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On Forecast Performance Using a Class of Weighted Moving Average Processes for Time Series

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Abstract
In this paper, we consider a class of weighted moving average models called the k-th moving average, the k-th weighted moving average and the k-th exponential weighted moving average models for modeling and forecasting economic time series data. Using real time series data set, we compare the ability of these various models to smooth the available data and also use an out of sample forecast performance to determine the best model among the various competitive models. Our findings is that the k-th exponential weighted moving average model performed best when all the three models where used to smooth our time series data, while the k-th simple moving average model outperformed the others in terms of future forecasting.
Keywords: k-th moving average; k-th weighted moving average; k-th exponential weighted moving average; forecasting

1 Introduction
A class of weighted moving average processes such as the simple moving average, weighted moving average and exponential weighted moving average processes has been applied to economic time series for so many years. These methods are widely used in industry and commerce because of their ability to remove trend and seasonal effects in time series data, Chatfield (2001). The exponential weighted moving average has been successfully used for many years by many authors: Winters (1960), Makridakis et. al (1984), Hunter (1986), Lucas and Saccucci (1990) and a host of others.

Recently, Tsokos (2010) introduced a class of weighted methods for forecasting nonstationary time series which is called the k-th moving average, k-th weighted moving average and k-th exponential weighted moving average processes. These weighted methods that is introduced by this author, appear to be a generalization of each of the weighted methods in the literature. However, these weighted methods when applied to original nonstationary time series will also be nonstationary. The new transformed nonstationary time series can be reduced to a stationary series using a differencing filter and can thereafter be modeled using the method due to Box and Jenkins (1976).

Tsokos (2010) however, did not compare the ability of these various weighted methods to smooth the data or forecast future observations. A purpose of this study therefore, is to investigate the ability of these various weighted methods to smooth the given data set and also to forecast future observations. We do not intend to transform the data to meet stationary condition. The forecast shall be based on out of sample performance. The data shall be divided into two parts so that the first part is used for modeling and the second part shall be used to compare our forecast values. A measure of forecast performance used in this paper is the forecast mean squared error (FMSE).

2 Review of literature
In this Section, we review briefly the k-th simple moving average, the k-th weighted moving average and the k-th exponential moving average using the idea of Tsokos (2010).

2.1 The k-th Simple Moving Average
The k-th simple moving average process of a time series $\{x_t\}$ is defined by

$$y_t = \frac{1}{k}\sum_{j=0}^{k-1} x_{t-k+1-j} \qquad (1)$$

where $t = k, k+1, \cdots, n.$

As k increases in (1), the number of observations $\{y_t\}$ decreases and the series $\{x_t\}$ gets closer and closer to the mean of the series $\{x_t\}$. Similarly if k = n, the series $\{y_t\}$ reduces to a single observation which is equal to the true mean μ. A consistent estimator of this μ is the sample mean ($\bar{x}$) and it is given defined by:

$$y_n = \frac{1}{n}\sum_{j=1}^{n} x_j = \bar{x} \qquad (2)$$

On the other hand, if a fairly small *k* is selected, one can smooth the edges of the series without losing much of the general information.

2.2 k-th Weighted Moving Average Time Series Model

The k-th weighted moving average process of a given time series $\{x_t\}$ is defined as follows:

$$z_t = \frac{2}{k(k+1)} \sum_{j=0}^{k-1} (j+1) x_{t-k+1-j} \tag{3}$$

where $t = k, k+1, \cdots, n.$ Similar to the moving average process, as *k* increases, the number of realization of the series $\{z_t\}$ decreases, as $k \to n$. From (3) the new time series $\{z_t\}$ becomes

$$z_n = \frac{2}{n(n+1)} \sum_{j=0}^{k-1} j x_j \tag{4}$$

For a small *k*, we can smooth the edges of the time series, and the new realization $\{z_t\}$ is closer to the actual series $\{x_t\}$. The k-th weighted moving average process put more weights on the most recent observation and therefore captures the original series better than the moving average.

2.3 The k-th Exponential Weighted Moving Average Time Series Model

The *k−th* exponential weighted moving average process of a given time series $\{x_t\}$ is defined by:

$$v_t = \frac{1}{\sum_{j=0}^{k-1} (1-\alpha)^j} \sum_{j=0}^{k-1} (1-\alpha)^{k-j-1} x_{t-k+1-j} \tag{5}$$

where $t = k, k+1, \cdots, n$ and the smoothing factor *α* is defined as $\alpha = \frac{2}{k+1}$. If *k* = *n*, $\alpha = \frac{2}{n+1}$. Moreover, $\sum_{j=0}^{k-1} (1-\alpha)^j$ reaches its maximum when *k* = 3, and it gets closer and closer to 1 as *k* increases. As *k* increases, the number of observations of the new time series $\{v_t\}$ decreases, and it eventually reduces to a single observation when k = n. As k → n, the time series $\{v_t\}$ becomes

$$v_n = \frac{1}{\sum_{j=0}^{n-1} (1-\alpha)^j} \sum_{j=0}^{n-1} (1-\alpha)^{n-j} x_{j+1} \tag{6}$$

The k-th exponential weighted moving average process, in addition to what the previous two models offer, instead of decreasing weight consistently as the weighted moving average method does, it decreases the weight exponentially. That is, much more weights are put on the most recent observations and older observations do not benefit. It is clear that the exponential weighted moving average process weights heavily on the most recent observation, and decreases the weight exponentially as time decreases. So if a fairly small k is chosen, then one can smooth the edges of the time series, as $\{v_t\}$ would be fairly close to the original time series.

3 Data presentation and methodology

The data used in this paper, comprises of the daily stock price of Access Bank PLC for 500 days. The plot of these observations is given in Figure 1. From this graph it is evident that there was a decreasing stochastic linear trend for the first one hundred and twelve days or so which was also followed by an increasing stochastic linear trend for another 50 days and thereafter the movement is characterize by a trough and pick which appears like a kind of seasonal variation. This data movement is difficult to model in practice. A common way of analyzing this data is to smooth so as to eliminate the trend and seasonal variation inherent in the observations. It is in this direction we apply these classes of weighted moving average models.

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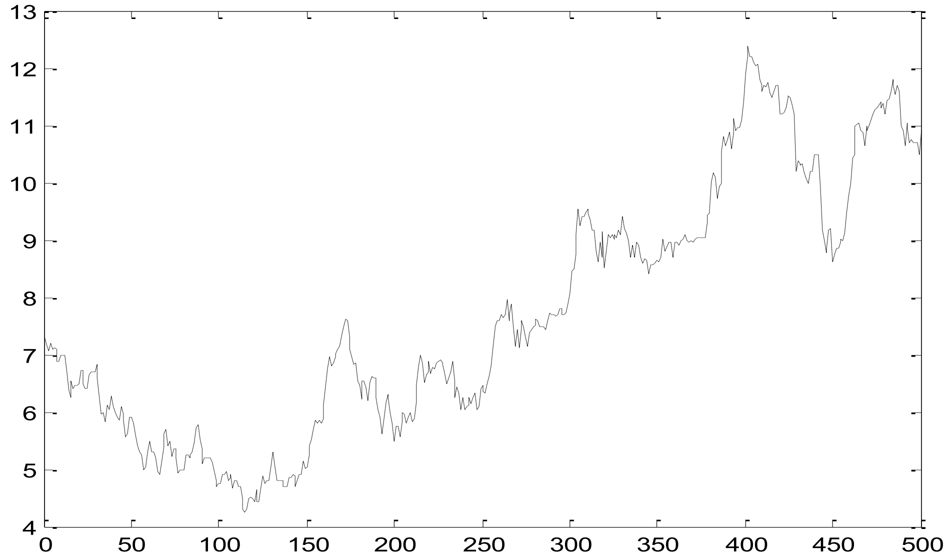

Figure 1: Daily Stock price movement for Access Bank PLC for 500 days.

The method used in this paper, is first to consider smoothing the time series observations using the k-th moving average, k-th weighted moving average and k-th exponential weighted moving average, where k=3 in all the three methods considered. We applied these methods of smoothing step by step to (1), (3) and (5) respectively to construct another set of time series data. These weighted moving average processes when applied to this original data $\{x_t\}$ can in general be represented by:

$$x_t = \hat{f}(t) + \varepsilon_t \tag{6}$$

where $\hat{f}(t)$ is the smooth version obtained as a result of application of the methods described in (1), (3) and (5) respectively while ε_t is a fall off of the smooth data from the actual data. Basic assumptions on $\hat{f}(t)$ is that it is a continuous function of time (t) while ε_t is a zero-mean error series. There abound various ways of analyzing ε_t. It can be modeled or used as a base for further analysis or even for forecasting future time series data. A common measure of ε_t is the mean square error (MSE) which is defined by:

$$MSE = n^{-1}\sum_{t=1}^{n}\hat{\varepsilon}_t^2 \tag{7}$$

From (7) one can obtain the standard deviation (STD) given by $\sqrt{MSE}$ and the standard error (SDE) given by $\sqrt{\frac{MSE}{n}}$. The results of these applications in terms of MSE, STD and SDE for the three methods considered are given in Table 1 while Figures 2, 3 and 4 gives the graphical representation of the actual data with the smoothed data superimposed respectively.

In order to forecast future observations, we re-define (1), (3) and (5) where k =3 in all cases.

For a 3-Day simple moving average (SMA), our forecast equation is given by

$$\hat{y}_t = \frac{x_{t-2} + x_{t-1} + x_t}{3} \tag{8}$$

For a 3-day weighted moving average (WMA), the forecast equation is also given by

$$\hat{y}_t = \frac{1}{6}(x_{t-2} + 2x_{t-1} + 3x_t) \tag{9}$$

Lastly, for a 3-Day exponential weighted moving average (EWMA), the forecast equation is

$$\hat{y}_t = \frac{4}{7}(0.25x_{t-2} + 0.5x_{t-1} + x_t) \tag{10}$$

As noted by Franses (1998), a typical method of evaluating forecast is to keep m observations to evaluate forecasts from models which are fitted to the first n observations. Another method is to check whether 95 percent of the forecasts lie within the 95 percent forecast interval. Typically, evaluation can be based on forecast mean squared error (FMSE) which are given by:

$$FMSE(h) = \sqrt{\frac{1}{m}\sum_{h=1}^{m}(\hat{y}_{t+h} - y_{t+h})^2} \tag{11}$$

where $\hat{y}_{t+h}$ is the forecast made at time t for a specific horizon h and y_{t+h} is the expected future value of the time series.

For simplicity, in this study, we set m = 25 and n = 475, so that the expected future value is the m values kept for out of sample performance. In this case we make use of the three forecasting methods given in (8), (9) and (10) and compare their results using RMSE given by (11). The results obtained using these forecasting methods are given in Table 2.

Table 1. Basic summary statistics for evaluating the error generated from actual data versus smoothed data.

Weighted Method	MSE	STD	SDE
3-Day SMA	0.0217	0.1473	0.0066
3-Day WMA	0.0107	0.1034	0.0046
3-Day EWMA	0.0078	0.0886	0.0040

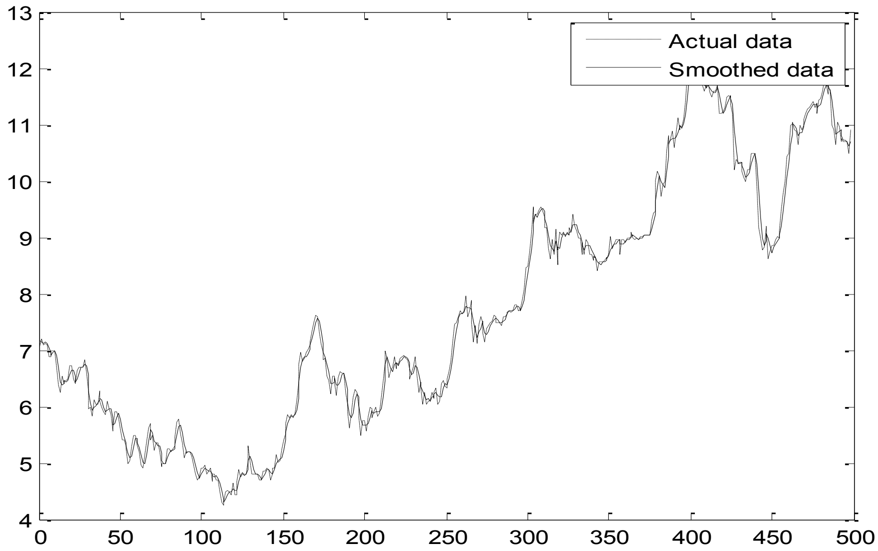

Figure 2: Comparison of the 3-Day SMA and the actual data. The actual data is represented by broken lines while the 3-Day SMA is represented by the smooth line

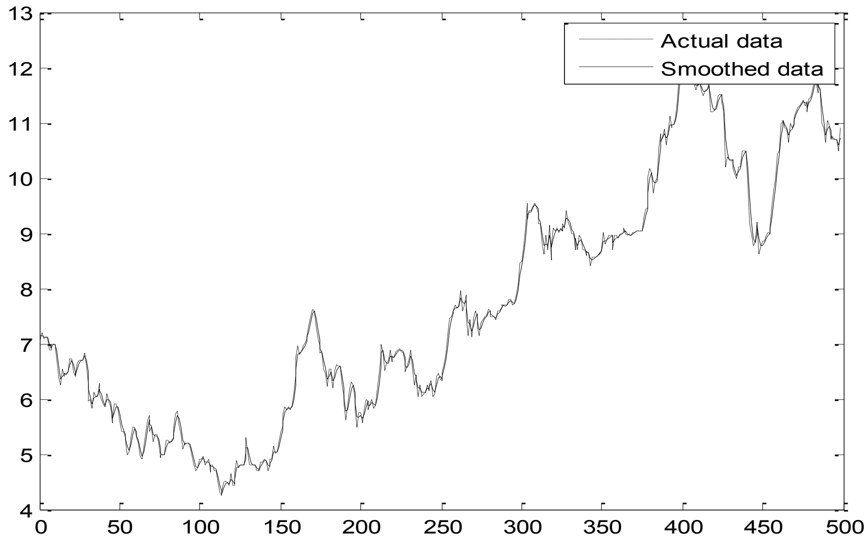

Figure 3: Comparison of the 3-Day WMA and the actual data. The actual data is represented by broken lines while the 3-Day WMA is represented by the smooth line

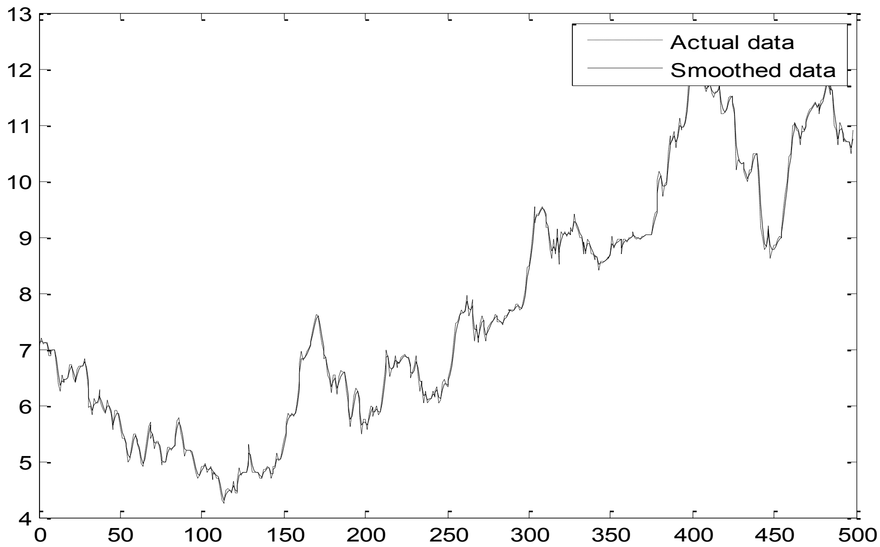

Figure 4: Comparison of the 3-Day EWMA and the actual data. The actual data is represented by broken lines while the 3-Day EWMA is represented by the smooth line

Table 2: Summary statistic for evaluating the residuals generated from the out of sample performance

Forecast Method	*FMSE*
3-Day SMA	0.4154
3-Day WMA	0.4176
3-Day EWMA	0.4181

5 Discussion of results

The graphs of the plotted actual data versus smoothed data presented in Figures 2, 3 and 4 for the 3-Day SMA, 3-Day WMA and 3-Day EWMA respectively appear very close in all. However, summary statistics displayed in Table 1, reveal that the 3-Day EWMA outperformed the 3-Day WMA and the 3-Day SMA. Similarly, summary statistics presented in Table 2 when the three methods were subjected to out-of-sample performance reveals that the 3-Day SMA performed better than the others.

6 Conclusion

In this paper, we show that the 3-Day EWMA has the ability to smooth our economic time series data more than the 3-Day WMA or the 3-Day SMA. We also show that when these three methods were subjected to out-of-sample forecast, the 3-Day SMA performed better than the rest two methods. This study further suggest that a particular weighted method which is good for smoothing an economic time series, may not necessarily be a better choice when it comes to forecasting.

REFERENCES

Box, G. E. P & Jenkins, G. M. (1976). Time series analysis, forecasting and control, Holden-Day, San Francisco.

Chatfield, C. , Koehler , A. B., Keith Ord, J. & Snyder, R. D. (2001). A New Look at Models for Exponential Smoothing .Journal of the Royal Statistical Society. Series D , Vol. 50, No. 2, pp. 147-159

Franses, P. H. (1998). Time series models for business and Ecconomic forecasting, Cambridge University press.

Hunter , J. S. (1986). The exponentially weighted moving average. Journal of Quality Technology, 18, 203-210

Lucas , J. M. & Saccucci , M. S. (1990). Exponentially Weighted Moving Average Control Schemes: Properties and Enhancements. Technometrics, 32(1),1-12

Makridakis, S., Andersen, A., Carbone, R., Fildes, R., Hibon, M., Lewandowski, R., Newton, J., Parzen, E., & Winkler, R. (1984). The Forecasting Accuracy of Major Time Series Methods, Wiley, Chichester.

Tsokos, C. P. (2010). K-th Moving, Weighted and Exponential Moving Average for Time Series Forecasting Models, European Journal of pure and applied Mathematics**.** Vol. 3, No. 3, 406-416.

Winters, P. R. (1960). Forecasting Sales by Exponentially Weighted Moving Averages. Management Science, Vol. 6, No. 3 (Apr., 1960), pp. 324-342.