

A Novel Algorithm for Capacitor Placement to improve Voltage stability of Radial and Meshed Power Systems

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Abstract

The objective of this paper is to present a novel method for determining the optimum location and amount of reactive power to be injected to improve the voltage stability of entire power system or set of buses that are prone to voltage instability. A new sensitivity matrix named L-Index sensitivity matrix is been proposed and the same is considered for identifying the buses, at which the reactive power is to be injected. The proposed procedure has been tested for practical examples of radial networks and IEEE-14 bus system. Test results demonstrate the effectiveness of the developed algorithm.

Keywords: L-index matrix, voltage stability, Jacobian matrix, sensitivity

1. Introduction

Voltage stability is concerned with the ability of the power system to maintain acceptable voltages at all the buses in the system under the normal conditions and after being subjected to a disturbance. Once associated primarily with weak systems and long lines, voltage problems are now also a source of concern in highly developed networks as a result of heavier loadings.The review paper by Ajjarapu and Lee [8] presents an exhaustive list of work done in the area of voltage stability till 1998. The phenomena which contributes to the voltage stability have been described, the various countermeasures to avert it have been enumerated and the various computer analysis methods used or proposed so far have been presented in a coherent way in [16].

2. Motivation

In 1997, a voltage instability problem in a distribution network, which spread to a corresponding transmission system, had caused a major blackout in the S/SE Brazilian system [44]. Therefore over the years, voltage stability of distribution systems has received great attention with a need for both analysis and enhancement of the operating conditions. The Voltage Stability problem of radial distribution system from its single line equivalent has been investigated and the voltage stability index (VSI) for identifying the node that is most sensitive to voltage collapse has been developed in [20], [21] and [25]. The determination of the location, size, number and type of capacitors to be placed are of great significance, as it reduces power and energy losses, increases the available capacity of the feeders and improves the feeder voltage profile. Numerous methods for solving this problem in view of minimizing losses have been suggested in the literature [[30]- [34]]. Algorithms for enhancing voltage stability of transmission systems by optimal capacitor placement have been discussed [[35]- [36]]. A relationship between voltage stability and loss minimization has been developed and the concept of maximizing voltage stability through loss minimization has been outlined [[37]- [38]]. Algorithms for enhancing voltage stability of distribution systems by network reconfiguration that alters the topological structure of the distribution feeders by rearranging the status of switches have been suggested [[39]- [41]]. However, there is no work till date to improve the stability of the system as a whole or to improve the stability of particular buses which are in our interest. In the literature several indices are been proposed to indicate the voltage stability of power systems. The L-Index method is proposed in [3] which attempts to provide a measure of the stability of the load buses in a system by ranking them according to a parameter(L-Index). The eigenvalues and eigenvectors of the power flow jacobian have been used in [22] to characterize the stability margin in a system. In this paper we are using L-Index [3] and Jacobin matrix [7] to derive the L-Index sensitivity matrix denoted as (L_q), which is used to calculate the optimal location of the capacitors. In this paper the affect of placing a capacitor at a bus on the remaining buses for radial and meshed systems is found out. L-Index Sensitivities(L_q)matrix which gives the information of the change in value of L-Index [3] with change in reactive power injection at any bus in the system has been proposed. A new method is developed to improve the stability of the system using L-Index sensitivities approach(L_q) which is applicable to improve the stability of radial and meshed systems. We have used linear programming optimization technique to get location and amount of reactive power to be injected.

3. Literature Review

3.1 L Index

This method is proposed in [3] to find the buses which are most prone to voltage instability. In this method the Y_{bus} matrix of the system is split into rows and columns of generators and load buses.

$$\begin{bmatrix} I_G \\ I_L \end{bmatrix} = \begin{pmatrix} Y_{GG} & Y_{GL} \\ Y_{LG} & Y_{LL} \end{pmatrix} \begin{bmatrix} V_G \\ V_L \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} V_L \\ I_G \end{bmatrix} = \begin{pmatrix} Z_{LL} & F_{LG} \\ K_{GL} & Y_{GG} \end{pmatrix} \begin{bmatrix} I_L \\ V_G \end{bmatrix} \tag{2}$$

$$F_{LG} = -[Y_{LL}]^{-1}[Y_{LG}] \tag{3}$$

$$L_j = \left| 1 - \sum_{i=1}^{i=g} F_{ji} \frac{V_i}{V_j} \right| \tag{4}$$

where the subscript

"G" :- refers to the generator buses in the system

"L" :- refers to the load buses in the system

A L-index value away from 1 and close to 0 indicates a large voltage stability margin. The maximum of L-indices(L_{max})of the buses to which it corresponds is the most critical bus. Also the summation of the squares of the L indices of the individual buses($\sum L^2$)is used as a relative indicator of the overall voltage stability of the system at different operating conditions [4].

3.2 Paticipation Factor

The participation factor has been proposed in [22]. It is developed as follows. Consider the load flow jacobian

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{pmatrix} H & N \\ M & L \end{pmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} \tag{5}$$

Although both P and Q changes affect system conditions, it is possible to study the effects of reactive power injections on the voltage stability by setting ΔP (P constant) and deriving the Q-V sensitivities at different loads. Thus the 5 can be written as

$$\Delta Q = \left[L - MH^{-1}N\right]\Delta V = J_R \Delta V \tag{6}$$

$$J_R = \xi \Lambda \eta \tag{7}$$

where

Λ :- left eigen matrix of J_R

η :- right eigen matrix of J_R

ξ :- eigenvalues of J_R

The participation factors for the bus k and the critical mode i are defined as $\xi_{ki}\eta_{ik}$

4. Derivation of L-Index sensitivities

L-Index as introduced in section. 3.1 would predict voltage stability.

To derive L-Index sensitivities Let {1,2,3..........g} be no of generator nodes

{g+1,g+2,g+3..........n} be no of load nodes

From Equ. 8 we get

$$L_j = \left| 1 - \sum_{i=1}^{i=g} F_{ji} \frac{V_i}{V_j} \right| \tag{8}$$

where the subscript

"G" :- refers to the generator buses in the system

"L" :- refers to the load buses in the system

Squaring the equation Equ. 8 we get

$$\begin{aligned} L_j^2 &= \left(\left| 1 - \sum_{i=1}^{i=g} F_{ji} \frac{V_i}{V_j} \right| \right)^2 \\ &= \left(1 - \sum_{i=1}^{i=g} F_{ji} \frac{V_i}{V_j} \right)^2 \end{aligned} \tag{9}$$

Let $K_j = L_j^2$

$$K_j = \left(1 - \sum_{i=1}^{i=g} F_{ji} \frac{V_i}{V_j} \right)^2 \tag{10}$$

differentiating Equ. 10 we get

$$\frac{\Delta K_j}{\Delta V_j} = -2\left(1-\sum_{i=1}^{i=g} F_{ji}\frac{V_i}{V_j}\right)\left(\sum_{i=1}^{i=g} F_{ji}\frac{V_i}{V_j^2}\right) \tag{11}$$

Taking the as already discussed in section we get

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix} = \begin{pmatrix} H & N \\ M & L\end{pmatrix}\begin{bmatrix}\Delta\theta \\ \Delta V\end{bmatrix} \tag{12}$$

As the inclusion of an extra capacitor would change only the reactive power, we would make ΔP in Equ.12 equal zero. From this we get

$$\Delta Q = \left[L - MH^{-1}N\right]\Delta V = J_R \Delta V \tag{13}$$

$$\Delta V = [J_R]^{-1}\Delta Q \tag{14}$$

where $[J_R]$ is the reduced jacobian.

where ΔQ is a vector of change in reactive power injections

ΔQ=[ΔQ_{g+1} ΔQ_{g+2} ΔQ_{g+3} ....]'

where ΔV is a vector of change in voltages

ΔV =[ΔV_{g+1} ΔV_{g+2} ΔV_{g+3} ....]'

Equ. 14 is very important as it gives us the relationship between the amount of reactive power injected at any particular or set of buses and the change in voltage at all buses. Let

$$J_R^{-1} = \begin{pmatrix} a_{g+1,g+1} & a_{g+1,g+2} & \cdots & a_{g+1,n} \\ a_{g+2,g+1} & a_{g+2,g+2} & \cdots & a_{g+2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,g+1} & a_{n,g+2} & \cdots & a_{n,n} \end{pmatrix} \tag{15}$$

From the Equ. 15, the voltage at a bus is given as

$$\Delta V_j = \sum_{i=g+1}^{i=n} a_{ji}\Delta Q_i \tag{16}$$

manipulating Equ. 11 to get the change in voltage in terms of change in K

$$\Delta V_j = -\frac{\Delta K_j}{2\left(1-\sum_{i=1}^{i=g} F_{ji}\frac{V_i}{V_j}\right)\left(\sum_{i=1}^{i=g} F_{ji}\frac{V_i}{V_j^2}\right)} \tag{17}$$

Substituting Equ. 21 in Equ. 20 we have

$$\Delta K_j = -2\left(1-\sum_{i=1}^{i=g} F_{ji}\frac{V_i}{V_j}\right)\left(\sum_{i=1}^{i=g} F_{ji}\frac{V_i}{V_j^2}\right)\sum_{i=1}^{i=g} a_{ji}\Delta Q_i \qquad (18)$$

As $\Delta K_j = 2L_j \ \Delta L_j$

$$\Delta L_j = -\frac{1}{L_j}\left(1-\sum_{i=1}^{i=g} F_{ji}\frac{V_i}{V_j}\right)\left(\sum_{i=1}^{i=g} F_{ji}\frac{V_i}{V_j^2}\right)\sum_{i=1}^{i=g} a_{ji}\Delta Q_i \qquad (19)$$

$$-\left|\left\{\frac{1}{L_j}\left(1-\sum_{i=1}^{i=g} F_{ji}\frac{V_i}{V_j}\right)\left(\sum_{i=1}^{i=g} F_{ji}\frac{V_i}{V_j^2}\right)\sum_{i=1}^{i=g} a_{ji}\right\}\right| = L_q$$

This is called L-Index sensitivity matrix matrix.

5 New Algorithm for Capacitor Placement

1. Calculate L-Index of all the load buses as given in section 3.1.

2. Calculate the sensitivity matrix as given in section 4.

3. Choose a limit for the L-Index, let it be L_{limit} .

4. Find the buses which is having L-Index more than L_{limit} , let the buses be [B_2 B_5 B_7].

5. Find ΔL_{red} , ΔL_{red} is taken for the buses which are exceeding L_{limit} . ΔL_{red} = [ΔL_2 ΔL_5 ΔL_7]. 6. Find the reduced sensitivity matrix(L_{rq}), reduced sensitivity matrix would relate ΔL_{rq} and ΔQ_{red} where ΔQ_{red} =[Q_2 Q_5 Q_7].

7. $\Delta L_{red} = \Delta L_{rq} \ \Delta Q_{red}$

8. Take $\sum \Delta Q_{red}$ as the objective function

9. Now perform optimization minimizing objective function satisfying the constraints $\Delta L_{min} \leq \Delta L \leq \Delta L_{max}$

   where

   $\Delta L_{min} = L_{actual}$ - L_{limit}

   ΔL_{max} = [1]- L_{actual} .

10. From the above algorithm we would get the value of capacitors to be placed.

6 Experimental Results

In order to illustrate the effectiveness of the algorithm we would implement the proposed method on 15 bus system with its loads increased four times of the base case given in Appendix A Table 7.

Table 1: L-Index values for the above system

From the above we get

$$\Delta V_j = \sum_{i=g+1}^{i=n} a_{ji}\Delta Q_i \tag{20}$$

From Equ. 11

$$\Delta V_j = \frac{\Delta K_j}{2\left(1-\sum_{i=1}^{i=g} F_{ji}\frac{V_i}{V_j}\right)\left(\sum_{i=1}^{i=g} F_{ji}\frac{V_i}{V_j^2}\right)} \tag{21}$$

Substituting Equ. 21in Equ. 20 we have

$$\Delta K_j = 2\left(1-\sum_{i=1}^{i=g} F_{ji}\frac{V_i}{V_j}\right)\left(\sum_{i=1}^{i=g} F_{ji}\frac{V_i}{V_j^2}\right)\sum_{i=1}^{i=g} a_{ji}\Delta Q_i \tag{22}$$

Table 1:L Index values for the above system

Bus no	L-Index
2	0.24756
3	0.43962
4	0.53262
5	0.54971
6	0.40118
7	0.41826
8	0.27086
9	0.28973
10	0.55208
11	0.63254
12	0.66014
13	0.57359
14	0.57676
15	0.43143

Let us take limit for L-Index(L_{limit}) as 0.5

Buses exceeding L_{limit} are [B_4 B_5 B_{10} B_{11} B_{12} B_{13} B_{14}].

ΔL_{red} =[ΔL_4 ΔL_5 ΔL_{10} ΔL_{11} ΔL_{12} ΔL_{13} ΔL_{14}].

Table 2: Reduced Sensitivity matrix of 15 bus experiment system

	4	5	10	11	12	13	14
4	0.18241	0.18657	0.16042	0.17813	0.18433	0.1925	0.1933
5	0.18861	0.22497	0.16587	0.18418	0.19059	0.19903	0.19986
10	0.16299	0.16671	0.20907	0.23326	0.24174	0.172	0.17272
11	0.19035	0.19469	0.24416	0.33598	0.34864	0.20087	0.20171
12	0.20023	0.20479	0.25683	0.35342	0.41922	0.2113	0.21218
13	0.19758	0.20208	0.17377	0.19295	0.19966	0.25867	0.20937
14	0.1988	0.20333	0.17484	0.19414	0.2009	0.2098	0.23782

ΔL_{max} = [0.53262 0.54971 0.55208 0.63254 0.66014 0.57359 0.57676]

ΔL_{min} =[0.032616 0.049709 0.052083 0.13254 0.16014 0.073588 0.076765]

Imposing the condition $\Delta L_{min} \leq \Delta L \leq \Delta L_{max}$ and running optimization program we get values of capacitors as given in Table 5. From the sensitivity matrix it can be seen that each row is having a maximum element in that row. As each row is having a maximum element in it we can say that in order to have maximum influence on the bus corresponding to that row we have to keep capacitor at the bus corresponding to the column of maximum element. This would reduce the value of capacitor to be placed. To illustrate this let's assume that bus no 11 in the system whose sensitivity matrix is given above is more prone to voltage instability. Checking the row corresponding to 11th bus in the sensitivity matrix it can be seen that 12th bus column element is having maximum value. Then keep the capacitor at 12th bus to have maximum influence on 11th bus. If we keep capacitor at 12th bus as ΔL_{11} = $L_q(11,12)\Delta Q(12)$ so if $L_q(11,12)$ is maximum then $\Delta Q(12)$ will be minimum to get the same ΔL_{11}. Minimum value of $\Delta Q(12)$ would correspond to minimum value of capacitor. so this would reduce the capacitor value if we put the capacitor at 12th bus then on any other buses in the system.

Table 3: Sensitivity matrix of 15 bus experiment system

	2	3	4	5	6	7	8	9	10	11	12	13	14
2	0.033928	0.047929	0.054724	0.05597	0.04346	0.044548	0.03524	0.036313	0.056215	0.06242	0.064592	0.057749	0.057989
3	0.055273	0.11262	0.12859	0.13152	0.070802	0.072575	0.05741	0.059158	0.1321	0.14668	0.15178	0.1357	0.13626
4	0.067127	0.13678	0.18241	0.18657	0.085986	0.088139	0.069722	0.071846	0.16042	0.17813	0.18433	0.1925	0.1933
5	0.069407	0.14142	0.18861	0.22497	0.088907	0.091133	0.07209	0.074286	0.16587	0.18418	0.19059	0.19903	0.19986
6	0.048995	0.069213	0.079026	0.080825	0.10904	0.11204	0.050889	0.052439	0.081179	0.090139	0.093276	0.083394	0.083741
7	0.050815	0.071785	0.081963	0.083829	0.1131	0.13642	0.05278	0.054388	0.084196	0.093489	0.096743	0.086493	0.086853
8	0.035891	0.050702	0.057891	0.059209	0.045975	0.047126	0.057056	0.058953	0.059468	0.066032	0.06833	0.061091	0.061345
9	0.037522	0.053007	0.060522	0.0619	0.048065	0.049268	0.059649	0.086174	0.062171	0.069033	0.071436	0.063867	0.064133
10	0.07006	0.14275	0.16299	0.16671	0.089743	0.091991	0.072769	0.074985	0.20907	0.23326	0.24174	0.172	0.17272
11	0.081819	0.16671	0.19035	0.19469	0.10481	0.10743	0.084982	0.08757	0.24416	0.33598	0.34864	0.20087	0.20171
12	0.086065	0.17537	0.20023	0.20479	0.11025	0.11301	0.089393	0.092115	0.25683	0.35342	0.41922	0.2113	0.21218
13	0.072709	0.14815	0.19758	0.20208	0.093138	0.09547	0.075521	0.077821	0.17377	0.19295	0.19966	0.25867	0.20937
14	0.073159	0.14907	0.1988	0.20333	0.093714	0.09606	0.075988	0.078302	0.17484	0.19414	0.2009	0.2098	0.23782
15	0.052262	0.073829	0.084296	0.086216	0.11632	0.11951	0.054283	0.055936	0.086593	0.096151	0.099497	0.088956	0.089326

In order to show the consistency of the L-Index sensitivities method we have calculated ΔL for the 15 bus system given in Fig.1 keeping the capacitor of 120kVAR at bus 12 from L-Index sensitivities method. ΔL is calculated as

L-Index(with capacitor)-L-Index(without capacitor) from the conventional method as given in Section ??. Values of ΔL from both methods is tabulated in Table 4.

Table 4: Comparision of ΔL from both methods

Bus no	ΔL from L-Index sensitivities	ΔL from conventional method
2	0.001561375	0.00153235
3	0.002982529	0.002926712
4	0.003036517	0.002977799
5	0.003045882	0.002986667
6	0.001625685	0.00159306
7	0.001632134	0.001599146
8	0.001577317	0.001547467
9	0.001582524	0.001552402
10	0.004477482	0.004394593
11	0.006484893	0.006368077
12	0.008109174	0.007966572
13	0.003058382	0.002998492
14	0.003059996	0.003000017
14	0.001636937	0.001603675

From the Table 4 it can be seen that the values of ΔL as obtained from the both methods are very much close to each other so we can conclude that L-Index sensitivities method derived in section 4 is correct.

Table 5: Optimum values of capacitor to be placed in 15 bus radial system

Bus no	Value of Capacitor
4	0
5	0
10	0
11	0
12	0.38191
13	0
14	0.00016339

Table 6: Optimum values of capacitor to be placed in IEEE 14 bus system

Bus no	Value of Capacitor
4	0
5	0.22485
7	0.20017
9	0.15241
10	0.24133
11	0.16807
12	0.096743
13	0.12373
14	0.15785

7 Remarks and Future Scope

In this chapter a new and efficient approach has been developed for the placement capacitor to make the system voltage stable. A new L-Index real power sensitivities matrix is proposed to see the influence of real power on voltage stability of system.

8 Conclusions

This paper is essentially concerned with the analyzing and improvement of voltage stability of radial power systems. Most of the indices which have been proposed for assessing voltage stability are studied. Shortcomings in Line Flow Index are identified. Patterns of voltage stability indices are been observed and their dependence on the electrical distance has been studied. A better comparison has been made by exploiting several unique features of radial power systems. Effect of loads, distance of the load bus from the slack bus and structure of the network on the voltage stability is investigated.

A new algorithm is found to optimally place the capacitors to improve the voltage stability to the required level. The effect of reactive power injections at a bus on the entire power system in consideration is studied. A new sensitivity matrix(L_p) which would relate the change in real power effect on the voltage stability is proposed. Buses at which the capacitors have to be placed to improve the voltage stability are identified.

The algorithms which have been proposed for radial power systems can be suitably modified and can be applied to meshed systems. The effect of real power on the L-Index has to be studied in more detail. We can a develop an index which can predict the maximum column element in a given row of L-Index sensitivity matrix(L_q) with out looking at the matrix.

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