Journal of Natural Sciences Research www.iiste.org
ISSN 2224-3186 (Paper) ISSN 2225-0921 (Online)
Vol.5, No.3, 2015

Preference of Estimation Approach for Rayleigh Progressive Type II Data

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Abstract.
This paper compare the performance of the empirical Bayes and generalized maximum likelihood estimation approaches in context of progressively Type II censored data from one parameter Rayleigh distribution. The generalized maximum likelihood and empirical Bayes estimates of scale parameter, reliability function, and failure rate function are compared using risk efficiency criterion. The empirical Bayes estimates are considered with respect to squared error loss function. The wind speed data is presented to illustrate the proposed estimation approaches, and an extensive Monte Carlo simulated study is done to compare the empirical Bayes and Generalized maximum likelihood estimates. The study indicates that the empirical Bayesian approach using squared error loss function is preferable than the generalized maximum likelihood approach for the estimation of reliability performances.
Keywords: Progressively Type II censored samples, generalized maximum likelihood estimation, squared error loss function, empirical Bayes estimation, Risk efficiency, Monte Carlo simulation.

1. Introduction

Rayleigh distribution is one of the most popular and widely used distributions in reliability and life testing analysis. Lord Rayleigh (1880) invented this distribution from the amplitude of sound resulting from many important sources. Polovko (1968) demonstrated the importance of the distribution in communication engineering and electro vacuum devices. Siddique (1962) has used this distribution as a radio wave power distribution. Bhattacharya and Tyagi (1990) applied this distribution in some clinical studies dealing with cancer patients.

In lifetime analysis, the most popular censoring schemes among the various types of censoring schemes is Type II censoring. Under this censoring scheme, the life testing experiment continues until a pre-specified number of failures occurs. However, in the above conventional schemes, a researcher cannot remove experimental units at points other than the terminal point of the experiment. Cohen (1963) generalized the conventional Type II censoring scheme in a manner that removal of the units are allowed in between also. This generalization is referred to as progressive Type II censoring scheme, which is useful in many practical situations where budget constraints are in place or there is a demand for rapid testing. It is known that this censoring scheme significantly improve upon conventional Type II censoring, and therefore received a significant importance in the last few decades. Several authors have discussed statistical inference problems for various distributions under progressive Type II censoring (Balakrishnan & Aggarwala 2000, Wu *et al.* 2006, Patel & Patel 2007).

Empirical Bayes approach is commonly used to make a data-driven choice of hyper-parameter (parameter of prior distribution). In practice rather than specifying the hyper-parameter, researcher often tempted to use some estimate of the hyper-parameter for expressing honest prior information. Many authors have described this approach extensively (Robbins 1964, Maritz & Lewin 1989, Casella 1992, Carlin & Louis 1996, Lehmann & Casella 1998). In order to obtain EB estimates, the choice of an appropriate loss function is essential, and depends on financial consideration only. One of the most popular symmetric loss functions is the squared error loss function (SELF), proposed by Legendre (1805) and Gauss (1810) to develop the least square theory. Most of the Bayesian inference procedures have been developed under the usual SELF, which gives an equal weight to over-estimation and under-estimation because of its symmetrical nature.

Based on risk efficiency criterion, the performance of Bayes estimates has been studied by several authors (Al-Nachawati & Abu-Youssef 2009, Dey 2011, Dey 2012, Barot & Patel 2014). However, up to now, the performance of empirical Bayes estimates of reliability performances relative to the SELF are not compared with that of generalized maximum likelihood (GML) estimates using risk efficiency criterion when the data are progressively censored from the Rayleigh distribution. The aim of the paper is to examine and compare the GML and empirical Bayes estimates of scale parameter, reliability function, and failure rate function of Rayleigh model under progressively Type II censoring. Section 2 describes the GML estimation and empirical Bayes estimation relative to SELF. In Section 3, the risk functions of GML and empirical Bayes estimates of reliability parameters relative to SELF are obtained under SELF. In Section 4, the wind speed data is analysed for an illustrative purpose. In Section 5, an extensive Monte Carlo simulation study is carried out to examine and compare the performance of the derived estimates. The paper concludes with a brief discussion in Section 6.

2. Estimation of Reliability Parameters of the Rayleigh Model

Under progressive Type II censoring scheme, let n units are placed on a life-testing experiment and only $m(<n)$ are completely observed until failure. At the time of each failure occurring prior to the termination point, one or more surviving units are removed from the test. Let r_i be the withdrawn units at i^{th} failure, $i=1,2,\dots,m$; and $x_{(i)}$ be the lifetimes of completely observed units following one parameter Rayleigh distribution with the probability density, cumulative distribution, reliability, and failure rate functions, respectively,

$$f(x\mid\theta)=\frac{x}{\theta^2}e^{-\frac{x^2}{2\theta^2}}\ ;\ F(x\mid\theta)=1-e^{-\frac{x^2}{2\theta^2}}\ ;R(t)=e^{-\frac{t^2}{2\theta^2}}\ ;\ \text{and}\ \ \lambda(t)=\frac{t}{\theta^2}\ ;\ x>0,\ \theta>0,\ t>0. \tag{1}$$

The likelihood function based the progressive Type II censored sample $\underset{\sim}{x}=\left(x_{(1)},x_{(2)},\ \dots\ ,x_{(m)}\right)$ can be obtained as (Barot & Patel 2014)

$$L(\underset{\sim}{x}\mid\theta)=\frac{A\prod_{i=1}^{m}x_{(i)}}{\theta^{2m}}e^{-\frac{T}{2\theta^2}}, \tag{2}$$

where

$$A=n(n-1-r_1)(n-2-r_1-r_2)\ .....\ \left(n-m+1-\sum_{i=1}^{m-1}r_i\right)\ \text{and}\ T=\sum_{i=1}^{m}(1+r_i)x_{(i)}^2.$$

It is straightforward to show that T has the gamma distribution with the shape parameter m, scale parameter $2\theta^2$ and probability density function

$$h(T)=\frac{e^{-\frac{T}{2\theta^2}}T^{m-1}}{\left(2\theta^2\right)^m\Gamma(m)}\quad ;\quad T>0. \tag{3}$$

To describe the uncertainty surrounding the unknown quantity θ of the model, we require assigning its prior distribution. In the previous couple of decades, many types of discrete, continuous, and mixed prior distributions have been proposed to consider subjective inputs from experienced experts or summary judgments of past research that yielded similar results. An inverted gamma distribution is one of most prominent random probability distributions, and its good mathematical properties facilitate insight and computational reduction. In reliability analysis and life testing, it is preferred over many other distributions due to its richness, computational ease, better fit to the failure data, analytical tractability, and easy interpretability. To ease the computational burden and get computable closed form expression for the posterior distribution of θ, it is assumed that θ follows an inverted gamma distribution with the probability density function

$$\pi(\theta\ \mid\ b)\ =\ \frac{\theta^{-2b-1}\ e^{-\frac{1}{2\theta^2}}}{\Gamma(b)\ 2^{b-1}}, \tag{4}$$

Where b is an unknown positive hyper-parameter chosen to reflect prior beliefs on θ. Since the prior density (4) belongs to a parametric family, the maximum likelihood estimate of the hyper-parameter, denoted by $\hat{b}_{ML}$, can be used for expressing honest prior information. Following the idea of Barot & Patel (2014), the estimate $\hat{b}_{ML}$ can be obtained by solving the log likelihood equation

$$\ln\left[\frac{b+m}{b(1+T)}\right]+\frac{m}{2b(b+m)}=0. \tag{5}$$

2.1 GML Estimation

The GML estimate of the parameter θ, denoted by $\hat{\theta}_{GML}$, is the value of θ at which the posterior density of θ given $\underset{\sim}{x}$ is maximal. The invariance property of GML estimation enables one to obtain the GML estimates $\hat{R}(t)_{GML}$ and $\hat{h}(t)_{GML}$ by substituting $\hat{\theta}_{GML}$ for θ in (1). Barot & Patel (2014) have obtained GML estimates of reliability parameters, respectively, as

$$\hat{\theta}_{GML}=\sqrt{\frac{1+T}{2\left(\hat{b}_{ML}+m\right)+1}}\ ;\ \hat{R}(t)_{GML}=e^{-\frac{t^2}{2\hat{\theta}^2_{GML}}}\ ;\ \text{and}\ \lambda(t)=\frac{t}{\hat{\theta}^2_{GML}}. \tag{6}$$

2.2 Empirical Bayes Estimation

As the performance of Bayes estimate depends upon the loss function, its choice is an integral part of Bayes estimation procedure. The SELF is one of the most popular symmetrical loss functions due to its mathematical simplicity and relevance with classical procedures. It is in the form $L(\hat{\phi},\phi)=(\hat{\phi}-\phi)^2$ and symmetrical in nature, that is, gives equal importance to the losses due to overestimation and underestimation of equal magnitude. This loss function is appropriate when decisions become gradually more damaging for large errors. Under the SELF, the usual Bayes estimate of scale parameter is the posterior mean. It is more appropriate when decisions become gradually more damaging for large errors. Following the idea of Lehmann and Casella (1998), the empirical Bayes estimates of scale parameter, reliability function, and failure rate function under SELF are obtained, respectively, as

$$\hat{\theta}_{ESE}=\frac{\Gamma\left(\hat{b}_{ML}+m-\frac{1}{2}\right)}{\Gamma\left(\hat{b}_{ML}+m\right)}\sqrt{\frac{1+T}{2}}\ ;\ \hat{R}(t)_{ESE}=\left(1+\frac{t^2}{1+T}\right)^{-\left(\hat{b}_{ML}+m\right)}\ ;\ \text{and}\ \hat{h}(t)_{ESE}=\frac{2\left(\hat{b}_{ML}+m\right)t}{1+T}. \tag{7}$$

3. Risk functions of GML and empirical Bayes estimates of reliability parameters under SELF

At this point, we obtain the risk functions of GML and empirical Bayes estimates of reliability parameters under *SELF*. The risk function of the estimate $\hat{\phi}$ under *SELF* is given by

$$R_L(\hat{\phi})=E^h\left[L(\hat{\phi},\phi)\right]==\int_0^\infty(\hat{\phi}-\phi)^2h(T)\,dT. \tag{8}$$

The risk functions of $\hat{\theta}_{ESE}$, and $\hat{\theta}_{GML}$ under *SELF* are given, respectively, by

$$R_L\left(\hat{\theta}_{ESE}\right)=\begin{cases}\left[\dfrac{\Gamma\left(\hat{b}_{ML}+m-\frac{1}{2}\right)}{\Gamma\left(\hat{b}_{ML}+m\right)}\right]^2\left(\dfrac{1+2m\theta^2}{2}\right)+\dfrac{\sqrt{2}\,\theta\,\Gamma\left(\hat{b}_{ML}+m-\frac{1}{2}\right)}{\Gamma\left(\hat{b}_{ML}+m\right)\Gamma(m)}S_1+\theta^2 & \text{when } T<1\\ \left[\dfrac{\Gamma\left(\hat{b}_{ML}+m-\frac{1}{2}\right)}{\Gamma\left(\hat{b}_{ML}+m\right)}\right]^2\left(\dfrac{1+2m\theta^2}{2}\right)+\dfrac{2\theta^2\,\Gamma\left(\hat{b}_{ML}+m-\frac{1}{2}\right)}{\Gamma\left(\hat{b}_{ML}+m\right)\Gamma(m)}S_2+\theta^2 & \text{when } T>1\end{cases}, \tag{9}$$

and

$$R_L\left(\hat{\theta}_{GML}\right)=\begin{cases}\dfrac{1}{\sqrt{2\hat{b}_{ML}+2m+1}}\left\{\dfrac{1+2m\theta^2}{\sqrt{2\hat{b}_{ML}+2m+1}}+\dfrac{2\theta}{\Gamma(m)}S_1\right\}+\theta^2, & \text{when } T<1\\ \dfrac{1}{\sqrt{2\hat{b}_{ML}+2m+1}}\left\{\dfrac{1+2m\theta^2}{\sqrt{2\hat{b}_{ML}+2m+1}}+\dfrac{2\sqrt{2}\,\theta^2}{\Gamma(m)}S_2\right\}+\theta^2, & \text{when } T>1\end{cases}, \tag{10}$$

where

$$S_1=\sum_{k=0}^{\infty}\binom{1/2}{k}\Gamma(m+k)\left(2\theta^2\right)^k\ ;\ S_2=\sum_{k=0}^{\infty}\binom{1/2}{k}\Gamma\left(m-k+\frac{1}{2}\right)\left(2\theta^2\right)^{-k}.$$

The risk functions of $\hat{R}(t)_{ESE}$ and $\hat{R}(t)_{GML}$ under SELF are given, respectively, by

$$R_L\left(\hat{R}(t)_{ESE}\right)=\begin{cases}\dfrac{1}{\Gamma(m)}\left[R_1-2\exp\left(-\dfrac{t^2}{2\theta^2}\right)R_3\right]+\exp\left(-\dfrac{t^2}{\theta^2}\right), & \text{when } T<1\\ \dfrac{1}{\Gamma(m)}\left[R_2-2\exp\left(-\dfrac{t^2}{2\theta^2}\right)R_4\right]+\exp\left(-\dfrac{t^2}{\theta^2}\right), & \text{when } T>1\end{cases}\quad \text{if } \frac{t^2}{1+T}<1, \tag{11}$$

and

$$R_L\left(\hat{R}(t)_{GML}\right)=\begin{cases}\dfrac{1}{\Gamma(m)}\left[R_5-2\exp\left(-\dfrac{t^2}{2\theta^2}\right)R_7\right]+\exp\left(-\dfrac{t^2}{\theta^2}\right), & \text{when } T<1\\ \dfrac{1}{\Gamma(m)}\left[R_6-2\exp\left(-\dfrac{t^2}{2\theta^2}\right)R_8\right]+\exp\left(-\dfrac{t^2}{\theta^2}\right), & \text{when } T>1\end{cases}, \tag{12}$$

where

$$R_1=\sum_{k=0}^{\infty}\sum_{p=0}^{\infty}\binom{-2(\hat{b}_{ML}+m)}{k}\binom{-k}{p}t^{2k}(2\theta^2)^p\,\Gamma(m+p);\quad R_2=\sum_{k=0}^{\infty}\sum_{p=0}^{\infty}\binom{-2(\hat{b}_{ML}+m)}{k}\binom{-k}{p}t^{2k}(2\theta^2)^{-(k+p)}\,\Gamma(m-p-k);$$

$$R_3=\sum_{k=0}^{\infty}\sum_{p=0}^{\infty}\binom{-(\hat{b}_{ML}+m)}{k}\binom{-k}{p}t^{2k}(2\theta^2)^p\,\Gamma(m+p);\quad R_4=\sum_{k=0}^{\infty}\sum_{p=0}^{\infty}\binom{-(\hat{b}_{ML}+m)}{k}\binom{-k}{p}t^{2k}(2\theta^2)^{-(k+p)}\,\Gamma(m-p-k);$$

$$R_5=\sum_{k=0}^{\infty}\sum_{p=0}^{\infty}\binom{-k}{p}(-1)^k\left[(2\hat{b}_{ML}+2m+1)t^2\right]^k\left[\frac{\Gamma(m+p)}{\Gamma(k+1)}\right](2\theta^2)^p;$$

$$R_6=\sum_{k=0}^{\infty}\sum_{p=0}^{\infty}\binom{-k}{p}(-1)^k\left[(2\hat{b}_{ML}+2m+1)t^2\right]^k\left[\frac{\Gamma(m-k-p)}{\Gamma(k+1)}\right](2\theta^2)^{-(k+p)};$$

$$R_7=\sum_{k=0}^{\infty}\sum_{p=0}^{\infty}\binom{-k}{p}(-1)^k\left[\left(\hat{b}_{ML}+m+\frac{1}{2}\right)t^2\right]^k\left[\frac{\Gamma(m+p)}{\Gamma(k+1)}\right](2\theta^2)^p;$$

$$R_8=\sum_{k=0}^{\infty}\sum_{p=0}^{\infty}\binom{-k}{p}(-1)^k\left[\left(\hat{b}_{ML}+m+\frac{1}{2}\right)t^2\right]^k\left[\frac{\Gamma(m-k-p)}{\Gamma(k+1)}\right](2\theta^2)^{-(k+p)}.$$

The risk functions of $\hat{\lambda}(t)_{ESE}$ and $\hat{\lambda}(t)_{GML}$ under SELF are given, respectively, by

$$R_L\left(\hat{\lambda}(t)_{ESE}\right)=\begin{cases}\dfrac{4(\hat{b}_{ML}+m)t^2}{\Gamma(m)}\left[(\hat{b}_{ML}+m)S_3-\dfrac{1}{\theta^2}S_5\right]+\dfrac{t^2}{\theta^4}, & \text{when } T<1\\ \dfrac{(\hat{b}_{ML}+m)t^2}{\theta^4\,\Gamma(m)}\left[(\hat{b}_{ML}+m)S_4-2S_6\right]+\dfrac{t^2}{\theta^4}, & \text{when } T>1\end{cases}, \tag{13}$$

and

$$R_L\left(\hat{\lambda}(t)_{GML}\right)=\begin{cases}\dfrac{\sqrt{2\hat{b}_{ML}+2m+1}\,t^2}{\Gamma(m)}\left[\sqrt{2\hat{b}_{ML}+2m+1}\,S_3-\dfrac{2}{\theta^2}S_5\right]+\dfrac{t^2}{\theta^4}, & \text{when } T<1\\ \dfrac{\sqrt{2\hat{b}_{ML}+2m+1}\,t^2}{\Gamma(m)\theta^4}\left[\dfrac{\sqrt{2\hat{b}_{ML}+2m+1}}{4}S_4-S_6\right]+\dfrac{t^2}{\theta^4}, & \text{when } T>1\end{cases}, \tag{14}$$

where

$$S_3=\sum_{k=0}^{\infty}\binom{-2}{k}(2\theta^2)^k\,\Gamma(m+k)\ ;\ S_4=\sum_{k=0}^{\infty}\binom{-2}{k}(2\theta^2)^{-k}\,\Gamma(m-k-2)\ ;$$

$$S_5=\sum_{k=0}^{\infty}\binom{-1}{k}(2\theta^2)^k\,\Gamma(m+k)\ ;\ S_6=\sum_{k=0}^{\infty}\binom{-1}{k}(2\theta^2)^{-k}\,\Gamma(m-k-1).$$

The risk efficiency is usually computed to see whether one estimate outperforms another estimate or not. The risk efficiency of estimate $\hat{\phi}_1$ with respect to estimate $\hat{\phi}_2$ under S*ELF*, denoted by $RE_L(\hat{\phi}_1,\hat{\phi}_2)$, can be defined as the ratio of $R_L(\hat{\phi}_2)$ to $R_L(\hat{\phi}_1)$. If $RE_L(\hat{\phi}_1,\hat{\phi}_2)$ is more than one then $\hat{\phi}_1$ outperforms $\hat{\phi}_2$. Under SELF, the risk efficiencies $RE_L(\hat{\theta}_{ESE},\hat{\theta}_{GML})$, $RE_L(\hat{R}(t)_{ESE},\hat{R}(t)_{GML})$ and $RE_L(\hat{\lambda}(t)_{ESE},\hat{\lambda}(t)_{GML})$ can be obtained from the results (9) - (14).

4. Numerical example (Real data)

The real data set consisting of average daily wind speeds (in meter/sec) at Elanora Heights during November 2007 (Best *et al.* 2010), is presented by making some modification to compare the performance of empirical Bayes estimates with that of GML estimates. The average daily wind speeds were as follows:

0.5833	0.6667	0.6944	0.7222	0.7500	0.7778	0.8056	0.8056	0.8611	0.8889
0.9167	1.0000	1.0278	1.0278	1.1111	1.1111	1.1111	1.1667	1.1667	1.1944
1.2778	1.2778	1.3056	1.3333	1.3333	1.3611	1.4444	2.1111	2.1389	2.7778

Barot & Patel (2014) have performed Kolmogorov-Smirnov and Anderson-Darling tests and suggested that the one-parameter Rayleigh distribution provides an adequate fit to this data set.

As a numerical illustration, the artificial progressive and conventional Type II censored samples of size $m = 20$ have been generated from the given data set. In the first case, let the vector of observed average wind speeds be $\underline{x}_1$ = (0.5833, 0.6667, 0.7222, 0.7500, 0.7778, 0.8056, 0.8889, 0.9167, 1.0000, 1.0278, 1.1111, 1.1111, 1.1667, 1.1944, 1.2778, 1.3333, 1.3611, 1.4444, 2.1389, 2.7778) with the progressive Type II censoring scheme $\underline{r}_1$ = (1*5, 0*10, 1*5). In the second case, let the vector of observed average wind speeds be $\underline{x}_2$ = (0.5833, 0.6667, 0.6944, 0.7222, 0.7500, 0.7778, 0.8056, 0.8056, 0.8611, 0.8889, 0.9167, 1.0000, 1.0278, 1.0278, 1.1111, 1.1111, 1.1111, 1.1667, 1.1667, 1.1944) with the conventional Type II censoring scheme $\underline{r}_2$ = (0*19, 10).

The risk efficiencies $RE_L\left(\hat{\theta}_{ESE}, \hat{\theta}_{GML}\right)$, $RE_L\left(\hat{R}(t)_{ESE}, \hat{R}(t)_{GML}\right)$ and $RE_L\left(\hat{\lambda}(t)_{ESE}, \hat{\lambda}(t)_{GML}\right)$ were computed at $t = 0.04$ using the results outlined in section 3. For progressive Type II censored sample, the risk efficiencies were, respectively, 1.00008, 1.00199, and 1.20468; and for conventional Type II censored sample, the risk efficiencies were, respectively, 1.00105, 1.00302, and 1.11089. This indicates that the empirical Bayes estimation outperforms the GML estimation. Moreover, the risk efficiencies $RE_L\left(\hat{\theta}_{ESE}, \hat{\theta}_{GML}\right)$ and $RE_L\left(\hat{R}(t)_{ESE}, \hat{R}(t)_{GML}\right)$ for the progressive Type II censored sample are smaller than that for the conventional Type II censored sample while the risk efficiencies $RE_L\left(\hat{\lambda}(t)_{ESE}, \hat{\lambda}(t)_{GML}\right)$ for the progressive Type II sample is greater than that for the conventional Type II censored sample.

5. Simulation study

Since the performance of GML and empirical Bayes estimates cannot be judged theoretically, we have performed an extensive Monte Carlo simulation study to examine and compare the performance of empirical Bayes estimates with that of GML estimates for different values of hyper-parameter (b), sample size (n), effective sample size (m), and progressive Type II censoring scheme $(\underline{r})$. The risk efficiencies $RE_L\left(\hat{\theta}_{ESE}, \hat{\theta}_{GML}\right)$, $RE_L\left(\hat{R}(t)_{ESE}, \hat{R}(t)_{GML}\right)$ and $RE_L\left(\hat{\lambda}(t)_{ESE}, \hat{\lambda}(t)_{GML}\right)$ were computed to see whether empirical Bayes estimates outperforms GML estimates or not. As one data set does not help to clarify the performance of the estimates, we have computed the risk efficiencies at $t = 0.04$ by averaging over 2,000 simulated progressively Type II censored samples of size m. These samples were generated from the Rayleigh distribution according to the algorithm given in Balakrishnan and Sandhu (1995).

Table 1 summarized the different progressive Type II censoring schemes applied in the simulation study. The risk efficiencies were computed at $t = 0.04$ using the results outlined in section 3. The simulated results are reported in Tables 2 - 4 respectively. From the simulation results, the following points can be drawn:

1. The risk efficiencies $RE_L\left(\hat{\theta}_{ESE}, \hat{\theta}_{GML}\right)$, $RE_L\left(\hat{R}(t)_{ESE}, \hat{R}(t)_{GML}\right)$ and $RE_L\left(\hat{\lambda}(t)_{ESE}, \hat{\lambda}(t)_{GML}\right)$ are greater than one for all the considered cases, which indicates that the empirical Bayes estimates are preferable than the GML estimates.
2. The risk efficiencies $RE_L\left(\hat{\theta}_{ESE}, \hat{\theta}_{GML}\right)$ and $RE_L\left(\hat{R}(t)_{ESE}, \hat{R}(t)_{GML}\right)$ for progressive Type II censored samples are smaller than that for conventional Type II censored samples while the risk efficiencies $RE_L\left(\hat{\lambda}(t)_{ESE}, \hat{\lambda}(t)_{GML}\right)$ for progressive Type II samples are greater than those for the conventional Type II censored samples.
3. The risk efficiencies under SELF are very sensitive to variation in hyper-parameter b for all the considered cases.

6. Conclusion

The present paper proposes the risk efficiency criterion for the comparison of GML and empirical Bayes estimates the unknown scale parameter, reliability function, and failure rate function of the Rayleigh model under progressive Type II censored data. The use of an inverted gamma distribution for the scale parameter resulted in a closed form expression for the posterior pdf. The risk functions of GML and empirical Bayes estimates are obtained under SELF; and compared using risk efficiency criterion with the help of simulation study and wind speed data application. The findings from the analysis of wind speed data are in accordance with those of simulation study, suggesting empirical Bayesian approach is superior to GML approach. As the empirical Bayes estimates outperforms the GML estimates, we recommend empirical Bayesian approach for estimating reliability parameters of the Rayleigh model.

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Table 1. Progressively Type II Censoring Schemes (C.S.) applied in the simulation study

n	m	C.S. No.	$\underline{r}=(r_1,r_2,\ldots,r_m)$
100	50	[1]	(2*12, 1, 0*26, 1, 2*12)
		[2]	(0*49, 50)
	34	[3]	(2*16, 1*2, 2*16)
		[4]	(0*33, 66)
90	50	[5]	(1*20, 0*10, 1*20)
		[6]	(0*49, 40)
	34	[7]	(2*14, 0*6, 2*14)
		[8]	(0*33, 56)
80	50	[9]	(1*15, 0*20, 1*15)
		[10]	(0*49, 30)
80	34	[11]	(2*11, 1, 0*10, 1, 2*11)
		[12]	(0*33, 46)
70	50	[13]	(1*10, 0*30, 1*10)
		[14]	(0*49, 20)
	34	[15]	(2*9, 0*16, 2*9)
		[16]	(0*33, 36)
60	50	[17]	(1*5, 0*40, 1*5)
		[18]	(0*49, 10)
	34	[19]	(1*13, 0*8, 1*13)
		[20]	(0*33, 26)

Table 2. Risk efficiency of $\hat{\theta}_{ESE}$ with respect to $\hat{\theta}_{GML}$ under SELF

C.S.	$b=25$	$b=30$	$b=35$
[1]	1.00052	1.00068	1.00024
[2]	1.00085	1.00089	1.00088
[3]	1.00009	1.00026	1.10331
[4]	1.00015	1.00038	1.46327
[5]	1.00013	1.00025	1.00035
[6]	1.00061	1.00084	1.00059
[7]	1.00006	1.00007	1.20818
[8]	1.00014	1.00090	1.69757
[9]	1.00040	1.00058	1.00092
[10]	1.00074	1.00075	1.00114
[11]	1.00009	1.00066	1.04622
[12]	1.00030	1.00070	1.05032
[13]	1.00011	1.00015	1.00013
[14]	1.00061	1.00078	1.00036
[15]	1.00002	1.00017	1.18351
[16]	1.00011	1.00037	1.24999
[17]	1.00015	1.00068	1.00053
[18]	1.00029	1.00087	1.00065
[19]	1.00007	1.00058	1.01941
[20]	1.00022	1.00077	1.02002

Table 3. Risk efficiency of $\hat{R}(t)_{ESE}$ with respect to $\hat{R}(t)_{GML}$ under SELF

C.S.	$b=25$	$b=30$	$b=35$
[1]	1.11102	1.12610	1.19014
[2]	1.11865	1.13533	1.20568
[3]	1.13845	1.23416	1.24010
[4]	1.14607	1.23425	1.24085
[5]	1.10754	1.12188	1.18254
[6]	1.11717	1.13355	1.20313
[7]	1.13743	1.23336	1.24060
[8]	1.14578	1.23777	1.24098
[9]	1.10593	1.11992	1.17898
[10]	1.11501	1.13093	1.19876
[11]	1.13591	1.23092	1.24088
[12]	1.14494	1.23938	1.24782
[13]	1.10325	1.11667	1.17302
[14]	1.11153	1.12672	1.19126
[15]	1.13374	1.22676	1.23072
[16]	1.14412	1.24115	1.23089
[17]	1.09837	1.11073	1.16203
[18]	1.10499	1.11878	1.17690
[19]	1.12717	1.21231	1.22076
[20]	1.14127	1.23851	1.22743

Table 4. Risk efficiency of $\hat{\lambda}(t)_{ESE}$ with respect to $\hat{\lambda}(t)_{GML}$ under SELF

C.S.	$b=25$	$b=30$	$b=35$
[1]	1.15589	1.02687	1.01362
[2]	1.05002	1.01765	1.01230
[3]	1.01898	1.01821	1.01594
[4]	1.01746	1.01669	1.01449
[5]	1.08421	1.03350	1.01432
[6]	1.07314	1.01906	1.01254
[7]	1.01914	1.01838	1.01611
[8]	1.01760	1.01682	1.01462
[9]	1.00085	1.03754	1.01467
[10]	1.00034	1.02141	1.01290
[11]	1.01937	1.01861	1.01634
[12]	1.01779	1.01702	1.01492
[13]	1.17944	1.04632	1.01528
[14]	1.15958	1.02609	1.01353
[15]	1.01971	1.01896	1.01669
[16]	1.01808	1.01730	1.01508
[17]	1.00076	1.07607	1.01655
[18]	1.00024	1.04036	1.01488
[19]	1.02082	1.02010	1.01788
[20]	1.01855	1.01777	1.01563