The General Theory of Space Time, Mass, Energy, Quantum Gravity, Perception, Four Fundamental Forces, Vacuum Energy, Quantum Field

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Abstract
Essentially GUT and Vacuum Field are related to Quantum Field where Quantum entanglement takes place. Mass energy equivalence and its relationship with Quantum Computing are discussed in various papers by the author. Here we finalize a paper on the relationship of GUT on one hand and space-time, mass-energy, Quantum Gravity and Vacuum Field with Quantum Field. In fact, noise, discordant notes also are all related to subjective theory of Quantum Mechanics which is related to Quantum Entanglement and Quantum computing.

Key words: Quantum Mechanics, Quantum computing, Quantum entanglement, vacuum energy

Introduction:
Physicists have always thought quantum computing is hard because quantum states are incredibly fragile. But could noise and messiness actually help things along? (Zeeya Merali) Quantum computation, attempting to exploit subatomic physics to create a device with the potential to outperform its best macroscopic counterparts IS A Gordian knot with the Physicists. . Quantum systems are fragile, vulnerable and susceptible both in its thematic and discursive form and demand immaculate laboratory conditions to survive long enough to be of any use. Now White was setting out to test an unorthodox quantum algorithm that seemed to turn that lesson on its head. Energetic franticness, ensorcelled frenzy, entropic entrepotishness, Ergodic erythrism messiness and disorder would be virtues, not vices — and perturbations in the quantum system would drive computation, not disrupt it.

Conventional view is that such devices should get their computational power from quantum entanglement — a phenomenon through which particles can share information even when they are separated by arbitrarily large distances. But the latest experiments suggest that entanglement might not be needed after all. Algorithms could instead tap into a quantum resource called discord, which would be far cheaper and easier to maintain in the lab.

Classical computers have to encode their data in an either/or fashion: each bit of information takes a value of 0 or 1, and nothing else. But the quantum world is the realm of both/and. Particles can exist in ‘superposition’s’ — occupying many locations at the same time, say, or simultaneously (e&eb)spinning clockwise and anticlockwise.
So, Feynman argued, computing in that realm could use quantum bits of information — qubits — that exist as superpositions of 0 and 1 simultaneously. A string of 10 such qubits could represent all 1,024 10-bit numbers simultaneously. And if all the qubits shared information through entanglement, they could race through myriad calculations in parallel — calculations that their classical counterparts would have to plod through in a languorous, lugubrious and lachrymosish manner sequentially (see ‘Quantum computing’).

The notion that quantum computing can be done only through entanglement was cemented in 1994, when Peter Shor, a mathematician at the Massachusetts Institute of Technology in Cambridge, devised an entanglement-based algorithm that could factorize large numbers at lightning speed — potentially requiring only seconds to break the encryption currently used to send secure online communications, instead of the years required by ordinary computers. In 1996, Lov Grover at Bell Labs in Murray Hill, New Jersey, proposed an entanglement-based algorithm that could search rapidly through an unsorted database; a classical algorithm, by contrast, would have to laboriously search the items one by one.

But entanglement has been the bane of many a quantum experimenter’s life, because the slightest interaction of the entangled particles with the outside world — even with a stray low-energy photon emitted by the warm walls of the laboratory — can destroy it. Experiments with entanglement demand ultra-low temperatures and careful handling. "Entanglement is hard to prepare, hard to maintain and hard to manipulate," says Xiaosong Ma, a physicist at the Institute for Quantum Optics and Quantum Information in Vienna. Current entanglement record-holder intertwines just 14 qubits, yet a large-scale quantum computer would need several thousand. Any scheme that bypasses entanglement would be warmly welcomed, without any hesitation, reservation, regret, remorse, compunction or contrition. Says Ma.

Clues that entanglement isn’t essential after all began to trickle in about a decade ago, with the first examples of rudimentary regimentation and seriotological sermonisations and padagouelogical pontifications quantum computation. In 2001, for instance, physicists at IBM’s Almaden Research Center in San Jose and Stanford University, both in California, used a 7-qubit system to implement Shor’s algorithm, factorizing the number 15 into 5 and 3. But controversy erupted over whether the experiments deserved to be called quantum computing, says Carlton Caves, a quantum physicist at the University of New Mexico (UNM) in Albuquerque.

The trouble was that the computations were done at room temperature, using liquid-based nuclear magnetic resonance (NMR) systems, in which information is encoded in atomic nuclei using(e) an internal quantum property known as spin. Caves and his colleagues had already shown that entanglement could not be sustained in these conditions. "The nuclear spins would be jostled about too much for them to stay lined up neatly,” says Caves. According to the orthodoxy, no entanglement meant any quantum computation. The NMR community gradually accepted that they had no entanglement, yet the computations were producing real results. Experiments were explicitly performed for a quantum search without (e(e))exploiting entanglement. These experiments really called into question what gives quantum
computing its power.

Order Out of Disorder

Discord, an obscure measure of quantum correlations. Discord quantifies (≈) how much a system can be disrupted when people observe it to gather information. Macroscopic systems are not (e&eb) affected by observation, and so have zero discord. But quantum systems are unavoidably (e&eb) affected because measurement forces them to settle on one of their many superposition values, so any possible quantum correlations, including entanglement, give (eb) a positive value for discord. Discord is connected (e&eb) to quantum computing. "An algorithm challenged the idea that quantum computing requires (e) to painstakingly prepare(eb) a set of pristine qubits in the lab.

In a typical optical experiment, the pure qubits might (e) consist of horizontally polarized photons representing 1 and vertically polarized photons representing 0. Physicists can entangle a stream of such pure qubits by passing them through a (e&eb) processing gate such as a crystal that alters (e&eb) the polarization of the light, and then read off the state of the qubits as they exit. In the real world, unfortunately, qubits rarely stay pure. They are far more likely to become messy, or 'mixed' — the equivalent of unpolarized photons. The conventional wisdom is that mixed qubits are(e) useless for computation because they e(e&eb) cannot be entangled, and any measurement of a mixed qubit will yield a random result, providing little or no useful information.

If a mixed qubit was sent through an entangling gate with a pure qubit. The two could not become entangled but, the physicists argued, their interaction might be enough to carry (eb)out a quantum computation, with the result read from the pure qubit. If it worked, experimenters could get away with using just one tightly controlled qubit, and letting the others be badly battered sadly shattered by environmental noise and disorder. "It was not at all clear why that should work," says White. "It sounded as strange as saying they wanted to measure someone's speed by measuring the distance run with a perfectly metered ruler and measuring the time with a stopwatch that spits out a random answer."

Datta supplied an explanation he calculated that the computation could be(eb) driven by the quantum correlation between the pure and mixed qubits — a correlation given mathematical expression by the discord. "It's true that you must have entanglement to compute with idealized pure qubits. But when you include mixed states, the calculations look very different. 'Quantum computation without (e) the hassle of entanglement,' seems to have become a point where the anecdote of life had met the aphorism of thought. Discord could be like sunlight, which is plentiful but has to be harnessed in a certain way to be useful.

The team confirmed that the qubits were not entangled at any point. Intriguingly, when the researchers tuned down the polarization quality of the one pure qubit, making (eb) it almost mixed, the computation still worked. "Even when you have a system with just a tiny fraction of purity, that is (≈) vanishingly close to classical, it still has power," says White. "That just blew our minds." The computational power only disappeared when the amount of discord in the system reached zero. "It's counter-intuitive, but it seems that putting noise and disorder in your system gives you power," says White. "Plus, it's easier to achieve."

For Ma, White's results provided the "wow! Moment" that made him take discord seriously. He was keen to test discord-based algorithms that used more than the two qubits used by White, and that could perform more glamorous tasks, but he had none to test. "Before I can carry out any experiments, I need the recipe of what to prepare from theoreticians," he explains, and those instructions were not forthcoming.

Although it is easier for experimenters to handle noisy real-world systems than pristinely glorified ones, it is a lot harder for theoretical physicists to analyse them mathematically. "We're talking about messy physical systems, and the equations are even messier," says Modi. For the past few years, theoretical physicists interested in discord have been trying to formulate prescriptions for new tests. It is not proved that discord is (eb) essential to computation — just that it is there. Rather than being the engine behind computational power, it could just be along for the ride, he argues. Last year, Acín and his colleagues calculated that almost every quantum system contains discord. "It's basically everywhere," he says. "That makes it difficult to explain why it causes power in specific situations and not others." It is almost like we can perform our official tasks amidst all noise, subordination pressure, superordinational scatological pontification, coordination dialectic deliberation, but when asked to do something different we want
“peace”, “Silence”, “No disturbance”. Personally, one thinks it is a force of habit. And habits die hard. Modi shares the concern. "Discord could be like sunlight, which is plentiful but has to be harnessed in a certain way to be useful. We need to identify what that way is," he says. Du and Ma are independently conducting experiments to address these points. Both are attempting to measure the amount of discord at each stage of a computation — Du using liquid NMR and electron-spin resonance systems, and Ma using photons. The very ‘importance giving’, attitude itself acts as an anathema, a misnomer.

A finding that quantifies how and where discord acts would strengthen the case for its importance, says Acín. We suspect it acts only in cases where there is ‘speciality’ like in quantum level. Other ‘mundane’ ‘world’ happenings take place amidst all discord and noise. Nobody bothers because it is ‘run of the mill’ But for ‘selective and important issues’ one needs ‘calm’ and ‘non disturbance’ and doing ‘all things’ amidst this worldly chaos we portend is ‘Khuda’ ‘Allah’ or ‘Brahman’. And we feel that Quantum Mechanics is a subjective science and teaches this philosophy much better than others. But if these tests find discord wanting, the mystery of how entanglement-free computation works will be reopened. "The search would have to begin for yet another quantum property," he adds. Vedral notes that even if Du and Ma’s latest experiments are a success, the real game-changer will be discord-based algorithms for factorization and search tasks, similar to the functions devised by Shor and Grover that originally ignited the field of quantum computing. "My gut feeling is that tasks such as these will ultimately need entanglement," says Vedral. "Though as yet there is no proof that they can’t be done with discord alone."

Zurek says that discord can be thought of as a complement to entanglement, rather than as a usurper. “There is no longer a question that discord works,” he declares. "The important thing now is to find out when discord without entanglement can be (e)xploited most usefully, and when entanglement is essential, and produces 'Quantum Computation'."

Notation:

Space And Time

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<thead>
<tr>
<th>Category One Of Time</th>
<th>Category Two Of Time</th>
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<td>G_{13}</td>
<td>G_{14}</td>
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<th>Category One Of Space</th>
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<tr>
<td>T_{13}</td>
<td>T_{14}</td>
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Mass And Energy

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<th>Category One Of Energy</th>
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<tr>
<td>G_{16}</td>
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<th>Category One Of Matter</th>
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<td>T_{16}</td>
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Quantum Gravity And Perception:

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<th>Category One Of Perception</th>
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<tbody>
<tr>
<td>G_{20}</td>
<td>G_{21}</td>
<td>G_{22}</td>
</tr>
</tbody>
</table>
\( T_{20} \): Category One Of Quantum Gravity
\( T_{21} \): Category Two Of Quantum Gravity
\( T_{22} \): Category Three Of Quantum Gravity

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**Strong Nuclear Force And Weak Nuclear Force:**

\( G_{14} \): Category One Of Weak Nuclear Force
\( G_{25} \): Category Two Of Weak Nuclear Force

\( G_{26} \): Category Three Of Weak Nuclear Force
\( T_{26} \): Category Three Of Strong Nuclear Force

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**Electromagnetism And Gravity:**

\( G_{28} \): Category One Of Gravity
\( G_{29} \): Category Two Of Gravity
\( G_{30} \): Category Three Of Gravity

\( T_{28} \): Category One Of Electromagnetism
\( T_{29} \): Category Two Of Electromagnetism
\( T_{29} \): Category Three Of Electromagnetism

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**Vacuum Energy And Quantum Field:**

\( G_{32} \): Category One Of Quantum Field
\( G_{33} \): Category Two Of Quantum Field
\( G_{34} \): Category Three Of Quantum Field

\( T_{32} \): Category One Of Vacuum Energy
\( T_{33} \): Category Two Of Vacuum Energy
\( T_{34} \): Category Three Of Vacuum Energy

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**Accentuation Coefficients:**

\( \{a_{13}\}^{(1)}, \{a_{14}\}^{(1)}, \{a_{15}\}^{(1)}, \{b_{13}\}^{(1)}, \{b_{14}\}^{(1)}, \{b_{15}\}^{(1)}, \{a_{16}\}^{(2)}, \{a_{17}\}^{(2)}, \{a_{18}\}^{(2)}, \{b_{16}\}^{(2)}, \{b_{17}\}^{(2)}, \{b_{18}\}^{(2)}; \{a_{20}\}^{(3)}, \{a_{21}\}^{(3)}, \{a_{22}\}^{(3)}, \{b_{20}\}^{(3)}, \{b_{21}\}^{(3)}, \{b_{22}\}^{(3)}, \{a_{24}\}^{(4)}, \{a_{25}\}^{(4)}, \{a_{26}\}^{(4)}, \{b_{24}\}^{(4)}, \{b_{25}\}^{(4)}, \{b_{26}\}^{(4)}, \{b_{28}\}^{(5)}, \{b_{29}\}^{(5)}, \{b_{30}\}^{(5)}, \{a_{28}\}^{(5)}, \{a_{29}\}^{(5)}, \{a_{30}\}^{(5)}, \{a_{32}\}^{(6)}, \{a_{33}\}^{(6)}, \{a_{34}\}^{(6)}, \{b_{32}\}^{(6)}, \{b_{33}\}^{(6)}, \{b_{34}\}^{(6)} \)

---

**Dissipation Coefficients**

\( \\)
\[
\begin{align*}
(a_{13}^{(1)}(t), a_{14}^{(1)}(t), a_{15}^{(1)}(t), b_{13}^{(1)}(t), b_{14}^{(1)}(t), b_{15}^{(1)}(t), a_{16}^{(2)}(t), a_{17}^{(2)}(t), a_{18}^{(2)}(t),
& b_{16}^{(2)}(t), b_{17}^{(2)}(t), b_{18}^{(2)}(t), a_{20}^{(3)}(t), a_{21}^{(3)}(t), a_{22}^{(3)}(t), b_{20}^{(3)}(t), b_{21}^{(3)}(t), b_{22}^{(3)}(t),
& a_{24}^{(4)}(t), a_{25}^{(4)}(t), a_{26}^{(4)}(t), b_{24}^{(4)}(t), b_{25}^{(4)}(t), b_{26}^{(4)}(t), b_{28}^{(5)}(t), b_{29}^{(5)}(t), b_{30}^{(5)}(t),
& a_{28}^{(5)}(t), a_{29}^{(5)}(t), a_{30}^{(5)}(t), a_{32}^{(6)}(t), a_{34}^{(6)}(t), b_{32}^{(6)}(t), b_{33}^{(6)}(t), b_{34}^{(6)}(t)
\end{align*}
\]

**Governing Equations: For The System Space And Time:**

The differential system of this model is now

\[
\begin{align*}
\frac{dG_{13}}{dt} &= (a_{13}^{(1)}(t))T_{14} - \left(\left[a_{13}^{(1)}(t) + (a_{13}^{(1)}(t))(T_{14}, t)\right]G_{13}
\end{align*}
\]

\[
\begin{align*}
\frac{dG_{14}}{dt} &= (a_{14}^{(1)}(t))T_{13} - \left(\left[a_{14}^{(1)}(t) + (a_{14}^{(1)}(t))(T_{14}, t)\right]G_{14}
\end{align*}
\]

\[
\begin{align*}
\frac{dG_{15}}{dt} &= (a_{15}^{(1)}(t))T_{14} - \left(\left[a_{15}^{(1)}(t) + (a_{15}^{(1)}(t))(T_{14}, t)\right]G_{15}
\end{align*}
\]

\[
\begin{align*}
\frac{dT_{13}}{dt} &= (b_{13}^{(1)}(t))T_{14} - \left(\left[b_{13}^{(1)}(t) - (b_{13}^{(1)}(t))(G, t)\right]T_{13}
\end{align*}
\]

\[
\begin{align*}
\frac{dT_{14}}{dt} &= (b_{14}^{(1)}(t))T_{13} - \left(\left[b_{14}^{(1)}(t) - (b_{14}^{(1)}(t))(G, t)\right]T_{14}
\end{align*}
\]

\[
\begin{align*}
\frac{dT_{15}}{dt} &= (b_{15}^{(1)}(t))T_{14} - \left(\left[b_{15}^{(1)}(t) - (b_{15}^{(1)}(t))(G, t)\right]T_{15}
\end{align*}
\]

\[
\begin{align*}
+a_{13}^{(1)}(T_{14}, t) &= \text{First augmentation factor}
\end{align*}
\]

\[
\begin{align*}
-b_{13}^{(1)}(G, t) &= \text{First detriments factor}
\end{align*}
\]

**Governing Equations: Of The System Mass (Matter) And Energy**

The differential system of this model is now

\[
\begin{align*}
\frac{dG_{16}}{dt} &= (a_{16}^{(2)}(G)T_{17} - \left(\left[a_{16}^{(2)}(G) + (a_{16}^{(2)}(G))(T_{17}, t)\right]G_{16}
\end{align*}
\]

\[
\begin{align*}
\frac{dG_{17}}{dt} &= (a_{17}^{(2)}(G)T_{16} - \left(\left[a_{17}^{(2)}(G) + (a_{17}^{(2)}(G))(T_{17}, t)\right]G_{17}
\end{align*}
\]

\[
\begin{align*}
\frac{dG_{18}}{dt} &= (a_{18}^{(2)}(G)T_{16} - \left(\left[a_{18}^{(2)}(G) + (a_{18}^{(2)}(G))(T_{17}, t)\right]G_{18}
\end{align*}
\]

\[
\begin{align*}
\frac{dT_{16}}{dt} &= (b_{16}^{(2)}(G)T_{17} - \left(\left[b_{16}^{(2)}(G) - (b_{16}^{(2)}(G))(G, t)\right]T_{16}
\end{align*}
\]

\[
\begin{align*}
\frac{dT_{17}}{dt} &= (b_{17}^{(2)}(G)T_{16} - \left(\left[b_{17}^{(2)}(G) - (b_{17}^{(2)}(G))(G, t)\right]T_{17}
\end{align*}
\]

\[
\begin{align*}
\frac{dT_{18}}{dt} &= (b_{18}^{(2)}(G)T_{17} - \left(\left[b_{18}^{(2)}(G) - (b_{18}^{(2)}(G))(G, t)\right]T_{18}
\end{align*}
\]

\[
\begin{align*}
+a_{16}^{(2)}(T_{17}, t) &= \text{First augmentation factor}
\end{align*}
\]

\[
\begin{align*}
-b_{16}^{(2)}(G, t) &= \text{First detriments factor}
\end{align*}
\]

**Governing Equations: Of The System Quantum Gravity And Perception**

The differential system of this model is now

\[
\begin{align*}
\frac{dG_{20}}{dt} &= (a_{20}^{(3)}(G)T_{21} - \left(\left[a_{20}^{(3)}(G) + (a_{20}^{(3)}(G))(T_{21}, t)\right]G_{20}
\end{align*}
\]

\[
\begin{align*}
\frac{dG_{21}}{dt} &= (a_{21}^{(3)}(G)T_{20} - \left(\left[a_{21}^{(3)}(G) + (a_{21}^{(3)}(G))(T_{21}, t)\right]G_{21}
\end{align*}
\]

\[
\begin{align*}
\frac{dG_{22}}{dt} &= (a_{22}^{(3)}(G)T_{21} - \left(\left[a_{22}^{(3)}(G) + (a_{22}^{(3)}(G))(T_{21}, t)\right]G_{22}
\end{align*}
\]
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)} T_{21} - [(b_{20}^{\prime})^{(3)} - (b_{20}^{\prime\prime})^{(3)} (G_{23}, t)] T_{20}$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)} T_{20} - [(b_{21}^{\prime})^{(3)} - (b_{21}^{\prime\prime})^{(3)} (G_{23}, t)] T_{21}$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)} T_{21} - [(b_{22}^{\prime})^{(3)} - (b_{22}^{\prime\prime})^{(3)} (G_{23}, t)] T_{22}$$

$$+(a_{20}^{\prime\prime})^{(3)} (T_{23}, t) = \text{First augmentation factor}$$

$$-(b_{20}^{\prime\prime})^{(3)} (G_{23}, t) = \text{First detritions factor}$$

**Governing Equations: Of The System Strong Nuclear Force And Weak Nuclear Force:**

The differential system of this model is now

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)} G_{25} - [(a_{24}^{\prime})^{(4)} + (a_{24}^{\prime\prime})^{(4)} (T_{25}, t)] G_{24}$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)} G_{24} - [(a_{25}^{\prime})^{(4)} + (a_{25}^{\prime\prime})^{(4)} (T_{25}, t)] G_{25}$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)} G_{25} - [(a_{26}^{\prime})^{(4)} + (a_{26}^{\prime\prime})^{(4)} (T_{25}, t)] G_{26}$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)} T_{25} - [(b_{24}^{\prime})^{(4)} - (b_{24}^{\prime\prime})^{(4)} (G_{27}, t)] T_{24}$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)} T_{24} - [(b_{25}^{\prime})^{(4)} - (b_{25}^{\prime\prime})^{(4)} (G_{27}, t)] T_{25}$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)} T_{25} - [(b_{26}^{\prime})^{(4)} - (b_{26}^{\prime\prime})^{(4)} (G_{27}, t)] T_{26}$$

$$+(a_{24}^{\prime\prime})^{(4)} (T_{25}, t) = \text{First augmentation factor}$$

$$-(b_{24}^{\prime\prime})^{(4)} (G_{27}, t) = \text{First detritions factor}$$

**Governing Equations: Of The System Electromagnetism And Gravity:**

The differential system of this model is now

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)} G_{29} - [(a_{28}^{\prime})^{(5)} + (a_{28}^{\prime\prime})^{(5)} (T_{29}, t)] G_{28}$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)} G_{28} - [(a_{29}^{\prime})^{(5)} + (a_{29}^{\prime\prime})^{(5)} (T_{29}, t)] G_{29}$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)} G_{29} - [(a_{30}^{\prime})^{(5)} + (a_{30}^{\prime\prime})^{(5)} (T_{29}, t)] G_{30}$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)} T_{29} - [(b_{28}^{\prime})^{(5)} - (b_{28}^{\prime\prime})^{(5)} (G_{31}, t)] T_{28}$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)} T_{28} - [(b_{29}^{\prime})^{(5)} - (b_{29}^{\prime\prime})^{(5)} (G_{31}, t)] T_{29}$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)} T_{29} - [(b_{30}^{\prime})^{(5)} - (b_{30}^{\prime\prime})^{(5)} (G_{31}, t)] T_{30}$$

$$+(a_{28}^{\prime\prime})^{(5)} (T_{29}, t) = \text{First augmentation factor}$$

$$-(b_{28}^{\prime\prime})^{(5)} (G_{31}, t) = \text{First detritions factor}$$

**Governing Equations: Of The System Vacuum Energy And Quantum Field:**

The differential system of this model is now

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)} G_{33} - [(a_{32}^{\prime})^{(6)} + (a_{32}^{\prime\prime})^{(6)} (T_{33}, t)] G_{32}$$
\[
\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{33} - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right)G_{33}
\]

\[
\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{34} - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right)G_{34}
\]

\[
\frac{df_{33}}{dt} = (b_{33})^{(6)}T_{33} - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{33}, t)\right)T_{33}
\]

\[
\frac{df_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{33}, t)\right)T_{33}
\]

\[
+(a''_{33})^{(6)}(T_{33}, t) = \text{First augmentation factor}
\]

\[
-(b''_{34})^{(6)}(G_{33}, t) = \text{First detritions factor}
\]


\[
\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{13} - \left((a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right)G_{13}
\]

\[
\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{14} - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right)G_{14}
\]

\[
\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{15} - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right)G_{15}
\]

Where

\[
(a''_{13})^{(1)}(T_{14}, t), (a''_{14})^{(1)}(T_{14}, t), (a''_{15})^{(1)}(T_{14}, t)
\]

are first augmentation coefficients for category 1, 2 and 3

\[
(a'_{13})^{(2)}(T_{14}, t), (a'_{14})^{(2)}(T_{14}, t), (a'_{15})^{(2)}(T_{14}, t)
\]

are second augmentation coefficient for category 1, 2 and 3

\[
(a''_{13})^{(2,2)}(T_{14}, t), (a''_{14})^{(2,2)}(T_{14}, t), (a''_{15})^{(2,2)}(T_{15}, t)
\]

are third augmentation coefficient for category 1, 2 and 3

\[
(a''_{13})^{(4,4,4)}(T_{25}, t), (a''_{14})^{(4,4,4)}(T_{25}, t), (a''_{15})^{(4,4,4)}(T_{25}, t)
\]

are fourth augmentation coefficient for category 1, 2 and 3

\[
(a''_{13})^{(5,5,5,5)}(T_{25}, t), (a''_{14})^{(5,5,5,5)}(T_{25}, t), (a''_{15})^{(5,5,5,5)}(T_{25}, t)
\]

are fifth augmentation coefficient for category 1, 2 and 3

\[
(a''_{13})^{(6,6,6,6)}(T_{25}, t), (a''_{14})^{(6,6,6,6)}(T_{25}, t), (a''_{15})^{(6,6,6,6)}(T_{25}, t)
\]

are sixth augmentation coefficient for category 1, 2 and 3
Where \(- (b_{16}^{(2)}(G_{16}, t))\), \(- (b_{16}^{(2)}(G_{16}, t))\), \(- (b_{16}^{(2)}(G_{16}, t))\) are first detrition coefficients for category 1, 2 and 3
\(- (b_{16}^{(2)}(G_{16}, t))\), \(- (b_{16}^{(2)}(G_{16}, t))\), \(- (b_{16}^{(2)}(G_{16}, t))\) are second detrition coefficients for category 1, 2 and 3
\(- (b_{16}^{(2)}(G_{16}, t))\), \(- (b_{16}^{(2)}(G_{16}, t))\), \(- (b_{16}^{(2)}(G_{16}, t))\) are third detrition coefficients for category 1, 2 and 3
\(- (b_{16}^{(2)}(G_{16}, t))\), \(- (b_{16}^{(2)}(G_{16}, t))\), \(- (b_{16}^{(2)}(G_{16}, t))\) are fourth detrition coefficients for category 1, 2 and 3
\(- (b_{16}^{(2)}(G_{16}, t))\), \(- (b_{16}^{(2)}(G_{16}, t))\), \(- (b_{16}^{(2)}(G_{16}, t))\) are fifth detrition coefficients for category 1, 2 and 3
\(- (b_{16}^{(2)}(G_{16}, t))\), \(- (b_{16}^{(2)}(G_{16}, t))\), \(- (b_{16}^{(2)}(G_{16}, t))\) are sixth detrition coefficients for category 1, 2 and 3

\[
\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[ (a_{17}^{(3)}+a_{19}^{(3)}+a_{23}^{(3)})(T_{17}, t) + (a_{20}^{(3)}+a_{24}^{(3)})(T_{17}, t) + (a_{21}^{(3)}+a_{25}^{(3)})(T_{17}, t) + (a_{22}^{(3)}+a_{26}^{(3)})(T_{17}, t) \right]
\]

\[
\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[ (a_{18}^{(3)}+a_{20}^{(3)}+a_{24}^{(3)})(T_{17}, t) + (a_{21}^{(3)}+a_{25}^{(3)})(T_{17}, t) + (a_{22}^{(3)}+a_{26}^{(3)})(T_{17}, t) \right]
\]

\[
\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[ (a_{19}^{(3)}+a_{21}^{(3)}+a_{25}^{(3)})(T_{17}, t) + (a_{20}^{(3)}+a_{25}^{(3)})(T_{17}, t) + (a_{21}^{(3)}+a_{26}^{(3)})(T_{17}, t) \right]
\]

Where \(+ (a_{16}^{(2)})(T_{17}, t)\), \(+ (a_{17}^{(2)})(T_{17}, t)\), \(+ (a_{18}^{(2)})(T_{17}, t)\) are first augmentation coefficients for category 1, 2 and 3
\(+ (a_{16}^{(2)})(T_{17}, t)\), \(+ (a_{17}^{(2)})(T_{17}, t)\), \(+ (a_{18}^{(2)})(T_{17}, t)\) are second augmentation coefficients for category 1, 2 and 3
\(+ (a_{16}^{(2)})(T_{17}, t)\), \(+ (a_{17}^{(2)})(T_{17}, t)\), \(+ (a_{18}^{(2)})(T_{17}, t)\) are third augmentation coefficients for category 1, 2 and 3
\(+ (a_{16}^{(2)})(T_{17}, t)\), \(+ (a_{17}^{(2)})(T_{17}, t)\), \(+ (a_{18}^{(2)})(T_{17}, t)\) are fourth augmentation coefficients for category 1, 2 and 3
\(+ (a_{16}^{(2)})(T_{17}, t)\), \(+ (a_{17}^{(2)})(T_{17}, t)\), \(+ (a_{18}^{(2)})(T_{17}, t)\) are fifth augmentation coefficients for category 1, 2 and 3
\(+ (a_{16}^{(2)})(T_{17}, t)\), \(+ (a_{17}^{(2)})(T_{17}, t)\), \(+ (a_{18}^{(2)})(T_{17}, t)\) are sixth augmentation coefficients for category 1, 2 and 3

\[
\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[ (b_{16}^{(2)})(G_{17}, t) - (b_{16}^{(2)})(G_{17}, t) - (b_{16}^{(2)})(G_{17}, t) - (b_{16}^{(2)})(G_{17}, t) \right]
\]

\[
\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[ (b_{17}^{(2)})(G_{16}, t) - (b_{17}^{(2)})(G_{16}, t) - (b_{17}^{(2)})(G_{16}, t) - (b_{17}^{(2)})(G_{16}, t) \right]
\]

\[
\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[ (b_{18}^{(2)})(G_{17}, t) - (b_{18}^{(2)})(G_{17}, t) - (b_{18}^{(2)})(G_{17}, t) - (b_{18}^{(2)})(G_{17}, t) \right]
\]

Where \(- (b_{16}^{(3)})(G_{16}, t)\), \(- (b_{16}^{(3)})(G_{16}, t)\), \(- (b_{16}^{(3)})(G_{16}, t)\) are first detrition coefficients for category 1, 2 and 3
\(- (b_{16}^{(3)})(G_{16}, t)\), \(- (b_{16}^{(3)})(G_{16}, t)\), \(- (b_{16}^{(3)})(G_{16}, t)\) are second detrition coefficients for category 1, 2 and 3
\(- (b_{16}^{(3)})(G_{16}, t)\), \(- (b_{16}^{(3)})(G_{16}, t)\), \(- (b_{16}^{(3)})(G_{16}, t)\) are third detrition coefficients for category 1, 2 and 3
\(- (b_{16}^{(3)})(G_{16}, t)\), \(- (b_{16}^{(3)})(G_{16}, t)\), \(- (b_{16}^{(3)})(G_{16}, t)\) are fourth detrition coefficients for category 1, 2 and 3
\(- (b_{16}^{(3)})(G_{16}, t)\), \(- (b_{16}^{(3)})(G_{16}, t)\), \(- (b_{16}^{(3)})(G_{16}, t)\) are fifth detrition coefficients for category 1, 2 and 3
\(- (b_{16}^{(3)})(G_{16}, t)\), \(- (b_{16}^{(3)})(G_{16}, t)\), \(- (b_{16}^{(3)})(G_{16}, t)\) are sixth detrition coefficients for category 1, 2 and 3

\[
\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[ (a_{17}^{(3)}+a_{19}^{(3)}+a_{23}^{(3)})(T_{17}, t) + (a_{20}^{(3)}+a_{24}^{(3)})(T_{17}, t) + (a_{21}^{(3)}+a_{25}^{(3)})(T_{17}, t) + (a_{22}^{(3)}+a_{26}^{(3)})(T_{17}, t) \right]
\]
\[
\begin{align*}
\frac{dg_{20}}{dt} &= (a_{20})^3 G_{20} - \\
&= (a_{14}^{(1)} + (a_{20}^{(3)})(T_{25}, t) + (a_{16}^{(2,2,2)})(T_{31}, t) + (a_{12}^{(1,1,1)})(T_{15}, t)) - \\
&(a_{24}^{(4)} + (a_{20}^{(3)})(T_{25}, t) + (a_{20}^{(5,5,5,5,5)})(T_{29}, t) + (a_{26}^{(6,6,6,6,6)})(T_{33}, t)) \\
G_{20} \\
\frac{dg_{21}}{dt} &= (a_{21})^3 G_{21} - \\
&= (a_{14}^{(1)} + (a_{20}^{(3)})(T_{25}, t) + (a_{16}^{(2,2,2)})(T_{31}, t) + (a_{12}^{(1,1,1)})(T_{15}, t)) - \\
&(a_{24}^{(4)} + (a_{20}^{(3)})(T_{25}, t) + (a_{20}^{(5,5,5,5,5)})(T_{29}, t) + (a_{26}^{(6,6,6,6,6)})(T_{33}, t)) \\
G_{21} \\
\frac{dg_{22}}{dt} &= (a_{22})^3 G_{22} - \\
&= (a_{14}^{(1)} + (a_{20}^{(3)})(T_{25}, t) + (a_{16}^{(2,2,2)})(T_{31}, t) + (a_{12}^{(1,1,1)})(T_{15}, t)) - \\
&(a_{24}^{(4)} + (a_{20}^{(3)})(T_{25}, t) + (a_{20}^{(5,5,5,5,5)})(T_{29}, t) + (a_{26}^{(6,6,6,6,6)})(T_{33}, t)) \\
G_{22}
\end{align*}
\]

\((a_{20}^{(3)})(T_{25}, t) + (a_{20}^{(3)})(T_{25}, t) + (a_{20}^{(3)})(T_{25}, t) + (a_{20}^{(3)})(T_{25}, t)\) are first augmentation coefficients for category 1, 2 and 3.

\((a_{14}^{(1)} + (a_{16}^{(2,2,2)})(T_{31}, t) + (a_{12}^{(1,1,1)})(T_{15}, t))\) are second augmentation coefficients for category 1, 2 and 3.

\((a_{14}^{(1)} + (a_{16}^{(2,2,2)})(T_{31}, t) + (a_{12}^{(1,1,1)})(T_{15}, t))\) are third augmentation coefficients for category 1, 2 and 3.

\((a_{14}^{(1)} + (a_{16}^{(2,2,2)})(T_{31}, t) + (a_{12}^{(1,1,1)})(T_{15}, t))\) are fourth augmentation coefficients for category 1, 2 and 3.

\((a_{14}^{(1)} + (a_{16}^{(2,2,2)})(T_{31}, t) + (a_{12}^{(1,1,1)})(T_{15}, t))\) are fifth augmentation coefficients for category 1, 2 and 3.

\((a_{14}^{(1)} + (a_{16}^{(2,2,2)})(T_{31}, t) + (a_{12}^{(1,1,1)})(T_{15}, t))\) are sixth augmentation coefficients for category 1, 2 and 3.

\((a_{14}^{(1)} + (a_{16}^{(2,2,2)})(T_{31}, t) + (a_{12}^{(1,1,1)})(T_{15}, t))\) are first detrition coefficients for category 1, 2 and 3.

\((a_{14}^{(1)} + (a_{16}^{(2,2,2)})(T_{31}, t) + (a_{12}^{(1,1,1)})(T_{15}, t))\) are second detrition coefficients for category 1, 2 and 3.

\((a_{14}^{(1)} + (a_{16}^{(2,2,2)})(T_{31}, t) + (a_{12}^{(1,1,1)})(T_{15}, t))\) are third detrition coefficients for category 1, 2 and 3.

\((a_{14}^{(1)} + (a_{16}^{(2,2,2)})(T_{31}, t) + (a_{12}^{(1,1,1)})(T_{15}, t))\) are fourth detrition coefficients for category 1, 2 and 3.

\((a_{14}^{(1)} + (a_{16}^{(2,2,2)})(T_{31}, t) + (a_{12}^{(1,1,1)})(T_{15}, t))\) are fifth detrition coefficients for category 1, 2 and 3.

\((a_{14}^{(1)} + (a_{16}^{(2,2,2)})(T_{31}, t) + (a_{12}^{(1,1,1)})(T_{15}, t))\) are sixth detrition coefficients for category 1, 2 and 3.
\[
d \frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{26} - \left[ (a_{26})^{(4)} + (a_{26}')^{(3)}(T_{25}, t) + (a_{30})^{(5,5)}(T_{29}, t) + (a_{34})^{(6,6)}(T_{33}, t) \right] G_{26}
\]

Where \( (a_{26})^{(4)}(T_{25}, t) \), \( (a_{26}')^{(3)}(T_{25}, t) \), \( (a_{30})^{(5,5)}(T_{29}, t) \), \( (a_{34})^{(6,6)}(T_{33}, t) \) are first augmentation coefficients for category 1, 2 and 3

\[
+ (a_{26})^{(5,5)}(T_{29}, t) + (a_{30})^{(5,5)}(T_{29}, t) + (a_{34})^{(6,6)}(T_{33}, t) \]

are second augmentation coefficient for category 1, 2 and 3

\[
+ (a_{26}')^{(1,1,1)}(T_{14}, t) + (a_{16}')^{(2,2,2)}(T_{17}, t) + (a_{22}')^{(3,3,3)}(T_{21}, t)
\]

are third augmentation coefficient for category 1, 2 and 3

\[
+ (a_{16}')^{(1,1,1)}(T_{14}, t) + (a_{14}')^{(1,1,1)}(T_{14}, t) \]

are fourth augmentation coefficients for category 1, 2, and 3

\[
+ (a_{22}')^{(1,1,1)}(T_{14}, t) + (a_{22}')^{(3,3,3)}(T_{14}, t)
\]

are fifth augmentation coefficients for category 1, 2, and 3

\[
+ (a_{22}')^{(3,3,3)}(T_{21}, t)
\]

are sixth augmentation coefficients for category 1, 2, and 3

\[
d \frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[ (b_{24})^{(4)} + (b_{24}')^{(4)}(G_{27}, t) + (b_{29})^{(5,5)}(G_{33}, t) + (b_{33})^{(6,6)}(G_{35}, t) \right] T_{24}
\]

\[
d \frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{25} - \left[ (b_{25})^{(4)} + (b_{25}')^{(4)}(G_{27}, t) + (b_{29})^{(5,5)}(G_{31}, t) + (b_{33})^{(6,6)}(G_{35}, t) \right] T_{25}
\]

\[
d \frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[ (b_{26})^{(4)} + (b_{26}')^{(4)}(G_{27}, t) + (b_{30})^{(5,5)}(G_{31}, t) + (b_{34})^{(6,6)}(G_{45}, t) \right] T_{26}
\]

Where \( (b_{24})^{(4)}(G_{27}, t) \), \( (b_{24}')^{(4)}(G_{27}, t) \), \( (b_{29})^{(5,5)}(G_{33}, t) \), \( (b_{33})^{(6,6)}(G_{35}, t) \) are first detrition coefficients for category 1, 2 and 3

\[
- (b_{24})^{(3,3,3)}(G_{27}, t) - (b_{24}')^{(3,3,3)}(G_{27}, t)
\]

are second detrition coefficients for category 1, 2 and 3

\[
- (b_{29})^{(5,5)}(G_{31}, t) - (b_{29})^{(5,5)}(G_{31}, t)
\]

are third detrition coefficients for category 1, 2 and 3

\[
- (b_{30})^{(5,5)}(G_{31}, t) - (b_{30})^{(5,5)}(G_{31}, t)
\]

are fourth detrition coefficients for category 1, 2 and 3

\[
- (b_{34})^{(6,6)}(G_{45}, t) - (b_{34})^{(6,6)}(G_{35}, t)
\]

are fifth detrition coefficients for category 1, 2, and 3

\[
- (b_{30})^{(3,3,3)}(G_{27}, t) - (b_{30})^{(3,3,3)}(G_{27}, t)
\]

are sixth detrition coefficients for category 1, 2, and 3

\[
d \frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{28} - \left[ (a_{28})^{(5)} + (a_{28}')^{(5)}(T_{29}, t) + (a_{29})^{(4,4)}(T_{25}, t) + (a_{32})^{(6,6)}(T_{33}, t) \right] G_{28}
\]

\[
d \frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{29} - \left[ (a_{29})^{(5)} + (a_{29}')^{(5)}(T_{29}, t) + (a_{29}')^{(4,4)}(T_{25}, t) + (a_{33})^{(6,6)}(T_{33}, t) \right] G_{29}
\]

\[
d \frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{30} - \left[ (a_{30})^{(5)} + (a_{30}')^{(5)}(T_{29}, t) + (a_{30}')^{(4,4)}(T_{25}, t) + (a_{34})^{(6,6)}(T_{33}, t) \right] G_{30}
\]

Where \( (a_{28})^{(5)}(T_{29}, t) \), \( (a_{28}')^{(5)}(T_{29}, t) \), \( (a_{30})^{(5)}(T_{29}, t) \), \( (a_{30}')^{(5)}(T_{29}, t) \) are first augmentation coefficients for category 1, 2 and 3

And \( (a_{28}')^{(4,4)}(T_{29}, t) \), \( (a_{29}')^{(4,4)}(T_{25}, t) \), \( (a_{30}')^{(4,4)}(T_{25}, t) \) are second augmentation coefficient for category 1, 2 and 3

\[
+ (a_{28})^{(6,6)}(T_{29}, t) + (a_{30})^{(6,6)}(T_{33}, t) \]

are third augmentation coefficient for category 1, 2

\[
+ (a_{28})^{(6,6)}(T_{29}, t) + (a_{30})^{(6,6)}(T_{33}, t) + (a_{34})^{(6,6)}(T_{33}, t)
\]
and 3
\[ \begin{align*}
+ (a_{12}^{(1,1,1,1)}(T_{14}, t)) & + (a_{14}^{(1,1,1,1)}(T_{14}, t)) + (a_{16}^{(1,1,1,1)}(T_{14}, t)) \\
\end{align*} \]
are fourth augmentation coefficients for category 1, 2, and 3

+ (a_{22}^{(2,2,2,2)}(T_{17}, t) + (a_{24}^{(2,2,2,2)}(T_{17}, t) + (a_{26}^{(2,2,2,2)}(T_{17}, t) are fifth augmentation coefficients for category

1,2, and 3

+ (a_{23}^{(3,3,3,3)}(T_{21}, t) + (a_{25}^{(3,3,3,3)}(T_{21}, t) + (a_{27}^{(3,3,3,3)}(T_{21}, t) are sixth augmentation coefficients for category 1, 2, and 3

\[
\begin{align*}
dT_{28} \over dt &= (b_{28})^{(5)}(T_{29} - \begin{align*}
- (b_{28}^{(1,1,1,1)}(G_{21}, t) & - (b_{28}^{(4,4,4)}(G_{27}, t) - (b_{28}^{(6,6,6)}(G_{35}, t) \\
- (b_{13}^{(1,1,1,1)}(G_{19}, t) & - (b_{13}^{(2,2,2,2)}(G_{19}, t) - (b_{21}^{(3,3,3,3)}(G_{23}, t) \end{align*} \\
& = T_{28} \end{align*} \]
\end{align*} \]

\[
\begin{align*}
dT_{29} \over dt &= (b_{29})^{(5)}(T_{29} - \begin{align*}
- (b_{29}^{(1,1,1,1)}(G_{21}, t) & - (b_{29}^{(4,4,4)}(G_{27}, t) - (b_{29}^{(6,6,6)}(G_{35}, t) \\
- (b_{14}^{(1,1,1,1)}(G_{19}, t) & - (b_{14}^{(2,2,2,2)}(G_{19}, t) - (b_{21}^{(3,3,3,3)}(G_{23}, t) \end{align*} \\
& = T_{29} \end{align*} \]
\end{align*} \]

\[
\begin{align*}
dT_{30} \over dt &= (b_{30})^{(5)}(T_{29} - \begin{align*}
- (b_{30}^{(1,1,1,1)}(G_{21}, t) & - (b_{30}^{(4,4,4)}(G_{27}, t) - (b_{30}^{(6,6,6)}(G_{35}, t) \\
- (b_{15}^{(1,1,1,1)}(G_{19}, t) & - (b_{15}^{(2,2,2,2)}(G_{19}, t) - (b_{22}^{(3,3,3,3)}(G_{23}, t) \end{align*} \\
& = T_{30} \end{align*} \]
\end{align*} \]

where

- (b_{28}^{(5)}(G_{21}, t) - (b_{28}^{(5)}(G_{21}, t) - (b_{28}^{(5)}(G_{21}, t)\]

are first detrition coefficients

- (b_{29}^{(4,4,4)}(G_{27}, t) - (b_{29}^{(4,4,4)}(G_{27}, t) - (b_{29}^{(4,4,4)}(G_{27}, t) \]

are second detrition coefficients for category 1, 2, and 3

- (b_{28}^{(6,6,6)}(G_{35}, t) - (b_{28}^{(6,6,6)}(G_{35}, t) - (b_{28}^{(6,6,6)}(G_{35}, t) \]

are third detrition coefficients for category 1, 2, and 3

- (b_{23}^{(1,1,1,1)}(G_{23}, t) - (b_{23}^{(1,1,1,1)}(G_{23}, t) - (b_{23}^{(1,1,1,1)}(G_{23}, t) \]

are fourth detrition coefficients for category 1, 2, and 3

- (b_{24}^{(2,2,2,2)}(G_{19}, t) - (b_{24}^{(2,2,2,2)}(G_{19}, t) - (b_{24}^{(2,2,2,2)}(G_{19}, t) \]

are fifth detrition coefficients for category 1, 2, and 3

- (b_{25}^{(3,3,3,3)}(G_{23}, t) - (b_{25}^{(3,3,3,3)}(G_{23}, t) - (b_{25}^{(3,3,3,3)}(G_{23}, t) \]

are sixth detrition coefficients for category 1, 2, and 3

\[
\begin{align*}
dG_{32} \over dt &= (a_{32})^{(6)}(G_{32} - \begin{align*}
+ (a_{32}^{(6)}(T_{32}, t) & + (a_{32}^{(5,5,5)}(T_{29}, t) + (a_{32}^{(4,4,4)}(T_{25}, t) \\
+ (a_{32}^{(1,1,1,1)}(T_{14}, t) & + (a_{32}^{(2,2,2,2)}(T_{21}, t) + (a_{32}^{(3,3,3,3)}(T_{21}, t) \\
& = G_{32} \end{align*} \]
\end{align*} \]

\[
\begin{align*}
dG_{33} \over dt &= (a_{33})^{(6)}(G_{33} - \begin{align*}
+ (a_{33}^{(6)}(T_{33}, t) & + (a_{33}^{(5,5,5)}(T_{29}, t) + (a_{33}^{(4,4,4)}(T_{25}, t) \\
+ (a_{33}^{(1,1,1,1)}(T_{14}, t) & + (a_{33}^{(2,2,2,2)}(T_{21}, t) + (a_{33}^{(3,3,3,3)}(T_{21}, t) \\
& = G_{33} \end{align*} \]
\end{align*} \]

\[
\begin{align*}
dG_{34} \over dt &= (a_{34})^{(6)}(G_{34} - \begin{align*}
+ (a_{34}^{(6)}(T_{34}, t) & + (a_{34}^{(5,5,5)}(T_{29}, t) + (a_{34}^{(4,4,4)}(T_{25}, t) \\
+ (a_{34}^{(1,1,1,1)}(T_{14}, t) & + (a_{34}^{(2,2,2,2)}(T_{21}, t) + (a_{34}^{(3,3,3,3)}(T_{21}, t) \\
& = G_{34} \end{align*} \]
\end{align*} \]

+ (a_{32}^{(5,5,5)}(T_{32}, t) + (a_{32}^{(5,5,5)}(T_{32}, t) + (a_{32}^{(5,5,5)}(T_{32}, t) \]

are first augmentation coefficients for category 1, 2, and 3

+ (a_{33}^{(5,5,5)}(T_{29}, t) + (a_{33}^{(5,5,5)}(T_{29}, t) + (a_{33}^{(5,5,5)}(T_{29}, t) \]

are second augmentation coefficients for category 1, 2, and 3

+ (a_{34}^{(5,5,5)}(T_{25}, t) + (a_{34}^{(5,5,5)}(T_{25}, t) + (a_{34}^{(5,5,5)}(T_{25}, t) \]

are third augmentation coefficients for category 1, 2, and 3

+ (a_{32}^{(1,1,1,1)}(T_{14}, t) + (a_{32}^{(1,1,1,1)}(T_{14}, t) + (a_{32}^{(1,1,1,1)}(T_{14}, t) \]

- are fourth augmentation coefficients

+ (a_{33}^{(2,2,2,2)}(T_{21}, t) + (a_{33}^{(2,2,2,2)}(T_{21}, t) + (a_{33}^{(2,2,2,2)}(T_{21}, t) \]

- are fifth augmentation coefficients

+ (a_{34}^{(3,3,3,3)}(T_{21}, t) + (a_{34}^{(3,3,3,3)}(T_{21}, t) + (a_{34}^{(3,3,3,3)}(T_{21}, t) \]

- are sixth augmentation coefficients

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With the Lipschitz condition, we place a restriction on the behavior of functions. They satisfy the Lipschitz condition:

\[ |(a''_i)(T_{14}, t) - (a''_i)(T_{14}, t)| \leq (k_{13})^{(1)} |T_{14} - T'_{14}| e^{-\tilde{M}_{13}}^{(1)} t \]

\[ |(b''_i)(G', t) - (b''_i)(G, t)| \leq (k_{13})^{(1)} |G' - G| e^{-\tilde{M}_{13}}^{(1)} t \]

With the Lipschitz condition, we place a restriction on the behavior of functions:

\[(a''_i)(T_{14}, t) \text{ and } (a''_i)(T_{14}, t) \text{ and } (T_{14}, t) \text{ and } (T_{14}, t) \text{ are points belonging to the interval} \]

\[ (\tilde{t}_{13})^{(1)}, (\tilde{M}_{13})^{(1)} \]. It is to be noted that \((a''_i)(T_{14}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\tilde{M}_{13})^{(1)} = 1\) then the function \((a''_i)(T_{14}, t)\), the first augmentation coefficient

Where we suppose

\[ (a_i)^{(1)}, (a''_i)^{(1)}, (a''''_i)^{(1)}, (b_i)^{(1)}, (b''_i)^{(1)}, (b''''_i)^{(1)} > 0, \]

\[ i, j = 13, 14, 15 \]

**Definition of** \((p_i)^{(1)}, (r_i)^{(1)}\):

\[ (a''_i)^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (A_{13})^{(1)} \]

\[ (b''_i)^{(1)}(G, t) \leq (r_i)^{(1)} \leq (B_{13})^{(1)} \]

**Definition of** \((A_{13})^{(1)}, (B_{13})^{(1)}\):

\[ \text{lim}_{T_{14} \to \infty} (a''_i)^{(1)}(T_{14}, t) = (p_i)^{(1)} \]

\[ \text{lim}_{G \to \infty} (b''_i)^{(1)}(G, t) = (r_i)^{(1)} \]

Where \((A_{13})^{(1)}, (B_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}\) are positive constants and \(i = 13, 14, 15\)
would be absolutely continuous.

**Definition of (\(M_{13}\)) \(^{(1)}\), (\(k_{13}\)) \(^{(1)}\) :**

(D) \((a_i)^{(1)}\), \((b_i)^{(1)}\), are positive constants

\[ \frac{(a_i)^{(1)}}{(M_{13})^{(1)}} , \frac{(b_i)^{(1)}}{(M_{13})^{(1)}} < 1 \]

**Definition of (\(\hat{M}_{13}\)) \(^{(1)}\), (\(\hat{k}_{13}\)) \(^{(1)}\) :**

(E) There exists two constants (\(\hat{P}_{13}\)) \(^{(1)}\) and (\(\hat{Q}_{13}\)) \(^{(1)}\) which together with (\(M_{13}\)) \(^{(1)}\), (\(k_{13}\)) \(^{(1)}\), (\(\hat{A}_{13}\)) \(^{(1)}\) and (\(\hat{B}_{13}\)) \(^{(1)}\) and the constants (\(a_i\)) \(^{(1)}\), (\(a_i')\) \(^{(2)}\), (\(b_i\)) \(^{(1)}\), (\(b_i')\) \(^{(2)}\), (\(p_i\)) \(^{(1)}\), (\(r_j\)) \(^{(1)}\), \(i = 13, 14, 15\), satisfy the inequalities

\[ \frac{1}{(M_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(2)} + (\hat{A}_{13})^{(1)} + (\hat{B}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1 \]

\[ \frac{1}{(M_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(2)} + (\hat{A}_{13})^{(1)} + (\hat{B}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1 \]

Where we suppose

(F) \((a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16, 17, 18\)

(G) The functions (\(a_i'')^{(2)}, (b_i'')^{(2)}) are positive continuous increasing and bounded.

**Definition of (\(p_i\)) \(^{(2)}\), (\(r_i\)) \(^{(2)}\) :**

\[ (a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \]

\[ (b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (\hat{B}_{16})^{(2)} \]

**Definition of (\(\hat{A}_{16}\)) \(^{(2)}\), (\(\hat{B}_{16}\)) \(^{(2)}\) :**

\[ \lim_{t \to \infty}(a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \]

\[ \lim_{t \to \infty}(b_i'')^{(2)}(G_{19}, t) = (r_i)^{(2)} \]

Where (\(\hat{A}_{16}\)) \(^{(2)}\), (\(\hat{B}_{16}\)) \(^{(2)}\), (\(p_i\)) \(^{(2)}\), (\(r_i\)) \(^{(2)}\) are positive constants and \(i = 16, 17, 18\)

They satisfy Lipschitz condition:

\[ |(a_i'')^{(2)}(T_{17}, t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{A}_{16})^{(2)}|T_{17} - T_{17}|e^{-2}\hat{A}_{16}t \]

\[ |(b_i'')^{(2)}(G_{19}, t) - (b_i'')^{(2)}(G_{19}, t)| \leq (\hat{B}_{16})^{(2)}|G_{19} - G_{19}|e^{-2}\hat{A}_{16}t \]

With the Lipschitz condition, we place a restriction on the behavior of functions (\(a_i''^{(2)}(T_{17}, t)\) and (\(a_i'')^{(2)}(T_{17}, t)\). It is to be noted that (\(a_i''^{(2)}(T_{17}, t)\) is uniformly continuous. In the eventuality of the fact, that if (\(\hat{M}_{16}\)) \(^{(2)}\) = 1 then the function (\(a_i'')^{(2)}(T_{17}, t)\), the SECOND first augmentation coefficient would be absolutely continuous.

**Definition of (\(M_{16}\)) \(^{(2)}\), (\(k_{16}\)) \(^{(2)}\) :**

(I) \((M_{16})^{(2)}, (k_{16})^{(2)}, \) are positive constants

\[ \frac{(a_i)^{(2)}}{(M_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(M_{16})^{(2)}} < 1 \]

**Definition of (\(\hat{M}_{16}\)) \(^{(2)}\), (\(\hat{k}_{16}\)) \(^{(2)}\) :**
There exists two constants \( (\hat{P}_{16})^{(2)} \) and \( (\hat{Q}_{16})^{(2)} \) which together with \( (\hat{R}_{16})^{(2)}, (\hat{K}_{16})^{(2)}, (\hat{A}_{16})^{(2)} \) and \( (\hat{B}_{16})^{(2)} \) and the constants \( (a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16, 17, 18, \) satisfy the inequalities

\[
\frac{1}{(\hat{R}_{16})^{(2)}} [ (a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{K}_{16})^{(2)} ] < 1
\]

\[
\frac{1}{(\hat{R}_{16})^{(2)}} [ (b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{K}_{16})^{(2)} ] < 1
\]

Where we suppose

\( (a_{i})^{(3)}, (a_{i}')^{(3)}, (a_{i}'')^{(3)}, (b_{i})^{(3)}, (b_{i}')^{(3)}, (b_{i}'')^{(3)} > 0, \ i, j = 20, 21, 22 \)

The functions \( (a_{i}'')^{(3)}, (b_{i}'')^{(3)} \) are positive continuous increasing and bounded.

**Definition of** \( (p_i)^{(3)}, (r_i)^{(3)}):\)

\[
(a_{i}'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}
\]

\[
(b_{i}'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (\hat{B}_{20})^{(3)}
\]

\[
\lim_{T_{21} \to \infty} (a_{i}'')^{(3)}(T_{21}, t) = (p_i)^{(3)}
\]

\[
\lim_{G_{23} \to \infty} (b_{i}'')^{(3)}(G_{23}, t) = (r_i)^{(3)}
\]

**Definition of** \( (\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}):\)

Where \( (\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)} \) are positive constants and \( i = 20, 21, 22 \)

They satisfy Lipschitz condition:

\[
|(a_{i}'')^{(3)}(T_{21}', t) - (a_{i}'')^{(3)}(T_{21}, t)| \leq (\hat{K}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{R}_{20})^{(3)}t}
\]

\[
|(b_{i}'')^{(3)}(G_{23}, t) - (b_{i}'')^{(3)}(G_{23}, t)| < (\hat{K}_{20})^{(3)}|G_{23} - G_{23}'|e^{-(\hat{R}_{20})^{(3)}t}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \( (a_{i}'')^{(3)}(T_{21}, t) \)

and \( (a_{i}'')^{(3)}(T_{21}, t) \cdot (T_{21}, t) \) And \( (T_{21}, t) \) are points belonging to the interval \( [\hat{K}_{20}^{(3)}, \hat{B}_{20}^{(3)}] \). It is to be noted that \( (a_{i}'')^{(3)}(T_{21}, t) \) is uniformly continuous. In the eventuality of the fact, that if \( (\hat{R}_{20})^{(3)} = 1 \) then the function \( (a_{i}'')^{(3)}(T_{21}, t) \), the THIRD augmentation coefficient attributable would be absolutely continuous.

**Definition of** \( (\hat{M}_{20})^{(3)}, (\hat{K}_{20})^{(3)}):\)

\( (\hat{M}_{20})^{(3)}, (\hat{K}_{20})^{(3)}, \) are positive constants

\[
\frac{(a_{i})^{(3)}}{(\hat{M}_{20})^{(3)}} \cdot \frac{(b_{i})^{(3)}}{(\hat{K}_{20})^{(3)}} < 1
\]

There exists two constants \( (\hat{P}_{20})^{(3)} \) and \( (\hat{Q}_{20})^{(3)} \) which together with

\( (\hat{M}_{20})^{(3)}, (\hat{K}_{20})^{(3)}, (\hat{A}_{20})^{(3)} \) and \( (\hat{B}_{20})^{(3)} \) and the constants

\( (a_{i})^{(3)}, (a_{i}')^{(3)}, (b_{i})^{(3)}, (b_{i}')^{(3)}, (p_{i})^{(3)}, (r_{i})^{(3)}, i = 20, 21, 22, \)

satisfy the inequalities

\[
\frac{1}{(\hat{M}_{20})^{(3)}} [ (a_{i})^{(3)} + (a_{i}')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{K}_{20})^{(3)} ] < 1
\]

\[
\frac{1}{(\hat{M}_{20})^{(3)}} [ (b_{i})^{(3)} + (b_{i}')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{K}_{20})^{(3)} ] < 1
\]
Where we suppose

(L) \((a_j^{(4)}(a_j')^{(4)}, (a_j'')^{(4)}, (b_j^{(4)}, (b_j')^{(4)}, (b_j'')^{(4)} > 0, \ i, j = 24, 25, 26\)

(M) The functions \((a''_j)^{(4)}, (b''_j)^{(4)}\) are positive continuous increasing and bounded.

**Definition of \((p_i)^{(4)}, (r_i)^{(4)}\):**

\[
(a''_j)^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}
\]

\[
(b''_j)^{(4)}((G_{27}), t) \leq (r_i)^{(4)} \leq (\hat{B}_{24})^{(4)}
\]

\[
\lim_{T_{25} \to 0} (a''_j)^{(4)}(T_{25}, t) = (p_i)^{(4)}
\]

\[
\lim_{G_{27} \to 0} (b''_j)^{(4)}((G_{27}), t) = (r_i)^{(4)}
\]

**Definition of \((\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}\):**

Where \((\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}\) are positive constants and \(i = 24, 25, 26\)

They satisfy Lipschitz condition:

\[
|a''_j(t') - a''_j(T_{25}, t)| \leq (\hat{A}_{24})^{(4)}|t' - T_{25}|e^{-\hat{\Theta}_{24}^{(4)}t}
\]

\[
|b''_j((G_{27}), t) - b''_j((G_{27}), t)| \leq (\hat{B}_{24})^{(4)}|G_{27} - G_{27}|e^{-\hat{\Theta}_{24}^{(4)}t}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a''_j)^{(4)}(T_{25}, t)\) and \((a''_j)^{(4)}(T_{25}, t)\). \((T_{25}, t)\) and \((T_{25}, t)\) are points belonging to the interval \([k_{24}^{(4)}, \hat{\Theta}_{24}^{(4)}]\). It is to be noted that \((a''_j)^{(4)}(T_{25}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\hat{\Theta}_{24}^{(4)} = 1)\) then the function \((a''_j)^{(4)}(T_{25}, t)\), the FOURTH augmentation coefficient would be absolutely continuous.

**Definition of \((\hat{\Theta}_{24}^{(4)}, (\hat{k}_{24})^{(4)}\):**

(N) \((\hat{\Theta}_{24}^{(4)}), (\hat{k}_{24})^{(4)}\), are positive constants

\[
\frac{\hat{\Theta}_{24}^{(4)}}{(\hat{\Theta}_{24}^{(4)})}, (\hat{k}_{24})^{(4)} < 1
\]

**Definition of \((\hat{P}_{24}^{(4)}, (\hat{Q}_{24})^{(4)}\):**

(O) There exists two constants \((\hat{P}_{24}^{(4)}, (\hat{Q}_{24})^{(4)})\) which together with \((\hat{\Theta}_{24}^{(4)}), (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)}\) and \((\hat{B}_{24})^{(4)}\) and the constants

\((a_i^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i^{(4)})^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24, 25, 26,\)

satisfy the inequalities

\[
\frac{1}{(\hat{\Theta}_{24}^{(4)})} \left[ (a_i^{(4)}(a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24}^{(4)})(\hat{k}_{24})^{(4)}\right] < 1
\]

\[
\frac{1}{(\hat{\Theta}_{24}^{(4)})} \left[ (b_i^{(4)}(b_i')^{(4)} + (\hat{B}_{24}^{(4)})(\hat{Q}_{24}^{(4)})(\hat{k}_{24}^{(4)})\right] < 1
\]

Where we suppose

(P) \((a_i^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i^{(5)})^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)} > 0, \ i, j = 28, 29, 30\)

(Q) The functions \((a''_i)^{(5)}, (b''_i)^{(5)}\) are positive continuous increasing and bounded.

**Definition of \((p_i)^{(5)}, (r_i)^{(5)}):**

\[
(a''_i)^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}
\]

\[
(b''_i)^{(5)}((G_{31}), t) \leq (r_i)^{(5)} \leq (\hat{B}_{28})^{(5)}
\]
They satisfy Lipschitz condition:

\[ \lim_{T_2 \rightarrow \infty} (a_i''(t))^{(5)} (T_{29}, t) = (p_i)^{(5)} \]
\[ \lim_{G \rightarrow \infty} (b_i''(t))^{(5)} (G_{31}, t) = (r_i)^{(5)} \]

**Definition of** \((\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}:\)

Where \((\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}\) are positive constants and \(i = 28, 29, 30\)

They satisfy Lipschitz condition:

\[ |(a_i'')^{(5)}(T_{29}, t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t} \]
\[ |(b_i'')^{(5)}((G_{31}), t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)}|((G_{31}) - (G_{31})'|e^{-(\hat{M}_{28})^{(5)}t} \]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i'')^{(5)}(T_{29}, t)\)

and \((a_i'')^{(5)}(T_{29}, t) \cdot T_{29}'\) and \((T_{29}, t)\) are points belonging to the interval \([(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]\).

It is to be noted that \((a_i'')^{(5)}(T_{29}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\hat{M}_{28})^{(5)} = 1\)
then the function \((a_i'')^{(5)}(T_{29}, t)\), the FIFTH augmentation coefficient would be absolutely continuous.

**Definition of** \((\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}:\)

\((\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)}\) and \((\hat{B}_{28})^{(5)}\) which together with the constants

\((a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28, 29, 30\)

satisfy the inequalities

\[ \frac{1}{(\hat{M}_{28})^{(5)}} \left[ (a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{B}_{28})^{(5)} (\hat{k}_{28})^{(5)} \right] < 1 \]
\[ \frac{1}{(\hat{M}_{28})^{(5)}} \left[ (b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)} \right] < 1 \]

Where we suppose

\((a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, i, j = 32, 33, 34\)

**Definition of** \((p_i)^{(6)}, (r_i)^{(6)}:\)

\( (a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)} \)
\( (b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)} \)

\[ \lim_{T_2 \rightarrow \infty} (a_i')^{(6)}(T_{33}, t) = (p_i)^{(6)} \]
\[ \lim_{G \rightarrow \infty} (b_i')^{(6)}((G_{35}), t) = (r_i)^{(6)} \]

**Definition of** \((\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}:\)

Where \((\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}\) are positive constants and \(i = 32, 33, 34\)

They satisfy Lipschitz condition:
\[(a'_{i}(t)^{(6)}(T_{33}, t) - (a_{i}(t)^{(6)}(T_{33}, t) \leq (k_{32})^{(6)}|T_{33} - T_{33}'|e^{-\left(k_{32}\right)^{(6)}t}\)

\[(b'_{i}(t)^{(6)}(G_{33}, t) - (b_{i}(t)^{(6)}(G_{33}, t) < (k_{32})^{(6)}||G_{33}||e^{-\left(k_{32}\right)^{(6)}}t\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a'_{i}(t)^{(6)}(T_{33}, t)\) and\((a_{i}(t)^{(6)}(T_{33}, t)\). \((T_{33}, t)\) and \((T_{33}, t)\) are points belonging to the interval \([k_{32}]^{(6)}, (\bar{M}_{32})^{(6)}\). It is to be noted that \((a'_{i}(t)^{(6)}(T_{33}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\bar{M}_{32})^{(6)} = 1\) then the function \((a'_{i}(t)^{(6)}(T_{33}, t)\), the SIXTH augmentation coefficient would be absolutely continuous.

**Definition of** \((\bar{M}_{32})^{(6)}, (k_{32})^{(6)}\):

\((\bar{M}_{32})^{(6)}, (k_{32})^{(6)}\), are positive constants

\[
\frac{(a_{i})}{(\bar{M}_{32})^{(6)}}, \frac{(b_{i})}{(\bar{M}_{32})^{(6)}} < 1
\]

**Definition of** \((\bar{P}_{32})^{(6)}, (\bar{Q}_{32})^{(6)}\):

There exists two constants \((\bar{P}_{32})^{(6)}\) and \((\bar{Q}_{32})^{(6)}\) which together with \((\bar{M}_{32})^{(6)}, (k_{32})^{(6)}, (\bar{A}_{32})^{(6)}\) and \((\bar{B}_{32})^{(6)}\) and the constants \((a_{i})^{(6)}, (a_{i})^{(6)}, (b_{i})^{(6)}, (b_{i})^{(6)}, (p_{i})^{(6)}, (r_{i})^{(6)}, i = 32, 33, 34,\)
satisfy the inequalities

\[
\frac{1}{(\bar{M}_{32})^{(6)}} [(a_{i})^{(6)} + (a_{i})^{(6)} + (\bar{A}_{32})^{(6)} + (\bar{P}_{32})^{(6)} (\bar{A}_{32})^{(6)}] < 1
\]

\[
\frac{1}{(\bar{M}_{32})^{(6)}} [(b_{i})^{(6)} + (b_{i})^{(6)} + (\bar{B}_{32})^{(6)} + (\bar{Q}_{32})^{(6)} (\bar{B}_{32})^{(6)}] < 1
\]

**Theorem 1:** if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions

**Definition of** \(G_{i}(0), T_{i}(0)\):

\(G_{i}(t) \leq (p_{13})^{(1)} e^{(R_{13})^{(1)} t}, \quad G_{i}(0) = G_{i}^{0} > 0\)

\(T_{i}(t) \leq (Q_{13})^{(1)} e^{(R_{13})^{(1)} t}, \quad T_{i}(0) = T_{i}^{0} > 0\)

If the conditions (F)-(J) above are fulfilled, there exists a solution satisfying the conditions

**Definition of** \(G_{i}(0), T_{i}(0)\):

\(G_{i}(t) \leq (p_{16})^{(2)} e^{(R_{16})^{(2)} t}, \quad G_{i}(0) = G_{i}^{0} > 0\)

\(T_{i}(t) \leq (Q_{16})^{(2)} e^{(R_{16})^{(2)} t}, \quad T_{i}(0) = T_{i}^{0} > 0\)

If the conditions (K)-(O) above are fulfilled, there exists a solution satisfying the conditions

\(G_{i}(t) \leq (p_{20})^{(3)} e^{(R_{20})^{(3)} t}, \quad G_{i}(0) = G_{i}^{0} > 0\)

\(T_{i}(t) \leq (Q_{20})^{(3)} e^{(R_{20})^{(3)} t}, \quad T_{i}(0) = T_{i}^{0} > 0\)

If the conditions (P)-(T) above are fulfilled, there exists a solution satisfying the conditions

**Definition of** \(G_{i}(0), T_{i}(0)\):

\(G_{i}(t) \leq (p_{24})^{(4)} e^{(R_{24})^{(4)} t}, \quad G_{i}(0) = G_{i}^{0} > 0\)

\(T_{i}(t) \leq (Q_{24})^{(4)} e^{(R_{24})^{(4)} t}, \quad T_{i}(0) = T_{i}^{0} > 0\)
If the conditions (U)-(Y) above are fulfilled, there exists a solution satisfying the conditions

**Definition of** \( G_i(0), T_i(0) : \)

\[
G_i(t) \leq \left( \hat{P}_{28} \right)^{5} e^{(\theta_{28})^{5}t}, \quad G_i(0) = G_i^0 > 0
\]

\[
T_i(t) \leq \left( \hat{Q}_{28} \right)^{5} e^{(\theta_{28})^{5}t}, \quad T_i(0) = T_i^0 > 0
\]

**Theorem 1:** if the conditions (Y)-(X4) above are fulfilled, there exists a solution satisfying the conditions

**Definition of** \( G_i(0), T_i(0) : \)

\[
G_i(t) \leq \left( \hat{P}_{32} \right)^{6} e^{(\theta_{32})^{6}t}, \quad G_i(0) = G_i^0 > 0
\]

\[
T_i(t) \leq \left( \hat{Q}_{32} \right)^{6} e^{(\theta_{32})^{6}t}, \quad T_i(0) = T_i^0 > 0
\]

**Proof:** Consider operator \( \mathcal{A}^{(1)} \) defined on the space of sextuples of continuous functions \( G_i, T_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) which satisfy

\[
G_i(0) = G_i^0, \ T_i(0) = T_i^0, \ G_i^0 \leq \left( \hat{P}_{13} \right)^{(1)}, \ T_i^0 \leq \left( \hat{Q}_{13} \right)^{(1)}
\]

\[
0 \leq G_i(t) - G_i^0 \leq \left( \hat{P}_{13} \right)^{(1)} e^{(\theta_{13})^{1}t}
\]

\[
0 \leq T_i(t) - T_i^0 \leq \left( \hat{Q}_{13} \right)^{(1)} e^{(\theta_{13})^{1}t}
\]

By

\[
\bar{G}_{13}(t) = G_{13}(t) + \int_{0}^{t} \left[ \left( a_{13}(1) G_{14}(s_{13}) \right) - \left( (a'_{13}(1) + a''_{13}(1)) T_{14}(s_{13}) \right) \right] ds_{13}
\]

\[
\bar{G}_{14}(t) = G_{14}(t) + \int_{0}^{t} \left[ \left( a_{14}(1) G_{13}(s_{13}) \right) - \left( (a'_{14}(1) + a''_{14}(1)) T_{14}(s_{13}) \right) \right] ds_{13}
\]

\[
\bar{G}_{15}(t) = G_{15}(t) + \int_{0}^{t} \left[ \left( a_{15}(1) G_{14}(s_{13}) \right) - \left( (a'_{15}(1) + a''_{15}(1)) T_{14}(s_{13}) \right) \right] ds_{13}
\]

\[
\bar{T}_{13}(t) = T_{13}(t) + \int_{0}^{t} \left[ \left( b_{13}(1) T_{14}(s_{13}) \right) - \left( (b'_{13}(1) + b''_{13}(1)) G_{14}(s_{13}) \right) \right] ds_{13}
\]

\[
\bar{T}_{14}(t) = T_{14}(t) + \int_{0}^{t} \left[ \left( b_{14}(1) T_{13}(s_{13}) \right) - \left( (b'_{14}(1) + b''_{14}(1)) G_{14}(s_{13}) \right) \right] ds_{13}
\]

\[
\bar{T}_{15}(t) = T_{15}(t) + \int_{0}^{t} \left[ \left( b_{15}(1) T_{14}(s_{13}) \right) - \left( (b'_{15}(1) + b''_{15}(1)) G_{14}(s_{13}) \right) \right] ds_{13}
\]

Where \( s_{13} \) is the integrand that is integrated over an interval \((0, t)\)

**Proof:**

Consider operator \( \mathcal{A}^{(2)} \) defined on the space of sextuples of continuous functions \( G_i, T_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) which satisfy

\[
G_i(0) = G_i^0, \ T_i(0) = T_i^0, \ G_i^0 \leq \left( \hat{P}_{16} \right)^{(2)}, \ T_i^0 \leq \left( \hat{Q}_{16} \right)^{(2)}
\]

\[
0 \leq G_i(t) - G_i^0 \leq \left( \hat{P}_{16} \right)^{(2)} e^{(\theta_{16})^{2}t}
\]

\[
0 \leq T_i(t) - T_i^0 \leq \left( \hat{Q}_{16} \right)^{(2)} e^{(\theta_{16})^{2}t}
\]

By

\[
\bar{G}_{16}(t) = G_{16}(t) + \int_{0}^{t} \left[ \left( a_{16}(2) G_{17}(s_{16}) \right) - \left( (a'_{16}(2) + a''_{16}(2)) T_{17}(s_{16}) \right) \right] ds_{16}
\]

\[
\bar{G}_{17}(t) = G_{17}(t) + \int_{0}^{t} \left[ \left( a_{17}(2) G_{16}(s_{16}) \right) - \left( (a'_{17}(2) + a''_{17}(2)) T_{17}(s_{16}) \right) \right] ds_{16}
\]

\[
\bar{G}_{18}(t) = G_{18}(t) + \int_{0}^{t} \left[ \left( a_{18}(2) G_{17}(s_{16}) \right) - \left( (a'_{18}(2) + a''_{18}(2)) T_{17}(s_{16}) \right) \right] ds_{16}
\]

\[
\bar{T}_{16}(t) = T_{16}(t) + \int_{0}^{t} \left[ \left( b_{16}(2) T_{17}(s_{16}) \right) - \left( (b'_{16}(2) + b''_{16}(2)) G_{17}(s_{16}) \right) \right] ds_{16}
\]
\[ T_{17}(t) = T_{17}^0 + \int_0^t \left[ (b_{17}(s)) T_{16}(s) - \left( (b_{17}(s)) - (b_{17}')(s) \right) G(s, s) T_{17}(s) \right] ds \\
T_{18}(t) = T_{18}^0 + \int_0^t \left[ (b_{18}(s)) T_{17}(s) - \left( (b_{18}(s)) - (b_{18}')(s) \right) G(s, s) T_{18}(s) \right] ds \\
\]

Where \( s(16) \) is the integrand that is integrated over an interval \((0, t)\)

**Proof:**

Consider operator \( \mathcal{A}^{(3)} \) defined on the space of sextuples of continuous functions \( G_i, T_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) which satisfy

\[ G_i(0) = G_i^0, \quad T_i(0) = T_i^0, \quad G_i^0 \leq (\hat{P}_{20})^{(3)}, T_i^0 \leq (\hat{Q}_{20})^{(3)}, \]

\[ 0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(\hat{A}_{20})^{(3)} t} \]

\[ 0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(\hat{A}_{20})^{(3)} t} \]

By

\[ \begin{align*}
\tilde{G}_{20}(t) &= G_{20}^0 + \int_0^t \left[ (a_{20}(s)) G_{21}(s) - \left( (a_{20}(s)) + a_{20}''(s) \right) T_{21}(s) G_{20}(s) \right] ds \\
\tilde{G}_{21}(t) &= G_{21}^0 + \int_0^t \left[ (a_{21}(s)) G_{20}(s) - \left( (a_{21}(s)) + a_{21}''(s) \right) T_{21}(s) G_{21}(s) \right] ds \\
\tilde{G}_{22}(t) &= G_{22}^0 + \int_0^t \left[ (a_{22}(s)) G_{21}(s) - \left( (a_{22}(s)) + a_{22}''(s) \right) T_{21}(s) G_{22}(s) \right] ds \\
\tilde{T}_{20}(t) &= T_{20}^0 + \int_0^t \left[ (b_{20}(s)) T_{21}(s) - \left( (b_{20}(s)) - (b_{20}')(s) \right) G(s, s) T_{20}(s) \right] ds \\
\tilde{T}_{21}(t) &= T_{21}^0 + \int_0^t \left[ (b_{21}(s)) T_{20}(s) - \left( (b_{21}(s)) - (b_{21}')(s) \right) G(s, s) T_{21}(s) \right] ds \\
\tilde{T}_{22}(t) &= T_{22}^0 + \int_0^t \left[ (b_{22}(s)) T_{21}(s) - \left( (b_{22}(s)) - (b_{22}')(s) \right) G(s, s) T_{22}(s) \right] ds \\
\end{align*} \]

Where \( s(20) \) is the integrand that is integrated over an interval \((0, t)\)

**Proof:** Consider operator \( \mathcal{A}^{(4)} \) defined on the space of sextuples of continuous functions \( G_i, T_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) which satisfy

\[ G_i(0) = G_i^0, \quad T_i(0) = T_i^0, \quad G_i^0 \leq (\hat{P}_{24})^{(4)}, T_i^0 \leq (\hat{Q}_{24})^{(4)}, \]

\[ 0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{A}_{24})^{(4)} t} \]

\[ 0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{A}_{24})^{(4)} t} \]

By

\[ \begin{align*}
\tilde{G}_{24}(t) &= G_{24}^0 + \int_0^t \left[ (a_{24}(s)) G_{25}(s) - \left( (a_{24}(s)) + a_{24}''(s) \right) T_{25}(s) G_{24}(s) \right] ds \\
\tilde{G}_{25}(t) &= G_{25}^0 + \int_0^t \left[ (a_{25}(s)) G_{24}(s) - \left( (a_{25}(s)) + a_{25}''(s) \right) T_{25}(s) G_{25}(s) \right] ds \\
\tilde{G}_{26}(t) &= G_{26}^0 + \int_0^t \left[ (a_{26}(s)) G_{25}(s) - \left( (a_{26}(s)) + a_{26}''(s) \right) T_{25}(s) G_{26}(s) \right] ds \\
\tilde{T}_{24}(t) &= T_{24}^0 + \int_0^t \left[ (b_{24}(s)) T_{25}(s) - \left( (b_{24}(s)) - (b_{24}')(s) \right) G(s, s) T_{24}(s) \right] ds \\
\tilde{T}_{25}(t) &= T_{25}^0 + \int_0^t \left[ (b_{25}(s)) T_{24}(s) - \left( (b_{25}(s)) - (b_{25}')(s) \right) G(s, s) T_{25}(s) \right] ds \\
\tilde{T}_{26}(t) &= T_{26}^0 + \int_0^t \left[ (b_{26}(s)) T_{25}(s) - \left( (b_{26}(s)) - (b_{26}')(s) \right) G(s, s) T_{26}(s) \right] ds \\
\end{align*} \]

Where \( s(24) \) is the integrand that is integrated over an interval \((0, t)\)
Proof: Consider operator $A^{(5)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy
\[
G_i(0) = G_i^0, \quad T_i(0) = T_i^0, \quad G_i^0 \leq (\hat{P}_i)_{(5)}, \quad T_i^0 \leq (\hat{Q}_i)_{(5)},
\]
\[
0 \leq G_i(t) - G_i^0 \leq (\hat{P}_i)_{(5)} e^{(\hat{A}_i)_{(5)} t},
\]
\[
0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_i)_{(5)} e^{(\hat{A}_i)_{(5)} t}.
\]
By
\[
\tilde{G}_{28}(t) = G_{28}^0 + \int_0^t \left[ (a_{28})_{(5)} G_{28}(s_{(28)}) - \left( (a'_{28})_{(5)} + (a''_{28})_{(5)} (T_{28}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}
\]
\[
\tilde{G}_{29}(t) = G_{29}^0 + \int_0^t \left[ (a_{29})_{(5)} G_{29}(s_{(29)}) - \left( (a'_{29})_{(5)} + (a''_{29})_{(5)} (T_{29}(s_{(29)}), s_{(29)}) \right) G_{29}(s_{(29)}) \right] ds_{(29)}
\]
\[
\tilde{G}_{30}(t) = G_{30}^0 + \int_0^t \left[ (a_{30})_{(5)} G_{30}(s_{(30)}) - \left( (a'_{30})_{(5)} + (a''_{30})_{(5)} (T_{30}(s_{(30)}), s_{(30)}) \right) G_{30}(s_{(30)}) \right] ds_{(30)}
\]
\[
\tilde{T}_{28}(t) = T_{28}^0 + \int_0^t \left[ (b_{28})_{(5)} T_{28}(s_{(28)}) - \left( (b'_{28})_{(5)} - (b''_{28})_{(5)} (G(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}
\]
\[
\tilde{T}_{29}(t) = T_{29}^0 + \int_0^t \left[ (b_{29})_{(5)} T_{29}(s_{(29)}) - \left( (b'_{29})_{(5)} - (b''_{29})_{(5)} (G(s_{(29)}), s_{(29)}) \right) T_{29}(s_{(29)}) \right] ds_{(29)}
\]
\[
\tilde{T}_{30}(t) = T_{30}^0 + \int_0^t \left[ (b_{30})_{(5)} T_{30}(s_{(30)}) - \left( (b'_{30})_{(5)} - (b''_{30})_{(5)} (G(s_{(30)}), s_{(30)}) \right) T_{30}(s_{(30)}) \right] ds_{(30)}
\]
Where $s_{(28)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:
Consider operator $A^{(6)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy
\[
G_i(0) = G_i^0, \quad T_i(0) = T_i^0, \quad G_i^0 \leq (\hat{P}_{i3})_{(6)}, \quad T_i^0 \leq (\hat{Q}_{i3})_{(6)},
\]
\[
0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{i3})_{(6)} e^{(\hat{A}_{i3})_{(6)} t},
\]
\[
0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{i3})_{(6)} e^{(\hat{A}_{i3})_{(6)} t}.
\]
By
\[
\tilde{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})_{(6)} G_{32}(s_{(32)}) - \left( (a'_{32})_{(6)} + (a''_{32})_{(6)} (T_{32}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}
\]
\[
\tilde{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})_{(6)} G_{33}(s_{(33)}) - \left( (a'_{33})_{(6)} + (a''_{33})_{(6)} (T_{33}(s_{(33)}), s_{(33)}) \right) G_{33}(s_{(33)}) \right] ds_{(33)}
\]
\[
\tilde{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})_{(6)} G_{34}(s_{(34)}) - \left( (a'_{34})_{(6)} + (a''_{34})_{(6)} (T_{34}(s_{(34)}), s_{(34)}) \right) G_{34}(s_{(34)}) \right] ds_{(34)}
\]
\[
\tilde{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})_{(6)} T_{32}(s_{(32)}) - \left( (b'_{32})_{(6)} - (b''_{32})_{(6)} (G(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}
\]
\[
\tilde{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})_{(6)} T_{33}(s_{(33)}) - \left( (b'_{33})_{(6)} - (b''_{33})_{(6)} (G(s_{(33)}), s_{(33)}) \right) T_{33}(s_{(33)}) \right] ds_{(33)}
\]
\[
\tilde{T}_{34}(t) = T_{34}^0 + \int_0^t \left[ (b_{34})_{(6)} T_{34}(s_{(34)}) - \left( (b'_{34})_{(6)} - (b''_{34})_{(6)} (G(s_{(34)}), s_{(34)}) \right) T_{34}(s_{(34)}) \right] ds_{(34)}
\]
Where $s_{(32)}$ is the integrand that is integrated over an interval $(0,t)$

(a) The operator $A^{(1)}$ maps the space of functions satisfying 3 into itself. Indeed it is obvious that
\[
G_{13}(t) \leq G_{13}^0 + \int_0^t \left[ (a_{13})_{(1)} (G_{14}^0 + (\hat{P}_{13})_{(1)} e^{(\hat{A}_{13})_{(1)} t_{(13)}}) + (1 + (a_{13})_{(1)} t) G_{14}^0 + \frac{(a_{13})_{(1)} (\hat{A}_{13})_{(1)}}{(M_{13})_{(1)}} (e^{(\hat{A}_{13})_{(1)} t} - 1) \right] ds_{(13)}
\]
From which it follows that

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\[(G_{13}(t) - G_{13}^0)e^{-(\hat{\theta}_{13})^{(1)}t} \leq \frac{(\hat{\theta}_{13})^{(1)}}{(M_{13})^{(1)}} \left[ (\hat{\theta}_{13})^{(1)} + G_{14}^0 \right] e^{\frac{-(\hat{\theta}_{13})^{(1)}G_{14}^0}{G_{14}^0}} + (\hat{\theta}_{13})^{(1)} \]

\((G^0_t)\) is as defined in the statement of theorem 1

Analogous inequalities hold also for \(G_{14}, G_{15}, T_{13}, T_{14}, T_{15}\)

(b) The operator \(\mathcal{A}^{(2)}\) maps the space of functions satisfying into itself. Indeed it is obvious that

\[G_{16}(t) \leq G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} \left[ (G_{17})^{(6)}e^{(\hat{\theta}_{16})^{(2)}x_{16}} \right) \right] ds_{(16)} = \left(1 + (a_{16})^{(2)} \right) G_{17}^0 + \frac{(a_{16})^{(2)}(\hat{\theta}_{16})^{(2)}}{(M_{16})^{(2)}} \left[ e^{(\hat{\theta}_{16})^{(2)}t} - 1 \right] \]

From which it follows that

\[(G_{16}(t) - G_{16}^0)e^{-(\hat{\theta}_{16})^{(2)}t} \leq \frac{(a_{16})^{(2)}}{(M_{16})^{(2)}} \left[ (\hat{\theta}_{16})^{(2)} + G_{21}^0 \right] e^{\frac{-(\hat{\theta}_{16})^{(2)}G_{21}^0}{G_{21}^0}} + (\hat{\theta}_{16})^{(2)} \]

Analogous inequalities hold also for \(G_{17}, G_{18}, T_{16}, T_{17}, T_{18}\)

(a) The operator \(\mathcal{A}^{(3)}\) maps the space of functions satisfying into itself. Indeed it is obvious that

\[G_{20}(t) \leq G_{20}^0 + \int_0^t \left[ (a_{20})^{(3)} \left[ (G_{21})^{(3)}e^{(\hat{\theta}_{20})^{(3)x_{20}}} \right) \right] ds_{(20)} = \left(1 + (a_{20})^{(3)} \right) G_{21}^0 + \frac{(a_{20})^{(3)}(\hat{\theta}_{20})^{(3)}}{(M_{20})^{(3)}} \left[ e^{(\hat{\theta}_{20})^{(3)}t} - 1 \right] \]

From which it follows that

\[(G_{20}(t) - G_{20}^0)e^{-(\hat{\theta}_{20})^{(3)}t} \leq \frac{(a_{20})^{(3)}}{(M_{20})^{(3)}} \left[ (\hat{\theta}_{20})^{(3)} + G_{25}^0 \right] e^{\frac{-(\hat{\theta}_{20})^{(3)}G_{25}^0}{G_{25}^0}} + (\hat{\theta}_{20})^{(3)} \]

Analogous inequalities hold also for \(G_{21}, G_{22}, T_{20}, T_{21}, T_{22}\)

(b) The operator \(\mathcal{A}^{(4)}\) maps the space of functions satisfying into itself. Indeed it is obvious that

\[G_{24}(t) \leq G_{24}^0 + \int_0^t \left[ (a_{24})^{(4)} \left[ (G_{24})^{(4)}e^{(\hat{\theta}_{24})^{(4)}x_{24}} \right) \right] ds_{(24)} = \left(1 + (a_{24})^{(4)} \right) G_{25}^0 + \frac{(a_{24})^{(4)}(\hat{\theta}_{24})^{(4)}}{(M_{24})^{(4)}} \left[ e^{(\hat{\theta}_{24})^{(4)}t} - 1 \right] \]

From which it follows that

\[(G_{24}(t) - G_{24}^0)e^{-(\hat{\theta}_{24})^{(4)}t} \leq \frac{(a_{24})^{(4)}}{(M_{24})^{(4)}} \left[ (\hat{\theta}_{24})^{(4)} + G_{25}^0 \right] e^{\frac{-(\hat{\theta}_{24})^{(4)}G_{25}^0}{G_{25}^0}} + (\hat{\theta}_{24})^{(4)} \]

\((G_t^0)\) is as defined in the statement of theorem NUMBERED ONE

(c) The operator \(\mathcal{A}^{(5)}\) maps the space of functions satisfying into itself. Indeed it is obvious that

\[G_{28}(t) \leq G_{28}^0 + \int_0^t \left[ (a_{28})^{(5)} \left[ (G_{28})^{(5)}e^{(\hat{\theta}_{28})^{(5)x_{28}}} \right) \right] ds_{(28)} = \left(1 + (a_{28})^{(5)} \right) G_{29}^0 + \frac{(a_{28})^{(5)}(\hat{\theta}_{28})^{(5)}}{(M_{28})^{(5)}} \left[ e^{(\hat{\theta}_{28})^{(5)}t} - 1 \right] \]

From which it follows that

\[(G_{28}(t) - G_{28}^0)e^{-(\hat{\theta}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(M_{28})^{(5)}} \left[ (\hat{\theta}_{28})^{(5)} + G_{29}^0 \right] e^{\frac{-(\hat{\theta}_{28})^{(5)}G_{29}^0}{G_{29}^0}} + (\hat{\theta}_{28})^{(5)} \]

\((G_t^0)\) is as defined in the statement of theorem 1

(d) The operator \(\mathcal{A}^{(6)}\) maps the space of functions satisfying into itself. Indeed it is obvious that
$G_{32}(t) \leq G_{32}^0 + \int_0^t \left[ (a_{32})^6 \left( G_{33}^0 + (\tilde{P}_{32})^6 e^{(\tilde{A}_{32})(6) S_{32}} \right) \right] dS_{32} = (1 + (a_{32})^6 e^{(\tilde{P}_{32})^6 t - 1}) G_{33}^0 + \frac{(a_{32})^6 (\tilde{P}_{32})^6}{(\tilde{A}_{32})(6)} e^{(\tilde{A}_{32})(6) t - 1}

From which it follows that

$$(G_{32}(t) - G_{32}^0) e^{-(\tilde{A}_{32})(6)t} \leq \frac{(a_{32})^6}{(\tilde{A}_{32})(6)} \left[ (\tilde{P}_{32})^6 + G_{33}^0 \left( \frac{-(\tilde{P}_{32})^6 + G_{33}^0}{G_{33}^0} \right) + (\tilde{P}_{32})^6 \right]$$

$(G_{32}^0)$ is as defined in the statement of theorem ONE

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

It is now sufficient to take $\frac{(a_{13})^{(1)}}{(\tilde{A}_{13})(1)} , \frac{(b_{13})^{(1)}}{(\tilde{M}_{13})(1)} < 1$ and to choose

$(\tilde{P}_{13})^{(1)}$ and $(\tilde{Q}_{13})^{(1)}$ large to have

$\frac{(a_{13})^{(1)}}{(\tilde{M}_{13})(1)} \left[ (\tilde{P}_{13})^{(1)} + ((\tilde{P}_{13})^{(1)} + G_{33}^0) e^{-(\tilde{P}_{13})(1) \gamma^0} \right] \leq (\tilde{P}_{13})^{(1)}$

$\frac{(b_{13})^{(1)}}{(\tilde{M}_{13})(1)} \left[ ((\tilde{Q}_{13})^{(1)} + T_{13}^0) e^{-(\tilde{Q}_{13})(1) \gamma^0} + (\tilde{Q}_{13})^{(1)} \right] \leq (\tilde{Q}_{13})^{(1)}$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying 34,35,36 into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric

$d \left( (G_{1}^{(1)}, T_{1}^{(1)}), (\bar{G}_{1}^{(2)}, T_{1}^{(2)}) \right) = \sup_{t \in \mathbb{R}^+} \max \left| G_{1}^{(1)}(t) - \bar{G}_{1}^{(2)}(t) \right| e^{-(\tilde{A}_{13})(1) t}, \max \left| T_{1}^{(1)}(t) - T_{1}^{(2)}(t) \right| e^{-(\tilde{M}_{13})(1) t}$

Indeed if we denote

**Definition of $\bar{G}, \bar{T}$ :**

$\bar{G}, \bar{T} = \mathcal{A}^{(1)}(G, T)$

It results

$|\tilde{G}_{13}^{(1)} - \tilde{G}_{13}^{(2)}| \leq \int_0^t (a_{13})^{(1)} \left| G_{1+4}^{(1)} - G_{1+4}^{(2)} \right| e^{-(\tilde{R}_{13})(1) \gamma_{13}} e^{(\tilde{R}_{13})(1) \gamma_{13}} dS_{13} +$ \n
$\int_0^t ((a_{13}^{(1)})^{(1)} | \tilde{G}_{13}^{(1)} - \tilde{G}_{13}^{(2)} | e^{-(\tilde{R}_{13})(1) \gamma_{13}} e^{-(\tilde{R}_{13})(1) \gamma_{13}} +$ \n
$(a_{13}^{(1)})^{(1)} (T_{14}^{(1)} + S_{13}^{(1)}) | \tilde{G}_{13}^{(1)} - \tilde{G}_{13}^{(2)} | e^{-(\tilde{R}_{13})(1) \gamma_{13}} e^{(\tilde{R}_{13})(1) \gamma_{13}} +$ \n
$G_{13}^{(2)} (a_{13}^{(1)})^{(1)} (T_{14}^{(1)} + S_{13}^{(1)}) - (a_{13}^{(1)})^{(1)} (T_{14}^{(2)} + S_{13}^{(1)}) | e^{-(\tilde{R}_{13})(1) \gamma_{13}} e^{(\tilde{R}_{13})(1) \gamma_{13}} dS_{13}$

Where $S_{13}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$|G_{1}^{(1)} - G_{1}^{(2)}| e^{-(\tilde{R}_{13})(1) t} \leq \frac{1}{(\tilde{M}_{13})(1)} \left( (a_{13})^{(1)} + (a_{13}^{(1)})^{(1)} + (\tilde{A}_{13})^{(1)} + (\tilde{P}_{13})^{(1)} (\tilde{K}_{13})^{(1)} \right) d \left( (G_{1}^{(1)}, T_{1}^{(1)}), (G_{1}^{(2)}, T_{1}^{(2)}) \right)$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a_{13}^{(1)})^{(1)}$ and $(b_{13})^{(1)}$ depending also on $t$ can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition
necessary to prove the uniqueness of the solution bounded by \((P_{13})^{(1)} e^{(M_{13})^{(1)} t}\) and \((Q_{13})^{(1)} e^{(M_{13})^{(1)} t}\) respectively of \(\mathbb{R}_+\).

If instead of proving the existence of the solution on \(\mathbb{R}_+\), we have to prove it only on a compact then it suffices to consider that \((a_i^{(i)})^{(1)}\) and \((b_i^{(i)})^{(1)}\), \(i = 13, 14, 15\) depend only on \(T_{14}\) and respectively on \(G(\text{and not on } t)\) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \(t\) where \(G_i(t) = 0\) and \(T_i(t) = 0\)

From 19 to 24 it results

\[
G_i(t) \geq G_i^0 e^{\int_{t_0}^{t} [a_i^{(i)}(s) - (a_i^{(i)})^{(3)} (T_{14}^{(3)}(x(s))) ds]} \geq 0
\]

\[
T_i(t) \geq T_i^0 e^{(b_i^{(i)})^{(1)}(1)} > 0 \quad \text{for } t > 0
\]

**Definition of** \((M_{13})^{(1)}_1, (M_{13})^{(1)}_2\) and \((M_{13})^{(1)}_3\):

**Remark 3:** if \(G_{13}\) is bounded, the same property have also \(G_{14}\) and \(G_{15}\) - indeed if \(G_{13} < (M_{13})^{(1)}\) it follows \(\frac{dG_{14}}{dt} \leq (M_{13})^{(1)}_1 - (a_{14}^{(1)}) G_{14}\) and by integrating

\[
G_{14} \leq (M_{13})^{(1)}_2 = G_{14}^0 + 2(a_{14})^{(1)} (M_{13})^{(1)}_1 / (a_{14})^{(1)}
\]

In the same way, one can obtain

\[
G_{15} \leq (M_{13})^{(1)}_3 = G_{15}^0 + 2(a_{15})^{(1)} (M_{13})^{(1)}_1 / (a_{15})^{(1)}
\]

If \(G_{14}\) or \(G_{15}\) is bounded, the same property follows for \(G_{13}, G_{15}\) and \(G_{13}, G_{14}\) respectively.

**Remark 4:** If \(G_{13}\) is bounded, from below, the same property holds for \(G_{14}\) and \(G_{15}\). The proof is analogous with the preceding one. An analogous property is true if \(G_{14}\) is bounded from below.

**Remark 5:** If \(T_{13}\) is bounded from below and \(\lim_{t \to \infty} (b_{13}^{(1)})(G(t), t) = (b_{13}^{(1)})\) then \(T_{14} \to \infty\).

**Definition of** \((m)^{(1)}\) and \(\varepsilon_1\):

Indeed let \(t_1\) be so that for \(t > t_1\)

\[
(b_{14})^{(1)} - (b_{13}^{(1)})(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}
\]

Then \(\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}\) which leads to

\[
T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right) \left(1 - e^{-\varepsilon_1 t}\right) + T_{14}^0 e^{-\varepsilon_1 t}. \quad \text{If we take } t \text{ such that } e^{-\varepsilon_1 t} = \frac{1}{2} \text{ it results}
\]

\[
T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = \log \frac{2}{\varepsilon_1} \text{ By taking now } \varepsilon_1 \text{ sufficiently small one sees that } T_{14} \text{ is unbounded.}
\]

The same property holds for \(T_{15}\) if \(\lim_{t \to \infty} (b_{15}^{(1)})(G(t), t) = (b_{15}^{(1)})\)

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take \(\frac{(a_{12})^{(2)}}{(M_{12})^{(2)}} \cdot \frac{(b_{12})^{(2)}}{(M_{12})^{(2)}} < 1\) and to choose

\((P_{16})^{(2)}\) and \((Q_{16})^{(2)}\) large to have

\[
\left(\frac{(a_{12})^{(2)}}{(M_{12})^{(2)}} \right) \left[ (P_{16})^{(2)} + ((P_{16})^{(2)} + G_1^0 e^{-\frac{(P_{16})^{(2)} + a_{12}^{(2)}(P_{16})^{(2)}}{a_1}} \right] \leq (P_{16})^{(2)}
\]
In order that the operator $A^{(2)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying 34, 35, 36 into itself

The operator $A^{(2)}$ is a contraction with respect to the metric

\[ d \left( \left( (G_{19})^{(2)}, (T_{19})^{(1)} \right), \left( (G_{19})^{(2)}, (T_{19})^{(2)} \right) \right) = \]

\[ \sup_i \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(R_{16})^{(2)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(R_{16})^{(2)}t} \]

Indeed if we denote

**Definition of $G_{19}, T_{19}$**: $\left( G_{19}, T_{19} \right) = A^{(2)}(G_{19}, T_{19})$

It results

\[ |G_{16}^{(1)} - G_{16}^{(2)}| \leq \int_0^t (a_{16}^{(2)}) G_{16}^{(1)} - G_{16}^{(2)} |e^{-(R_{16})^{(2)}s_{16}}| e^{(R_{16})^{(2)}s_{16}} dS_{16} + \]

\[ \int_0^t (a_{16}^{(2)}) G_{16}^{(1)} - G_{16}^{(2)} |e^{-(R_{16})^{(2)}s_{16}}| e^{(R_{16})^{(2)}s_{16}} dS_{16} + \]

\[ (a_{16}^{(2)}) (T_{19}^{(1)} - T_{19}^{(2)}) |G_{16}^{(1)} - G_{16}^{(2)} |e^{-(R_{16})^{(2)}s_{16}}| e^{(R_{16})^{(2)}s_{16}} + \]

\[ G_{16}^{(2)} (a_{16}^{(2)} (T_{19}^{(1)})) |G_{16}^{(1)} - G_{16}^{(2)} |e^{-(R_{16})^{(2)}s_{16}}| e^{(R_{16})^{(2)}s_{16}} dS_{16} \]

Where $s_{16}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

\[ |(G_{19})^{(1)} - (G_{19})^{(2)}|e^{-(R_{16})^{(2)}t} \leq \]

\[ \frac{1}{(M_{16})^{(2)}} \left( (a_{16}^{(2)}) + (a_{16}^{(2)}) + (A_{16})^{(2)} + (P_{16})^{(2)} (K_{16})^{(2)} \right) d \left( ((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}; (T_{19})^{(2)}) \right) \]

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis (34, 35, 36) the result follows

**Remark 1**: The fact that we supposed $(a_{16}^{(2)})$ and $(b_{16}^{(2)})$ depending also on $t$ can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(P_{16})^{(2)} e^{(M_{16})^{(2)}t}$ and $(Q_{16})^{(2)} e^{(R_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_{16}^{(2)})$ and $(b_{16}^{(2)})$, $i = 16, 17, 18$ depend only on $T_{17}$ and respectively on $(G_{19})$ and on $t$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2**: There does not exist any $t$ where $G_i(t) = 0$ and $T_i(t) = 0$

From global equations it results

\[ G_i(t) \geq G_i^0 e^{-(a_{16}^{(2)} - (T_{17}(s_{16}) s_{16})) dS_{16}} \geq 0 \]

\[ T_i(t) \geq T_i^0 e^{-(b_{16}^{(2)}t)} > 0 \text{ for } t > 0 \]

**Definition of** $(\bar{M}_{16})^{(2)}_1, (\bar{M}_{16})^{(2)}_2$ and $(\bar{M}_{16})^{(2)}_3$:

**Remark 3**: if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$ . Indeed if
\( G_{16} < (\overline{M}_{16})^{(2)} \) it follows \( \frac{d G_{17}}{dt} \leq ((\overline{M}_{16})^{(2)})_1 - (a_{17})^{(2)} G_{17} \) and by integrating
\[ G_{17} \leq ((\overline{M}_{16})^{(2)})_2 = G_{17}^0 + 2(a_{17})^{(2)}((\overline{M}_{16})^{(2)})_1/(a_{17})^{(2)} \]
In the same way, one can obtain
\[ G_{18} \leq ((\overline{M}_{16})^{(2)})_3 = G_{18}^0 + 2(a_{18})^{(2)}((\overline{M}_{16})^{(2)})_2/(a_{18})^{(2)} \]
If \( G_{17} \) or \( G_{18} \) is bounded, the same property follows for \( G_{16}, G_{18} \) and \( G_{16}, G_{17} \) respectively.

**Remark 4:** If \( G_{16} \) is bounded, from below, the same property holds for \( G_{17} \) and \( G_{18} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{17} \) is bounded from below.

**Remark 5:** If \( T_{16} \) is bounded from below and \( \lim_{t \to \infty} (b_i)^{(2)}((G_{19})(t), t) = (b_i)^{(2)} \) then \( T_{17} \to \infty \).

**Definition of** \((m)^{(2)}\) and \( \varepsilon_2 \):

Indeed let \( t_2 \) be so that for \( t > t_2 \)
\[ (b_{17})^{(2)} - (b_{17})^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)} \]
Then \( \frac{dt_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17} \) which leads to
\[ T_{17} \geq \left( \frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2} \right) \left( 1 - e^{-\varepsilon_2 t} \right) T_{17} e^{-\varepsilon_2 t} \]
If we take \( t \) such that \( e^{-\varepsilon_2 t} = \frac{1}{2} \) it results
\[ T_{17} \geq \left( \frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2} \right), \ t = \log \frac{1}{2} \varepsilon_2 \]
By taking now \( \varepsilon_2 \) sufficiently small one sees that \( T_{17} \) is unbounded.

The same property holds for \( T_{18} \) if \( \lim_{t \to \infty} (b_{18})^{(2)}((G_{19})(t), t) = (b_{18})^{(2)} \)

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42.

It is now sufficient to take \( \frac{(a_{20})^{(3)}}{(M_{20})^{(3)}} < 1 \) and to choose
\[ (\hat{P}_{20})^{(3)} \text{ and } (\hat{Q}_{20})^{(3)} \text{ large to have} \]
\[ \frac{(a_{20})^{(3)}}{(M_{20})^{(3)}} \left[ (\hat{P}_{20})^{(3)} + (G_{20})^{(3)} + G_{20}^{(4)} e^{-\left(\frac{(\hat{P}_{20})^{(3)}+\delta_{20}}{\delta_{20}}\right)} \right] \leq (\hat{P}_{20})^{(3)} \]
\[ \frac{(b_{20})^{(3)}}{(M_{20})^{(3)}} \left[ (\hat{Q}_{20})^{(3)} + T_{20}^{(3)} e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+\delta_{20}}{\delta_{20}}\right)} \right] \leq (\hat{Q}_{20})^{(3)} \]

In order that the operator \( \mathcal{A}^{(3)} \) transforms the space of sextuples of functions \( G_i, T_i \) satisfying 34,35,36 into itself.

The operator \( \mathcal{A}^{(3)} \) is a contraction with respect to the metric
\[ d \left( (G_{23})^{(1)}, (T_{23})^{(1)}, (G_{23})^{(2)}, (T_{23})^{(2)} \right) = \sup_{t \in \mathbb{R}_{\geq0}} \left[ |G_{i}^{(1)}(t) - G_{i}^{(2)}(t)| + |T_{i}^{(1)}(t) - T_{i}^{(2)}(t)| e^{-\left(\frac{(M_{20})^{(3)}t}{\delta_{20}}\right)} \right] \]

Indeed if we denote

**Definition of** \( \hat{G}_{23}, \hat{T}_{23} : (G_{23}), (T_{23}) = \mathcal{A}^{(3)}((G_{23}), (T_{23})) \)

It results
\[|\mathcal{G}_{20}^{(1)} - \mathcal{G}_{i}^{(2)}| \leq \int_0^t (a_{20})^{(3)} |\mathcal{G}_{21}^{(1)} - G_{21}^{(2)}| e^{-((\mathcal{M}_{20})^{(3)})(\xi_{20})} e^{((\mathcal{M}_{20})^{(3)})(\xi_{20})} dS_{20}(t) + \int_0^t ((a_{20}^{(3)})(G_{20}^{(1)} - G_{20}^{(2)})) e^{-((\mathcal{M}_{20})^{(3)})(\xi_{20})} e^{((\mathcal{M}_{20})^{(3)})(\xi_{20})} + (a_{20}^{(3)})(T_{21}^{(1)})(S_{20}(t)) | G_{20}^{(1)} - G_{20}^{(2)} | e^{-((\mathcal{M}_{20})^{(3)})(\xi_{20})} e^{(\mathcal{M}_{20})^{(3)})(\xi_{20})} + \]

Remark 1: The fact that we supposed \((a_{20}^{(3)})\) and \((b_{20}^{(3)})\) depending also on \(t\) can be considered as not conform with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((\mathcal{P}_{20})^{(3)} e^{(\mathcal{M}_{20})^{(3)}(t)}\) and \((\mathcal{Q}_{20})^{(3)} e^{(\mathcal{M}_{20})^{(3)}(t)}\) respectively of \(\mathbb{R}_+\).

From the hypotheses, it follows
\[|G^{(1)} - G^{(2)}| e^{-((\mathcal{M}_{20})^{(3)}(t))} \leq \frac{1}{((\mathcal{M}_{20})^{(3)}(t))^3} ((a_{20}^{(3)}(t) + (a_{20}^{(3)}(t) + (\mathcal{M}_{20})^{(3)}(t) + (\mathcal{P}_{20})^{(3)}(T_{20})(t))) \int ((G_{20}^{(1)}), (T_{20})(t), (G_{20}^{(2)}), (T_{20})(t))) \]

And analogous inequalities for \(G_i\) and \(T_i\). Taking into account the hypothesis \((34,35,36)\) the result follows

Remark 2: There does not exist any \(t\) where \(G_i(t) = 0\) and \(T_i(t) = 0\)

From 19 to 24 it results
\[G_1(t) \geq G_0^0 e^{-\int_0^t [(a_1^{(3)} - (a_1^{(3)}(T_{21}^{(1)}(\xi_{20})])) d\xi_{20}(t)]} \geq 0\]
\[T_1(t) \geq T_0^1 e^{-[(b_1^{(3)}(t))] > 0 \text{ for } t > 0}\]

Definition of \((\mathcal{M}_{20})^{(3)}(1), (\mathcal{M}_{20})^{(3)}(2)\) and \((\mathcal{M}_{20})^{(3)}(3)\): 

Remark 3: if \(G_{20}\) is bounded, the same property have also \(G_{21}\) and \(G_{22}\) - indeed if \(G_{20} < ((\mathcal{M}_{20})^{(3)}(t))\) it follows \(\frac{dG_{21}}{dt} \leq ((\mathcal{M}_{20})^{(3)}(t)) - (a_{21}^{(3)}) G_{21}\) and by integrating
\[G_{21} \leq ((\mathcal{M}_{20})^{(3)}(t)) = G_{20}^0 + 2(a_{21}^{(3)})((\mathcal{M}_{20})^{(3)}(t)) / (a_{21}^{(3)})\]

In the same way, one can obtain
\[G_{22} \leq ((\mathcal{M}_{20})^{(3)}(t)) = G_{20}^0 + 2(a_{22}^{(3)})((\mathcal{M}_{20})^{(3)}(t)) / (a_{22}^{(3)})\]

If \(G_{21}\) or \(G_{22}\) is bounded, the same property follows for \(G_{20}, G_{22}\) and \(G_{20}, G_{21}\) respectively.

Remark 4: If \(G_{20}\) is bounded, from below, the same property holds for \(G_{21}\) and \(G_{22}\). The proof is analogous with the preceding one. An analogous property is true if \(G_{21}\) is bounded from below.

Remark 5: If \(T_{20}\) is bounded from below and \(\lim_{t \to \infty} ((b_{21}^{(3)})(t), (t)) = (b_{21}^{(3)})(t)\) then \(T_{21} \to \infty\).

Definition of \((m)^{(3)}\) and \(\varepsilon_3\):

Indeed let \(t_3\) be so that for \(t > t_3\)
\[(b_{21}^{(3)} - (b_{21}^{(3)}((G_{22})(t), t) < \varepsilon_3, T_{20}(t) > (m)^{(3)})\]
Then \( \frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_5 T_{21} \) which leads to

\[
T_{21} \geq \left( \frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_5} \right) (1 - e^{-\varepsilon_5 t}) + T_{21}^0 e^{-\varepsilon_5 t}
\]

If we take \( t \) such that \( e^{-\varepsilon_5 t} = \frac{1}{2} \) it results

\[
T_{21} \geq \left( \frac{(a_{21})^{(3)}(m)^{(3)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_5}
\]

By taking now \( \varepsilon_5 \) sufficiently small one sees that \( T_{21} \) is unbounded.

The same property holds for \( T_{22} \) if \( \lim_{t \to +\infty} (b''_{22})^{(3)} ((G_{22})(t), t) = (b''_{22})^{(3)} \)

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take \( \frac{(a_j^{(4)})}{(M_{24})^{(4)}} \) and \( \frac{(b_j^{(4)})}{(M_{24})^{(4)}} \) to be large to have

\[
\left( \cap_{j=1}^4 \right) \left( \frac{(a_j^{(4)})}{(M_{24})^{(4)}} + \frac{(b_j^{(4)})}{(M_{24})^{(4)}} \right) \leq \left( \frac{(P_{24})^{(4)}}{(M_{24})^{(4)}} \right)
\]

In order that the operator \( \mathcal{A}^{(4)} \) transforms the space of sextuples of functions \( G_i, T_i \) into itself

The operator \( \mathcal{A}^{(4)} \) is a contraction with respect to the metric

\[
d \left( \left( G_{22} \right)^{(1)}, \left( T_{22} \right)^{(1)} \right), \left( \left( G_{22} \right)^{(2)}, \left( T_{22} \right)^{(2)} \right) = \sup \left( \max_{t \in \mathbb{R}^+} \left| G_i^{(1)}(t) - G_i^{(2)}(t) \right| e^{-\left( (M_{24})^{(4)} \right) t}, \max_{t \in \mathbb{R}^+} \left| T_i^{(1)}(t) - T_i^{(2)}(t) \right| e^{-\left( (M_{24})^{(4)} \right) t} \right)
\]

Indeed if we denote

**Definition of** \( \mathcal{A}^{(4)}((G_{27}), (T_{27})) : \quad (G_{27}), (T_{27}) = \mathcal{A}^{(4)}((G_{27}), (T_{27})) \)

It results

\[
\left| G_{24}^{(1)} - G_{24}^{(2)} \right| \leq \int_0^t (a_{24})^{(4)} \left| G_{25}^{(1)} - G_{25}^{(2)} \right| e^{-((M_{24})^{(4)})(24)} e^{((M_{24})^{(4)})(s)} ds +
\]

\[
\int_0^t ((a_{24})^{(4)} G_{24}^{(1)} - G_{24}^{(2)})(t) e^{-((M_{24})^{(4)})(24)} e^{((M_{24})^{(4)})(s)} ds +
\]

\[
(a_{24}^{(4)})(T_{25}^{(1)}, s_{(24)}) (G_{25}^{(1)} - G_{25}^{(2)}) e^{-((M_{24})^{(4)})(24)} e^{((M_{24})^{(4)})(s)} ds +
\]

Where \( s_{(24)} \) represents integrand that is integrated over the interval \( [0, t] \)

From the hypotheses it follows

\[
\left| (G_{27})^{(1)} - (G_{27})^{(2)} \right| e^{-((M_{24})^{(4)})(1)} \leq \frac{1}{(M_{24})^{(4)}} (a_{24})^{(4)} + (a_{24}^{(4)}) + (a_{24}^{(4)}) + (P_{24})^{(4)} \left( (G_{27})^{(1)}, (T_{27})^{(1)} \right)
\]

And analogous inequalities for \( G_i \) and \( T_i \). Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed \( (a''_{24})^{(4)} \) and \( (b''_{24})^{(4)} \) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \( \left( P_{24} \right)^{(4)} e^{((M_{24})^{(4)})(1)} \) and \( \left( Q_{24} \right)^{(4)} e^{((M_{24})^{(4)})(1)} \)
respectively of \( \mathbb{R}_+ \).

If instead of proving the existence of the solution on \( \mathbb{R}_+ \), we have to prove it only on a compact then it suffices to consider that \((a_i^{(4)}) \) and \((b_i^{(4)}) \), \(i = 24, 25, 26 \) depend only on \( T_25 \) and respectively on \((G_27)\) (and not on \( t \)) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \( t \) where \( G_1 (t) = 0 \) and \( T_1 (t) = 0 \)

From 19 to 24 it results

\[
G_1 (t) \geq G_1^0 e^{\left[ - \int_0^t (a_j^{(4)} - a_j^{(4)} (T_25 (x_25) \alpha (x_25) )) dx_25 \right]} \geq 0
\]

\[
T_1 (t) \geq T_1^0 e^{(-b_j^{(4)})} > 0 \quad \text{for } t > 0
\]

**Definition of \((\overline{M_{24}})^{(4)}_1, (\overline{M_{24}})^{(4)}_2 \) and \((\overline{M_{24}})^{(4)}_3 \):**

**Remark 3:** If \( G_{24} \) is bounded, the same property have also \( G_{25} \) and \( G_{26} \). Indeed if

\[
G_{24} \leq (\overline{M_{24}})^{(4)}_1 \leq (\overline{M_{24}})^{(4)}_2 - (a_{25})^{(4)} G_{25} \quad \text{and} \quad G_{25} \leq (\overline{M_{24}})^{(4)}_2 = G_{25}^0 + 2(a_{25})^{(4)} (\overline{M_{24}})^{(4)}_2 / (a_{26})^{(4)}
\]

In the same way one can obtain

\[
G_{26} \leq (\overline{M_{24}})^{(4)}_3 = G_{26}^0 + 2(a_{26})^{(4)} (\overline{M_{24}})^{(4)}_3 / (a_{26})^{(4)}
\]

If \( G_{25} \) or \( G_{26} \) is bounded, the same property follows for \( G_{24} \), \( G_{25} \) and \( G_{24}, G_{25} \) respectively.

**Remark 4:** If \( G_{24} \) is bounded, from below, the same property holds for \( G_{25} \) and \( G_{26} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{25} \) is bounded from below.

**Remark 5:** If \( T_{24} \) is bounded from below and \( \lim_{t \to \infty} ((b_i^{(4)}) \ (G_{27} (t), t)) = (b_{25}^{(4)}) \) then \( T_{25} \to \infty \).

**Definition of \((m)^{(4)} \) and \( \epsilon_4 \):**

Indeed let \( t_4 \) be so that for \( t > t_4 \)

\[
(b_{25})^{(4)} - (b_i^{(4)}) (G_{27}) (t), t) < \epsilon_4, T_{24} (t) > (m)^{(4)}
\]

Then \( \frac{dT_{25}}{dt} \geq (a_{25})^{(4)} (m)^{(4)} - \epsilon_4 T_{25} \) which leads to

\[
T_{25} \geq \left( \frac{(a_{25})^{(4)} (m)^{(4)}}{\epsilon_4} \right) (1 - e^{-\epsilon_4 t}) + T_{25}^0 e^{-\epsilon_4 t}
\]

If we take \( t \) such that \( e^{-\epsilon_4 t} = \frac{1}{2} \) it results

\[
T_{25} \geq \left( \frac{(a_{25})^{(4)} (m)^{(4)}}{2} \right), \quad t = \log \frac{2}{\epsilon_4}
\]

By taking now \( \epsilon_4 \) sufficiently small one sees that \( T_{25} \) is unbounded.

The same property holds for \( T_{26} \) if \( \lim_{t \to \infty} (b_i^{(4)}) (G_{27}) (t), t) = (b_{26}^{(4)}) \)

We now state a more precise theorem about the behaviors at infinity of the solutions ANALOGOUS inequalities hold also for \( G_{29}, G_{30}, T_{29}, T_{29}, T_{30} \)

It is now sufficient to take \( \frac{(a_i^{(5)})}{(M_{28})^{(5)}} < \frac{(b_i^{(5)})}{(M_{28})^{(5)}} \) and to choose

\[
\left( \tilde{P}_{28} \right)^{(5)} \text{and} \left( \tilde{Q}_{28} \right)^{(5)} \text{large to have}
\]

\[
\frac{(a_i^{(5)})}{(M_{28})^{(5)}} \left[ \left( \tilde{P}_{28} \right)^{(5)} + \left( \tilde{P}_{28} \right)^{(5)} + C_{ij}^{(5)} e^{-\frac{\left( Q_{28}^{(5)} + T_{28}^{(5)} \left( Q_{28}^{(5)} + T_{28}^{(5)} \right) \right)}{\tau_{ij}}} \right] \leq \left( \tilde{P}_{28} \right)^{(5)}
\]

\[
\frac{(b_i^{(5)})}{(M_{28})^{(5)}} \left[ \left( \tilde{Q}_{28} \right)^{(5)} + T_{ij}^{(5)} e^{-\frac{\left( Q_{28}^{(5)} + T_{28}^{(5)} \left( Q_{28}^{(5)} + T_{28}^{(5)} \right) \right)}{\tau_{ij}}} + \left( \tilde{Q}_{28} \right)^{(5)} \right] \leq \left( \tilde{Q}_{28} \right)^{(5)}
\]
In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric

$$d \left( (G_{31}^{(1)}, T_{31}^{(1)}), (G_{31}^{(2)}, T_{31}^{(2)}) \right) =$$

$$\sup_{t \in [a, b]} \left| g_1^{(1)}(t) - g_1^{(2)}(t) \right| e^{-\left(\mathcal{M}_{28}^{(5)}\right) t}, \sup_{t \in [a, b]} \left| g_2^{(1)}(t) - g_2^{(2)}(t) \right| e^{-\left(\mathcal{M}_{28}^{(5)}\right) t}$$

Indeed if we denote

**Definition of** $\left(G_{31}, T_{31}\right): \left( (G_{31}), (T_{31}) \right) = \mathcal{A}^{(5)}\left( (G_{31}), (T_{31}) \right)$

It results

$$\left| \tilde{G}_{28}^{(1)} - \tilde{G}_{28}^{(2)} \right| \leq \int_{0}^{T} \left( a_{29}(s) \right)^{(5)} \left| g_{29}^{(1)} - g_{29}^{(2)} \right| e^{-\left(\mathcal{M}_{28}^{(5)}\right) s_{28}} e^{\left(\mathcal{M}_{28}^{(5)}\right) s_{28}} ds_{28} +$$

$$\int_{0}^{\infty} \left( b_{29}(s) \right)^{(5)} \left| g_{29}^{(1)} - g_{29}^{(2)} \right| e^{-\left(\mathcal{M}_{28}^{(5)}\right) s_{28}} e^{\left(\mathcal{M}_{28}^{(5)}\right) s_{28}} +$$

$$\int_{0}^{T} \left( \tilde{g}_{29}^{(1)} - \tilde{g}_{29}^{(2)} \right) e^{-\left(\mathcal{M}_{28}^{(5)}\right) s_{28}} e^{\left(\mathcal{M}_{28}^{(5)}\right) s_{28}} ds_{28} =$$

$$\int_{0}^{T} \left( \tilde{g}_{29}^{(1)} - \tilde{g}_{29}^{(2)} \right) e^{-\left(\mathcal{M}_{28}^{(5)}\right) s_{28}} e^{\left(\mathcal{M}_{28}^{(5)}\right) s_{28}} ds_{28}$$

Where $s_{28}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\frac{1}{\left(\mathcal{M}_{28}^{(5)}\right)^{(s)}} \left( a_{29}^{(5)} + (a_{29}^{(5)} + (A_{29}^{(5)} + (\tilde{G}_{28}^{(5)} + (\tilde{k}_{28}^{(5)}}) d \left( (G_{31}^{(1)}, T_{31}^{(1)}), (G_{31}^{(2)}, T_{31}^{(2)}) \right) \right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a_{29}^{(5)}$ and $(b_{29}^{(5)}$ depending also on $t$ can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\tilde{G}_{28}^{(5)} e^{\left(\mathcal{M}_{28}^{(5)}\right) t}$ and $(\tilde{G}_{28}^{(5)} e^{\left(\mathcal{M}_{28}^{(5)}\right) t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_{29}^{(5)}$ and $(b_{29}^{(5)}$, $i = 28, 29, 30$ depend only on $T_{29}$ and respectively on $(G_{31})$ and not on $t$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0$ and $T_i(t) = 0$

From 19 to 28 it results

$$g_i(t) \geq g_i^0 e^{-\int_{0}^{t} \left( a_{29}^{(5)} - (a_{29}^{(5)} \right) ds_{28}) |ds_{28}|} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-\left( b_{29}^{(5)} - (b_{29}^{(5)} \right) |t| > 0 \ for \ t > 0$$

**Definition of** $\left( (\mathcal{M}_{28}^{(5)})_1, (\mathcal{M}_{28}^{(5)})_2 \text{ and } (\mathcal{M}_{28}^{(5)})_3 \right)$

**Remark 3:** if $G_{28}$ is bounded, the same property have also $G_{29}$ and $G_{30}$ - indeed if

$$G_{28} < (\mathcal{M}_{28}^{(5)}) \text{ it follows } \frac{dG_{28}}{dt} \leq (\mathcal{M}_{28}^{(5)})_1 + (a_{29}^{(5)} G_{29} \text{ and by integrating }$$

$$G_{29} \leq (\mathcal{M}_{28}^{(5)})_2 = G_{29} + 2(a_{29}^{(5)} (\mathcal{M}_{28}^{(5)})_1 / (a_{29}^{(5)}$$

In the same way, one can obtain
If \( G_{30} \) or \( G_{30} \) is bounded, the same property follows for \( G_{28}, G_{30} \) and \( G_{28}, G_{29} \) respectively.

**Remark 4:** If \( G_{28} \) is bounded, from below, the same property holds for \( T_{29} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{29} \) is bounded from below.

**Remark 5:** If \( G_{30} \) is bounded from below and \( \lim_{t \to \infty} (b_{30})^{(5)} \left(G_{31}(t), t\right) = (b_{30})^{(5)} \) then \( T_{29} \to \infty \).

**Definition of** \( (m)^{(5)} \) and \( \varepsilon_{5} \):

Indeed let \( t_{5} \) be so that for \( t > t_{5} \)

\[
(b_{29})^{(5)} - (b_{29})^{(5)}(\left(G_{31}(t), t\right) < \varepsilon_{5}, T_{29}(t) > (m)^{(5)}
\]

Then \( \frac{dT_{29}}{dt} \geq \left(a_{29}(m)^{(5)}(m)^{(5)} - \varepsilon_{5}T_{29}\right) \) which leads to

\[T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)} - \varepsilon_{5}}{\varepsilon_{5}}\right)(1 - e^{-\varepsilon_{5}t}) + T_{29}^{0}e^{-\varepsilon_{5}t}
\]

If we take \( t \) such that \( e^{-\varepsilon_{5}t} = \frac{1}{2} \) it results

\[T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)} - \varepsilon_{5}}{\varepsilon_{5}}\right), t \to \log \frac{2}{\varepsilon_{5}}\]

By taking now \( \varepsilon_{5} \) sufficiently small one sees that \( T_{29} \) is unbounded.

The same property holds for \( T_{30} \) if \( \lim_{t \to \infty} (b_{30})^{(5)} \left(G_{31}(t), t\right) = (b_{30})^{(5)} \)

We now state a more precise theorem about the behaviors at infinity of the solutions ANALOGOUS inequalities hold also for \( G_{33}, G_{34}, T_{32}, T_{33}, T_{34} \)

It is now sufficient to take \( \left(\frac{(a_{i})^{(6)}}{(M_{i})^{(6)}}\right), \left(\frac{(b_{j})^{(6)}}{(M_{j})^{(6)}}\right) < 1 \) and to choose

\((\tilde{P}_{32})^{(6)} + (\tilde{Q}_{32})^{(6)}\) large to have

\[
\left(\frac{(a_{i})^{(6)}}{(M_{i})^{(6)}}\right) \left(\tilde{P}_{32}^{(6)} + \left(\frac{(a_{i})^{(6)}}{(M_{i})^{(6)}} + (\tilde{P}_{32}^{(6)} + \tilde{Q}_{32}^{(6)}) \right)^{\frac{1}{2}}\right) \leq \left(\tilde{P}_{32}^{(6)}\right)^{(6)}
\]

\[
\left(\frac{(b_{j})^{(6)}}{(M_{j})^{(6)}}\right) \left(\tilde{Q}_{32}^{(6)} + \tilde{T}_{j}^{(6)}(\frac{(D_{i})^{(6)} + \tilde{T}_{j}^{(6)}(\frac{1}{\tilde{T}_{j}^{(6)}})}{\tilde{T}_{j}^{(6)}} + (\tilde{Q}_{32}^{(6)})^{(6)}\right) \leq \left(\tilde{Q}_{32}^{(6)}\right)^{(6)}
\]

In order that the operator \( \mathcal{A}^{(6)} \) transforms the space of sextuples of functions \( G_{i}, T_{i} \) into itself

The operator \( \mathcal{A}^{(6)} \) is a contraction with respect to the metric

\[
d \left(\left(G_{33}^{(1)}, T_{35}^{(1)}\right), \left(G_{35}^{(2)}, T_{35}^{(2)}\right)\right) = \sup \left(\max_{t \in \mathbb{R}_{+}} \left|G_{i}^{(1)}(t) - G_{i}^{(2)}(t)\right| e^{-\left(M_{i}\right)^{(6)}t}, \max_{t \in \mathbb{R}_{+}} \left|T_{i}^{(1)}(t) - T_{i}^{(2)}(t)\right| e^{-\left(M_{i}\right)^{(6)}t}\right)\]

Indeed if we denote

**Definition of** \( (G_{35}, T_{35}) : (G_{35}, T_{35}) = \mathcal{A}^{(6)}(G_{35}, T_{35})\)

It results

\[
\int_{0}^{t} \left(\frac{(a_{i})^{(6)}}{(M_{i})^{(6)}}\right) \left[G_{i}^{(1)} - G_{i}^{(2)} \right] e^{-\left(M_{i}\right)^{(6)}t} e^{\left(M_{i}\right)^{(6)}t} ds_{i}^{(6)} + \left(\frac{(a_{i})^{(6)}}{(M_{i})^{(6)}}\right) \left[T_{i}^{(1)} - T_{i}^{(2)} \right] e^{-\left(M_{i}\right)^{(6)}t} e^{\left(M_{i}\right)^{(6)}t} ds_{i}^{(6)} + \left(\frac{(a_{i})^{(6)}}{(M_{i})^{(6)}}\right) \left[T_{i}^{(1)} - T_{i}^{(2)} \right] e^{-\left(M_{i}\right)^{(6)}t} e^{\left(M_{i}\right)^{(6)}t} ds_{i}^{(6)}
\]

Indeed if we denote

**Definition of** \( (G_{35}, T_{35}) : (G_{35}, T_{35}) = \mathcal{A}^{(6)}(G_{35}, T_{35})\)
Where \( s_{i(0)} \) represents integrand that is integrated over the interval \([0,t]\)

From the hypotheses it follows

\[
\left| (G_{35})^{(1)} - (G_{35})^{(2)} \right| e^{-\left(\theta_{32}\right)^{6} t} \leq \frac{1}{(\theta_{32})^{6}} \left( (a_{32})^{(6)} + (a_{32}''')^{(6)} + (a_{32}''')^{(6)} + (\theta_{32})^{6} (\theta_{32})^{(6)} d \left( ((G_{35})^{(1)}; (T_{33})^{(1)}; (G_{35})^{(2)}; (T_{33})^{(2)}) \right) \right)
\]

And analogous inequalities for \( G_{i} \) and \( T_{i} \). Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed \((a_{32}''')^{(6)} \) and \((b_{32}''')^{(6)} \) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((\theta_{32})^{6} e^{-\left(\theta_{32}\right)^{6} t} \) and \((\theta_{32})^{6} e^{-\left(\theta_{32}\right)^{6} t} \) respectively of \( \mathbb{R}_{+} \).

If instead of proving the existence of the solution on \( \mathbb{R}_{+} \), we have to prove it only on a compact then it suffices to consider that \((a_{i''})^{(6)} \) and \((b_{i''})^{(6)} \), \( i = 32,33,34 \) depend only on \( T_{33} \) and respectively on \((G_{35})\) (and not on \( t \)) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \( t \) where \((a_{i''})^{(6)} \) and \((b_{i''})^{(6)} \) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((\theta_{32})^{6} e^{-\left(\theta_{32}\right)^{6} t} \) and \((\theta_{32})^{6} e^{-\left(\theta_{32}\right)^{6} t} \) respectively of \( \mathbb{R}_{+} \).

From 69 to 32 it results

\[
G_{i}(t) \geq G_{i}^{0} e^{-\left| \int_{0}^{t} (a_{i''})^{(6)} - (a_{i''})^{(6)} (\theta_{32}^{6} (s_{32}) \theta_{32}^{6}) ds_{32} \right|} \geq 0
\]

\[
T_{i}(t) \geq T_{i}^{0} e^{-\left| (b_{i''})^{(6)} \right|} > 0 \quad \text{for } t > 0
\]

**Definition of** \((\bar{M}_{32})^{(6)}_{1}, (\bar{M}_{32})^{(6)}_{2} \) and \((\bar{M}_{32})^{(6)}_{3} \):

**Remark 3:** If \( G_{32} \) is bounded, the same property have also \( G_{33} \) and \( G_{34} \). Indeed if

\[
G_{32} < (\bar{M}_{32})^{(6)} \text{ it follows } \frac{dG_{32}}{dt} \leq (\bar{M}_{32})^{(6)} \text{ and by integrating}
\]

\[
G_{33} \leq (\bar{M}_{32})^{(6)}_{2} = G_{33}^{0} + 2(a_{33})^{(6)} (\bar{M}_{32})^{(6)}_{1} / (a_{33}''')^{(6)}
\]

In the same way, one can obtain

\[
G_{34} \leq (\bar{M}_{32})^{(6)}_{3} = G_{34}^{0} + 2(a_{34})^{(6)} (\bar{M}_{32})^{(6)}_{2} / (a_{34}''')^{(6)}
\]

If \( G_{32} \) or \( G_{34} \) is bounded, the same property follows for \( G_{32}, G_{34} \) and \( G_{32}, G_{33} \) respectively.

**Remark 4:** If \( G_{32} \) is bounded, from below, the same property holds for \( G_{33} \) and \( G_{34} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{33} \) is bounded from below.

**Remark 5:** If \( T_{32} \) is bounded from below and \( \lim_{t \to \infty} (b_{i''})^{(6)} ((G_{35})^{(t)} t, t) = (b_{i''})^{(6)} \) then \( T_{33} \to \infty \).
We now state a more precise theorem about the behaviors at infinity of the solutions.

**Behavior of the solutions**

**Theorem 2:** If we denote and define

- **Definition of** $(\sigma_1)(1)$, $(\sigma_2)(1)$, $(\tau_1)(1)$, $(\tau_2)(1)$:
  \[ \sigma_3(1), \sigma_2(1), \tau_1(1), \tau_2(1) \]

- By 
  \[ \sqrt{a_{13}(1)} \leq -(a_{14}(1) - (a_{13}(1)(T_{14}, t) + (a_{14}(1)(T_{14}, t) - (a_{13}(1))^2) \leq -\sigma_1(1) \]

- \[ (\tau_2)(1) \leq -b_{14}(1) - b_{13}(1)(G, t) - b_{14}(1)(G, t) \leq -\tau_1(1) \]

**Definition of** $(v_1)(1)$, $(v_2)(1)$, $(u_1)(1)$, $(u_2)(1)$:

- If we denote and define 
  \[ (a_{14}(1)(v_1)(1)^2 + (a_{1}(1)(v_1)(1) - (a_{13}(1)^2 + (b_{14}(1)(u_1)(1)^2 + (\tau_1)(1)^2u_1)(1) - (b_{13}(1)) \]

**Definition of** $(\bar{y}_1)(1)$, $(\bar{y}_2)(1)$, $(\bar{u}_1)(1)$, $(\bar{u}_2)(1)$:

- By 
  \[ a_{14}(1)(v_1)(1)^2 + (a_{1}(1)(v_1)(1) - (a_{13}(1)^2 + (b_{14}(1)(u_1)(1)^2 + (\tau_1)(1)^2u_1)(1) - (b_{13}(1)) \]

**Definition of** $(m_1)(1)$, $(m_2)(1)$, $(\mu_1)(1)$, $(\mu_2)(1)$, $(v_0)(1)$:

- If we define 
  \[ m_1(1) = (v_0)(1), m_2(1) = (v_1)(1), m_3(1) = (v_2)(1) \]

- By 
  \[ (v_0)(1) = \frac{v_0}{e_{14}}, (v_1)(1) = (v_1)(1) \]

and analogously

**Definition of** $(\bar{v}_1)(1)$, $(\bar{v}_2)(1)$, $(\bar{u}_1)(1)$, $(\bar{u}_2)(1)$:

- \[ (\bar{v}_0)(1) = \frac{\bar{v}_0}{e_{14}}, (\bar{v}_1)(1) = (\bar{v}_1)(1) \]

and analogously

- \[ (\mu_1)(1) = (u_0)(1), (\mu_2)(1) = (u_1)(1), (\mu_3)(1) = (u_2)(1) \]

- \[ (\bar{v}_0)(1) = \frac{u_0}{e_{14}}, (\bar{v}_1)(1) = (\bar{v}_1)(1) \]

- \[ (\bar{u}_0)(1) = \frac{u_0}{e_{14}}, (\bar{u}_1)(1) = (\bar{u}_1)(1) \]

- \[ (\mu_2)(1) = (u_2)(1), (\mu_3)(1) = (u_0)(1), (\mu_4)(1) = (u_1)(1) \]

are defined by 59 and 61 respectively.

Then the solution satisfies the inequalities

\[ G_{13} e^{((s_1)(1) - (p_1)(1))t} \leq G_{13}(t) \leq G_{13} e^{((s_1)(1) + (p_1)(1))t} \]

where $(p_j)(1)$ is defined by equation 25

\[ \frac{1}{(m_1)(1)^2} G_{13} e^{((s_1)(1) - (p_1)(1))t} \leq G_{14}(t) \leq \frac{1}{(m_2)(1)^2} G_{13} e^{((s_1)(1) + (p_1)(1))t} \]

\[ G_{13} e^{((s_1)(1) - (p_1)(1))t} \leq G_{15}(t) \leq G_{13} e^{((s_1)(1) + (p_1)(1))t} \]

\[ T_{13}(R_1)(1) \leq T_{13}(t) \leq T_{13} e^{((R_1)(1) + (R_1)(1))t} \]
\[
\frac{1}{(\mu_1)^2} T_{11}^0 e^{(R_1(1))t} \leq T_{13}(t) \leq \frac{1}{(\mu_2)^2} T_{12}^0 e^{((R_1(1))+(R_3(1)))t} \\
\]
\[
\frac{b_{13}^0}{(\mu_1)^3} \left[ e^{(R_1(1))t} - e^{-(b_{13}(1))t} \right] + T_{15}^0 e^{-(b_{13}(1))t} \leq T_{15}(t) \leq T_{15}^0 e^{-(b_{13}(1))t} + \frac{b_{13}^0}{(\mu_1)^3} \left[ e^{((R_1(1))+(R_3(1)))t} - e^{-(R_2(1))t} \right] \\
\]

**Definition of \((S_1)(1), (S_2)(1), (R_1)(1), (R_2)(1)\):**

Where \((S_1)(1) = (a_{13}(1)(m_2(1) - (a_{13}(1)(1) \left( S_2(1) = (a_{13}(1) - (p_{13}(1)(1) \left( R_2(1) = (b_{13}(1) - (r_{13}(1)(1) \\
\]

**Behavior of the solutions**

If we denote and define

**Definition of \((\sigma_1)(2), (\sigma_2)(2), (\tau_1)(2), (\tau_2)(2)\):**

\(\sigma_1)(2), (\sigma_2)(2), (\tau_1)(2), (\tau_2)(2)\) four constants satisfying

\[-(\sigma_1)(2) \leq -(a_{16}(2) + (a_{17}(2) - (a_{16}(2)\left(T_{17}, t) + (a_{17}(2)\left(T_{17}, t) \leq -(\sigma_1)(2) \\
-(\tau_1)(2) \leq -(b_{16}(2) + (b_{17}(2) - (b_{16}(2)\left(G_{16}, t) - (b_{17}(2)\left(G_{16}, t) \leq -(\tau_1)(2) \\
\]

**Definition of \((v_1)(2), (v_2)(2), (u_1)(2), (u_2)(2)\):**

By \((v_2)(2) > 0, (v_2)(2) < 0 \text{ and respectively } (u_1)(2) > 0, (u_2)(2) < 0 \text{ the roots}\)

\[
(e) \quad \text{of the equations } (a_{17}(2)(v(2))^2 + (\sigma_1)(2)v(2) - (a_{16}(2) = 0 \\
\text{and } (b_{16}(2))^2 + (\tau_1)(2)u(2) - (b_{16}(2) = 0 \text{ and} \\
\]

**Definition of \((\tilde{v}_1)(2), (\tilde{v}_2)(2), (\tilde{u}_1)(2), (\tilde{u}_2)(2)\):**

By \((\tilde{v}_2)(2) > 0, (\tilde{v}_2)(2) < 0 \text{ and respectively } (\tilde{u}_1)(2) > 0, (\tilde{u}_2)(2) < 0 \text{ the roots}\)

\[
\text{of the equations } (a_{17}(2)(v(2))^2 + (\sigma_2)(2)v(2) - (a_{16}(2) = 0 \\
\text{and } (b_{17}(2))^2 + (\tau_2)(2)u(2) - (b_{16}(2) = 0 \text{ and} \\
\]

**Definition of \((m_1)(2), (m_2)(2), (\mu_1)(2), (\mu_2)(2)\):**

\[(m_2)(2) = (v_0)(2), (m_1)(2) = (v_1)(2), \text{ if } (v_0)(2) < (v_1)(2) \\
(m_2)(2) = (v_0)(2), (m_1)(2) = (\tilde{v}_1)(2), \text{ if } (v_0)(2) < (v_0)(2) < (\tilde{v}_1)(2), \\
\text{and } (V_0)(2) = \frac{1}{a_{16}} \frac{1}{a_{17}} \\
(m_2)(2) = (v_1)(2), (m_1)(2) = (v_0)(2), \text{ if } (\tilde{v}_1)(2) < (v_0)(2) \\
\text{and analogously} \\
(\mu_2)(2) = (u_0)(2), (\mu_1)(2) = (u_1)(2), \text{ if } (u_0)(2) < (u_1)(2) \\
(\mu_2)(2) = (u)(2), (\mu_1)(2) = (\tilde{u})(2), \text{ if } (u)(2) < (u_0)(2) < (\tilde{u})(2), \\
\text{and } (U_0)(2) = \frac{1}{a_{16}} \frac{1}{a_{17}} \\
\]
If we define 
\[ (\mu_2) = (u_2) (2), (\mu_0) = (u_0) (2), \]
then the solution of 19, 20, 21, 22, 23 and 24 satisfies the inequalities
\[ G_{16}^0 ((S_1) - (p_{16}))^2 t \leq G_{16}(t) \leq G_{16}^0 ((S_2) - (p_{16}))^2 t \]
\( (p_1) \) is defined by equation

\[
\frac{1}{(m_1)^2} G_{16}^0 ((S_1) - (p_{16}))^2 t \leq G_{17}(t) \leq \frac{1}{(m_2)^2} G_{16}^0 ((S_2) - (p_{16}))^2 t
\]

\[
\left( (a_{16})^2 G_{16} \right) \left[ e^{((S_1) - (p_{16}))^2 t} - e^{-(S_2) - (p_{16})^2 t} \right] + G_{14}^0 e^{-\frac{(S_2)}{2} t} \leq G_{14}(t) \leq \frac{1}{(m_2)^2} G_{16}^0 \left( (a_{16})^2 \right) \left[ e^{((S_1) - (p_{16}))^2 t} - e^{-(a_{16})^2 t} \right] + G_{14}^0 e^{-\frac{(a_{16})^2}{2} t}
\]

\[
\frac{1}{(m_1)^2} T_{16}^0 e^{(R_1)^2 t} \leq T_{16}(t) \leq \frac{1}{(m_2)^2} T_{16}^0 e^{(R_2)^2 t}
\]

\[
\left( b_{16}^2 \right)^2 T_{16}^0 \left[ e^{(R_1)^2 t} - e^{-(b_{16})^2 t} \right] + T_{18}^0 e^{-\frac{(b_{16})^2}{2} t} \leq T_{18}(t) \leq \left( b_{16}^2 \right)^2 T_{16}^0 \left[ e^{(R_2)^2 t} - e^{-(b_{16})^2 t} \right] + T_{18}^0 e^{-\frac{(b_{16})^2}{2} t}
\]

**Definition of \((S_1)^2, (S_2)^2, (R_1)^2, (R_2)^2\):**

Where \((S_1)^2 = (a_{16})^2 (m_2)^2 - (p_{16})^2 \)
\((S_2)^2 = (a_{16})^2 - (p_{16})^2 \)
\((R_1)^2 = (b_{16})^2 (\mu_2)^2 - (b_{16})^2 \)
\((R_2)^2 = (b_{16})^2 - (\tau_{16})^2 \)

**Behavior of the solutions**

**Theorem 2:** If we denote and define

\[ \sigma_1^3, \sigma_2^3, (\tau_1)^3, (\tau_2)^3 \]

\[-(\sigma_1)^3 \leq -(a_{20})^3 + (a_{12})^3 - (a_{02})^3 (T_{21}, t) + (a_{20})^3 (T_{21}, t) \leq -(\sigma_1)^3 \]

\[-(\tau_2)^3 \leq -(b_{20})^3 + (b_{21})^3 - (b_{20})^3 (G, t) - (b_{21})^3 (G_{23}, t) \leq -(\tau_2)^3 \]

**Definition of \((v_1)^3, (v_2)^3, (u_1)^3, (u_2)^3\):**

By \((v_1)^3 > 0, (v_2)^3 < 0\) and respectively \((u_1)^3 > 0, (u_2)^3 < 0\) the roots of the equations
\[(a_{21})^3 (v)^3 + (a_{20})^3 (v)^3 = 0 \]
and \[(b_{21})^3 (u)^3 + (b_{20})^3 (u)^3 = 0 \]
By \((\tilde{v}_1)^3 > 0, (\tilde{v}_2)^3 < 0\) and respectively \((\tilde{u}_1)^3 > 0, (\tilde{u}_2)^3 < 0\) the roots of the equations
\[(a_{21})^3 (v)^3 + (b_{20})^3 (v)^3 = 0 \]
and \[(b_{21})^3 (u)^3 + (b_{20})^3 (u)^3 = 0 \]

**Definition of \((m_1)^3, (m_2)^3, (\mu_1)^3, (\mu_2)^3\):**

If we define \((m_1)^3, (m_2)^3, (\mu_1)^3, (\mu_2)^3\) by
\[(m_2)^3 = (v_0)^3, (m_1)^3 = (v_1)^3, \quad \text{if} \quad (v_0)^3 < (v_1)^3\]
(m_2)(3) = (v_1)(3), (m_1)(3) = (v_2)(3), if (v_1)(3) < (v_0)(3) < (v_1)(3),
and
(v_0)(3) = \frac{r_2^0}{r_{21}}
(m_2)(3) = (v_1)(3), (m_1)(3) = (v_0)(3), if (v_2)(3) < (v_0)(3)
and analogously
(\mu_2)(3) = (u_0)(3), (\mu_1)(3) = (u_1)(3), if (u_0)(3) < (u_1)(3)

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

\begin{align*}
G_{20}^0 & \leq G_{21}(t) \leq G_{20}^0 e^{(S_1)(3)t} \\
\frac{1}{(m_2)(3)}G_{20}^0 e^{((S_1)(3) - p_{20}(3))t} & \leq G_{21}(t) \leq \frac{1}{(m_2)(3)}G_{20}^0 e^{(S_1)(3)t} \\
(\frac{a_20(3)u_0(3)}{(m_2)(3)((S_1)(3) - p_{20}(3)) - (S_2)(3))} & \left[ e^{((S_1)(3) - p_{20}(3))t} - e^{-(S_2)(3)t} \right] + G_{22}^0 e^{-(S_2)(3)t} \leq G_{20}^0 e^{(R_1)(3)t} \leq T_{20}(t) & \leq T_{20}^0 e^{((R_1)(3) + (R_2)(3))t} \\
\frac{1}{(m_2)(3)}T_{20}^0 & \leq T_{20}(t) \leq \frac{1}{(m_2)(3)}T_{20}^0 e^{((R_1)(3) + (R_2)(3))t} \\
(\frac{a_20(3)u_0(3)}{(m_2)(3)((R_1)(3) - p_{20}(3)) + (R_2)(3))} & \left[ e^{((R_1)(3) + (R_2)(3))t} - e^{-(R_2)(3)t} \right] + T_{22}^0 e^{-(R_2)(3)t} \leq T_{20}(t) \leq T_{20}^0 e^{((R_1)(3) + (R_2)(3))t} \\
(\frac{a_20(3)u_0(3)}{(m_2)(3)((R_1)(3) + (R_2)(3))} & \left[ e^{((R_1)(3) + (R_2)(3))t} - e^{-(R_2)(3)t} \right] + T_{22}^0 e^{-(R_2)(3)t} \leq T_{20}(t) \leq T_{20}^0 e^{((R_1)(3) + (R_2)(3))t}
\end{align*}

**Definition of \((S_1)(3), (S_2)(3), (R_1)(3), (R_2)(3)\):**

Where \((S_1)(3) = (a_20(3)(m_2)(3) - (a_20)(3))\)
\((S_2)(3) = (a_20(3) - (p_{22})(3))\)
\((R_1)(3) = (b_{20}(3)(\mu_2)(3) - (b_{20}(3))\)
\((R_2)(3) = (b_{20}(3) - (r_{22})(3))\)

**Behavior of the solutions**

**Theorem 2:** If we denote and define

**Definition of \((\sigma_1)(4), (\sigma_2)(4), (\tau_1)(4), (\tau_2)(4)\):**

\((\sigma_1)(4), (\sigma_2)(4), (\tau_1)(4), (\tau_2)(4)\) four constants satisfying
\(- (\sigma_1)(4) \leq - (a_{24}(4)) + (a_{25}(4) - (a_{26}(4))(T_{25}, t) + (a_{25}(4))(T_{25}, t) \leq - (\sigma_1)(4)\)
\(- (\tau_2)(4) \leq - (b_{24}(4)) + (b_{25}(4) - (b_{26}(4))((G_{27}, t) - (b_{25}(4))((G_{27}, t) \leq - (\tau_2)(4)\)

**Definition of \((v_1)(4), (v_2)(4), (u_1)(4), (u_2)(4), (v)(4), (u)(4)\):**

By \((v_1)(4) > 0, (v_2)(4) < 0 \) and respectively \((u_1)(4) > 0, (u_2)(4) < 0 \) the roots of the equations
\((a_{23}(4)(v)(4))^2 + (\sigma_2(4)v)(4) - (a_{24}(4)) = 0\)
and \((b_{23})^4(u^{(4)})^2 + (r_4)^4u^{(4)} - (b_{24})^4 = 0\) and

**Definition of** \((\tilde{v}_1^{(4)}), (\tilde{v}_2^{(4)}), (\tilde{u}_1^{(4)}), (\tilde{u}_2^{(4)})^4:\)**

By \((\tilde{v}_1^{(4)}) > 0, (\tilde{v}_2^{(4)}) < 0\) and respectively \((\tilde{u}_1^{(4)}) > 0, (\tilde{u}_2^{(4)}) < 0\) the roots of the equations \((a_{25})^4(v^{(4)})^2 + (\sigma_2)^4v^{(4)} - (a_{24})^4 = 0\)

and \((b_{25})^4(u^{(4)})^2 + (r_2)^4u^{(4)} - (b_{24})^4 = 0\)

**Definition of** \((m_1^{(4)}), (m_2^{(4)}), (\mu_1^{(4)}), (\mu_2^{(4)}), (v_0^{(4)})^4: -\)**

(f) If we define \((m_1^{(4)}), (m_2^{(4)}), (\mu_2^{(4)}), (\mu_2^{(4)})\) by

\[(m_2^{(4)}) = (v_0^{(4)}), (m_1^{(4)}) = (v_1^{(4)}), \text{if } (v_0^{(4)}) < (v_1^{(4)})\]

\[(m_2^{(4)}) = (v_1^{(4)}), (m_1^{(4)}) = (v_0^{(4)}), \text{if } (v_0^{(4)}) < (v_1^{(4)})\]

and analogously

\[(\mu_2^{(4)}) = (u_0^{(4)}), (\mu_1^{(4)}) = (u_1^{(4)}), \text{if } (u_0^{(4)}) < (u_1^{(4)})\]

\[(\mu_2^{(4)}) = (u_1^{(4)}), (\mu_1^{(4)}) = (u_0^{(4)}), \text{if } (u_0^{(4)}) < (u_1^{(4)})\]

are defined respectively.

Then the solution satisfies the inequalities

\[G_26^0e^{((r_1)^4-(r_2)^4)t} \leq G_{24}(t) \leq G_24^0e^{((s_1)^4)^4}t\]

where \((p_i)^4\) is defined by equation IN THE FOREGOING:

\[
\frac{1}{(m_2)^4}G_24^0e^{((s_1)^4-(r_2)^4)4t} \leq G_24(t) \leq \frac{1}{(m_2)^4}G_24^0e^{((s_1)^4)^4}t \\
\left(\frac{(a_{23})^4(a_{24})^4}{(m_1)^4((s_1)^4-(r_2)^4)^4} \left[\left(\frac{(s_1)^4-(r_2)^4}{(s_1)^4} e^{((s_1)^4-(r_2)^4)4t} - e^{((s_1)^4)^4}t\right) + G_{26}^0e^{-(s_1)^4}t \right] \right) + G_{26}^0e^{-(s_1)^4}t \leq G_{26}(t) \leq \left(\frac{(a_{23})^4(a_{24})^4}{(m_2)^4((s_1)^4-(r_2)^4)^4} \left[\left(\frac{(s_1)^4-(r_2)^4}{(s_1)^4} e^{((s_1)^4-(r_2)^4)4t} - e^{((s_1)^4)^4}t\right) + G_{26}^0e^{-(s_1)^4}t \right] \right) + G_{26}^0e^{-(s_1)^4}t \\
T_{24}^0e^{((r_1)^4)4t} \leq T_{24}(t) \leq T_{24}^0e^{((r_1)^4+(r_2)^4)4t}t \\
\frac{1}{(r_2)^4}T_{24}^0e^{((r_1)^4)4t}t \leq T_{24}(t) \leq \frac{1}{(r_2)^4}T_{24}^0e^{((r_1)^4+(r_2)^4)4t}t \\
\left(\frac{(b_{23})^4(b_{24})^4}{(r_2)^4((r_1)^4+(r_2)^4)^4} \left[\left(\frac{(r_1)^4}{(r_1)^4} e^{((r_1)^4+(r_2)^4)4t} - e^{-(r_2)^4}t\right) + T_{26}^0e^{-(r_2)^4}t \right] \right) + T_{26}^0e^{-(r_2)^4}t \leq T_{26}(t) \leq \left(\frac{(b_{23})^4(b_{24})^4}{(r_2)^4((r_1)^4+(r_2)^4)^4} \left[\left(\frac{(r_1)^4}{(r_1)^4} e^{((r_1)^4+(r_2)^4)4t} - e^{-(r_2)^4}t\right) + T_{26}^0e^{-(r_2)^4}t \right] \right) + T_{26}^0e^{-(r_2)^4}t \\
\text{Definition of } (S_1)^4, (S_2)^4, (R_1)^4, (R_2)^4:\)

Where \((S_1)^4 = (a_{23})^4(m_2)^4 - (a_{24})^4\)

\[(S_2)^4 = (a_{23})^4 - (p_2)^4\]

\[(R_1)^4 = (b_{23})^4(m_2)^4 - (b_{24})^4\]
\[(R_2)^{(4)} = (b_{28})^{(4)} - (r_{28})^{(4)}\]

**Behavior of the solutions**

**Theorem 2:** If we denote and define

**Definition of** \((\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)} :\)

\[(\sigma_2)^{(5)} \leq - (a_{29}^{(5)}) + (a_{29}^{(5)}) - (a_{29}^{(5)})(T_{29}, t) + (a_{29}^{(5)})(T_{29}, t) \leq - (\sigma_1)^{(5)}\]

\[ - (\tau_2)^{(5)} \leq - (b_{29}^{(5)}) + (b_{29}^{(5)}) - (b_{29}^{(5)})(G_{31}, t) - (b_{29}^{(5)})(G_{31}, t) \leq - (\tau_1)^{(5)}\]

**Definition of** \((v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)} :\)

\[(h) \quad \text{By} \quad (v_1)^{(5)} > 0, (v_2)^{(5)} < 0 \quad \text{and respectively} \quad (u_1)^{(5)} > 0, (u_2)^{(5)} < 0 \quad \text{the roots of the equations} \]

\[(a_{29}^{(5)})(v^{(5)})^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28}^{(5)}) = 0\]

and \((b_{29}^{(5)})(u^{(5)})^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28}^{(5)}) = 0\)

**Definition of** \((\tilde{v}_1)^{(5)}, (\tilde{v}_2)^{(5)}, (\tilde{u}_1)^{(5)}, (\tilde{u}_2)^{(5)} :\)

\[\text{By} \quad (\tilde{v}_1)^{(5)} > 0, (\tilde{v}_2)^{(5)} < 0 \quad \text{and respectively} \quad (\tilde{u}_1)^{(5)} > 0, (\tilde{u}_2)^{(5)} < 0 \quad \text{the roots of the equations} \]

\[(a_{29}^{(5)})(v^{(5)})^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28}^{(5)}) = 0\]

and \((b_{29}^{(5)})(u^{(5)})^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28}^{(5)}) = 0\)

**Definition of** \((m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)} :\)

\[(i) \quad \text{If we define} \quad (m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)} \quad \text{by} \]

\[(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}, \quad \text{if} \quad (v_0)^{(5)} < (v_1)^{(5)}\]

\[(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}, \quad \text{if} \quad (v_1)^{(5)} < (v_0)^{(5)} < (v_1)^{(5)}, \quad \text{and} \quad (v_0)^{(5)} = \frac{g_0}{g_29}\]

\[(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (v_0)^{(5)}, \quad \text{if} \quad (v_1)^{(5)} < (v_0)^{(5)}\]

and analogously

\[(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \quad \text{if} \quad (u_0)^{(5)} < (u_1)^{(5)}\]

\[(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \quad \text{if} \quad (u_0)^{(5)} < (u_0)^{(5)} < (u_1)^{(5)}, \quad \text{and} \quad (u_0)^{(5)} = \frac{g_0}{g_29}\]

\[(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \quad \text{if} \quad (u_0)^{(5)} < (u_0)^{(5)}\]

where \((u_0)^{(5)}, (\tilde{u}_1)^{(5)}\) are defined by respectively

Then the solution satisfies the inequalities

\[G_{28}^0 e^{((s_1)^{(5)} - (p_{28})^{(5)}) t} \leq G_{29}^0 (t) \leq G_{28}^0 e^{((s_1)^{(5)}) t}\]

where \((p_i)^{(5)}\) is defined by equation IN THE FOREGOING

\[\frac{1}{(m_2)^{(5)}} G_{28}^0 e^{((s_1)^{(5)} - (p_{28})^{(5)}) t} \leq G_{29}^0 (t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{((s_1)^{(5)}) t}\]

\[\frac{(a_{29})^{(5)} g_{29}}{(m_2)^{(5)} ((s_1)^{(5)} - (p_{28})^{(5)} - (s_2)^{(5)}))} e^{((s_1)^{(5)} - (p_{28})^{(5)}) t - e^{((s_2)^{(5)} t)} + G_{30}^0 e^{-(s_2)^{(5)} t} \leq G_{30}^0 (t) \leq}

\[\frac{(g_{29})^{(5)} g_{29}}{(m_2)^{(5)} ((s_1)^{(5)} - (a_{28})^{(5)}) e^{((s_1)^{(5)} t} - e^{-(a_{28})^{(5)} t}) + G_{30}^0 e^{-(a_{28})^{(5)} t}}\]

\[T_{28}^0 e^{((s_1)^{(5)} + (p_{28})^{(5)}) t} \leq T_{28}^0 \leq T_{28}^0 e^{((s_1)^{(5)} + (p_{28})^{(5)}) t}\]
Then the solution satisfies the inequalities respectively and analogously (l)

Definition of \( (\sigma_1)^{(6)} \), \( (\sigma_2)^{(6)} \), \( (\tau_1)^{(6)} \), \( (\tau_2)^{(6)} \): four constants satisfying

\[-(\sigma_2)^{(6)} \leq -a_{32}^{(6)} + (a_{33}^{(6)} - a_{32}^{(6)}) (T_{33}, t) + (a_{33}^{(6)} - a_{32}^{(6)}) (T_{33}, t) \leq -(\sigma_1)^{(6)}\]

\[-(\tau_2)^{(6)} \leq -(b_{32}^{(6)} + b_{33}^{(6)} - b_{32}^{(6)}) (T_{33}, t) - (b_{33}^{(6)} (T_{33}, t)) \leq -(\tau_1)^{(6)}\]

Definition of \( (v_1)^{(6)} \), \( (v_2)^{(6)} \), \( (u_1)^{(6)} \), \( (u_2)^{(6)} \), \( (v)^{(6)} \), \( (u)^{(6)} \):

By \( (v_1)^{(6)} > 0 \), \( (v_2)^{(6)} < 0 \) and respectively \( (u_1)^{(6)} > 0 \), \( (u_2)^{(6)} < 0 \) the roots of the equations

\[ (a_{33}^{(6)} (v^{(6)})^2 + (\sigma_1)^{(6)} (v^{(6)}) - (a_{32}^{(6)}) = 0 \]

and \( (b_{33}^{(6)} (u^{(6)})^2 + (\tau_1)^{(6)} (u^{(6)}) - (b_{32}^{(6)}) = 0 \)

Definition of \( (\bar{v}_1)^{(6)} \), \( (\bar{v}_2)^{(6)} \), \( (\bar{u}_1)^{(6)} \), \( (\bar{u}_2)^{(6)} \):

By \( (\bar{v}_1)^{(6)} > 0 \), \( (\bar{v}_2)^{(6)} < 0 \) and respectively \( (\bar{u}_1)^{(6)} > 0 \), \( (\bar{u}_2)^{(6)} < 0 \) the roots of the equations

\[ (a_{33}^{(6)} (v^{(6)})^2 + (\sigma_2)^{(6)} (v^{(6)}) - (a_{32}^{(6)}) = 0 \]

and \( (b_{33}^{(6)} (u^{(6)})^2 + (\tau_2)^{(6)} (u^{(6)}) - (b_{32}^{(6)}) = 0 \)

Definition of \( (m_1)^{(6)} \), \( (m_2)^{(6)} \), \( (\mu_1)^{(6)} \), \( (\mu_2)^{(6)} \), \( (v_0)^{(6)} \):

(i) If we define \( (m_1)^{(6)} \), \( (m_2)^{(6)} \), \( (\mu_1)^{(6)} \), \( (\mu_2)^{(6)} \), \( (v_0)^{(6)} \) by

\[ (m_2)^{(6)} = (v_0)^{(6)} (m_1)^{(6)} = (v_1)^{(6)} \text{ and } (v_0)^{(6)} < (v_1)^{(6)} \]

\[ (m_2)^{(6)} = (v_1)^{(6)} (m_1)^{(6)} = (\bar{v}_1)^{(6)} \text{ and } (v_0)^{(6)} < (\bar{v}_1)^{(6)} \]

\[ (m_2)^{(6)} = (v_2)^{(6)} (m_1)^{(6)} = (v_0)^{(6)} \text{ and } (v_0)^{(6)} < (v_2)^{(6)} \]

and analogously

\[ (\mu_2)^{(6)} = (u_0)^{(6)} (\mu_1)^{(6)} = (u_1)^{(6)} \text{ and } (u_0)^{(6)} < (u_1)^{(6)} \]

\[ (\mu_2)^{(6)} = (u_1)^{(6)} (\mu_1)^{(6)} = (\bar{u}_1)^{(6)} \text{ and } (u_0)^{(6)} < (\bar{u}_1)^{(6)} \]

\[ (\mu_2)^{(6)} = (u_2)^{(6)} (\mu_1)^{(6)} = (u_0)^{(6)} \text{ and } (u_0)^{(6)} < (u_2)^{(6)} \]

Then the solution satisfies the inequalities

\[ \frac{1}{\mu^{(5)}_{28}} T_{28}^{(5)} e^{(R_{1})^{(5)} t} \leq T_{28}^{(5)} e^{((R_{1})^{(5)} + (R_{2})^{(5)} t) t} \]

\[ \frac{1}{\mu^{(5)}_{30}} T_{30}^{(5)} e^{(R_{1})^{(5)} t} \leq T_{30}^{(5)} e^{((R_{1})^{(5)} + (R_{2})^{(5)} t) t} \]

\[ \frac{1}{\mu^{(5)}_{30}} T_{30}^{(5)} e^{(R_{1})^{(5)} t} \leq T_{30}^{(5)} e^{((R_{1})^{(5)} + (R_{2})^{(5)} t) t} \]
\[ G^{(6)}_{a2}e^{((S_1)^{(6)}-(p_{32})^{(6)})t} \leq G^{(6)}_{a2}e^{(S_1)^{(6)}t} \]

where \((p_{j})^{(6)}\) is defined by equation IN THE FOREGOING

\[ \frac{1}{(m_{1})^{6}}G^{0}_{32}e^{((S_1)^{(6)}-(p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_{2})^{6}}G^{0}_{32}e^{(S_1)^{(6)}t} \]

\[ \left(\frac{(a_{34})^{(6)}G_{22}^{(6)}}{(m_{2})^{6}}(S_1)^{(6)}-(a_{34})^{(6)}\right) \left(e^{(S_1)^{(6)}t}-(p_{32})^{(6)}t) - e^{-(S_2)^{(6)}t} \right) + G^{0}_{34}e^{-(S_2)^{(6)}t} \leq G_{34}(t) \leq \]

\[ \frac{(a_{34})^{(6)}G_{22}^{(6)}}{(m_{2})^{6}} \left(\left(e^{(S_1)^{(6)}t}-(a_{34})^{(6)}t \right) + G^{0}_{34}e^{-(a_{34})^{(6)}t} \right) \]

\[ \frac{T_{32}^{0}e^{(R_{1})^{(6)}t}}{e} \leq \frac{T_{32}^{0}e^{((R_{1})^{(6)}+r_{32})^{(6)}t}}{e} \]

\[ \frac{(b_{24})^{(6)}r_{32}^{(6)}}{(\mu_{2})^{6}} \left[e^{(R_{1})^{(6)}t} - e^{-(b_{24})^{(6)}t} \right] + T_{34}^{0}e^{-(b_{24})^{(6)}t} \leq T_{34}(t) \leq \frac{(b_{24})^{(6)}r_{32}^{(6)}}{(\mu_{2})^{6}} \left[e^{(R_{1})^{(6)}+(r_{32})^{(6)}t} - e^{-(R_{2})^{(6)}t} \right] + T_{34}^{0}e^{-(R_{2})^{(6)}t} \]

**Definition of \((S_{1})^{(6)}, (S_{2})^{(6)}, (R_{1})^{(6)}, (R_{2})^{(6)}\):**

Where \((S_{1})^{(6)} = (a_{32})^{(6)}(m_{2})^{6} - (a_{32})^{(6)}\)

\[(S_{2})^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}\]

\[(R_{1})^{(6)} = (b_{32})^{(6)}(\mu_{2})^{6} - (b_{32})^{(6)}\]

\[(R_{2})^{(6)} = (b_{34})^{(6)} - (r_{32})^{(6)}\]

**Proof:** From GLOBAL EQUATIONS we obtain

\[ \frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - ((a_{13})^{(1)}(T_{14}, t)) - (a_{14})^{(1)}(T_{14}, t) - (a_{14})^{(1)}v^{(1)} \]

**Definition of \(v^{(1)}\):**  \[ v^{(1)} = \frac{a_{13}}{a_{14}} \]

It follows

\[ -(a_{13})^{(1)}(v^{(1)})^{2} + (a_{14})^{(1)}v^{(1)} - (a_{14})^{(1)} \leq \frac{dv^{(1)}}{dt} \leq -(a_{14})^{(1)}(v^{(1)})^{2} + (a_{14})^{(1)}v^{(1)} - (a_{13})^{(1)} \]

From which one obtains

**Definition of \((\bar{v}_{1})^{(1)}, (v_{0})^{(1)}\):**

(a) For \(0 < \frac{(v_{0})^{(1)}}{G_{14}} < (v_{1})^{(1)} < (\bar{v}_{1})^{(1)}\)

\[ v^{(1)}(t) \geq \frac{(v_{1})^{(1)}(\bar{v}_{1})^{(1)}}{1 + (C)^{(1)}[(v_{1})^{(1)}(\bar{v}_{1})^{(1)}]e^{[-(a_{14})^{(1)}(v_{1})^{(1)}-(v_{0})^{(1)}]t]} \]

it follows \( (v_{0})^{(1)} \leq v^{(1)}(t) \leq (v_{1})^{(1)} \)

In the same manner, we get

\[ v^{(1)}(t) \leq \frac{(v_{1})^{(1)}(\bar{v}_{1})^{(1)}}{1 + (C)^{(1)}[(v_{1})^{(1)}(\bar{v}_{1})^{(1)}]e^{[-(a_{14})^{(1)}(v_{1})^{(1)}-(v_{0})^{(1)}]t]} \]

\[ \bar{v}^{(1)} = \frac{(v_{1})^{(1)}(\bar{v}_{1})^{(1)}}{1 + (C)^{(1)}[(v_{1})^{(1)}(\bar{v}_{1})^{(1)}]e^{[-(a_{14})^{(1)}(v_{1})^{(1)}-(v_{0})^{(1)}]t]} \]

From which we deduce \((v_{0})^{(1)} \leq v^{(1)}(t) \leq (v_{1})^{(1)}\)

(b) If \(0 < (v_{1})^{(1)} < (v_{0})^{(1)} = \frac{G_{14}}{G_{14}} < (\bar{v}_{1})^{(1)}\) we find like in the previous case,
\((v_1)^{(1)} \leq \frac{(v_1)^{(1)} + (c_1)^{(1)}(v_2)^{(1)}e^{-[\alpha_14](v_1)^{(1)} - (v_2)^{(1)}]}{1 + (c_1)^{(1)}e^{-[\alpha_14](v_1)^{(1)} - (v_2)^{(1)}]} \leq (v_1)^{(1)} \leq \frac{(v_1)^{(1)} + (c_2)^{(1)}(v_2)^{(1)}e^{-[\alpha_14](v_1)^{(1)} - (v_2)^{(1)}]}{1 + (c_2)^{(1)}e^{-[\alpha_14](v_1)^{(1)} - (v_2)^{(1)}]} \leq (\bar{v}_1)^{(1)} \)

(c) \quad \text{If } 0 < (v_1)^{(1)} \leq (\bar{v}_1)^{(1)} \leq (v_0)^{(1)} = \frac{\bar{v}_1}{\bar{v}_1} \text{, we obtain}

\[(v_1)^{(1)} \leq (v_1)^{(1)} \leq \frac{(v_1)^{(1)} + (c_1)^{(1)}(v_2)^{(1)}e^{-[\alpha_14](v_1)^{(1)} - (v_2)^{(1)}]}{1 + (c_1)^{(1)}e^{-[\alpha_14](v_1)^{(1)} - (v_2)^{(1)}]} \leq (v_0)^{(1)} \]

And so with the notation of the first part of condition (c), we have

**Definition of** \((v_1)^{(1)}(t) \): 

\[(m_2)^{(1)} \leq (v_1)^{(1)}(t) \leq (m_1)^{(1)} \]

In a completely analogous way, we obtain

**Definition of** \((u_1)^{(1)}(t) \): 

\[(\mu_2)^{(1)} \leq (u_1)^{(1)}(t) \leq (\mu_1)^{(1)} \]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case:** If \((a_1)^{(1)} = (a_1)^{(1)} \quad \text{then} \quad \sigma_1)^{(1)} = (\sigma_2)^{(1)} \quad \text{and in this case} \quad (v_1)^{(1)} = (\bar{v}_1)^{(1)} \quad \text{if in addition} \quad (v_0)^{(1)} = (v_1)^{(1)} \quad \text{then} \quad (v_1)^{(1)} = (v_0)^{(1)} \quad \text{and as a consequence} \quad G_13(t) = G_13(t) \quad G_14(t) \quad \text{this also defines} \quad (v_0)^{(1)} \quad \text{for the special case}

Analogously if \((b_1)^{(1)} = (b_1)^{(1)} \quad \text{then} \quad (\tau_1)^{(1)} = (\bar{\tau}_1)^{(1)} \) and then \((u_1)^{(1)} = (\bar{u}_1)^{(1)} \) if in addition \((u_0)^{(1)} = (u_1)^{(1)} \quad \text{then} \quad T_13(t) = (u_0)^{(1)}T_14(t) \text{ This is an important consequence of the relation between} \quad (v_1)^{(1)} \quad \text{and} \quad (\bar{v}_1)^{(1)} \quad \text{and definition of} \quad (u_0)^{(1)} \).

**Proof:** From GLOBAL EQUATIONS we obtain

\[
\frac{dv^{(2)}}{dt} = (a_1)^{(2)} - ((a_1)^{(2)} - (a_1)^{(2)} + (a_1)^{(2)}(T_17,t) - (a_1)^{(2)}(T_17,t)v^{(2)} - (a_1)^{(2)}v^{(2)}
\]

**Definition of** \((v^{(2)} \text{:-})

\[
(v^{(2)}) = \frac{G_16}{G_17}
\]

It follows

\[-((a_1)^{(2)}v^{(2)})^2 + (\sigma_2)^{(2)}v^{(2)} - (a_1)^{(2)} \leq \frac{dv^{(2)}}{dt} \leq -((a_1)^{(2)}v^{(2)})^2 + (\sigma_2)^{(2)}v^{(2)} - (a_1)^{(2)}
\]

From which one obtains

**Definition of** \((\bar{v}_1)^{(2)} (v_0)^{(2)} \text{:-}

\[(d) \quad \quad \text{For } 0 < (v_0)^{(2)} = \frac{\bar{v}_1}{\bar{v}_1} < (v_1)^{(2)} < (\bar{v}_1)^{(2)}
\]

\[
(v^{(2)}) \leq \frac{(v_0)^{(2)} + (c_2)^{(2)}(v_2)^{(2)}e^{-[\alpha_14](v_1)^{(2)} - (v_2)^{(2)}]}{1 + (c_2)^{(2)}e^{-[\alpha_14](v_1)^{(2)} - (v_2)^{(2)}]} \leq (v^{(2)}) \leq \frac{(v_0)^{(2)} + (c_2)^{(2)}(v_2)^{(2)}e^{-[\alpha_14](v_1)^{(2)} - (v_2)^{(2)}]}{1 + (c_2)^{(2)}e^{-[\alpha_14](v_1)^{(2)} - (v_2)^{(2)}]} \leq (\bar{v}_1)^{(2)}
\]

\[
(v^{(2)}) \leq \frac{(v_0)^{(2)} + (c_2)^{(2)}(v_2)^{(2)}e^{-[\alpha_14](v_1)^{(2)} - (v_2)^{(2)}]}{1 + (c_2)^{(2)}e^{-[\alpha_14](v_1)^{(2)} - (v_2)^{(2)}]} \leq (v^{(2)}) \leq \frac{(v_0)^{(2)} + (c_2)^{(2)}(v_2)^{(2)}e^{-[\alpha_14](v_1)^{(2)} - (v_2)^{(2)}]}{1 + (c_2)^{(2)}e^{-[\alpha_14](v_1)^{(2)} - (v_2)^{(2)}]} \leq (\bar{v}_1)^{(2)}
\]

From which one obtains

**Definition of** \((\bar{v}_1)^{(2)} (v_0)^{(2)} \text{:-}

\[(d) \quad \quad \text{For } 0 < (v_0)^{(2)} = \frac{\bar{v}_1}{\bar{v}_1} < (v_1)^{(2)} < (\bar{v}_1)^{(2)}
\]

\[
(v^{(2)}) \leq \frac{(v_0)^{(2)} + (c_2)^{(2)}(v_2)^{(2)}e^{-[\alpha_14](v_1)^{(2)} - (v_2)^{(2)}]}{1 + (c_2)^{(2)}e^{-[\alpha_14](v_1)^{(2)} - (v_2)^{(2)}]} \leq (v^{(2)}) \leq \frac{(v_0)^{(2)} + (c_2)^{(2)}(v_2)^{(2)}e^{-[\alpha_14](v_1)^{(2)} - (v_2)^{(2)}]}{1 + (c_2)^{(2)}e^{-[\alpha_14](v_1)^{(2)} - (v_2)^{(2)}]} \leq (\bar{v}_1)^{(2)}
\]

\[
(v^{(2)}) \leq \frac{(v_0)^{(2)} + (c_2)^{(2)}(v_2)^{(2)}e^{-[\alpha_14](v_1)^{(2)} - (v_2)^{(2)}]}{1 + (c_2)^{(2)}e^{-[\alpha_14](v_1)^{(2)} - (v_2)^{(2)}]} \leq (v^{(2)}) \leq \frac{(v_0)^{(2)} + (c_2)^{(2)}(v_2)^{(2)}e^{-[\alpha_14](v_1)^{(2)} - (v_2)^{(2)}]}{1 + (c_2)^{(2)}e^{-[\alpha_14](v_1)^{(2)} - (v_2)^{(2)}]} \leq (\bar{v}_1)^{(2)}
\]

In the same manner, we get
\[ v^{(2)}(t) \leq \frac{(\bar{v}_2(2)+(\bar{C}(2)\bar{v}_2(2))e^{-[(a_{17}(2))\bar{v}_2(2)]t}}{1+(\bar{C}(2)\bar{v}_2(2))e^{-[(a_{17}(2))\bar{v}_2(2)]t}} \leq \frac{(\bar{v}_4(2)-v_0(2))}{(\bar{v}_2(2))} \]

From which we deduce \( (v_0(2)) \leq v^{(2)}(t) \leq (\bar{v}_4(2)) \)

(e) If 0 < \( (v_1(2)) \leq (v_0(2)) = \frac{G_{16}}{G_{17}} < (\bar{v}_4(2)) \) we find like in the previous case,

\[ (v_1(2)) \leq v^{(2)}(t) \leq \frac{(v_4(2)+(\bar{C}(2)\bar{v}_2(2))e^{-[(a_{17}(2))\bar{v}_2(2)]t}}{1+(\bar{C}(2)\bar{v}_2(2))e^{-[(a_{17}(2))\bar{v}_2(2)]t}} \leq (v_4(2)) \]

(f) If 0 < \( (v_1(2)) \leq (\bar{v}_4(2)) \leq (v_0(2)) = \frac{G_{16}}{G_{17}} \), we obtain

\[ (v_1(2)) \leq v^{(2)}(t) \leq \frac{(v_4(2)+(\bar{C}(2)\bar{v}_2(2))e^{-[(a_{17}(2))\bar{v}_2(2)]t}}{1+(\bar{C}(2)\bar{v}_2(2))e^{-[(a_{17}(2))\bar{v}_2(2)]t}} \leq (v_0(2)) \]

And so with the notation of the first part of condition (c), we have

**Definition of** \( v^{(2)}(t) \) :-

\[ (m_2(2)) \leq v^{(2)}(t) \leq (m_1(2)) \]

\[ v^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)} \]

In a completely analogous way, we obtain

**Definition of** \( u^{(2)}(t) \) :-

\[ (\mu_2(2)) \leq u^{(2)}(t) \leq (\mu_1(2)) \]

\[ u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)} \]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case**:
If \( (a_{16}^{(2)}) = (a_{17}^{(2)}) \), then \( (\sigma_1(2)) = (\sigma_2(2)) \) and in this case \( (v_1(2)) = (\bar{v}_4(2)) \) if in addition \( (v_0(2)) = (v_2(2)) \) then \( v^{(2)}(t) = (v_0(2)) \) and as a consequence \( G_{16}(t) = (v_0(2))G_{17}(t) \)

Analogously if \( (b_{16}^{(2)}) = (b_{17}^{(2)}) \), then \( (\tau_1(2)) = (\tau_2(2)) \) and then

\[ (u_1(2)) = (\bar{u}_4(2)) \] in addition \( (u_0(2)) = (u_1(2)) \) then \( T_{16}(t) = (u_0(2))T_{17}(t) \) This is an important consequence of the relation between \( (v_1(2)) \) and \( (\bar{v}_4(2)) \)

**Proof**: From GLOBAL EQUATIONS we obtain

\[ \frac{dv^{(3)}}{dt} = (a_{20}(3)) - (a_{21}^{(3)}(3) - (a_{21}^{(3)}(3) + (a_{20}^{(3)}(3)T_{21}(3))) - (a_{21}^{(3)}(3)T_{21}(3))v^{(3)} - (a_{21}^{(3)}(3)\nu^{(3)}) \]

**Definition of** \( v^{(3)} \) :

\[ v^{(3)} = \frac{\bar{v}_{20}}{\bar{v}_{21}} \]

It follows

\[ -((a_{21}^{(3)}(3)v^{(3)})^2 + (a_{21}^{(3)}(3)v^{(3)}) \leq \frac{dv^{(3)}}{dt} \leq -((a_{21}^{(3)}(3)v^{(3)})^2 + (a_{21}^{(3)}(3)v^{(3)}) - (a_{20}^{(3)}(3)) \]

From which one obtains

For 0 < \( (v_0(3)) = \frac{\bar{v}_{20}}{\bar{v}_{21}} < (v_1(3)) < (\bar{v}_4(3)) \)
it follows \( v^{(3)}(t) \leq (v_i)^{(3)} \)

In the same manner, we get

\[
\frac{(v_j)^{(3)}}{1+(\tilde{C})^{(3)}} \leq \frac{(v_j)^{(3)}}{1+(\tilde{C})^{(3)}} - \frac{(v_i)^{(3)}}{1+(\tilde{C})^{(3)}} \leq 0
\]

**Definition of** \( (\tilde{v}_i)^{(3)} \) :-

From which we deduce \( (v_0)^{(3)} \leq v^{(3)}(t) \leq (\tilde{v}_i)^{(3)} \)

If \( 0 < (v_i)^{(3)} < (v_0)^{(3)} = \frac{g_0}{g_{21}} < (\tilde{v}_i)^{(3)} \) we find like in the previous case,

\[
(v_j)^{(3)} \leq \frac{(v_j)^{(3)}}{1+(\tilde{C})^{(3)}} - \frac{(v_i)^{(3)}}{1+(\tilde{C})^{(3)}} \leq v^{(3)}(t) \leq \frac{(v_j)^{(3)}}{1+(\tilde{C})^{(3)}} - \frac{(v_i)^{(3)}}{1+(\tilde{C})^{(3)}} \leq (v_0)^{(3)}
\]

If \( 0 < (v_i)^{(3)} < (v_0)^{(3)} = \frac{g_0}{g_{21}} \), we obtain

\[
(v_j)^{(3)} \leq (t) \leq (v_j)^{(3)} \leq (v_0)^{(3)} \leq (v_0)^{(3)}
\]

And so with the notation of the first part of condition (c), we have

**Definition of** \( v^{(3)}(t) :-\)

\[
(m_2)^{(3)} \leq v^{(3)}(t) \leq (m_1)^{(3)}, \quad v^{(3)}(t) = \frac{g_{20}(t)}{g_{21}(t)}
\]

In a completely analogous way, we obtain

**Definition of** \( u^{(3)}(t) :-\)

\[
(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}
\]

Now, using this result and replacing it in GLOBAL EQUATIONMS we get easily the result stated in the theorem.

**Particular case :**

If \( (a_{20}^{(3)}) = (a_{21}^{(3)}) \), then \( (\sigma_1)^{(3)} = (\sigma_2)^{(3)} \) and in this case \( v^{(3)}(t) = (\tilde{v}_i)^{(3)} \) if in addition \( (v_0)^{(3)} = (v_1)^{(3)} \) then \( v^{(3)}(t) = (v_0)^{(3)} \) and as a consequence \( G_{20}(t) = (v_0)^{(3)}G_{21}(t) \)

Analogously if \( (b_{20}^{(3)}) = (b_{21}^{(3)}) \), then \( (\tau_1)^{(3)} = (\tau_2)^{(3)} \) and then

\( (u_2)^{(3)} = (\tilde{u}_1)^{(3)} \)

If in addition \( (u_0)^{(3)} = (u_1)^{(3)} \) then \( T_{20}(t) = (u_0)^{(3)}T_{21}(t) \) This is an important consequence of the relation between \( (v_1)^{(3)} \) and \( (\tilde{v}_1)^{(3)} \)

**Proof :** From GLOBAL EQUATIONS we obtain

\[
\frac{dv^{(3)}}{dt} = (a_{24})^{(4)} - \left( (a_{24})^{(4)} - (a_{25})^{(4)} + (a_{24})^{(4)}(T_{25}, t) \right) - (a_{25})^{(4)}(T_{25}, t)v^{(4)} - (a_{25})^{(4)}v^{(4)}
\]

**Definition of** \( v^{(4)} :-\)

\[
v^{(4)} = \frac{g_{24}}{g_{25}}
\]

It follows
- \((a_{25})^4(v_0^{(4)})^2 + (a_{24})^4v_0^{(4)} - (a_{24})^4\) \(\frac{dv_0^{(4)}}{dt}\) \(\leq -((a_{25})^4(v_0^{(4)})^2 + (a_{24})^4v_0^{(4)} - (a_{24})^4)\)

From which one obtains

**Definition of** \((\bar{v}_0)^{(4)}, (v_0)^{(4)} :\)

\(\begin{align*}
& (d) \quad \text{For } 0 < \left(\frac{v_0^{(4)}}{\bar{v}_0^{(4)}}\right) = \frac{2}{\sqrt{5}} < (v_1)^{(4)} < (\bar{v}_1)^{(4)} \\
& \quad v_0^{(4)}(t) \geq \frac{(v_2)^{(4)} + (C)^{(4)}(v_2)^{(4)}}{4 + (C)^{(4)}} \left[-(a_{25})^4((v_1)^{(4)} - (v_2)^{(4)})\right] \\
& \quad \text{it follows } (v_0)^{(4)} \leq v_0^{(4)}(t) \leq (\bar{v}_1)^{(4)} \\
\end{align*}\)

In the same manner, we get

\(\begin{align*}
& (e) \quad \text{If } 0 < (v_1)^{(4)} < (v_0)^{(4)} = \frac{2}{\sqrt{5}} < (\bar{v}_2)^{(4)} \text{ we find like in the previous case,} \\
& \quad (v_1)^{(4)} \leq \frac{(v_2)^{(4)} + (C)^{(4)}(v_2)^{(4)}}{4 + (C)^{(4)}} \left[-(a_{25})^4((v_1)^{(4)} - (v_2)^{(4)})\right] \leq (v_0)^{(4)}(t) \leq \frac{(v_2)^{(4)} + (C)^{(4)}(v_2)^{(4)}}{4 + (C)^{(4)}} \left[-(a_{25})^4((v_1)^{(4)} - (v_2)^{(4)})\right] \leq (\bar{v}_2)^{(4)} \\
\end{align*}\)

And so with the notation of the first part of condition (c), we have

**Definition of** \(v_0^{(4)}(t) :\)

\(\begin{align*}
& (m_2)^{(4)} \leq v_0^{(4)}(t) \leq (m_1)^{(4)}, \quad v_0^{(4)}(t) = \frac{24(\bar{t})}{G_{25}(\bar{t})} \\
\end{align*}\)

In a completely analogous way, we obtain

**Definition of** \(u_0^{(4)}(t) :\)

\(\begin{align*}
& (m_2)^{(4)} \leq u_0^{(4)}(t) \leq (m_1)^{(4)}, \quad u_0^{(4)}(t) = \frac{T_{24}(\bar{t})}{T_{25}(\bar{t})} \\
\end{align*}\)

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case :**

If \((a_{24}^{(4)})^4 = (a_{25}^{(4)})^4\), then \((\sigma_1)^{(4)} = (\sigma_2)^{(4)}\) and in this case \((v_1)^{(4)} = (\bar{v}_2)^{(4)}\) if in addition \((v_0)^{(4)} = (v_1)^{(4)}\) then \(v_0^{(4)}(t) = (v_0)^{(4)}\) and as a consequence \(G_{24}(t) = (v_0)^{(4)}G_{25}(t)\) **this also defines** \((v_0)^{(4)}\) **for the special case.**

Analogously if \((b_{24}^{(4)})^4 = (b_{25}^{(4)})^4\), then \((\tau_1)^{(4)} = (\tau_2)^{(4)}\) and then \((u_0)^{(4)} = (u_0)^{(4)}\) if in addition \((u_0)^{(4)} = (u_0)^{(4)}\) then \(T_{24}(t) = (u_0)^{(4)}T_{25}(t)\) This is an important consequence of the relation between \((v_1)^{(4)}\) and \((\bar{v}_2)^{(4)}\), *and definition of* \((u_0)^{(4)}\).
\[ \frac{dv^{(5)}}{dt} = (a_{28}^{(5)}) - (a_{29}^{(5)}) - (a_{28}^{(5)}(T_{29}^{(5)}(t))) - (a_{29}^{(5)}(T_{29}^{(5)}))v^{(5)} - (a_{29}^{(5)}v^{(5)}) \]

**Definition of** \( v^{(5)} \) :-

\[ v^{(5)} = \frac{g_{28}}{g_{29}} \]

It follows

\[ -(a_{28}^{(5)}(v^{(5)})^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28}^{(5)})) \leq \frac{dv^{(5)}}{dt} \leq -(a_{29}^{(5)}(v^{(5)})^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{29}^{(5)}) \]

From which one obtains

**Definition of** \((\bar{v})^{(5)}, (v_0)^{(5)} :) -\)

\[ (g) \quad \text{For } 0 < (v_0)^{(5)} = \frac{g_{28}}{g_{29}} < (v_1)^{(5)} < (\bar{v})^{(5)} \]

\[ v^{(5)}(t) \geq \frac{(y_1)^{(5)}(v_0)^{(5)}e^{-a_{29}^{(5)}(v_1)^{(5)}(v_0)^{(5)}t}}{1 + (C)^{(5)}e^{-a_{29}^{(5)}(v_1)^{(5)}(v_0)^{(5)}t}}} \]

\[ (C)^{(5)} = \frac{(y_1)^{(5)}(v_0)^{(5)} - (v_2)^{(5)}}{(y_0)^{(5)}(v_0)^{(5)} - (v_2)^{(5)} \]

it follows \((v_0)^{(5)} \leq v^{(5)}(t) \leq (v_1)^{(5)} \]

In the same manner, we get

\[ v^{(5)}(t) \leq \frac{(y_1)^{(5)}(v_0)^{(5)}e^{-a_{29}^{(5)}(v_1)^{(5)}(v_0)^{(5)}t}}{1 + (C)^{(5)}e^{-a_{29}^{(5)}(v_1)^{(5)}(v_0)^{(5)}t}}} \]

\[ (C)^{(5)} = \frac{(y_1)^{(5)}(v_0)^{(5)} - (v_2)^{(5)}}{(y_0)^{(5)}(v_0)^{(5)} - (v_2)^{(5)} \]

From which we deduce \((v_0)^{(5)} \leq v^{(5)}(t) \leq (\bar{v})^{(5)} \]

\[ (h) \quad \text{If } 0 < (v_1)^{(5)} < (v_0)^{(5)} = \frac{g_{28}}{g_{29}} < (\bar{v})^{(5)} \]

we find like in the previous case,

\[ (v_1)^{(5)} \leq \frac{(y_1)^{(5)}(v_0)^{(5)}e^{-a_{29}^{(5)}(v_1)^{(5)}(v_0)^{(5)}t}}{1 + (C)^{(5)}e^{-a_{29}^{(5)}(v_1)^{(5)}(v_0)^{(5)}t}}} \leq v^{(5)}(t) \leq \]

\[ \frac{(y_1)^{(5)}(v_0)^{(5)}e^{-a_{29}^{(5)}(v_1)^{(5)}(v_0)^{(5)}t}}{1 + (C)^{(5)}e^{-a_{29}^{(5)}(v_1)^{(5)}(v_0)^{(5)}t}}} \leq (\bar{v})^{(5)} \]

\[ (i) \quad \text{If } 0 < (v_1)^{(5)} \leq (\bar{v})^{(5)} \leq (v_0)^{(5)} = \frac{g_{28}}{g_{29}} \]

we obtain

\[ (v_1)^{(5)} \leq v^{(5)}(t) \leq \frac{(y_1)^{(5)}(v_0)^{(5)}e^{-a_{29}^{(5)}(v_1)^{(5)}(v_0)^{(5)}t}}{1 + (C)^{(5)}e^{-a_{29}^{(5)}(v_1)^{(5)}(v_0)^{(5)}t}}} \leq (v_0)^{(5)} \]

And so with the notation of the first part of condition (c), we have

**Definition of** \( v^{(5)}(t) :) -\)

\[ (m_2)^{(5)} \leq v^{(5)}(t) \leq (m_1)^{(5)}, \quad v^{(5)}(t) = \frac{g_{28}(t)}{g_{29}(t)} \]

In a completely analogous way, we obtain

**Definition of** \( u^{(5)}(t) :) -\)

\[ (\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad u^{(5)}(t) = \frac{g_{28}(t)}{g_{29}(t)} \]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case:**

If \((a_{28}^{(5)}) = (a_{29}^{(5)})\), then \((\sigma_1)^{(5)} = (\sigma_2)^{(5)}\) and in this case \((v_1)^{(5)} = (\bar{v})^{(5)}\) if in addition \((v_0)^{(5)} = \]
(ν₁)₅(t) = (ν₀)₅ and as a consequence G₂₈(t) = (ν₀)₅G₂₉(t) this also defines (ν₀)₅ for the special case.

Analogously if (bᵢ₆)₅ = (b₉₄)₅, then (τ₁)₅ = (τ₂)₅ and then (u₄)₅ = (ū₄)₅ if in addition (u₀)₅ = (u₁)₅ then T₂₈(t) = (u₀)₅T₂₉(t) This is an important consequence of the relation between (ν₄)₅ and (ū₄)₅, and definition of (u₀)₅.

Proof: From GLOBAL EQUATIONS we obtain

\[ \frac{dv(t)}{dt} = (a₃₂)₆(ν₆) - (a₃₃)₆(ν₆)(T₃₃, t) \]

Definition of (ν₆)₆ := \[ ν₆ = \frac{G₃₃}{G₃₃} \]

It follows

\[ -(a₃₃)₆(ν₆)² + (σ₃)₆ν₆ - (a₃₂)₆ ≤ \frac{dv(t)}{dt} ≤ -(a₃₃)₆(ν₆)² + (σ₃)₆ν₆ - (a₃₂)₆ \]

From which one obtains

Definition of (v₆)(t), (u₀)(t) :=

(j) For 0 < \[ \frac{v₀}{G₃₃} \] < (v₁)₆ < (ū₆) then

\[ v₆(t) ≥ \frac{(v₀)₆ + (C)(v₀)₆e^{-(a₃₃)₆[(v₁)₆-(v₀)₆]t}}{1+|(C)|e^{-(a₃₃)₆[(v₁)₆-(v₀)₆]t}} \]

(k) If 0 < (v₁)₆ < (v₀)₆ = \[ \frac{G₃₃}{G₃₃} \] < (ū₆) we find like in the previous case,

\[ (v₁)₆ ≤ \frac{(v₀)₆ + (C)(v₀)₆e^{-(a₃₃)₆[(v₁)₆-(v₀)₆]t}}{1+|(C)|e^{-(a₃₃)₆[(v₁)₆-(v₀)₆]t}} ≤ v₆(t) ≤ \frac{(v₁)₆ + (C)(v₂)₆e^{-(a₃₃)₆[(v₂)₆-(v₁)₆]t}}{1+|(C)|e^{-(a₃₃)₆[(v₂)₆-(v₁)₆]t}} \]

(l) If 0 < (v₁)₆ ≤ (ū₁)₆ ≤ \[ \frac{v₀}{G₃₃} \] we obtain

\[ (v₁)₆ ≤ v₆(t) ≤ \frac{(v₁)₆ + (C)(v₂)₆e^{-(a₃₃)₆[(v₂)₆-(v₁)₆]t}}{1+|(C)|e^{-(a₃₃)₆[(v₂)₆-(v₁)₆]t}} ≤ (v₀)₆ \]

And so with the notation of the first part of condition (c), we have

Definition of v₆(t) :=

\[ m₂(t) ≤ v₆(t) ≤ m₁(t), \quad v₆(t) = \frac{G₃₃}{G₃₃} \]

In a completely analogous way, we obtain

Definition of u₆(t) :=

\[ m₂(t) ≤ v₆(t) ≤ m₁(t), \quad v₆(t) = \frac{G₃₃}{G₃₃} \]
(μ₂)^(6) ≤ u^(6)(t) ≤ (μ₁)^(6), \[ u^(6)(t) = \frac{T_{32}(t)}{T_{33}(t)} \]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case:**

If \((a''_{13})^{(6)} = (a''_{14})^{(6)}\) then \((σ₁)^(6) = (σ₂)^(6)\) and in this case \((v₁)^(6) = (v₂)^(6)\) if in addition \((v₀)^(6) = (ν₁)^(6)\) then \(ν^(6)(t) = (v₀)^(6)\) and as a consequence \(G_{32}(t) = (v₀)^(6)G_{33}(t)\) this also defines \((v₀)^(6)\)

for the special case.

Analogously if \((b''_{13})^{(6)} = (b''_{14})^{(6)}, then (τ₁)^(6) = (τ₂)^(6)\) and then

\((u₁)^(6) = (u₀)^(6)\) if in addition \((u₀)^(6) = (u₁)^(6)\) then \(T_{32}(t) = (u₀)^(6)T_{33}(t)\) This is an important consequence of the relation between \((v₁)^(6)\) and \((v₂)^(6)\), and definition of \((u₀)^(6)\).

We can prove the following

#### Theorem 3:

If \((a''_{1i})^{(1)}\) and \((b''_{i})^{(1)}\) are independent on \(t\), and the conditions

\[(a''_{13})^{(1)}(a''_{14})^{(1)} - (a''_{13})(a''_{14})^{(1)} < 0\]
\[(a''_{13})^{(1)}(a''_{14})^{(1)} - (a''_{13})(a''_{14})^{(1)} + (a''_{13})^{(1)}(p''_{13})^{(1)} + (a''_{14})^{(1)}(p''_{14})^{(1)} + (p''_{13})(p''_{14})^{(1)} > 0\]
\[(b''_{13})^{(1)}(b''_{14})^{(1)} - (b''_{13})(b''_{14})^{(1)} > 0\]
\[(b''_{13})^{(1)}(b''_{14})^{(1)} - (b''_{13})(b''_{14})^{(1)} + (b''_{13})^{(1)}(r''_{14})^{(1)} - (b''_{14})^{(1)}(r''_{14})^{(1)} + (r''_{13})(r''_{14})^{(1)} < 0\]

with \((p''_{13})^{(1)}, (r''_{14})^{(1)}\) as defined by equation IN THE FOREGOING are satisfied, then the system

If \((a''_{1i})^{(2)}\) and \((b''_{i})^{(2)}\) are independent on \(t\), and the conditions (SECOND MODULE)

\[(a''_{16})^{(2)}(a''_{17})^{(2)} - (a''_{16})(a''_{17})^{(2)} < 0\]
\[(a''_{16})^{(2)}(a''_{17})^{(2)} - (a''_{16})(a''_{17})^{(2)} + (a''_{16})^{(2)}(p''_{16})^{(2)} + (a''_{17})^{(2)}(p''_{17})^{(2)} + (p''_{16})(p''_{17})^{(2)} > 0\]
\[(b''_{16})^{(2)}(b''_{17})^{(2)} - (b''_{16})(b''_{17})^{(2)} > 0\]
\[(b''_{16})^{(2)}(b''_{17})^{(2)} - (b''_{16})(b''_{17})^{(2)} + (b''_{16})^{(2)}(r''_{17})^{(2)} + (b''_{17})^{(2)}(r''_{17})^{(2)} + (r''_{16})(r''_{17})^{(2)} < 0\]

with \((p''_{16})^{(2)}, (r''_{17})^{(2)}\) as defined by equation IN THE FOREGOING are satisfied, then the system

#### Theorem 3:

If \((a''_{1i})^{(3)}\) and \((b''_{i})^{(3)}\) are independent on \(t\), and the conditions (SECOND MODULE)

\[(a''_{20})^{(3)}(a''_{21})^{(3)} - (a''_{20})(a''_{21})^{(3)} < 0\]
\[(a''_{20})^{(3)}(a''_{21})^{(3)} - (a''_{20})(a''_{21})^{(3)} + (a''_{20})^{(3)}(p''_{20})^{(3)} + (a''_{21})^{(3)}(p''_{21})^{(3)} + (p''_{20})(p''_{21})^{(3)} > 0\]
\[(b''_{20})^{(3)}(b''_{21})^{(3)} - (b''_{20})(b''_{21})^{(3)} > 0\]
\[(b''_{20})^{(3)}(b''_{21})^{(3)} - (b''_{20})(b''_{21})^{(3)} + (b''_{20})^{(3)}(r''_{21})^{(3)} - (b''_{21})^{(3)}(r''_{21})^{(3)} + (r''_{20})(r''_{21})^{(3)} < 0\]

with \((p''_{20})^{(3)}, (r''_{21})^{(3)}\) as defined by equation IN THE FOREGOING are satisfied, then the system

We can prove the following (FOURTH MODULE CONSEQUENCES)

#### Theorem 3:

If \((a''_{1i})^{(4)}\) and \((b''_{i})^{(4)}\) are independent on \(t\), and the conditions

\[(a''_{24})^{(4)}(a''_{25})^{(4)} - (a''_{24})(a''_{25})^{(4)} < 0\]
\[(a''_{24})^{(4)}(a''_{25})^{(4)} - (a''_{24})(a''_{25})^{(4)} + (a''_{24})^{(4)}(p''_{24})^{(4)} + (a''_{25})^{(4)}(p''_{25})^{(4)} + (p''_{24})(p''_{25})^{(4)} > 0\]
\[(b''_{24})^{(4)}(b''_{25})^{(4)} - (b''_{24})(b''_{25})^{(4)} > 0\]
\[(b''_{24})^{(4)}(b''_{25})^{(4)} - (b''_{24})(b''_{25})^{(4)} - (b''_{24})^{(4)}(r''_{25})^{(4)} - (b''_{25})^{(4)}(r''_{25})^{(4)} + (r''_{24})(r''_{25})^{(4)} < 0\]
with \((p_{24})^4, (r_{25})^4\) as defined by equation IN THE FOREGOING are satisfied, then the system

**Theorem 3:** If \(a''''(t)\) and \(b''''(t)\) are independent on \(t\), and the conditions (FIFTH MODULE CONSEQUENCES)

\[
\begin{align*}
(a'_{28}^{(5)}(a_{29}^{(5)} - (a_{28}^{(5)}(a_{29}^{(5)} < 0 \\
(a'_{28}^{(5)}(a_{29}^{(5)} - (a_{28}^{(5)}(a_{29}^{(5)} + (a_{28}^{(5)}(p_{28}^{(5)} + (a_{29}^{(5)}(p_{28}^{(5)} + (p_{28}^{(5)}(p_{28}^{(5)} > 0 \\
(b'_{28}^{(5)}(b_{29}^{(5)} - (b_{28}^{(5)}(b_{29}^{(5)} - (b_{28}^{(5)}(b_{29}^{(5)} - (b_{28}^{(5)}(r_{29}^{(5)} - (b_{28}^{(5)}(r_{29}^{(5)} < 0
\end{align*}
\]

with \((p_{28})^5, (q_{29})^5\) as defined by equation IN THE EQUATION STATED IN THE FOREGOING are satisfied, then the system

**Theorem 3:** If \(a''''(t)\) and \(b''''(t)\) are independent on \(t\), and the conditions

\[
\begin{align*}
(a'_{32}^{(6)}(a_{33}^{(6)}} - (a_{32}^{(6)}(a_{33}^{(6)} < 0 \\
(a'_{32}^{(6)}(a_{33}^{(6)}} - (a_{32}^{(6)}(a_{33}^{(6)} + (a_{32}^{(6)}(p_{32}^{(6)} + (a_{33}^{(6)}(p_{33}^{(6)} + (p_{32}^{(6)}(p_{33}^{(6)} > 0 \\
(b'_{32}^{(6)}(b_{33}^{(6)}} - (b_{32}^{(6)}(b_{33}^{(6)} - (b_{32}^{(6)}(b_{33}^{(6)}(r_{33}^{(6)}} - (b_{32}^{(6)}(r_{33}^{(6)}(r_{33}^{(6)} + (r_{32}^{(6)}(r_{33}^{(6)} < 0
\end{align*}
\]

with \((p_{32})^6, (r_{33})^6\) as defined by equation IN THE FOREGOING are satisfied, then the system

\[
\begin{align*}
(a_{13}^{(1)}G_{14} - [(a'_{14}^{(1)} + (a''_{14}^{(1)}(T_{14})]G_{13} = 0 \\
(a_{14}^{(1)}G_{13} - [(a'_{14}^{(1)} + (a''_{14}^{(1)}(T_{14})]G_{14} = 0 \\
(a_{15}^{(1)}G_{14} - [(a'_{15}^{(1)} + (a''_{15}^{(1)}(T_{14})]G_{15} = 0 \\
(b_{13}^{(1)}T_{14} - [(b'_{14}^{(1)} + (b''_{14}^{(1)}(G)]T_{13} = 0 \\
(b_{14}^{(1)}T_{13} - [(b'_{14}^{(1)} + (b''_{14}^{(1)}(G)]T_{14} = 0 \\
(b_{15}^{(1)}T_{14} - [(b'_{15}^{(1)} + (b''_{15}^{(1)}(G)]T_{15} = 0
\end{align*}
\]

has a unique positive solution, which is an equilibrium solution for the system

\[
\begin{align*}
(a_{16}^{(2)}G_{17} - [(a'_{16}^{(2)} + (a''_{16}^{(2)}(T_{17})]G_{16} = 0 \\
(a_{17}^{(2)}G_{16} - [(a'_{17}^{(2)} + (a''_{17}^{(2)}(T_{17})]G_{17} = 0 \\
(a_{18}^{(2)}G_{17} - [(a'_{18}^{(2)} + (a''_{18}^{(2)}(T_{17})]G_{18} = 0 \\
(b_{16}^{(2)}T_{17} - [(b'_{16}^{(2)} + (b''_{16}^{(2)}(G)]T_{16} = 0 \\
(b_{17}^{(2)}T_{16} - [(b'_{17}^{(2)} + (b''_{17}^{(2)}(G)]T_{17} = 0 \\
(b_{18}^{(2)}T_{17} - [(b'_{18}^{(2)} + (b''_{18}^{(2)}(G)]T_{18} = 0
\end{align*}
\]

has a unique positive solution, which is an equilibrium solution

\[
\begin{align*}
(a_{20}^{(3)}G_{21} - [(a'_{20}^{(3)} + (a''_{20}^{(3)}(T_{21})]G_{20} = 0 \\
(a_{21}^{(3)}G_{20} - [(a'_{21}^{(3)} + (a''_{21}^{(3)}(T_{21})]G_{21} = 0 \\
(a_{22}^{(3)}G_{21} - [(a'_{22}^{(3)} + (a''_{22}^{(3)}(T_{21})]G_{22} = 0 \\
(b_{20}^{(3)}T_{21} - [(b'_{20}^{(3)} + (b''_{20}^{(3)}(G)]T_{20} = 0 \\
(b_{21}^{(3)}T_{20} - [(b'_{21}^{(3)} + (b''_{21}^{(3)}(G)]T_{21} = 0 \\
(b_{22}^{(3)}T_{21} - [(b'_{22}^{(3)} + (b''_{22}^{(3)}(G)]T_{22} = 0
\end{align*}
\]

has a unique positive solution, which is an equilibrium solution.
(a_{24})^{(4)}G_{25} - [(a_{24}')^{(4)} + (a_{24}'')^{(4)}(T_{25})]G_{24} = 0

(a_{25})^{(4)}G_{24} - [(a_{25}')^{(4)} + (a_{25}'')^{(4)}(T_{25})]G_{25} = 0

(a_{26})^{(4)}G_{25} - [(a_{26}')^{(4)} + (a_{26}'')^{(4)}(T_{25})]G_{26} = 0

(b_{24})^{(4)}T_{25} - [(b_{24}')^{(4)} - (b_{24}'')^{(4)}(G_{24})]T_{24} = 0

(b_{25})^{(4)}T_{24} - [(b_{25}')^{(4)} - (b_{25}'')^{(4)}(G_{25})]T_{25} = 0

(b_{26})^{(4)}T_{25} - [(b_{26}')^{(4)} - (b_{26}'')^{(4)}(G_{26})]T_{26} = 0

has a unique positive solution, which is an equilibrium solution

(a_{28})^{(5)}G_{29} - [(a_{28}')^{(5)} + (a_{28}'')^{(5)}(T_{29})]G_{28} = 0

(a_{29})^{(5)}G_{28} - [(a_{29}')^{(5)} + (a_{29}'')^{(5)}(T_{29})]G_{29} = 0

(a_{30})^{(5)}G_{29} - [(a_{30}')^{(5)} + (a_{30}'')^{(5)}(T_{29})]G_{30} = 0

(b_{28})^{(5)}T_{29} - [(b_{28}')^{(5)} - (b_{28}'')^{(5)}(G_{31})]T_{28} = 0

(b_{29})^{(5)}T_{28} - [(b_{29}')^{(5)} - (b_{29}'')^{(5)}(G_{31})]T_{29} = 0

(b_{30})^{(5)}T_{29} - [(b_{30}')^{(5)} - (b_{30}'')^{(5)}(G_{31})]T_{30} = 0

has a unique positive solution, which is an equilibrium solution

(a_{32})^{(6)}G_{33} - [(a_{32}')^{(6)} + (a_{32}'')^{(6)}(T_{33})]G_{32} = 0

(a_{33})^{(6)}G_{32} - [(a_{33}')^{(6)} + (a_{33}'')^{(6)}(T_{33})]G_{33} = 0

(a_{34})^{(6)}G_{33} - [(a_{34}')^{(6)} + (a_{34}'')^{(6)}(T_{33})]G_{34} = 0

(b_{32})^{(6)}T_{33} - [(b_{32}')^{(6)} - (b_{32}'')^{(6)}(G_{35})]T_{32} = 0

(b_{33})^{(6)}T_{32} - [(b_{33}')^{(6)} - (b_{33}'')^{(6)}(G_{35})]T_{33} = 0

(b_{34})^{(6)}T_{33} - [(b_{34}')^{(6)} - (b_{34}'')^{(6)}(G_{35})]T_{34} = 0

has a unique positive solution, which is an equilibrium solution

Proof:

(a) Indeed the first two equations have a nontrivial solution \( G_{13}, G_{14} \) if

\[
F(T) = (a_{13}')^{(1)}(a_{14}')^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13}')^{(1)}(a_{14}'')^{(1)}(T_{14}) + (a_{14}')^{(1)}(a_{13}'')^{(1)}(T_{14}) + (a_{13}'')^{(1)}(T_{14})(a_{14}')^{(1)}(T_{14}) = 0
\]

Proof:

(a) Indeed the first two equations have a nontrivial solution \( G_{16}, G_{17} \) if

\[
F(T_{16}) = (a_{16}')^{(2)}(a_{17}')^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16}')^{(2)}(a_{17}'')^{(2)}(T_{17}) + (a_{17}')^{(2)}(a_{16}'')^{(2)}(T_{17}) + (a_{16}'')^{(2)}(T_{17})(a_{17}')^{(2)}(T_{17}) = 0
\]

(a) Indeed the first two equations have a nontrivial solution \( G_{20}, G_{21} \) if

\[
F(T_{23}) = (a_{20}')^{(3)}(a_{21}')^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20}')^{(3)}(a_{21}'')^{(3)}(T_{21}) + (a_{21}')^{(3)}(a_{20}'')^{(3)}(T_{21}) + (a_{20}'')^{(3)}(T_{21})(a_{21}')^{(3)}(T_{21}) = 0
\]

(a) Indeed the first two equations have a nontrivial solution \( G_{24}, G_{25} \) if
F(T_{27}) = (a_{24}''')^{(4)}(a_{25}')^{(4)} - (a_{24}')^{(4)}(a_{25}''')^{(4)} + (a_{24}')^{(4)}(a_{25}')^{(4)}(T_{25}) + (a_{24}')^{(4)}(a_{25}')^{(4)}(T_{25}) = 0

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

F(T_{31}) = (a_{28}')^{(5)}(a_{29}')^{(5)} - (a_{28}')^{(5)}(a_{29}')^{(5)} + (a_{28}')^{(5)}(a_{29}')^{(5)}(T_{29}) + (a_{28}')^{(5)}(a_{29}')^{(5)}(T_{29}) = 0

**Proof:**

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

F(T_{33}) = (a_{32}''')^{(6)}(a_{33}')^{(6)} - (a_{32}')^{(6)}(a_{33}''')^{(6)} + (a_{32}')^{(6)}(a_{33}')^{(6)}(T_{33}) + (a_{32}')^{(6)}(a_{33}')^{(6)}(T_{33}) = 0

**Definition and uniqueness of T_{14} :-**

After hypothesis f(0) < 0, f(\infty) > 0 and the functions (a_i''')^{(1)}(T_{14}) being increasing, it follows that there exists a unique T_{14} for which f(T_{14}) = 0. With this value, we obtain from the three first equations

\[ G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a_{13}')^{(1)} + (a_{13}''')^{(1)}(T_{14})]} \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a_{15}')^{(1)} + (a_{15}''')^{(1)}(T_{14})]} \]

**Definition and uniqueness of T_{17} :-**

After hypothesis f(0) < 0, f(\infty) > 0 and the functions (a_i''')^{(2)}(T_{17}) being increasing, it follows that there exists a unique T_{17} for which f(T_{17}) = 0. With this value, we obtain from the three first equations

\[ G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a_{16}')^{(2)} + (a_{16}''')^{(2)}(T_{17})]} \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a_{18}')^{(2)} + (a_{18}''')^{(2)}(T_{17})]} \]

**Definition and uniqueness of T_{21} :-**

After hypothesis f(0) < 0, f(\infty) > 0 and the functions (a_i''')^{(1)}(T_{21}) being increasing, it follows that there exists a unique T_{21} for which f(T_{21}) = 0. With this value, we obtain from the three first equations

\[ G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a_{20}')^{(3)} + (a_{20}''')^{(3)}(T_{21})]} \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a_{22}')^{(3)} + (a_{22}''')^{(3)}(T_{21})]} \]

**Definition and uniqueness of T_{25} :-**

After hypothesis f(0) < 0, f(\infty) > 0 and the functions (a_i''')^{(4)}(T_{25}) being increasing, it follows that there exists a unique T_{25} for which f(T_{25}) = 0. With this value, we obtain from the three first equations

\[ G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a_{24}')^{(4)} + (a_{24}''')^{(4)}(T_{25})]} \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a_{26}')^{(4)} + (a_{26}''')^{(4)}(T_{25})]} \]

**Definition and uniqueness of T_{29} :-**

After hypothesis f(0) < 0, f(\infty) > 0 and the functions (a_i''')^{(5)}(T_{29}) being increasing, it follows that there exists a unique T_{29} for which f(T_{29}) = 0. With this value, we obtain from the three first equations

\[ G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a_{28}')^{(5)} + (a_{28}''')^{(5)}(T_{29})]} \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a_{30}')^{(5)} + (a_{30}''')^{(5)}(T_{29})]} \]
\textbf{Definition and uniqueness of } T^3_3 :\textbf{-}\n
After hypothesis \( f(0) < 0, f(\infty) > 0 \) and the functions \((a''r)(T_{33})\) being increasing, it follows that there exists a unique \( T^3_3 \) for which \( f(T^3_3) = 0 \). With this value, we obtain from the three first equations

\[
G_{32} = \frac{(a_{32})^{(6)}G_{33}}{(a_{32})^{(6)}+(a_{12})^{(6)}(T^3_3)} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{(a_{34})^{(6)}+(a_{14})^{(6)}(T^3_3)}
\]

\text{(e) By the same argument, \textbf{THE SOLUTIONAL EQUATIONS OF THE GLOBAL EQUATIONS}}

\textbf{ADMIT solutions } G_{13}, G_{14} \text{ if}

\[
\varphi(G) = (b_{13})^{(1)}(b_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - [\{b_{31}^{(2)}(b_{13})^{(3)}G_{19}\} + (b_{17})^{(2)}(b_{16})^{(2)}G_{19}] + (b_{16})^{(2)}(b_{17})^{(2)}G_{19} = 0
\]

Where in \( G(G_{13}, G_{14}, G_{15}) \), \( G_{13}, G_{15} \) must be replaced by their values. It is easy to see that \( \varphi \) is a decreasing function in \( G_{14} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G^*_4 \) such that \( \varphi(G^*_4) = 0 \)

\text{(f) By the same argument, \textbf{GLOBAL EQUATIONS \textbf{admit solutions}} } G_{16}, G_{17} \text{ if}

\[
\varphi(G_{19}) = (b_{16})^{(2)}(b_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - [\{b_{16}^{(2)}(b_{17})^{(2)}G_{19}\} + (b_{17})^{(2)}(b_{16})^{(2)}G_{19}] + (b_{16})^{(2)}(b_{17})^{(2)}G_{19} = 0
\]

Where in \( G_{19}(G_{16}, G_{17}, G_{18}) \), \( G_{16}, G_{18} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{16} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G^*_4 \) such that \( \varphi(G^*_4) = 0 \)

\text{(g) By the same argument, \textbf{SOLUTIONAL EQUATIONS \textbf{admit solutions} } G_{20}, G_{21} \text{ if}

\[
\varphi(G_{23}) = (b_{20})^{(3)}(b_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - [\{b_{20}^{(3)}(b_{21})^{(3)}G_{23}\} + (b_{21})^{(3)}(b_{20})^{(3)}G_{23}] + (b_{20})^{(3)}(b_{21})^{(3)}G_{23} = 0
\]

Where in \( G_{23}(G_{20}, G_{22}, G_{20}), G_{20}, G_{22} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{22} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G^*_4 \) such that \( \varphi(G^*_4) = 0 \)

\text{(h) By the same argument, \textbf{GLOBAL EQUATIONS \textbf{admit solutions} } G_{24}, G_{25} \text{ if}

\[
\varphi(G_{27}) = (b_{24})^{(4)}(b_{23})^{(4)} - (b_{24})^{(4)}(b_{23})^{(4)} - [\{b_{24}^{(4)}(b_{23})^{(4)}G_{27}\} + (b_{23})^{(4)}(b_{24})^{(4)}G_{27}] + (b_{24})^{(4)}(b_{23})^{(4)}G_{27} = 0
\]

Where in \( G_{27}(G_{24}, G_{25}, G_{26}), G_{24}, G_{26} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{25} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G^*_4 \) such that \( \varphi(G^*_4) = 0 \)

\text{(i) By the same argument, \textbf{GLOBAL EQUATIONS AND CONCOMITANT DERIVED}}

\textbf{ADMIT solutions } G_{26}, G_{29} \text{ if}

\[
\varphi(G_{31}) = (b_{28})^{(5)}(b_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - [\{b_{28}^{(5)}(b_{29})^{(5)}G_{31}\} + (b_{29})^{(5)}(b_{28})^{(5)}G_{31}] + (b_{28})^{(5)}(b_{29})^{(5)}G_{31} = 0
\]

Where in \( G_{31}(G_{26}, G_{29}, G_{30}), G_{26}, G_{30} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{29} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that
there exists a unique $G_{29}^*$ such that $\varphi((G_{31})^*) = 0$

By the same argument, the GLOBAL EQUATIONS admit solutions $G_{32}, G_{33}$ if

$$\varphi(G_{33}) = (b_{32j}^*)^6(b_{33}^*)^6 - (b_{322}^*)^6(b_{33}^*)^6 - [b_{32j}^*(G_{33}) + (b_{33}^*(b_{33}^*)^6(G_{33}) + (b_{322}^*)^6(G_{33})] = 0$$

Where in $(G_{33})(G_{32}, G_{34}), G_{32}, G_{34}$ must be replaced by their values. It is easy to see that $\varphi$ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^*$ such that $\varphi(G^*) = 0$

Finally we obtain the unique solution

$G_{14}^*$ given by $\varphi(G^*) = 0$, $T_{14}^*$ given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)}(G_{14})^*}{[a_{13}]^{(1)}(T_{14}^*)}, \quad G_{15}^* = \frac{(a_{15})^{(1)}(G_{14})^*}{[a_{15}]^{(1)}(T_{14}^*)}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}(T_{14}^*)}{[b_{13}]^{(1)}(T_{14}^*)}, \quad T_{15}^* = \frac{(b_{15})^{(1)}(T_{14}^*)}{[b_{15}]^{(1)}(T_{14}^*)}$$

Obviously, these values represent an equilibrium solution of THE SYSTEM

Finally we obtain the unique solution of THE SYSTEM

$G_{17}^*$ given by $\varphi(G_{19}^*) = 0$, $T_{17}^*$ given by $f(T_{17}^*) = 0$ and

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[a_{16}]^{(2)}(T_{17}^*)}, \quad G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[a_{18}]^{(2)}(T_{17}^*)}$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[b_{16}]^{(2)}(T_{17}^*)}, \quad T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[b_{18}]^{(2)}(T_{17}^*)}$$

Obviously, these values represent an equilibrium solution of THE GLOBAL EQUATIONS

Finally we obtain the unique solution of THE GLOBAL EQUATIONS

$G_{21}^*$ given by $\varphi(G_{23}^*) = 0$, $T_{21}^*$ given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[a_{20}]^{(3)}(T_{21}^*)}, \quad G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[a_{22}]^{(3)}(T_{21}^*)}$$

$$T_{20}^* = \frac{(b_{20})^{(3)}T_{21}^*}{[b_{20}]^{(3)}(T_{21}^*)}, \quad T_{22}^* = \frac{(b_{22})^{(3)}T_{21}^*}{[b_{22}]^{(3)}(T_{21}^*)}$$

Obviously, these values represent an equilibrium solution of GLOBAL SYSTEM

Finally we obtain the unique solution of THE SYSTEM

$G_{25}^*$ given by $\varphi(G_{27}^*) = 0$, $T_{25}^*$ given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)}G_{25}^*}{[a_{24}]^{(4)}(T_{25}^*)}, \quad G_{26}^* = \frac{(a_{26})^{(4)}G_{25}^*}{[a_{26}]^{(4)}(T_{25}^*)}$$

$$T_{24}^* = \frac{(b_{24})^{(4)}T_{25}^*}{[b_{24}]^{(4)}(T_{25}^*)}, \quad T_{26}^* = \frac{(b_{26})^{(4)}T_{25}^*}{[b_{26}]^{(4)}(T_{25}^*)}$$

Obviously, these values represent an equilibrium solution of THE GLOBAL SYSTEM

Finally we obtain the unique solution of THE GLOBAL SYSTEM

$G_{29}^*$ given by $\varphi(G_{31}^*) = 0$, $T_{29}^*$ given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)}G_{29}^*}{[a_{28}]^{(5)}(T_{29}^*)}, \quad G_{30}^* = \frac{(a_{30})^{(5)}G_{29}^*}{[a_{30}]^{(5)}(T_{29}^*)}$$

$$T_{28}^* = \frac{(b_{28})^{(5)}T_{29}^*}{[b_{28}]^{(5)}(T_{29}^*)}, \quad T_{30}^* = \frac{(b_{30})^{(5)}T_{29}^*}{[b_{30}]^{(5)}(T_{29}^*)}$$

Obviously, these values represent an equilibrium solution of THE SYSTEM
Finally we obtain the unique solution of THE DERIVED EQUATIONS OF THE GLOBAL EQUATIONS

\[ G_{33}^* \text{ given by } \varphi((G_{35})^*) = 0, \ T_{33}^* \text{ given by } f(T_{33}^*) = 0 \text{ and} \]

\[ G_{32}^* = \frac{(a_{32})^6 G_{32}}{[a_{32}]^6 + [a_{32}^4]^6} , \ G_{34}^* = \frac{(a_{34})^6 G_{34}}{[a_{34}]^6 + [a_{34}^4]^6} \]

\[ T_{32}^* = \frac{(b_{32})^6 r_{32}^*}{(b_{32})^6 - (b_{32}^4)^6} , \ T_{34}^* = \frac{(b_{34})^6 r_{34}^*}{(b_{34})^6 - (b_{34}^4)^6} \]

Obviously, these values represent an equilibrium solution of THE SYSTEM


Energy and Quantum Field

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions

\((a''')^{(1)} \text{ and } (b''')^{(1)} \) Belong to \( C^1(\mathbb{R}_+) \) then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of \( G_i, T_i \):

\[ G_i = G_i^* + G_i \quad , \quad T_i = T_i^* + T_i \]

\[ \frac{\partial (a''')^{(1)}}{\partial G_i} (T_{14}^*) = (q_{14})^{(1)} , \quad \frac{\partial (b''')^{(1)}}{\partial G_i} (G^*) = s_{ij} \]

Then taking into account DERIVED EQUATIONS OF THE GLOBAL EQUATIONS neglecting the terms of power 2, we obtain

\[ \frac{dG_{13}}{dt} = -((a_{13}^{(1)} + (p_{13})^{(1)}) G_{13} + (a_{13})^{(1)} G_{13} - (q_{13})^{(1)} G_{13}^* T_{14} \]

\[ \frac{dG_{14}}{dt} = -((a_{14}^{(1)} + (p_{14})^{(1)}) G_{14} + (a_{14})^{(1)} G_{14} - (q_{14})^{(1)} G_{14}^* T_{14} \]

\[ \frac{dG_{15}}{dt} = -((a_{15}^{(1)} + (p_{15})^{(1)}) G_{15} + (a_{15})^{(1)} G_{15} - (q_{15})^{(1)} G_{15}^* T_{14} \]

\[ \frac{dT_{13}}{dt} = -((b_{13}^{(1)} + (r_{13})^{(1)}) T_{13} + (b_{13})^{(1)} T_{14} + \sum_{j=1}^{15} (s_{13}(j) T_{15} G_{15}) \]

\[ \frac{dT_{14}}{dt} = -((b_{14}^{(1)} + (r_{14})^{(1)}) T_{14} + (b_{14})^{(1)} T_{13} + \sum_{j=1}^{15} (s_{14}(j) T_{15} G_{15}) \]

\[ \frac{dT_{15}}{dt} = -((b_{15}^{(1)} + (r_{15})^{(1)}) T_{15} + (b_{15})^{(1)} T_{13} + \sum_{j=1}^{15} (s_{15}(j) T_{15} G_{15}) \]

If the conditions of the previous theorem are satisfied and if the functions \((a''')^{(2)} \text{ and } (b''')^{(2)} \) Belong to \( C^2(\mathbb{R}_+) \) then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of \( G_i, T_i \):

\[ G_i = G_i^* + G_i \quad , \quad T_i = T_i^* + T_i \]

\[ \frac{\partial (a''')^{(2)}}{\partial G_i} (T_{17}^*) = (q_{17})^{(2)} , \quad \frac{\partial (b''')^{(2)}}{\partial G_i} (G_{19}^*) = s_{ij} \]

taking into account equations DERIVED EQUATIONS OF THE GLOBAL EQUATIONS and neglecting the terms of power 2, we obtain

\[ \frac{dG_{16}}{dt} = -((a_{16}^{(2)} + (p_{16})^{(2)}) G_{16} + (a_{16})^{(2)} G_{17} + (q_{16})^{(2)} G_{16}^* T_{17} \]
If the conditions of the previous theorem are satisfied and if the functions \(a_i''(3)\) and \(b_i''(3)\) belong to \(C^3(\mathbb{R}_+)^n\) then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** \(G_i, T_i^*\):

\[
G_i = G_i^* + \xi_i, \quad T_i = T_i^* + \xi_i
\]

\[
\frac{\partial a_i''(3)}{\partial x_i} (T_i^*) = (q_{2i})^{(3)}, \quad \frac{\partial b_i''(3)}{\partial \xi_j} (G_{2i})^* = s_{ij}
\]

Then taking into account equations DERIVED FROM THE GLOBAL EQUATIONS and neglecting the terms of power 2, we obtain

\[
\frac{dG_{20}}{dt} = -((a_{20}''(3) + (p_{20})^{(3)})G_{20} + (a_{20})^{(3)}G_{21} - (q_{20})^{(3)}G_{20}^* T_{21}
\]

\[
\frac{dG_{21}}{dt} = -((a_{21}''(3) + (p_{21})^{(3)})G_{21} + (a_{21})^{(3)}G_{20} - (q_{21})^{(3)}G_{21}^* T_{21}
\]

\[
\frac{dG_{22}}{dt} = -((a_{22}''(3) + (p_{22})^{(3)})G_{22} + (a_{22})^{(3)}G_{21} - (q_{22})^{(3)}G_{22}^* T_{21}
\]

\[
\frac{dG_{23}}{dt} = -((b_{2i}''(3) - (r_{2i})^{(3)})G_{23} + (b_{2i})^{(3)}G_{21} + \sum_{j=20}^{22}(G_{2j}^* T_{2j})
\]

If the conditions of the previous theorem are satisfied and if the functions \(a_i''(4)\) and \(b_i''(4)\) belong to \(C^4(\mathbb{R}_+)^n\) then the above equilibrium point is asymptotically stable.(FOURTH MODULE)

**Proof:** Denote

**Definition of** \(G_i, T_i^*\):

\[
G_i = G_i^* + \xi_i, \quad T_i = T_i^* + \xi_i
\]

\[
\frac{\partial a_i''(4)}{\partial x_i} (T_i^*) = (q_{2i})^{(4)}, \quad \frac{\partial b_i''(4)}{\partial \xi_j} (G_{2i})^* = s_{ij}
\]

Then taking into account equations DERIVED EQUATIONS OF THE GLOBAL EQUATIONS and neglecting the terms of power 2, we obtain

\[
\frac{dG_{24}}{dt} = -((a_{24}''(4) + (p_{24})^{(4)})G_{24} + (a_{24})^{(4)}G_{25} - (q_{24})^{(4)}G_{24}^* T_{25}
\]

\[
\frac{dG_{25}}{dt} = -((a_{25}''(4) + (p_{25})^{(4)})G_{25} + (a_{25})^{(4)}G_{24} - (q_{25})^{(4)}G_{25}^* T_{25}
\]

\[
\frac{dG_{26}}{dt} = -((a_{26}''(4) + (p_{26})^{(4)})G_{26} + (a_{26})^{(4)}G_{25} - (q_{26})^{(4)}G_{26}^* T_{25}
\]

\[
\frac{dG_{27}}{dt} = -((b_{2i}''(4) - (r_{2i})^{(4)})G_{27} + (b_{2i})^{(4)}G_{25} + \sum_{j=24}^{26}(G_{2j}^* T_{2j})
\]
\[
\frac{dT_{25}}{dt} = -((b_{25}(5) - r_{25}(4))T_{25} + (b_{25}(5)T_{24} + \sum_{j=24}^{26}\lambda_{(25,j)}T_{25}G_{j})
\]

\[
\frac{dT_{26}}{dt} = -((b_{26}(5) - r_{26}(4))T_{26} + (b_{26}(5)T_{25} + \sum_{j=24}^{26}\lambda_{(26,j)}T_{26}G_{j})
\]

**ASYMPTOTIC STABILITY ANALYSIS (FIFTH MODULE)**

If the conditions of the previous theorem are satisfied and if the functions \(a_i^{(5)}\) and \(b_i^{(5)}\) Belong to \(C^{(5)}(\mathbb{R}^+)\) then the above equilibrium point is asymptotically stable.

Denote

**Definition of** \(G_i, T_i :-\)

\[
G_i = G_i^* + \xi_i, \quad T_i = T_i^* + \xi_i
\]

\[
\frac{\partial a_i^{(5)}}{\partial \xi_i}(T_{29}^{(*)}) = (q_{29})^{(5)}, \quad \frac{\partial b_i^{(5)}}{\partial \xi_j}( (G_{31})^{*} ) = s_{ij}
\]

Then taking into account equations **DERIVED EQUATIONS OF THE GLOBAL EQUATIONS** and neglecting the terms of power 2, we obtain

\[
\frac{dG_{28}}{dt} = -((a_{28}^{(5)}) + (p_{28})^{(5)}G_{28} + (a_{28})^{(5)}G_{29} - (q_{28})^{(5)}G_{28} T_{29}
\]

\[
\frac{dG_{29}}{dt} = -((a_{29}^{(5)}) + (p_{29})^{(5)}G_{29} + (a_{29})^{(5)}G_{28} - (q_{29})^{(5)}G_{29} T_{29}
\]

\[
\frac{dG_{30}}{dt} = -((a_{30}^{(5)}) + (p_{30})^{(5)}G_{30} + (a_{30})^{(5)}G_{29} - (q_{30})^{(5)}G_{30} T_{29}
\]

\[
\frac{dG_{31}}{dt} = -((b_{31}^{(5)}) - (r_{31})^{(5)}T_{31} + (b_{31})^{(5)}T_{32} + \sum_{j=28}^{30}(\lambda_{(31,j)}T_{28}G_{j})
\]

\[
\frac{dG_{32}}{dt} = -((b_{32}^{(5)}) - (r_{32})^{(5)}T_{32} + (b_{32})^{(5)}T_{33} + \sum_{j=28}^{30}(\lambda_{(32,j)}T_{28}G_{j})
\]

**ASYMPTOTIC STABILITY ANALYSIS (SIXTH MODULE RAMIFICATIONS ON THE CONCATENATED GLOBAL EQUATIONS)**

If the conditions of the previous theorem are satisfied and if the functions \(a_i^{(6)}\) and \(b_i^{(6)}\) Belong to \(C^{(6)}(\mathbb{R}^+)\) then the above equilibrium point is asymptotically stable.

Denote

**Definition of** \(G_i, T_i :-\)

\[
G_i = G_i^* + \xi_i, \quad T_i = T_i^* + \xi_i
\]

\[
\frac{\partial a_i^{(6)}}{\partial \xi_i}(T_{33}^{(*)}) = (q_{33})^{(6)}, \quad \frac{\partial b_i^{(6)}}{\partial \xi_j}( (G_{35})^{*} ) = s_{ij}
\]

Then taking into account equations **DERIVED FROM THE CONCATENATED GLOBAL EQUATIONS** and neglecting the terms of power 2, we obtain

\[
\frac{dG_{32}}{dt} = -((a_{32}^{(6)}) + (p_{32})^{(6)}G_{32} + (a_{32})^{(6)}G_{33} - (q_{32})^{(6)}G_{32} T_{33}
\]

\[
\frac{dG_{33}}{dt} = -((a_{33}^{(6)}) + (p_{33})^{(6)}G_{33} + (a_{33})^{(6)}G_{32} - (q_{33})^{(6)}G_{33} T_{33}
\]

\[
\frac{dG_{34}}{dt} = -((a_{34}^{(6)}) + (p_{34})^{(6)}G_{34} + (a_{34})^{(6)}G_{33} - (q_{34})^{(6)}G_{34} T_{33}
\]

\[
\frac{dG_{35}}{dt} = -((b_{35}^{(6)}) - (r_{35})^{(6)}T_{32} + (b_{35})^{(6)}T_{33} + \sum_{j=32}^{34}(\lambda_{(35,j)}T_{32}G_{j})
\]

\[
\frac{dG_{32}}{dt} = -((b_{33}^{(6)}) - (r_{33})^{(6)}T_{33} + (b_{33})^{(6)}T_{32} + \sum_{j=32}^{34}(\lambda_{(33,j)}T_{32}G_{j})
\]
\[ \frac{dT_{34}}{dt} = -(b_{34}(6) - r_{34}(6)) T_{34} + (b_{34}(6) T_{33} + \sum_{j=12}^{34}(s_{34}(j) T_{34} G_{j}) \]

The characteristic equation of this system is

\[ ((\lambda)^{(1)} + (b_{34}(1)^{(1)} - (r_{34}(1)^{(1)}))((\lambda)^{(1)} + (a_{13}(1)^{(1)} + (p_{13}(1)^{(1)})) \]
\[ \left[ ((\lambda)^{(1)} + (a_{13}(1)^{(1)} + (p_{13}(1)^{(1)})(q_{14}(1)^{(1)} G_{14} + (a_{13}(1)^{(1)} G_{13}^*) \right] \]
\[ \left( ((\lambda)^{(1)} + (b_{34}(1)^{(1)} - (r_{34}(1)^{(1)})s_{(14),(14)} T_{14} + (b_{14}(1)^{(1)} s_{(13),(13)} T_{13} \right] \]
\[ + \left( ((\lambda)^{(1)} + (a_{13}(1)^{(1)} + (p_{14}(1)^{(1)})(q_{14}(1)^{(1)} G_{14}^*) + (a_{13}(1)^{(1)} G_{14}^*) \right] \]
\[ \left( ((\lambda)^{(1)} + (b_{13}(1)^{(1)} - (r_{13}(1)^{(1})s_{(14),(13)} T_{14} + (b_{14}(1)^{(1)} s_{(13),(13)} T_{13} \right] \]
\[ + ((\lambda)^{(1)} + (b_{13}(1)^{(1)} - (r_{13}(1)^{(1})s_{(14),(13)} T_{14} + (b_{14}(1)^{(1)} s_{(13),(13)} T_{13} \right] = 0 \]
\[ + ((\lambda)^{(2)} + (b_{18}(2)^{(2)} - (r_{18}(2)^{(2)}} \]
\[ \left[ ((\lambda)^{(2)} + (a_{18}(2)^{(2)} + (p_{18}(2)^{(2)})(q_{17}(2)^{(2)} G_{17}^* + (a_{17}(2)^{(2)} (q_{16}(2)^{(2)} G_{16}^* \right] \]
\[ \left( ((\lambda)^{(1)} + (b_{19}(2)^{(2)} - (r_{18}(2)^{(2)})s_{(17),(17)} T_{17} + (b_{17}(2)^{(2)} s_{(16),(16)} T_{16} \right] \]
\[ + ((\lambda)^{(2)} + (b_{17}(2)^{(2)} + (p_{17}(2)^{(2)})(q_{16}(2)^{(2)} G_{16}^* + (a_{16}(2)^{(2)} (q_{17}(2)^{(2)} G_{17}^* \right] \]
\[ \left( ((\lambda)^{(2)} + (b_{19}(2)^{(2)} - (r_{18}(2)^{(2)})s_{(17),(17)} T_{17} + (b_{17}(2)^{(2)} s_{(16),(16)} T_{16} \right] \]
\[ + ((\lambda)^{(2)} + (b_{17}(2)^{(2)} + (p_{17}(2)^{(2)})(q_{16}(2)^{(2)} G_{16}^* + (a_{16}(2)^{(2)} (q_{17}(2)^{(2)} G_{17}^* \right] \]
\[ + ((\lambda)^{(2)} + (b_{18}(2)^{(2)} - (r_{18}(2)^{(2)}} \]
\[ \left[ ((\lambda)^{(2)} + (a_{18}(2)^{(2)} + (p_{18}(2)^{(2)})(q_{17}(2)^{(2)} G_{17}^* + (a_{17}(2)^{(2)} (q_{16}(2)^{(2)} G_{16}^* \right] \]
\[ \left( ((\lambda)^{(2)} + (b_{19}(2)^{(2)} - (r_{18}(2)^{(2)})s_{(17),(17)} T_{17} + (b_{17}(2)^{(2)} s_{(16),(16)} T_{16} \right] = 0 \]
\[ + ((\lambda)^{(3)} + (b_{22}(3)^{(3)} - (r_{22}(3)^{(3)}} \]
\[ \left[ ((\lambda)^{(3)} + (a_{22}(3)^{(3)} + (p_{22}(3)^{(3)}) (q_{21}(3)^{(3)} G_{21}^* + (a_{21}(3)^{(3)} (q_{20}(3)^{(3)} G_{20}^* \right] \]
\[ \left( ((\lambda)^{(3)} + (b_{20}(3)^{(3)} - (r_{20}(3)^{(3)})s_{(21),(21)} T_{21} + (b_{21}(3)^{(3)} s_{(20),(21)} T_{21} \right] \]
\[ + ((\lambda)^{(3)} + (a_{21}(3)^{(3)} + (p_{21}(3)^{(3)})(q_{20}(3)^{(3)} G_{20}^* + (a_{20}(3)^{(3)} (q_{21}(3)^{(3)} G_{21}^* \right] \]
\[ \left( ((\lambda)^{(3)} + (b_{20}(3)^{(3)} - (r_{20}(3)^{(3)})s_{(21),(20)} T_{21} + (b_{21}(3)^{(3)} s_{(20),(20)} T_{21} \right] \]

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\[
\left( (\lambda)^{(3)} \right)^2 + \left( a_{20}^{(3)} + a_{21}^{(3)} + p_{20}^{(3)} + p_{21}^{(3)} \right) (\lambda)^{(3)}
\]
\[
\left( (\lambda)^{(3)} \right)^2 + \left( b_{20}^{(3)} + b_{21}^{(3)} - r_{20}^{(3)} + r_{21}^{(3)} \right) (\lambda)^{(3)}
\]
\[
+ \left( (\lambda)^{(3)} \right)^2 + \left( a_{20}^{(3)} + a_{21}^{(3)} + p_{20}^{(3)} + p_{21}^{(3)} \right) (\lambda)^{(3)}\right) (q_{22}^{(3)} G_{22}^*)
\]
\[
+ (\lambda)^{3} + (a_{20}^{(3)} + p_{20}^{(3)}) \left( a_{22}^{(3)} (q_{21}^{(3)} G_{21}^* + (a_{21}^{(3)} a_{22}^{(3)} (q_{20}^{(3)} G_{20}^*)
\]
\[
\left( (\lambda)^{(3)} + (b_{20}^{(3)} - (r_{20}^{(3)}) s_{(21),(22)} T_{21}^* + (b_{21}^{(3)} s_{(20),(22)} T_{20}^*) = 0 \right)
\]
\[
+ (\lambda)^{(4)} + (b_{26}^{(4)} - (r_{26}^{(4)}) (\lambda)^{(4)} + (a_{26}^{(4)} + p_{26}^{(4)}
\]
\[
\left[ ((\lambda)^{(4)} + (a_{24}^{(4)} + (p_{24}^{(4)} (q_{25}^{(4)} G_{25}^* + (a_{23}^{(4)} (q_{24}^{(4)} G_{24}^*)
\]
\[
\left( (\lambda)^{(4)} + (b_{24}^{(4)} - (r_{24}^{(4)} s_{(25),(25)} T_{25}^* + (b_{25}^{(4)} s_{(24),(24)} T_{24}^*)
\]
\[
+ ((\lambda)^{(4)} + (a_{25}^{(4)} + (p_{25}^{(4)} (q_{24}^{(4)} G_{24}^* + (a_{24}^{(4)} (q_{25}^{(4)} G_{25}^*)
\]
\[
\left( (\lambda)^{(4)} + (b_{24}^{(4)} - (r_{24}^{(4)} s_{(25),(24)} T_{25}^* + (b_{25}^{(4)} s_{(24),(24)} T_{24}^*)
\]
\[
\left( (\lambda)^{(4)} + (b_{24}^{(4)} + (b_{25}^{(4)} (a_{2}^{(4)} + (p_{24}^{(4)} (a_{25}^{(4)} + (p_{25}^{(4)} (a_{24}^{(4)}
\]
\[
\left( (\lambda)^{(4)} + (b_{24}^{(4)} - (r_{24}^{(4)} s_{(25),(26)} T_{25}^* + (b_{25}^{(4)} s_{(24),(26)} T_{24}^*) = 0 \right)
\]
\[
+ (\lambda)^{(5)} + (b_{26}^{(5)} - (r_{30}^{(5)}) (\lambda)^{(5)} + (a_{30}^{(5)} + (p_{30}^{(5)}
\]
\[
\left[ ((\lambda)^{(5)} + (a_{26}^{(5)} + (p_{28}^{(5)} (q_{29}^{(5)} G_{29}^* + (a_{24}^{(5)} (q_{28}^{(5)} G_{28}^*)
\]
\[
\left( (\lambda)^{(5)} + (b_{26}^{(5)} - (r_{28}^{(5)} s_{(29),(29)} T_{29}^* + (b_{29}^{(5)} s_{(28),(29)} T_{28}^*)
\]
\[
+ ((\lambda)^{(5)} + (a_{29}^{(5)} + (p_{29}^{(5)} (q_{28}^{(5)} G_{28}^* + (a_{28}^{(5)} (q_{29}^{(5)} G_{29}^*)
\]
\[
\left( (\lambda)^{(5)} + (b_{24}^{(5)} - (r_{28}^{(5)} s_{(29),(28)} T_{28}^* + (b_{29}^{(5)} s_{(28),(28)} T_{28}^*)
\]
\[
\left( (\lambda)^{(5)} + (b_{26}^{(5)} + (b_{29}^{(5)} (a_{2}^{(5)} + (p_{28}^{(5)} (a_{28}^{(5)} + (p_{29}^{(5)} (a_{27}^{(5)}
\]
\[
\left( (\lambda)^{(5)} + (b_{26}^{(5)} - (r_{28}^{(5)} s_{(29),(26)} T_{28}^* + (b_{29}^{(5)} s_{(28),(26)} T_{28}^*) = 0 \right)
\]
\[
+ (\lambda)^{(6)} + (b_{34}^{(6)} - (r_{34}^{(6)}) (\lambda)^{(6)} + (a_{34}^{(6)} + (p_{34}^{(6)}
\]
\[
\left[ ((\lambda)^{(6)} + (a_{32}^{(6)} + (p_{32}^{(6)} (q_{33}^{(6)} G_{33}^* + (a_{33}^{(6)} (q_{32}^{(6)} G_{32}^*)
\]
\[
((\lambda)^{(6)} + (b_{32}^{'})^{(6)} - (r_{32})^{(6)})S_{(33),(33)}T_{33}^{*} + (b_{33})^{(6)}S_{(32),(33)}T_{32}^{*}
\]
\[
+ ((\lambda)^{(6)} + (a_{33}^{'})^{(6)} + (p_{33})^{(6)})(q_{32})^{(6)}G_{32} + (a_{32})^{(6)}(q_{33})^{(6)}G_{32}^{*}
\]
\[
+ ((\lambda)^{(6)} + (b_{32}^{'})^{(6)} - (r_{32})^{(6)})S_{(33),(32)}T_{32}^{*} + (b_{33})^{(6)}S_{(32),(32)}T_{32}^{*}
\]
\[
[((\lambda)^{(6)} + (a_{32}^{'})^{(6)} + (a_{33}^{'})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)})(\lambda)^{(6)}
\]
\[
+ ((\lambda)^{(6)} + (a_{32}^{'})^{(6)} + (p_{32})^{(6)})((a_{34})^{(6)}(q_{33})^{(6)}G_{33} + (a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G_{32}^{*})
\]
\[
+ ((\lambda)^{(6)} + (b_{32}^{'})^{(6)} - (r_{32})^{(6)})S_{(33),(34)}T_{33}^{*} + (b_{33})^{(6)}S_{(32),(34)}T_{32}^{*}
\]
\[
= 0
\]
And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

Acknowledgments:

The introduction is a collection of information from various articles, Books, News Paper reports, Home Pages Of authors, Journal Reviews, the internet including Wikipedia. We acknowledge all authors who have contributed to the same. In the eventuality of the fact that there has been any act of omission on the part of the authors, we regret with great deal of compunction, contrition, and remorse. As Newton said, it is only because erudite and eminent people allowed one to piggy ride on their backs; probably an attempt has been made to look slightly further. Once again, it is stated that the references are only illustrative and not comprehensive.

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