

DEM-based Group Behaviour Model and Its Impact in Tsunami Evacuation Simulation

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Abstract

Most of the existing crowd behavior models simulate crowd motions based on interactions among isolated walking individuals. In reality, a crowd not only contains individuals who walk alone, but walk in groups as well. Hence, the interactions should be extended among individuals who belong to the same group as well as among individuals and different groups' persons. Limited number of works has incorporate group behavior in crowd behavior models. This topic is still new and its impact in crowd dynamics is more to be discovered. Driven by the belief of group behavior modeling can lead to a more realistic crowd motions, this paper is objectively to model group behavior in crowd behavior simulation and its impact in tsunami evacuation process. For this, observations and video footages of public area had been conducted as a basis of group behavior model development. Relying on observations made, the existing Distinct Element Method (DEM)-based Crowd Behavior Simulator for Disaster Evacuation (CBS-DE) is extended to incorporate group behavior by introducing in-group and out-group interactions. Validation on the extended CBS-DE was executed in two stages work; (1) looking for a suitable grouping scenario from the observations conducted; and (2) the simulation of a suitable scenario in (1). It shows that the CBS-DE with group behavior model can realistically simulate group behavior. Later, the new model was demonstrated in the simulation of tsunami evacuation process. The effects of the group behavior model were shown in the context of evacuation completion time by comparison with the CBS-DE without group behavior model.

Keywords: DEM, group behaviour, in-group interaction, out-group interaction.

1. Introduction

The study of crowd behavior has initiate great interest in pedestrian facilities and traffic management studies (Helbing *et al.*, 2001, Teknomo, 2006, Usher *et al.*, 2010, Gotoh *et al.*, 2012), and in the study of crowd safety during mass events or evacuation processes (Helbing *et al.*, 2000, Gawronski *et al.*, 2011, Kwak *et al.*, 2013). Different crowd behavior models have been developed to date (e.g., cellular automata-, social force-, agent-based- and DEM-based model). In this paper, the DEM-based model is a preferred model. It models each individual distinctly by tracking the trajectory and rotation of each individual in a domain in order to evaluate individual's position and orientation through the calculation of the interactions between individuals and between individual and boundaries. Hence, the Crowd Behavior Simulator for Disaster Evacuation (CBS-DE) has been developed by implementing the DEM. The capability of this simulator has been demonstrated in the previous study (Gotoh *et al.*, 2004, 2008 2009).

Most of the existing crowd behavior models simulate crowd motions based on interactions among isolated walking individuals. In reality, majority of individuals do not walk alone, but in groups who have social ties and purposely walk together. For example family members walk side-by-side at a park or in a shopping mall friends stay together and maintain the group during their motion. Grouping is a common phenomenon in a crowd and as discussed by Aveni (1997), crowds contained both isolated individuals and persons in groups. Moussaid *et al.* (2010) in their article claimed that about 70% of observed pedestrians in a commercial street are walking in group with the most frequent size of group is two to four members. Group behaviour is still new and has not been incorporated in the existing crowd behaviour models. Its impact on crowd dynamics is still more to be discovered.

Some of the works incorporates the influence of grouping in crowd dynamic model are Sarmady *et al.* (2009), Singh *et al.* (2009), Moussaid *et al.* (2010) and Qiu *et al.* (2010). The groups of persons may have an important effect to the flow of a crowd. Sarmady *et al.* (2009) reported that groups of persons act as a virtual barrier since a group normally move slower than individual person and therefore slow down the crowd. Similarly as discussed by Qiu *et al.* (2010), different group structures can result in slow movement and affect evacuation efficiency in

emergency situations. Meanwhile, Moussaid *et al.* (2010) discussed the group walking behavior from the sociological sense of view and its impact on the crowd dynamics by extending the social force model to include social interactions among people walking in groups. Singh *et al.* (2009) modified the existing DEM-based model, the CrowdDMX, in studying the subgroup behaviour in crowd dynamics simulation. Previously the CrowdDMX (Langston *et al.*, 2006) was limited to modelling individual move independently which led to discrepancies in the simulation with unrealistic splitting of subgroups.

The works mentioned are limited to the pedestrians' behavioral studies. Driven by the belief of group modelling can lead to a more realistic crowd behavior, this paper is objectively to model group behavior in crowd behavior simulation and its impact in tsunami evacuation process. In this contribution, the existing CBS-DE of DEM-based simulator is extended to incorporate group behavior on the basis of observations made. Part of activities involved were observations at different spotted locations of a crowd. Hence, Section 2 of this paper describes the outcomes of observations of crowd behavior at public area. Later, in Section 3, the outline of group behavior model is presented. This model is an individual-based model of person behavior which describes how a person interacts with other persons in the same group and with other persons in different groups. The validation between simulated scenario and the real situation was compared. This part of explanation is presented in Section 4. A two stages work were conducted in validation work; (1) looking for a suitable grouping scenario from the observations conducted; and (2) the simulation of a suitable scenario in (1). In Section 5, a further application of the extended CBS-DE with group behaviour model is demonstrated in simulation of tsunami evacuation process. The effect of the new model is shown in the context of evacuation completion time by comparison with the CBS-DE without group behaviour model. Finally in Section 6 the conclusion is presented.

2. Observations of the Crowd

2.1 Empirical Observations

The first part of this study involved observations and video recordings at public areas. The observations were made under normal condition at two locations; a park area (location **A**) and an area of travel activities near a train station (location **B**). Two locations are observed to ensure the data collected is reliable. The selected public areas also suitable to view persons walk for a sufficient length of time to evident their walking behaviour.

There were 1,325 people observed in this study with 162 people at location **A** and 1,163 people at location **B**. The proportions of people belong to a group at location **A** and **B** is 67.1% and 42.1%, respectively. Higher frequency of groups was found at location **A** than location **B**. This is possibly related to the different activities of locations. Location **A** is a sightseeing place where more groupings observed and location **B** involved travel activities where mostly single persons travel. Table 1 and Fig. 1 show the detail of frequency distribution of the group size observed. From the findings, groups composed of two persons are the most frequent, following by groups of three and four persons, whereas groups of size five and larger are infrequent. Further, the average walking velocity of observed people in groups were determined. With the growing size of a group, the average walking velocity of a group is decreasing. This is evident from the data summarized in Table 1.

3. DEM-based CBS-DE with group behaviour model

In this study, the model developed is an individual-based model of person behavior which describes how a person interacts with other persons in the same group and with other persons in different groups. A crowd is considered consists of single persons and groups of persons. An isolated walking person is considered to be a group of itself. The motion of a crowd is defined based on local interactions in a crowd. Local interactions

Table 1. Frequency distribution of group size

Group Size	No. of Group		Population		Group Percentage (%)		Group Proportion		Average Walking Velocity (m/s)	
	A	B	A	B	A	B	A	B	A	B
Single	28	435	28	435	32.9	57.9	0.33	0.58	1.19	1.21
2	45	242	90	484	52.9	32.2	0.53	0.32	1.15	1.17
3	6	55	18	165	7.1	7.3	0.07	0.07	0.95	1.02
4	4	16	16	64	4.7	2.1	0.05	0.02	0.86	0.86
> 5	2	3	10	15	2.4	0.4	0.02	0.00	0.80	0.70
Total	85	751	162	1163	100.00	100.00	1.00	1.00		

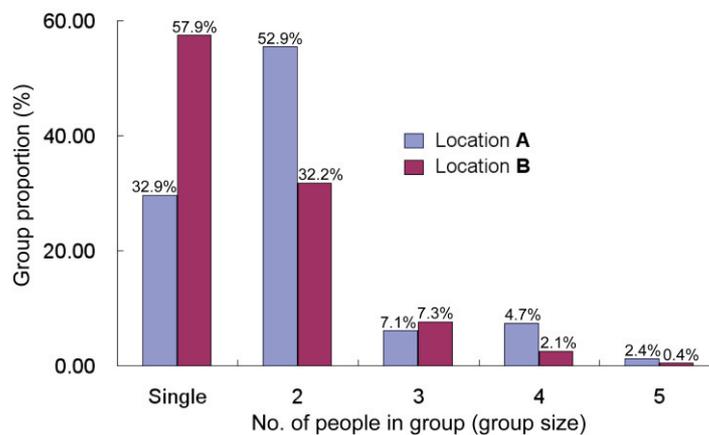


Figure 1. Group size proportion

were model from two aspects, in-group and out-group interactions. The in-group interaction refers to the interaction among persons who belong to the same group, while the out-group interaction refers to the interaction of a person with other persons of different groups and with environment. The existing simulator, the CBS-DE, is extended to include these in-group and out-group interactions.

Each person is governed by the translational and rotational equations of motion in which the motion of each person is described by the vector addition of the separate force terms: an autonomous driving force, F_{aw} and interaction forces, F_{int} and a torque, T_{int} due to interaction force. The complete governing equations are describes as follows;

$$m \frac{d^2 \mathbf{x}_i}{dt^2} = \mathbf{F}_{int} + \mathbf{F}_{aw} \quad \mathbf{F}_{aw} = m\mathbf{a} ; \quad (1)$$

$$I \frac{d\boldsymbol{\omega}_i}{dt} = \mathbf{T}_{int} \quad (2)$$

where m and I are the mass and moment of inertia of a person, respectively. The person is represented as a cylindrical element. \mathbf{x}_i is the positional vector of the person i and $\boldsymbol{\omega}_i$ is the angular velocity of the person i . \mathbf{a} is the acceleration vector. Each person i is assumed to accelerate up to the specific equilibrium velocity, v_{limit} by the autonomous walking force F_{aw} .

The interaction forces are defined as in Eq. 3;

$$\mathbf{F}_{inhi} = \mathbf{F}_{in} + \mathbf{F}_{out} \quad (3)$$

where \mathbf{F}_{in} is the in-group interaction force acting between a person and his/her relative neighbor in the same group and \mathbf{F}_{out} is the out-group interaction force acting between groups and group-wall element.

In the existing CBS-DE, the interaction forces only consider the repulsion effects which defined based on the overlapping between two isolated persons elements or between person element and wall element. Here

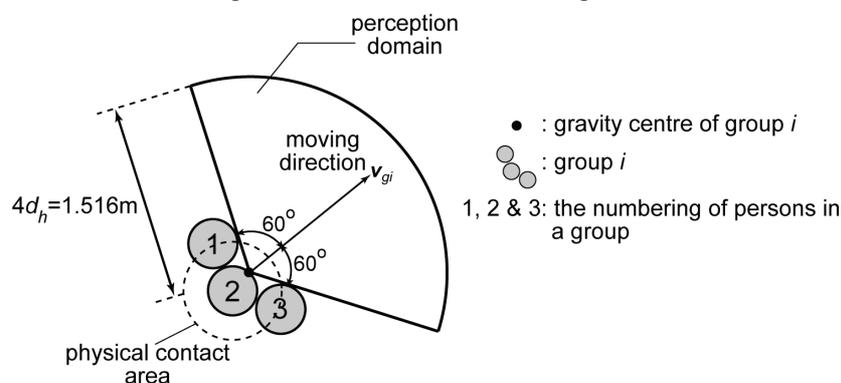


Figure 2. The domain of the interaction

the estimation of the in-group and out-group interactions forces are made by the Voigt model in reference to the

physical contact area introduced for each person to hold the group formation and the perception domain for each group to avoid collision with other groups. Figure 2 illustrates the domain of the interaction.

3.1 In-group interaction

The in-group interaction, F_{in} is defined by the attraction effects which postulated based on the separation between two person elements who belong to the same group. In addition, to avoid overlap between persons elements in the same group, the repulsion effect also included. To hold the group in formation, the reference is made based on the relative position of each person in the group to his/her relative neighbor who belongs to the same group. In reference to Fig. 2, the 2nd person is a relative neighbor to the 1st and the 3rd person.

Total the in-group interaction force on the person i is described as follows:

$$F_{in} = F_{rep} + F_{att} \quad (4)$$

$$= \sum_{j(\neq i)} f_{igrep} + \sum_{j(\neq i)} f_{igatt} \quad (5)$$

where F_{rep} is the net physical repulsive force and F_{att} is the net psychological attractive force, f_{igrep} is the local in-group physical repulsive force and f_{igatt} is the local in-group psychological attractive force. These local forces are given as follows;

$$\begin{aligned} f_{con} &= e + d \\ &= e^n + d^n + e^t + d^t \\ &= {}^{pre}e_s^n + k^n \Delta x_s^n + c^n \Delta x_s^n + {}^{pre}e_s^t + k^t \Delta x_s^t + c^t \Delta x_s^t \end{aligned} \quad (6)$$

where e^n and e^t denote the components of the repulsive or attractive force due to the spring and d^n and d^t denote the components of the repulsive or attractive force due to the dashpot. k^n and k^t are the spring constants in the normal and tangential direction, respectively, while c^n and c^t are the dashpot constants in the normal and tangential direction, respectively. Δx is the relative displacement between contact elements, “con.” indicates the in-group interaction, “pre” indicates the previous numerical time step and “*” indicates the contact elements.

Each person is having the physical or psychological contact area. The psychological attractive force will be triggered if the relative position between the person i and his/her relative neighbor j is satisfying the condition in Eq. 7. Meanwhile, if the person and his/her relative neighbor are in physical contact, the physical repulsive force is triggered, as in Eq. 8.

$$|x_i - x_j| > \frac{d_{hi} + d_{hj}}{2} \quad (7)$$

$$|x_i - x_j| \leq \frac{d_{hi} + d_{hj}}{2} \quad (8)$$

where x_i is the positional vector of person i , x_j is the positional vector of relative neighbor j and d_h is the diameter of person (0.379 m).

3.2 Out-group interaction

To avoid collision with other groups, the out-group interaction considers the repulsion effects which defined based on the overlapping between isolated group elements or between group elements and wall elements. For this, the formulation is developed by considering a group as a single entity (see Fig. 3). The gravity centre of each group i , x_{gi} is determined by calculating the arithmetic mean of positions of persons in a group i , as in Eq. 9. Besides, the mean velocity of each group i , v_{gi} is determined by finding the arithmetic mean velocity of persons in a group i , as in Eq. 10.

$$x_{gi} = \frac{1}{n} \sum_{k=1}^n x_k \quad (9)$$

$$v_{gi} = \frac{1}{n} \sum_{k=1}^n v_k \quad (10)$$

where x_k is the positional vector of a person in a group i , n is total number of persons in a group i . Each group is having the perception domain with the radius $4d_h$ and angle of vision 120° . The repulsive forces will only be

triggered if Eq. 11 is satisfied;

$$|\mathbf{x}_{gi} - \mathbf{x}_{gj}| \leq 4d_h \quad |\mathbf{x}_{gi} - \mathbf{x}_W| \leq 4d_W \quad (11)$$

where d_W is the diameter of the wall (0.379 m).

The interaction between isolated group elements or between group element and wall element is defined as the following;

$$\mathbf{F}_{out} = \mathbf{F}_{rep} \quad (\because \mathbf{F}_{att} = \mathbf{0}) \quad (12)$$

\mathbf{F}_{rep} is the net repulsive force between the group i and j . The net repulsive forces between groups and group-wall elements are quantified by the summation of the local repulsive forces between groups and group-wall elements.

$$\mathbf{F}_{rep} = \sum \mathbf{f}_{ogrep} \quad (13)$$

where \mathbf{f}_{ogrep} is the local out-group repulsive force. Similarly, the local out-group repulsive force is determined in the similar way in Eq. 6 above. And this time, “con.” indicates the out-group interaction.

3.3 Model parameter

In the CBS-DE, the model parameters for the acceleration as the driving force of a person need to be calibrated. In this study, the model parameters in the CBS-DE with self-evasive action model (Gotoh *et al.*, 2012) are used. For the normal case, the acceleration is taken as 0.837 m/s^2 . Meanwhile, the magnitude of specific equilibrium walking velocity, v_{limit} is taken as 1.47 m/s .

As for the interaction force, the Voigt model is a mechanical joint composed of a spring, dashpot and frictional slider is introduced. The spring and dashpot are arranged in both normal and tangential directions on the tangential plane. In the present study, the spring and dashpot constants using in the DEM are given according to the setup procedure proposed by Kiyono *et al.* (1996, 1998) as shown in Table 2.

Table 2 Model parameter

Physical			Psychological		
Parameter	Value	Unit	Parameter	Value	Unit
k_{phy}^n	1.26×10^4	N/m	k_{psy}^n	6.01×10	N/m
k_{phy}^t	6.30×10^2	N/m	k_{psy}^t	3.01	N/m
c_{phy}^n	1.69×10^3	N s/m	c_{psy}^n	1.17×10^2	N s/m
c_{phy}^t	3.77×10^2	N s/m	c_{psy}^t	2.61×10	N s/m

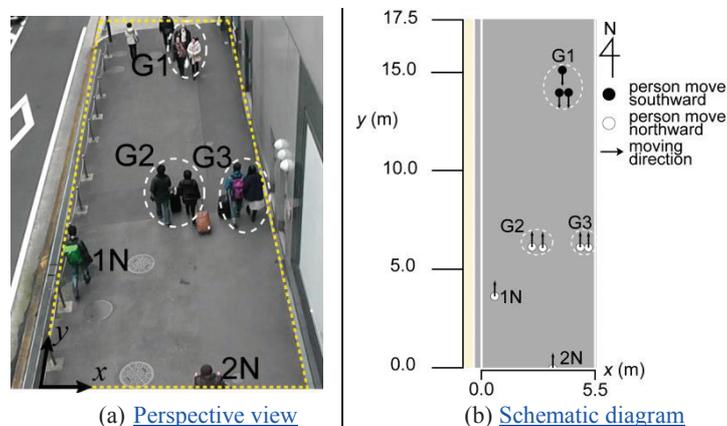


Figure 3. The observation trap and the initial position

4. Validation of Group Behaviour Model

Two procedures were performed in validation work and only location **B** was considered. The first procedure was looking for preferred scenario from the footages. The footages were manually examined. The criteria for the preferred scenario were contra-flow and involved multiple groups including persons who walk alone. The chosen scenario was then saved in sequence images of png format. Afterwards, the number of groups, number of people in a specific group and the average walking velocity of each group were determined.

In the second procedure, simulation of the chosen scenario was performed by employing the CBS-DE with group behaviour model. Concurrently, simulation also performed using the existing CBS-DE. Further the simulated results were compared to the footage from the observation.

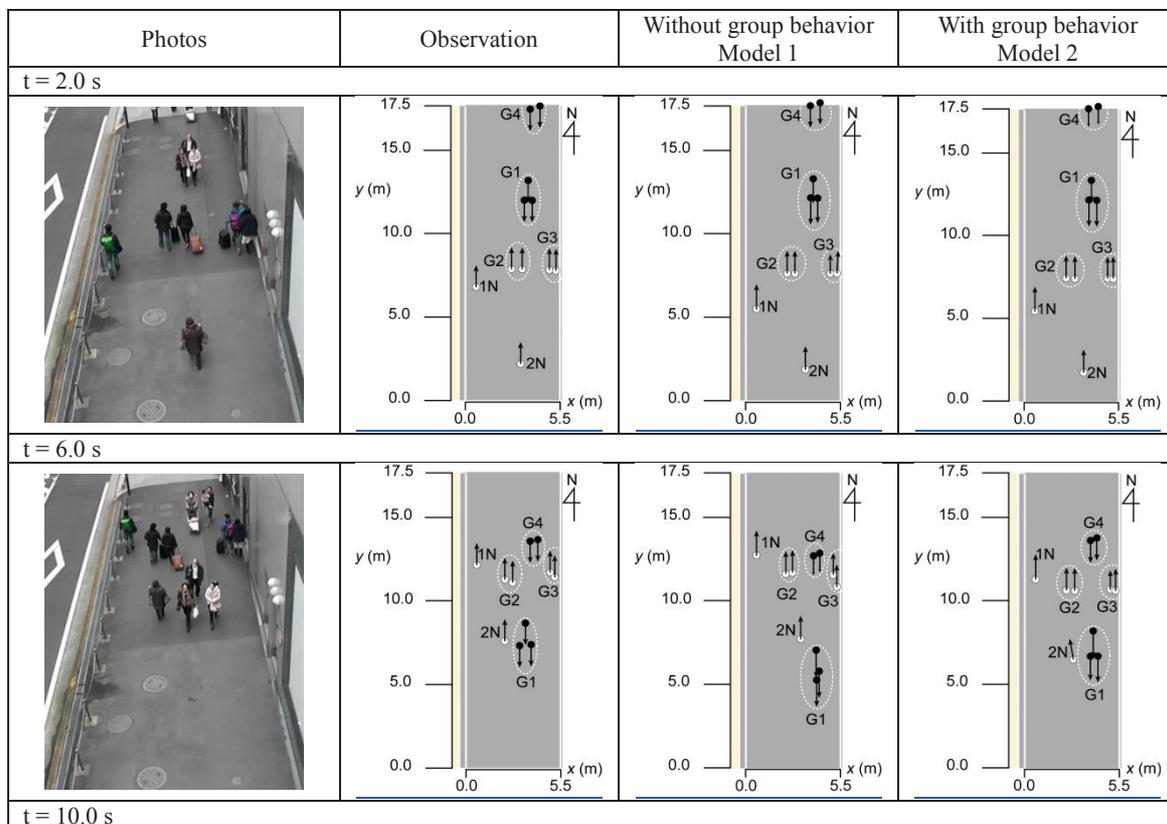
4.1 Preferred Scenario from the Footage

For validation work, the observation trap was as shown in Figure 3. Figure 3(a) shows the perspective view taken from the video footage and Fig. 3(b) shows the schematic diagram of the observation trap. The dimension of the observation trap is 17.5 m length and 5.5 m width.

Beforehand, the footage at location **B** was converted into sequence images of png format. After reviewing the entire video, one preferred scenario was considered. The scenario is in interval of 10 s length. Then, the snapshots (png format) of the preferred scenario were sorted for every 2 s. The initial position of persons in specific group and persons who walk alone were determined using the in-house code written in MAYA Embedded Language (MEL), named Human Behavior Simulator (HBS), which is a plug-in for Autodesk® MAYA® software to project the position of persons on a 2D plane. The initial position for preferred scenario is as shown in Figure 3.

4.2 Validation Outcomes

The outcome of the validation works for the preferred scenario can be evident in the snapshots shown in Figure 4. Model 1 is a model without group behavior and Model 2 is a model with group behavior. The simulations were conducted and comparisons were made with the observation. From Model 1, obviously



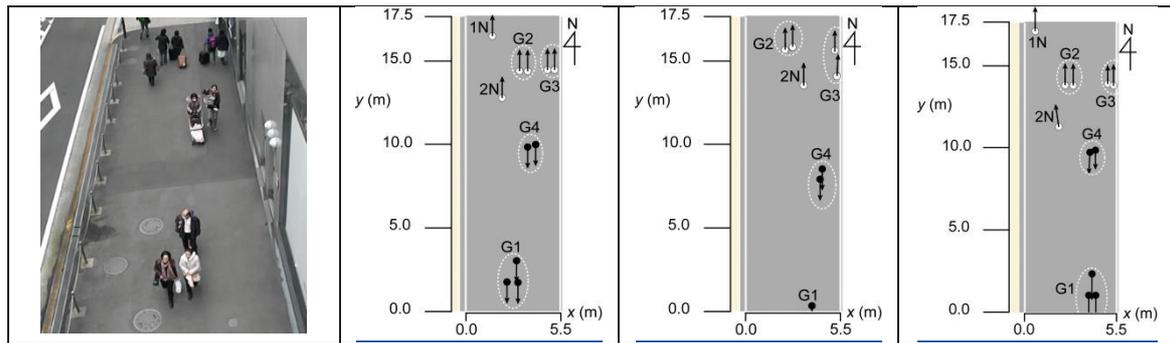


Figure 4. The snapshots of validations

the unrealistic situations can be seen clearly particularly at time $t = 6.0$ s where groups G1 and G3 began to split when met head-on with other person and group in contra-flow. Hence, Model 1 which being the conventional CBS-DE shows significant limitations in the group behavior.

In Model 2, the simulator models grouping realistically which show reasonably good agreement with the observation. These can be evident in the snapshots, where the groups formation were maintained during the motion.

5. Tsunami Evacuation Process

5.1 Simulation Setup

In this contribution, the CBS-DE with group behaviour model is demonstrated further in tsunami evacuation process. A local airport located at the Langkawi Island, Malaysia is chosen as the study area. The airport is facing the Andaman Sea. Historically, the giant earthquake off Sumatra Island caused serious damage by tsunami along the Andaman Sea coast. Hence, it is possible to study the tsunami evacuation process at the Langkawi Airport, Malaysia for the disaster prevention and mitigation.

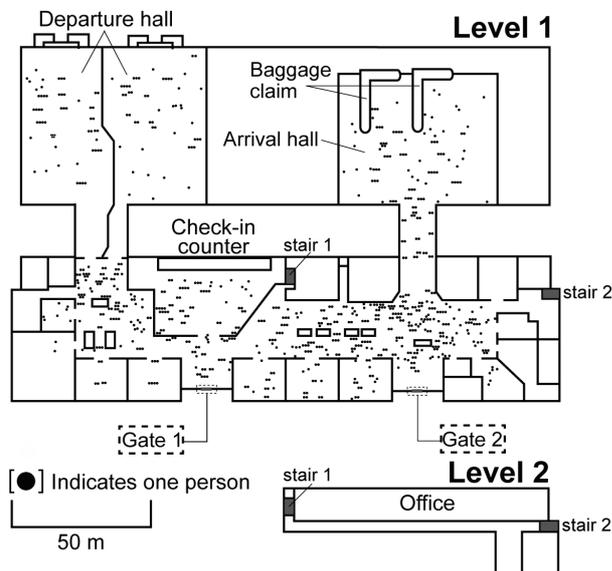


Figure 5. Floor plan and the initial position of person at the airport

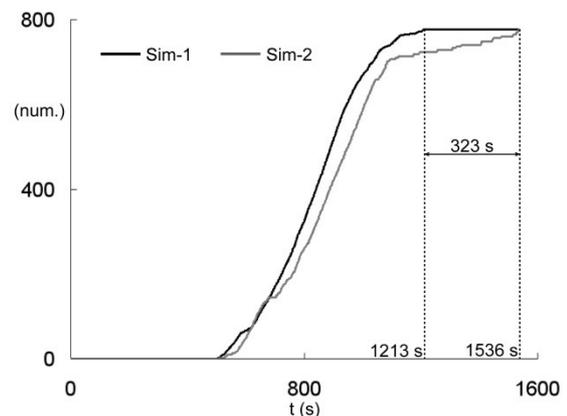


Figure 7. Time series of the evacuation completion time

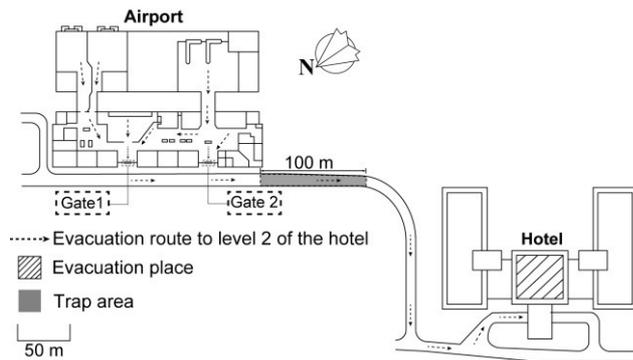


Figure 6. Evacuation route and the trap area

Table 3. The composition of the person distribution at the airport

Group size	No. of Group	No. of people	Group percentage	Group proportion	Velocity [m/s]
Single	232	232	51.9	0.52	1.21
2	136	272	30.4	0.30	1.17
3	51	153	11.4	0.11	1.02
4	20	80	4.5	0.05	0.86
5	8	40	1.8	0.02	0.70
Total	447	777	100	1.00	

Figure 5 shows the floor plan together with the initial position of persons in the airport. The composition of persons distribution was observed during survey conducted. The composition and the walking velocity applied to persons either walk alone or in groups are as shown in Table 3.

Two simulations (Sim-1 and Sim-2) of tsunami evacuation process were performed. Sim-1 refers to simulation using the CBE-DE without group behaviour model, while Sim-2 refers to the simulation of evacuation process by employing the CBS-DE with group behaviour model. For both simulations, the same persons composition and walking velocity (as in Table 3) are employed. The distinction between these two simulations was discussed from the viewpoint of the evacuation completion time and the grouping formation maintain. In this simulation, the evacuation place was set at the level 2 of the nearest hotel (as shown in Fig. 6).

5.2 Effect of Group Behavior Model

To demonstrate the effect of the group behaviour, the trap area is chosen as shown in Fig. 6. The length of the trap area is 100 m. The trap area is used to record the motions of the evacuees and the formation of groupings.

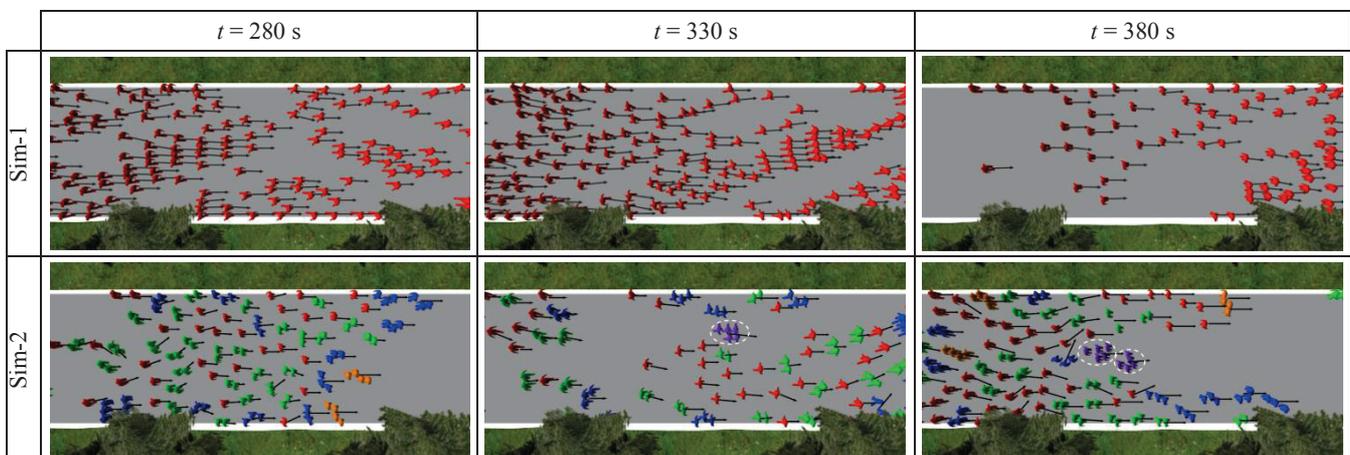


Figure 8. The top view snapshots of the trap area for Sim-1 and Sim-2

Figure 7 shows the time series of the evacuation completion time for the Sim-1 and Sim-2. From the figure it is clearly shows that the groups of persons have an important effect to the flow of a crowd. The time to complete the evacuation for the Sim-2 is 1,536 s which is 26.6% longer than Sim-1, 1,213 s. Groups move slower than individual person and therefore slow down the crowd, hence have a tendency to affect evacuation efficiency in emergency situations.

Figure 8 shows the snapshots at the trap area for the Sim-1 and Sim-2. To show the grouping phenomenon, the colours of persons for different group size were used. Persons who walk independently were denoted by red, while the groups of size 2 were coloured with green, the groups of size 3 were coloured with blue, the groups of size 4 in orange and the groups of size 5 were coloured with purple.

From the snapshots, it can be seen the splitting of the group occurred in the Sim-1 in which the person are walking independently. Meanwhile, in the Sim-2, the improvement of the CBS-DE can be evident. For instance, at the times $t=280s$, $330s$ and $380s$, the group formation was maintained during the motion. The group of size four and five were still walk in the group. These demonstrate the validity of the algorithmic changes made to the existing CBS-DE.

6. Conclusion

This paper is objectively to model group behavior in a crowd and to show its impact in tsunami evacuation process. The existing simulator, the CBS-DE, is extended to incorporate the group behavior. Previously, the CBS-DE was developed based on the interaction among the isolated individuals which bring significant limitations in simulate grouping phenomenon as can be evident in the validation work. The in-group and out-group interactions were introduced in the CBS-DE to overcome the limitations. The in-group interaction maintained a group so that persons who belong to the same group keep together while walking. And the out-group interaction was to ensure persons in the different groups are not colliding each other.

The improvements were validated on a visual basis through simulation and direct comparison to the filmed footage. The development is reasonably successful in simulating the group behavior. Further, the CBS-DE with group behavior was demonstrated in the tsunami evacuation process. The impact of the group behavior during evacuation can be seen in comparison to the CBS-DE without group behavior model. The groups of persons have an important effect to the flow of a crowd. Groups move slower than individual person and therefore slow down the crowd, hence affect evacuation efficiency in emergency situations.

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