# Optimum Production Planning Problem (A Case Study of Aspect Water Company Limited Intechiman Municipality) 

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#### Abstract

All production firms aim at maximizing profit after sales of their products but due to lack of technological and scientific approach in the production setting, many cannot achieve the stated objectives. This study showed the trend of production of sachet water at AWCL which gave the quantity of sachet water produced in each month for the year, 2011. The major objective of this study is to minimize the total cost of production at AWCL using Linear Programming model. The optimal solution to the production planning problem was generated by LP Solver and the demand and supply at each month were determined. The AWCL incurs cost of GHф 1.2355 when producing a bag of sachet water but with the use of linear Programming model, the cost of producing a bag of water was reduced to $\mathrm{GH} \not \subset 0.831519$. The analysis also showed that, increasing the wages of regular workers and reducing that of overtime help the company to produce more with minimum cost of production. AWCL should employ more overtime labour when it is necessary to meet the urgent demands from the customers. Instead of employing more manual labour force, the company could have used machinery that can do assembling and packaging of the sachet water. Computer - based planning (scheduling) help the manufacturers to attend to orders from their respective customers easily and to enhance on - time delivery of products. The computerized planning performs better and faster than manual scheduling tools. The analysis showed that the production planning can facilitate the production processes in a way that help the company to streamline the activities that go on during acquisition of raw materials for production and the demands from the customers could be met when the wages of regular labour force are increased.


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## 1. Introduction

Water is a basic requirement for life and when the resource is to be used for domestic purposes, it should meet some set standards.

In the past two decades, the adverse health among the populace in Ghana and the neighbouring countries were as the result of untreated and improper management of water system. This stems from the fact that water used in our homes and public places are not well treated. As a result of that the contaminated water for human consumption, many people suffered from Cholera and other water borne related diseases (www.WHO.int/water_health/disease/cholera/WWD, 2001)

At the United Nations Millennium summit in 2000 and the Johannesburg Earth summit in 2002, world leaders agreed on the set measurable development targets popularly known as Millennium Development Goals for 2015, which aim at a commitment to reduce to about half the population of people without access to safe drinking water, [ www.WHO.int/water_sanitation.../combating_Disease part1].

According to the World Health Organization (2004), 1 billion people did not have access to improved water supply in 2002, and 2.4 billion people suffered from diseases caused by contaminated water. About 1.8 million people die from diarrhoea disease and $90 \%$ of these deaths are of children under 5 years old (World Health Organization, 2004).

Assessment Report of United Nations Mid-term (United Nations International Children's Emergency Fund and World Health Organization, 2004), $80 \%$ of the world's population used an improved drinking water. As the population increases there will be some challenges as regards to the use of improved drinking water.

Besides mortality issues, water-related diseases also prevent people from working and having active lives. The problem of unsafe drinking water in the country is prevalent with associated diseases such as malaria, yellow fever, schistosomiasis (bilharzias), typhoid and diarrhoea.

The renewed global commitments towards the Millennium Development Goals marked for 2015, the importance of locally sourced, low-cost alternative drinking water schemes in contributing to increased sustainable access in rural and peri-urban settings of developing nations cannot be over-emphasized. One of such local interventions in Ghana, where public drinking water supply is endemic is packaged drinking water. This form of packaged water is usually distributed and sold in sachets. Packaged water refers to water that is packaged generally for consumption in a range of vessels including cans, laminated boxes, glass, plastic sachets and pouches, and an iced prepared for consumption .

The demand for sachet water nationwide is much considering the fact that majority of people drink (pure
water) sachet water. Most communities in Ghana are presently under-serviced by water utilities due to the inability of the designated Ghana water company limited to meet their needs. Households and public seek other alternative sources which are safe for human consumption. Prominent among these is the sachet water production companies.

The introduction of sachet water in Ghana has helped to solve the problem of contaminated and unhygienic water for public consumption. The production of sachet water is booming and many people are entering into this business that has created a lot of jobs for many people in Ghana. Now, no matter the number of production plants, Ghana cannot cover or meet the demand of sachet water. Considering the selling price of sachet water (Ghp10) which is affordable to every Ghanaian.

The introduction of sachet water system popularly known as "pure water", its production and distribution channels on the bases of market demands and public consumption are the major concern of every sachet water company in Ghana.

Quality of good safe drinking water is obtained by intense competition among the water companies. Profit is the main goal of every businessman, and demand of sachet water is due to its quality. It therefore encourages the management to develop modern technology for production methodologies in order to remain competitive.

Safe water is critical to maintaining the good health of people in every country. It is evident that the introduction of sachet water in Ghana, cholera disease has been reduced drastically in the country with good atmosphere. This is due to emergence of many sachet water companies in Ghana. One of such companies is the Aspect Water Company Limited (AWCL) which is located in Techiman in the Brong Ahafo Region of Ghana. Techiman is one of the municipalities in Brong Ahafo, with the human population of about two hundred and six thousand, eight hundred and fifty six $(206,856)$ and area of $1,053.5 \mathrm{~km}^{2}=196.4 \mathrm{inh} . / \mathrm{km}^{2}$. (Thomas Brinkoff, Ghana Statistical service/ www.statsghana.gov.gh/docfile/2010). The production of safe drinking water by Aspect Water Company in Techiman Municipality started in the year 2002 and the map of Ghana in Appendix II shows the exact location of town where Aspect Water Company limited is situated. The goal of AWCL is to provide safe drinking water for people in Techiman Municipality in order to reduce or eliminate water related diseases in the area.

AWCL produces sachet water and distributes them to depots in six districts and municipals from Brong Ahafo Region, namely Wenchi, Berekum, Dormaa Ahenkro, Goaso, Bechem and Sunyani. AWCL also supplies sachet water to retailed customers in small containers

The capacity at each destination is as follows: Wench -1500 bags per month, Sunyani -2000 bags per month, Berekum - 1500 bags per month, Dormaa Ahenkro - 1350 bags per month, Goaso -1400 bags per month and Bechem - 1300 bags per month.
ssssThe total capacity at these six depots is equal to 9050 bags of sachet water supply each month. In order to keep constant stock of sachet water in various depots, AWCL needs to produce maximum amount of products that can be supplied. The demand for sachet water for every month is between 400000 and 500000 . AWCL being a profitable company has employed more labourers in order to produce more to furnish the stock in various depots.

The production of sachet water in large quantities and its strong patronage by the public pose serious challenges to the manufacturers. The production in large quantities depends largely on the cost of materials for production, labour cost, inventory cost, managerial cost and control, transportation cost, housing and electricity etc.

## Methodology

## Mathematical Formulation

To minimize the cost of production at AWCL, the production setting requires the methods that will streamline the production cost. This can be done by minimizing the total costs of production and maximizing production. The cost of production is minimized by using Mathematical discipline called Linear Programming (LP).

LP involves the planning of activities to obtain an optimal result. Many problems can be formulated as maximizing or minimizing an objective function, given limited resources and competing constraints We wish to optimize a linear function subject to a set of linear inequalities. Given a set of real numbers: $a_{1}, a_{2}, a_{3}, \ldots, a_{n}$ and $a$ set of variables $x_{1}, x_{2}, x_{3}, \ldots, x_{n}$.
Linear function $f$ on those variables is defined by
$f\left(\mathrm{x}_{1}, \mathrm{x}_{2}, \ldots, \mathrm{x}_{\mathrm{n}}\right)=\mathrm{a}_{1} \mathrm{x}_{1}+\mathrm{a}_{2} \mathrm{x}_{2}+\mathrm{a}_{3} \mathrm{x}_{3}+\ldots+\mathrm{a}_{\mathrm{n}} \mathrm{x}_{\mathrm{n}}=$

$$
\sum_{j=1}^{n} a_{j} x_{j}
$$

If $b$ is a real number and $f$ is a linear function, then the equation $f\left(\mathrm{x}_{1}, \mathrm{x}_{2}, \ldots, \mathrm{x}_{\mathrm{n}}\right) \leq b$ and
$f\left(\mathrm{x}_{1}, \mathrm{x}_{2}, \ldots, \mathrm{x}_{\mathrm{n}}\right) \geq b$ are linear inequalities.
A Linear Programming problem is said be in standard form when it is written:

$$
\operatorname{maximize} \quad \sum_{j=1}^{n} C_{i j} X_{j}
$$

## Subject to

$$
\sum_{\substack{j=1}}^{n} a_{i j} x_{i j} \leq b_{i} \quad \quad i=1, \ldots, \boldsymbol{m}
$$

The problem has $m$ variables and $n$ constraints. It may be written using vector terminology as:

$$
\begin{array}{ll}
\text { Maximize } & \mathrm{C}^{T} \mathrm{X} \\
\text { Subject } & \mathrm{AX} \leq \mathrm{b} \\
& \mathrm{X} \geq 0
\end{array}
$$

In minimizing the cost function instead of maximizing, it may be rewritten in standard by negating the cost coefficient $\mathrm{C}_{\mathrm{j}}\left(\mathrm{C}^{\mathrm{T}}\right)$.
An LP can be expressed as follows:
Minimize $\quad \mathrm{C}_{1} \mathrm{X}_{1}+\mathrm{C}_{2} \mathrm{X}_{2}+\ldots+\mathrm{C}_{\mathrm{n}} \mathrm{X}_{\mathrm{n}}$
Subject to

$$
\begin{aligned}
& \mathrm{a}_{11} \mathrm{x}_{1}+\mathrm{a}_{12} \mathrm{x}_{2} \ldots+\mathrm{a}_{1 \mathrm{n}} \mathrm{x}_{\mathrm{n}} \leq \mathrm{b}_{1} \\
& \mathrm{a}_{21} \mathrm{X}_{1}+\mathrm{a}_{22} \mathrm{X}_{2}+\ldots+\mathrm{a}_{2 \mathrm{n}} \mathrm{x}_{\mathrm{n}} \leq \mathrm{b}_{2} \\
& \cdot \\
& \cdot \\
& \cdot \\
& \mathrm{a}_{\mathrm{m} 1} \mathrm{x}_{1}+\mathrm{a}_{\mathrm{m} 2} \mathrm{x}_{2}+\ldots+a_{\mathrm{mn}} x_{\mathrm{n}} \leq b_{\mathrm{n}} \\
& \quad \mathrm{X}_{\mathrm{j}} \geq 0 \text { for } j=1, \ldots, n .
\end{aligned}
$$

The objective is the minimization of costs. The vector $\mathrm{c}_{1}, \ldots, \mathrm{c}_{\mathrm{n}}$ vector is referred to as the cost vector. The variables $\mathrm{x}_{1}, \ldots, \mathrm{x}_{\mathrm{n}}$ have to be determined so that the objective function
$\mathrm{c}_{1} \mathrm{x}_{1}+\ldots \ldots+\mathrm{c}_{\mathrm{n}} \mathrm{x}_{\mathrm{n}}$ is minimized.
A general mathematical way of representing a Linear Programming Problem (L.P.P.) is as given below:
Objective function $Z=c_{1} x_{1}+c_{2} x_{2}+\ldots c_{n} x_{n}$
Subjects to
$a_{11} x_{1}+a_{12} x_{2}+a_{13} x_{3}+\ldots+a_{1 j} x_{j}+\ldots+a_{1 n} x_{n}(\geq,=, \leq) b_{1}$
$\mathrm{a}_{21} \mathrm{x}_{1}+\mathrm{a}_{22} \mathrm{x}_{2}+\mathrm{a}_{23} \mathrm{x}_{3}+\ldots+\mathrm{a}_{2 \mathrm{j}} \mathrm{x}_{\mathrm{j}}+\ldots+\mathrm{a}_{2 \mathrm{n}} \mathrm{x}_{\mathrm{n}}(\geq,=, \leq) b_{2}$
.
$\mathrm{a}_{\mathrm{m} 1} \mathrm{x}_{1}+\mathrm{a}_{\mathrm{m} 2} \mathrm{X}_{2}+\mathrm{a}_{\mathrm{m} 3} \mathrm{x}_{3}+\ldots+\mathrm{a}_{\mathrm{mj}} \mathrm{x}_{\mathrm{j}} \ldots+\mathrm{a}_{\mathrm{mn}} \mathrm{x}_{\mathrm{n}}(\geq,=, \leq) b_{m}$
and all $\mathrm{x}_{\mathrm{j}}$ 's are $=0$
Where $j=1,2,3, \ldots, n$
Where all $c_{j}$ ' $s, b_{i}$ 's and $a_{i j}$ 's are constants and $x_{j}$ 's are decision variables.
.The matrix form of LP model
A general LP model in the standard form is the vector $A x=b$ is written in the matrix form:
A general LP model in the standard form is the vector $A x=b$ is written in the matrix form:
$\left.\left(\begin{array}{lllllll}\mathbf{a}_{11} & \mathbf{a}_{12} & \mathbf{a}_{13} & \cdot & \cdot & \cdot & \mathbf{a}_{1 n} \\ \mathbf{a}_{21} & \mathbf{a}_{22} & \mathbf{a}_{22} & \cdot & \cdot & \cdot & \mathbf{a}_{2 n} \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \mathbf{a}_{m 1} & \mathbf{a}_{\mathrm{m} 2} & \mathbf{a}_{\mathrm{m} 3} & \cdot & \cdot & \cdot & \mathbf{a}_{\mathrm{mn}}\end{array}\right)\left(\begin{array}{l}\mathbf{x}_{1} \\ \mathbf{x}_{2} \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{x}_{n}\end{array}\right)=\begin{array}{l}\mathbf{b}_{1} \\ \mathbf{b}_{2} \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{b}_{n}\end{array}\right]$

## LP FOR TRANSPORTATION MODEL

Let $X_{i j}$ denote the number of units to be produced during time period i from Si for shipment during time period j to $W_{j}, i=1,2 \ldots \ldots$. n. Then $\mathrm{X} i j \geq 0$ for all $i$ and $j$.
For each $i$, the total amount $\sum_{i=1}^{n} X_{i j}$
We consider a set of $m$ supply points from which a unit of the product is produced. But since supply point $i$ can supply at most $\mathrm{a}_{\mathrm{i}}$, units in any given period.

We have $\quad \sum_{j=1}^{\boldsymbol{n}} \boldsymbol{X}_{i j} \leq \boldsymbol{a}_{\boldsymbol{i}} i=1,2, \ldots, m$ (Supply constraints)
We also consider a set of n demand points to which the product is shipped. Since demand points $j$ must receive at least $d_{j}$ units of the shipped products.

We have

$$
\sum_{i=1}^{m} X_{i j} \leq d_{j}
$$

$$
j=1,2, \ldots, \mathrm{n} \quad \text { (Demand constraints) }
$$

Since units produced cannot be shipped prior to being produced, $C_{i j}$ is prohibitively large for $i>j$ to force the corresponding $X_{i j}$ to be zero or if shipment is impossible between a given source and destination, a large cost of M is entered.
The total cost of production is given as

$$
\sum_{i=1}^{m} \sum_{j=1}^{n} C_{i j} X_{i j}
$$

The general formulation of a production problem is:
$\begin{array}{ll}\text { Minimize } & \sum_{i=1}^{m} \sum_{j=1}^{n} C_{i j} \boldsymbol{X}_{i j} \\ \text { Subject to } & \sum_{j=1}^{n} X_{i j} \leq a_{i}\end{array}$

$$
i=1,2, \ldots, \mathrm{~m} \quad \text { (Supply constraints) }
$$

$\sum_{i=1}^{m} X_{i j} \leq d_{j}$
$j=1,2, \ldots, \mathrm{n} \quad$ (Demand constraints)
$X i j \geq 0, \quad i=1,2, \ldots, \mathrm{~m} ; \mathrm{j}=1,2, \ldots, \mathrm{n}$
The balanced problem from the supply and demand constraints will be;

$$
\sum_{i=1}^{m} a_{i}=\sum_{j=1}^{n} d_{j}
$$

Then total supply equals total demand and the problem is said to be balanced production problem.
Thus, the balanced production problem may be written as:
Minimize

$$
\begin{aligned}
& \sum_{i=1}^{m} \sum_{j=1}^{n} C_{i j} X_{i j} \\
& \text { subject to: } \quad \sum_{\substack{j=1 \\
m}}^{n} X_{i j}=a_{i} \quad i=1,2, \ldots, m \quad \text { (Supply constraints) } \\
& \sum_{i=1}^{m} X_{i j}=d_{j} \quad j=1,2, \ldots, n \quad \text { (Demand constraints) } \\
& \text { Xij } \geq 0, \quad I=1,2, \ldots, m ; j=1,2, \ldots, n .
\end{aligned}
$$

## 3. Data Collection and Analysis

Aspect Water Company Limited produces and sells innovative, high quality and consumable sachet water products. AWCL supplies sachet water to its customers in the Brong Ahafo Region and the nation at large. The company produces 'pure water' based on orders from its registered customers and other retailers.

The quantity of sachet water produced per day depends on the number of workers at the production room and raw materials available. These jobs often have to be processed on the machines in a production room.

Unexpected events on the shop floor, such as machine breakdowns, reduction of human workforce due to sickness or absenteeism has to be taken into consideration, since they may reduce the quantity to be produced in a day. The variables are inventory $\left(\mathrm{x}_{1}\right)$, raw materials $\left(\mathrm{x}_{2}\right)$, the regular time labour $\left(\mathrm{x}_{3}\right)$, overtime labour $\left(\mathrm{x}_{4}\right)$ and transportation ( $\mathrm{x}_{5}$ ).

Computational Procedure and Data Analysis
Tables 4.1, 4.2 and 4.3 show the company's production capacity (regular and overtime) and expected demands (in bags) for sachet water from January - December, 2011.The variable quantities for production and the production cost for each variable.
Table 4.1: Expected demand and capacity of sachet water for the year, 2011

| Month | Sachet Water <br> Demand (bags) | Regular Time shift <br> Capacity (bags) | Overtime Shift <br> Capacity (bags) |
| :--- | :---: | :---: | :---: |
| January | 15000 | 6933 | 3467 |
| February | 15000 | 6667 | 3333 |
| March | 15000 | 6934 | 3466 |
| April | 14334 | 6915 | 3455 |
| May | 14034 | 6934 | 3436 |
| June | 14167 | 6778 | 3392 |
| July | 15000 | 7000 | 3500 |
| August | 15000 | 6895 | 3445 |
| September | 14834 | 6934 | 3466 |
| October | 14500 | 6933 | 3467 |
| November | 14667 | 6912 | 3458 |
| December | 14367 | 6933 | 3467 |

Source: Aspect Water Company Ltd
Table 4.2: The production quantity of sachet water for the year, 2011

| Month | Jan | Feb | March | April | May | June | July | Aug | Sept | Oct | Nov |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Dec |  |  |  |  |  |  |  |  |  |  |  |
| Inventory 10000 | 10100 | 10000 | 10000 | 10000 | 10000 | 11200 | 10000 | 12000 | 11000 | 11000 | 11200 |
| Raw material 1000 | 1100 | 1000 | 1120 | 1000 | 1150 | 1120 | 1000 | 1200 | 1000 | 1120 | 1300 |
| Regular labour 2080 1950 | 1976 | 2080 | 1950 | 2028 | 2080 | 2080 | 2080 | 1950 | 1820 | 2080 |  |
| Overtime labour 1040 1040 1040 | 1040 | 988 | 910 | 1040 | 1040 | 1040 | 1040 | 1040 | 1040 |  |  |
| Transportation 10400 10000 10400 | 10370 | 10400 | 10170 | 10500 | 10340 | 10370 | 10400 | 10370 | 10400 |  |  |

Source: Aspect Water Company Ltd
Table 4.3. The production cost in (Gh $\phi$ ) for the year, 2011

| Month Jan | Feb | March | April | May | June | July | Aug | Sept | Oct | Nov | Dec |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Inventory 550 | 600 | 550 | 560 | 550 | 600 | 700 | 580 | 650 | 600 | 610 | 650 |
| Raw material 8000 | 8100 | 8500 | 8960 | 8500 | 9200 | 9960 | 8800 | 10560 | 9000 | 10080 | 11700 |
| Regular labour 31200 | 31200 | 31200 | 31200 | 31200 | 31200 | 31200 | 31200 | 31200 | 31200 | 31200 | 31200 |
| Overtime labour 15600 | 15600 | 15600 | 15600 | 15600 | 15600 | 15600 | 15600 | 15600 | 15600 | 15600 | 15600 |
| Transportation 71760 | 71760 | 71760 | 71760 | 71760 | 71760 | 71760 | 71760 | 71760 | 71760 | 71760 | 71760 |

## Source: Aspect Water Company Ltd

The production takes place at both regular and overtime shifts for each of the twelve months. Since the demand for sachet water each month is greater than the supply, each of these months is a source. The inventory at the storage served as work in progress (WIP). The company works with maximum number of one hundred and twenty workers a day.

It was clear that the company incurred the total production costs from the following units: Regular unit cost of GH¢0.30, overtime unit cost of $\mathrm{GH} \not \subset 0.15$, raw material unit cost of, $\mathrm{GH} \Varangle 0.0897$, inventory unit cost of $\mathrm{GH} ¢ 0.0058$ and transportation unit cost of $\mathrm{GH} \not \subset 0.69$ giving the total production cost of $\mathrm{GH} \nless 1.2355$ per bag for
producing 124,150 bags of sachet water for the one year period. The company sells a bag of sachet water to its customers for $\mathrm{GH} \not \subset 1.50$.

## Implementation of Model

The imperial data in Table 4.2 is used to formulate the objective function.
The proposed model involves the planning( scheduling) formulation taking into account the unit cost of production, $C_{\mathrm{ij}}$, the supply at $\mathrm{a}_{\mathrm{i}}$ at source $\mathrm{S}_{\mathrm{i}}$ and the demand $d_{\mathrm{j}}$ at destinations(depots) for $\mathrm{i} \in(1,2, \ldots, 12)$. The problem is:
Minimize $Z=\sum_{j=1}^{n} C_{i j} X_{i j}$
Subject to: $\quad \sum_{j=1}^{n} a_{i j} x_{i j} \leq b_{i}$

$$
\mathrm{X} j \geq 0, \quad j=1,2, \ldots, \mathrm{n}
$$

The problem is formulated as:
Minimize $Z=0.0058 \mathrm{x}_{1}+0.0897 \mathrm{x} 2+0.30 \mathrm{x}_{3}+0.15 \mathrm{x}_{4}+0.69 \mathrm{x}_{5}$
Subject to:

| 10000x ${ }_{1}$ | + | $1000 \mathrm{x}_{2}$ | + | 2080x ${ }_{3}$ | $+$ | 1040x4 | + | $10400 x_{5}$ | $\leq$ | 15000 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10100x ${ }_{1}$ | + | $1100 x_{2}$ | + | 1950x3 | + | 1040x4 | + | 10000x5 | $\leq$ | 15000 |
| $10000 \mathrm{x}_{1}$ | + | $1000 \mathrm{x}_{2}$ | + | 1976x ${ }^{\text {a }}$ | + | 1040x4 | + | 10400x ${ }_{5}$ | $\leq$ | 15000 |
| $10000 \mathrm{x}_{1}$ | + | 1120x2 | + | 2080x ${ }_{3}$ | + | 1040x4 | + | 10370x5 | $\geq$ | 14334 |
| 10000x ${ }_{1}$ | + | $1000 x_{2}$ | + | 2080x ${ }_{3}$ | + | 988x ${ }_{4}$ | + | 10400x ${ }_{5}$ | $=$ | 14034 |
| 10000x ${ }_{1}$ | + | $1150 \mathrm{x}_{2}$ | + | 1950x ${ }_{3}$ | + | 910x4 | + | 10170x5 | $\leq$ | 14167 |
| $11200 \mathrm{x}_{1}$ | + | $1120 x_{2}$ | + | 2080x3 | + | 1040x4 | + | 10500x ${ }_{5}$ | $\leq$ | 15000 |
| $10000 \mathrm{x}_{1}$ | + | $1000 \mathrm{x}_{2}$ | + | 2080x ${ }_{3}$ | + | 1040x4 | + | 10340x ${ }_{5}$ | $\leq$ | 15000 |
| $12000 \mathrm{x}_{1}$ | + | 1200x ${ }_{2}$ | $+$ | 2080x ${ }_{3}$ | + | 1040x4 | + | 10400x ${ }_{5}$ | $\geq$ | 14834 |
| $11000 \mathrm{x}_{1}$ | + | $1000 \mathrm{x}_{2}$ | + | 1950x ${ }^{\text {d }}$ | + | 1040x4 | + | 10400x5 | $\geq$ | 14500 |
| $11000 \mathrm{x}_{1}$ | + | $1120 \mathrm{x}_{2}$ | + | 1820x ${ }_{3}$ | + | 1040x4 | + | 10370x5 | $=$ | 14667 |
| $11200 x_{1}$ |  | 1300x ${ }_{2}$ |  | 2080x |  | 1040x |  | 10400x5 |  |  |

## Solution of Production Planning Model

LP solver was used to find the solution of the planning model. LP solver is a windows package which can be used to obtain the optimal solution to production planning problem. It is an optimization package intended for solving linear, integer and other programming problems. LP solver is based on the efficient implementation of the modified Simplex method that solves large scale problems.

The total production output according to Table 4.5 is 173788 and the total demand is 175903 . Since the total supply is less than total demand, dummy supply of 49638(i.e. 173788-124150) is created to balance the production problem. The optimal solutions to the problem are shown in Table 4.5.
Table 4.5. Optimal Solutions generated by LP solver

| Decision variable | Solution Variable | Unit Cost | Total contribution | Reduced Cost |
| :--- | :---: | :--- | :---: | :---: |
| Inventory $\left(\mathrm{x}_{1}\right)$ | 0.508605 | 0.0058 | 0.002949909 | 0 |
| Raw material $\left(\mathrm{x}_{2}\right)$ | 2.23042 | 0.0897 | 0.2000686774 | 0 |
| Regular labour $\left(\mathrm{x}_{3}\right)$ | 0.675403 | 0.30 | 0.2026209 | 0 |
| Overtime labour $\left(\mathrm{x}_{4}\right)$ | 0.869185 | 0.15 | 0.13037775 | 0 |
| Transportation $\left(\mathrm{x}_{5}\right)$ | 0.428263 | 0.69 | 0.29550147 | 0 |

$\operatorname{Min}(Z)=0.831519$
From the Table 4.5, the decision variables are the inventory ( $\mathrm{x}_{1}$ ), raw material ( $\mathrm{x}_{2}$ ), regular labour ( $\mathrm{x}_{3}$ ), overtime labour ( $\mathrm{x}_{4}$ ) and transportation ( $\mathrm{x}_{5}$ ). $\mathrm{x}_{1}=0.508605, \mathrm{x}_{2}=2.23042$,
$\mathrm{x}_{3}=0.675403, \mathrm{x}_{4}=0.869185$ and $\mathrm{x}_{5}=0.428263$.
The optimal solution of the production problem is given by:
$0.0058(0.508605)+0.0897(2.23042)+0.30(0.675403)+0.15(0.869185)+0.69(0.428263)=0.831519$. Thus the cost of producing a bag of sachet water is GH0 $\propto 0.831519$ developed by the model and cost incurred by the company is GH1 $\propto 1.2355$ per bag.
The monthly production output and demands generated by LP solver are also shown in the Table 4.6.

Table 4.6: The optimal constraints (values) generated by LP Solver

| Month | Quantity Supplied | Quantity Demanded | Surplus | Shortage |
| :--- | :---: | :---: | :---: | :---: |
| January | 14079 | 15000 | - | 921 |
| February | 14094 | 15000 | - | 904 |
| March | 14009 | 15000 | - | 991 |
| April | 14334 | 14334 | - | - |
| May | 14034 | 14034 | - | - |
| June | 14115 | 14167 | - | 52 |
| July | 15000 | 15000 | - | - |
| August | 14054 | 15000 | - | 946 |
| September | 15543 | 14834 | 709 | - |
| October | 14500 | 14500 | - | - |
| November | 14667 | 14667 | - | - |
| December | 15359 | 14367 | 992 | - |

From Table 4.6, the company had shortages in January, February, March, June, and August.
However, there was some surplus in September and December. Changing the coefficients of Regular labour ( $\mathrm{x}_{3}$ ) and Overtime labour ( $\mathrm{x}_{4}$ ) in the objective function.
When the coefficients of the variables $\mathrm{x}_{3}$ and $\mathrm{x}_{4}$ of the objective function were changed the optimal solutions are shown in the table below.
Table 4.7: The optimal solution generated by the LP Solver

| Decision Variable | Solution Variable | Unit Cost | Total contribution | Reduced Cost |
| :--- | :---: | :---: | :---: | :---: |
| Inventory $\left(\mathrm{x}_{1}\right)$ | 0.508605 | 0.0058 | 0.002949909 | 0 |
| Raw material $\left(\mathrm{x}_{2}\right)$ | 2.23042 | 0.0897 | 0.200068674 | 0 |
| Regular labour $\left(\mathrm{x}_{3}\right)$ | 0.675403 | 0.35 | 0.23639105 | 0 |
| Overtime labour $\left(\mathrm{x}_{4}\right)$ | 0.869185 | 0.10 | 0.0869185 | 0 |
| Transportation $\left(\mathrm{x}_{5}\right)$ | 0.428263 | 0.69 | 0.29550147 | 0 |

$\operatorname{Min}(Z)=0.821830$
From the Table 4.7, the optimal solution given by the model when the variables $x_{3}$ and $x_{4}$ are perturbed. That is, $0.0058(0.508605)+0897(2.23042)+0.35(0.675403)+0.10(0.869185)+0.69(0.428263)=0.821830$. The monthly production output and demands generated by LP solver were not changed when the coefficients of $x_{3}$ and $\mathrm{x}_{4}$ were changed but there was significant changed in the cost of production as shown in the Table 4.7.

## Sensitivity Analysis of the Proposed Model

Sensitivity (or post - optimal) analysis of the proposed model allows us to observe the effect of changes in the parameters of the LP problem on the optimal solution.
The following table shows the post - optimal solution when the coefficients of the objective function changed.
Table 4.8: Post - Optimal Solution generated by LP Solver

| Decision Variable | Current Cost | Min Cost | Max Cost | Str Vector |
| :--- | :---: | :---: | :---: | :---: |
| Inventory $\left(\mathrm{x}_{1}\right)$ | 0.0058 | 0.0058 | 0.0058 | 0 |
| Raw material $\left(\mathrm{x}_{2}\right)$ | 0.0897 | 0.0897 | 0.0897 | 0 |
| Regular labour $\left(\mathrm{x}_{3}\right)$ | 0.35 | 0.269792 | 0.425679 | 0.35 |
| Overtime labour $\left(\mathrm{x}_{4}\right)$ | 0.10 | 0.0770835 | 0.121622 | 0.10 |
| Transportation $\left(\mathrm{x}_{5}\right)$ | 0.69 | 0.69 | 0.69 | 0 |

Scale Factor Range: Min $=-0.229165$ Max $=0.216224$

## Results/Findings

From the Table 4.8, changing the unit costs of regular labour and overtime labour gave the costs of 0.269792 and 0.0770835 respectively. This means the company could have minimized the cost further if the costs of regular and overtime labour were changed.

All constraints and optimality conditions were satisfied and a solution was found after eight (8) iterations. From Tables 4.6 and 4.8, the total production in September and December were greater than the demands. The company produced more than what was demanded from the customers. The higher quantities produced by AWCL in those months were as a result of large quantity of raw materials available for production. The months January, February, March, June, and August products were less than their demands but greater than the supplies. For the months April, May, July, October and November the demands were satisfied by producing the same quantities. Comparing the months April and June, the quantity of raw materials for production were in June was larger than that in April but the production output in April was bigger than that in June. This is because the number of workers employed in April was larger than that in June.

Thus the quantity of water to be produced by AWCL depends on the available raw materials and the number of labourers.

The optimal solution computed gave the total cost of production by GH¢0.831519.Thus: 0.0058(0.508605) $+0.0897(2.23042)+0.30(0.675405)+0.15(0.869185)+0.69(0.428263)=0.831519$.

When the coefficients of the unit costs of the regular labour and Overtime labour (i.e. regular labour unit cost of GH 0.30 changed to $\mathrm{GH} \Varangle 0.35$, overtime labour unit cost of $\mathrm{GH} \Varangle 0.15$ changed to GH 0.10 ) were changed, their total contributions were also changed but that of inventory, raw materials and transportation costs remained unchanged as shown in table 4.7.

The optimal solution generated by LP Solver during perturbation analysis gave the minimum cost of GH\& 0.821830 .

The total contribution for inventory is 0.002949909 which gives the same value when perturbed. Raw materials total contribution from the analysis is 0.2000686774 which has the same value during perturbation analysis. Regular time labour total contribution shows tremendous change in value when perturbed ( 0.2026209 is the contribution and 0.23639105 is the perturbation result). Overtime labour contribution from the analysis 0.13037775 and when perturbed gave the value 0.0869185 .

That means when the wages of overtime labour were reduced and the regular labour increased, the cost of producing a bag of sachet water would reduced.

The optimal solution generated by the LP Solver gives the total minimum cost for production thereby increasing the total production output for the company.

This study showed the trend of production of sachet water at AWCL in Table 4.6 which gave the quantity of sachet water produced in each month for the year, 2011.the cost of production has been reduced to the minimum. The major objective of this study is to minimize the total cost of production at AWCL using Linear Programming model.

The secondary data was collected on monthly production capacities and demands of sachet water from customers in bags. The data was formulated and analysed as Linear Programming model. The optimal solution to the production planning problem was generated by LP Solver. The demand and supply at each month were determined using the LP solver.

The AWCL incurs cost of GH\&1.2355 when producing a bag of sachet water but with the use of linear Programming model, the cost of producing a bag of water was reduced to GH\& 0.831519 . The analysis also showed that, increasing the wages of regular workers and reduce that of overtime helps the company to produce more with minimum cost of production.

The use of the model showed how the monthly production should be done in order to reduce the total cost of production. It also showed the number of bags of sachet water the company could have produced and supplied to the customers to satisfy the monthly demands.

The following were the production output for AWCL for the year, 2011 generated by LP Solver.
(i). 14079 bags of sachet water were used to satisfy the demand for January.
(ii). 14094 bags of sachet water were supplied to the customers in February.
(iii). In March, 14009 bags were produced instead of 15000 bags demanded.
(iv). The AWCL was able to meet the demand of 14334 bags of sachet water in April.
(v). 14034 bags were supplied to the customers in May. Here, the company was able to satisfy the demand from the customers.
(vi). 14115 bags were produced to clear the demand in June.
(vii). 15000 bags of sachet water were produced by AWCL to satisfy the demand of 15000 bags in July.
(viii). In August, the demand from the customers was 15000 but the company supplied 14054 bags.
s (ix). The demand for water in September was the highest production by the company. The company supplied 15543 bags of sachet water to satisfy the demand of 14834 bags.
(x). The demand in October was also met by the company. The supply was 14500 bags.
(xi). There were 14667 bags of sachet water were produced to balance the demand in November.
(xii). The month December was the second highest production by the company. The supply was greater than the demand from the customers. The supply was 1535 bags and the demand was 14367 bags of sachet water.

## Discussions and Conclusions

The total production cost for the company was $\mathrm{GH} \phi 214715.074$ and the minimum total cost of production cost from the findings gave GH\& 144508.024 , the percentage reduction of $32.70 \%$. From the findings, the Aspect Water Company Limited could have reduced the total production by GH\& $0.403981(32.70 \%$ ) gone by the model.

The cost of overtime labour was higher as expected. The company could maximize total profit after sales of its products if it reduces the number of workers for overtime labour and increases the wages of workers engaged in regular production, therefore ensures optimum utilization of human and plant capacities that would bring about some savings for the company thereby reducing the cost of labour to the minimum. AWCL should employ more
overtime labour when it is necessary to meet the urgent demands from the customers.
Instead of employing more manual labour force, the company could have used machinery that can do assembling and packaging of the sachet water.

Orders from customers could have been increased in April, May, September, October and December if proper communication has been done. The use of ICT for processing information from the customers helps to ascertain the required orders from the regular customers and also to study the market trend before production. This prompts the company to increase or reduce its production (That is, when and what to produce) satisfy the demands.

The use of technology in the manufacturing and production industries helps the companies or firms to produce more with minimum cost.

## Recommendations

Linear Programming models solve all the production planning problems by increasing the production capacities, minimizing the cost and hence maximizing profits in the production industries and firms.

It is recommended that production companies, including AWCL should incorporate the Linear Programming model in their production.

In line with the findings of this research, Sachet Water Companies should adopt ICT devices to enhance their performance. The expertise should be mandated to train the existing worker (especially those at the lower level) on the job - shop for maximum productivity and also to face the challenges of new technology.

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APPENDIX I
Expected demand and capacity of sachet water for the year, 2011

| Month | Sachet Water <br> Demand(bags) | Regular Time Shift <br> Capacity(bags) | Overtime Shift <br> Capacity(bags) |
| :--- | :--- | :--- | :--- |
| January | 15000 | 6933 | 3467 |
| February | 15000 | 6667 | 3333 |
| March | 15000 | 6934 | 3466 |
| April | 14334 | 6915 | 3455 |
| May | 14034 | 6934 | 3436 |
| June | 14167 | 6778 | 3392 |
| July | 15000 | 7000 | 3500 |
| August | 15000 | 6895 | 3445 |
| September | 14834 | 6934 | 3466 |
| October | 14500 | 6933 | 3467 |
| November | 14667 | 6912 | 3458 |
| December | 14367 | 6933 | 3467 |

## APPENDIX II

Map of Ghana showing where Techiman is located


