(12,1,1)
4	(17,1,1)	(18,1,1)
5	(16,1,1)	(15,1,1)

Using Johnsons procedure, the order of sequencing is,

4	5	3	2	1

Table-4

Job	Machine-1		Machine-2		Machine-3	
	Time in	Time out	Time in	Time out	Time in	Time out
4	(0,0,0)	(11,1,1)	(11,1,1)	(17,1,1)	(17,1,1)	(29,1,1)
5	(11,1,1)	(21,1,1)	(21,1,1)	(27,1,1)	(29,1,1)	(38,1,1)
3	(21,1,1)	(31,2,2)	(31,2,2)	(35,2,2)	(38,1,1)	(45,1,1,)
2	(31,2,2)	(44,2,2)	(44,2,2)	(51,2,2)	(51,2,2)	(56,2,2)
1	(44,2,2)	(52,2,2)	(52,2,2)	(59,2,2)	(59,2,2)	(63,2,2)

The minimum total elapsed time = (63, 2, 2) hrs.

Idle time on machine- I = (63, 2, 2) – (52, 2, 2) = (11, 2, 2) hrs.
Idle time on machine-II = (4,1,1) + (4,1,1) + (9,2,2) + (1,2,2) = (18,2,2) hrs.
Idle time on machine-III = (6, 2, 2) + (3, 2, 2) = (9, 2, 2) hrs.

Machine - I is used for (52, 2, 2) hrs.
Rental cost of machine - I = (52, 2, 2) x 4 = (208, 8, 8) units.

Machine -II is used for (59, 2, 2) - (18, 2, 2) = (41, 2, 2) hrs.
Rental cost of machine -II = (41, 2, 2) x 2 = (82, 4, 4) units.

Machine -III is used for (63, 2, 2) – (9, 2, 2) = (54, 2, 2) hrs.
Rental cost of machine -III = (54, 2, 2) x 3 = (162, 6, 6) units.

Total Rental cost = (208, 8, 8) + (82, 4, 4) + (162, 6, 6) = (452, 8, 8) units.
The Total Rental cost of the machines by the proposed algorithm is less compared to the algorithm proposed in [02].

5. Conclusion

This paper investigated the fuzzy n-jobs 3-machines flow shop scheduling problem with uncertain parameters. We have proposed a new approach to minimize the rental cost of the machines where processing times of the machines are uncertain. The validity of the proposed method is examined with numerical example. The algorithm proposed here is more cost effective as compared with the algorithm proposed by Deepak Sharma etc in [02]. The applicability of the above procedure is useful for various types of problems in which uncertain results are involved. The proposed algorithm can be extended to fuzzy n-jobs m- machines flow shop scheduling problem with uncertain parameters.

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