Mathematical Model of Refrigerants Boiling Process in the Partially Closed Volume

Dr. Abbas Alwi Sakhir Abed
University of al-Qadisiyah, College of engineering
(abbasabed59@gmail.com)

Abstract:
The aim of the work is the creation of the mathematical model of heat transfer process at boiling of a refrigerant on the tubes with the partially closed volume (PCV). The construction of heat transfer tubes with PCV for shell and tube evaporators, allowing to increase the evaporators performance, and the mathematical model of the refrigerant boiling process in the PCV channels are designed. The comparison of the results on the mathematical model with experimental data has shown their close fit.

Key words: boiling, finning, volume, heat transfer, apparatus.

Introduction
At the moment the most urgent is the problem of heat transfer intensification in the evaporators of refrigeration machines and systems. The evaporators work at low temperatures that always leads to the small values of heat transfer coefficients. The small values of heat transfer coefficients are due to at the low boiling temperatures the low heat transfer coefficients exist both on the side of boiling refrigerant and on the side of liquid coolant.

The principle diagram of the heat-exchange tube with partially closed volume is shown on figure 1. The heat-exchange tube 1 has finning on the outer surface 2 which perform the partially closed volume 3, the gap between the ribs 4 and two slits 5: in the upper part of the tube with width $b_1$ and in the lower part of the tube with width $b_2$, plus $b_1 = s$, $b_2 = (1.4\div1.6)b_1$, where $s$-fin pitch. It has been experimentally proved that for the large values of $b_1$ and $b_2$ the heat exchange decreases and even smaller values $b_1$ and $b_2$ are not sufficient to provide the heat transfer fluid and to remove the steam volume.

The heat transfer fluid, such as agent R407C, in the partially closed volume 3 is divided into the two phases: the vapor bubble and the liquid film.

The heat-exchange tube works as follows.
When the partially closed volume 3 is filled by the heat transfer fluid through the slits 5 and the gaps between the ribs 4 and during the heat input to the tube, on the heat-exchange surface in the evaporation centre, bubbles are formed. With growing of the bubbles the thickness of the surrounding liquid film is reduced and heat transfer coefficient, as experiments have shown, increases. In the presence of the slits, as they grow, the bubbles leave the partially closed volume not letting the liquid film to evaporate and to form a "dry wall" and letting the new portion of the heat transfer fluid to get in.

The proposed heat-exchanging tube is easy to manufacture. Do so tubes with standard rolling finning are used. Replacing of heat exchange tubes with outer finning on the proposed tubes in shell-and-tube evaporators will reduce their weight and increase cooling capacity by 10% with the same dimensions. This heat exchange surface allows not only increasing heat transfer coefficient, but also reducing dimensions of heat exchangers compared to standard finned tubes, due to their smaller outer diameter.

To create the mathematical model the following physical model has been chosen.
The steam bubble is formed in the center of the evaporation on the bottom of the heat exchange surface during the heat input. Then the bubble continues to grow, almost touching the walls of the partially closed volume, which does not allow the bubble to get beyond the ribs' contour. The bubble continuous to grow and occupies the entire partially closed volume and reaching a certain size leaving the partially closed volume through the upper slit and then the process of formation and growth of the bubble repeats. The process of the bubble behavior in the partially closed volume is shown on figure 2.

To create the mathematical model the following assumptions were made:
- the temperature of the ribs' base and the ribs themselves are the same and constant, the partially closed volume is fully closed, and the bubbles do not go through the gap between the ribs;
- the temperature and the pressure of boiling are constant; -the boiling process in the partially closed volume is seen as boiling in capillary tubes;
- the steam generating capillary acts as a heat pipe in the steady mode: the steam flow moves along the central part and the film with thickness $\Delta$ moves along the wall;
- the thickness of the film depends on the size of partially closed volume, the fluid properties and the process parameters;
-the basic mechanism of the heat removal is the evaporation of the liquid from the film to the steam flow, the film separates the flow from the side surface, the energy transfer through the film is like heat transfer through a cylindrical wall;
-the heat transmitted through the liquid film at the base of the shaft is considered as negligible;
-the flows of the liquid and the steam in the partially closed volume is laminar, the driving force is capillary pressure.

With the taken assumptions the boiling and movement processes of the liquid and the vapor in the partially closed volume is defined by the following equations.

**The equations of the heat transfer**

It is known that for cylindrical wall with length \(dz\) the amount of the transferred heat is defined by the equation [1]:

\[
dQ = 2\pi \lambda dz \frac{\Delta T}{\ln \frac{d_1}{d_2}}
\]

Where \(\pi = 3.14; \quad \lambda \) - heat conductivity coefficient; \(\Delta T\) - temperature difference: boiling temperature and temperature of the heat wall; \(d_1, d_2\) – inner and outer diameters of the liquid film respectively.

Apply the equation to the cylindrical liquid film with thickness \(\Delta\) [1]:

\[
dQ = 2\pi \lambda dz \frac{\Delta T}{\ln \frac{d_1}{d_2}}
\]

And applying dryness factor [1]:

\[
\varphi = \left(\frac{d - 2\Delta}{d}\right)^2
\]

\[
dQ = 2\pi \lambda dz \frac{\Delta T}{\ln \frac{d_1}{\varphi}}
\]

So, on the part \((0;z)\) the total transferred heat per unit time is:

\[
Q(z) = \int_0^z dQ = 2\pi \lambda \int_0^z \frac{\Delta T}{\ln \frac{1}{\varphi}} \; dz
\]

For the cases of the constant temperature lift the last expression for constant \(\Delta T\) takes the form:

\[
Q(z) = 2\pi \lambda' \frac{\Delta Tz}{\ln \frac{1}{\varphi_{av}}}
\]

Where \(\varphi\) - average dryness factor in the part \((0;z)\).

In the same time through the cross-section \(z\) per unit time the energy used for the evaporation is taken out by the steam at a rate of [1]:

\[
Q^{steam} = G^{steam} \varphi^{r} = \frac{1}{4} \pi dz^2 \varphi^{''}(z) \rho^{''} r
\]

Where \(\varphi^{''}(z)\) - velocity of the steam in the cross-section \(z\);
\(\rho^{''}\)-Density of the liquid; \(r\)-evaporation heat; \(G^{steam}\)-mass steam rate.

Equating the expressions for \(Q\) and \(Q^{steam}\), the values for the steam velocity at constant \(\Delta T\) is obtained [1]:

\[
w^{''} = \frac{8\lambda' \Delta Tz}{d2\varphi^{r} \ln \frac{1}{\varphi}}
\]

**Mass transfer equations**

For any capillary cross-section mass steam and liquid flows are eventually equal to[1]:

\[
G^{steam} - S^{steam} \rho^{''} w^{''}(z) = G^{liquid} - S^{liquid} \rho^{''} w^{''}(z)
\]

And it leads that:

\[
w' = w'' \frac{\varphi^{r}}{1 - \varphi^{r}}
\]

**The hydrodynamic equations**

Capillary head is spent to overcome hydraulic resistance of steam and liquid flows:
The rate of the capillary head is defined by the radius of the interface in the base and in the first approximation has the form [1]:

$$\Delta P_{\text{CAP}} = \Delta P_{\text{STEAM}} + \Delta P_{\text{LIQUID}}$$  \hspace{1cm} (11)

Where \( \sigma \) - superficial tension.

Hydraulic resistance of fluid and steam flows (at laminar motion) is determined from the equation [1]:

$$\Delta P = \frac{\xi dz}{4R \rho \frac{w^2}{2}}$$  \hspace{1cm} (12)

Where \( \xi \) - coefficient of hydraulic resistance; \( R \) - hydraulic radius.

Then:

For steam pressure in the capillary:

$$\Delta P = \frac{32}{d^2 \rho} \mu' \omega' dz$$  \hspace{1cm} (14)

For steam pressure in the liquid film:

$$dp = \frac{48}{d^2(1 - \sqrt{\phi})^2} \mu' \omega' dz$$  \hspace{1cm} (15)

The full hydraulic resistance in the whole capillary is obtained by integrating the last two expressions along the length of the capillary. At that for the constant temperature head:

$$\Delta P_{\text{STEAM}} = \frac{32 \mu' \frac{1}{2} {w'}_{\text{OUT}}^2}{d^2 \rho}$$  \hspace{1cm} (16)

$$\Delta P_{\text{STEAM}} = \frac{48 \mu' \frac{1}{2} {w'}_{\text{OUT}}^2}{d^2(1 - \sqrt{\phi})^2}$$  \hspace{1cm} (17)

Where \( {w'}_{\text{OUT}} \) and \( {w'}_{\text{OUT}} \) - average steam velocity in the liquid and in the outflow of the capillary (at \( z = \delta \) ) [1].

After the range of the substitutions the following equations for the constant temperature head have been obtained [1]:

$$\sqrt{\phi} \ln \phi \left[ \frac{v''}{\phi^2} + \frac{3}{2} \frac{v'}{(1 - \sqrt{\phi})(1 - \sqrt{\phi})^2} \right] = \frac{1}{64 \delta^2 \lambda T_o} \Delta W$$  \hspace{1cm} (18)

Take into account the following similarity numbers: the Jakobi number \( -Ja = \frac{\lambda \Delta T}{\rho \alpha} \); the Archimedean number \( -Ar = \frac{\rho d^3 (\rho - \rho')}{\sqrt{\phi}} \); the prandtl \( -\rho = \frac{\tau}{\phi \rho'} \) and the capillary constant \( l = \frac{\sigma}{\phi (\rho - \rho')} \).

Then the final equations will take the form [1]:

$$\sqrt{\phi} \ln \phi \left[ \frac{v''}{\phi^2} + \frac{3}{2} \frac{v'}{(1 - \sqrt{\phi})(1 - \sqrt{\phi})^2} \right] = C. K$$  \hspace{1cm} (19)

Where \( K = \frac{pr Ar}{l^2 \rho} \); \( v^* = \frac{v''}{v'} \); \( l^* = \frac{l}{d} \); \( L = \frac{\delta}{d} \); \( l = \frac{\rho'}{\rho} \); \( C = 1/64 \).

The calculations in [1] have shown that it is possible that the last equation: a) does not have a solution; b) has a unique solution, and c) has two solutions.

Thus, the case where the last equation has not got any solutions corresponds to the regime (to \( \Delta T \)) when the existing capillary pressure is not enough to supply the capillary by the liquid - the liquid film dries and the capillary dehydrates in the case of formation a steam embryo in the capillary. The case when the equation has a unique solution corresponds to the only possible mode in which the capillary pressure is equal to the hydraulic resistance. The case when the equation has two solutions corresponds to the regime where at the two values of dryness factor the equality of the existing capillary pressure and the hydraulic resistance is achieved. The carried analysis has shown that at the moment the liquid thick film is stable, in which \( \phi \) is a really less value.

Using the model, the range of the calculations of heat-exchange coefficient has been carried out as:

$$\alpha = \frac{\delta}{\delta}$$  \hspace{1cm} (20)

Comparing the model and the experimental data, the data for refrigerant R22 has been taken.

For this purpose the thermal characteristics of the refrigerant have been taken form [2]. The diameter of the partially closed volume of the channel has been set at \( d = 0.001 \text{ m} \), the boiling temperature \( t_o = -10, -20 \) and -30 °C, the temperature difference has been assumed as \( \Delta T = 1 \text{ °C} \), the length of the partially closed volume of the channel \( - z = 0.001 \text{ m} \).
The results are presented in table. 1.
In this case the total the heat transfer rate is defined from the equation [3 ]:
\[ q = k \Delta t \] (21)
Where \( k \) – thermal conductivity coefficient. In this case \( k \) is defined using [3 ] as:
\[ k = \frac{1}{\lambda \frac{d}{d \Delta \delta}} \] (22)
Then, using [3 ]:
\[ q = \frac{\alpha (d - \delta)}{\delta d} \Delta t \] (23)
From the last equation and the calculations presented above, the results of the calculations of heat flow rate \( q \) have been taken(table 2).
Using table 2 the graph has been plotted (figure 3) for obtaining the dependence kind of:
\[ \alpha(q) = Aq^{m} \] (24)
Then
\[ \alpha(q) = 0.0457q^{0.645} \] (25)
For the validation of the model the experiments have been arranged. The results are shown on figures 4, 5.
During the boiling it is possible to divide two zones of heat transfer on the smooth tube: undeveloped boiling – until \( q=3000\text{Watt/m}^{2} \) and developed – for the large values of the heat transfer rate.
The area of the underdeveloped boiling shifts to the lower heat transfer rates when using the pipes with special heat transfer surfaces. Thus, for the finned tubes the tipping point corresponds to \( q=2500 \text{ watt/m}^{2} \) for the pipes with the partially closed volume – \( q=2000 \text{ Watt/m}^{2} \) with growth of the pressure the transfer area leading to the developed boiling shifts toward the lower values of \( q \).
For comparison the data about boiling R12 on the smooth pipe is shown which within the measurement accuracy coincides with the data for boiling mixture R22/142b(60/40) on the same tube, although it is possible to note some influence of \( q \) on under the developed boiling.
The experiment has shown that the boiling intensity of the tube with the partially closed volume approximately 3 times and of the finned tube 2 times more than of the smooth tube.
It is possible to note the different influence of \( q \) on the in the dependence for the smooth pipes \( n=0.7 \), for the finned \( n=0.62 \), for the tubes with the partially closed volume \( n=0.56 \).
The graph on figure 5 shows that with the replacement of R12 to the mixture of R22/142b the evaporators with the smooth pipes do not change(within the measurement accuracy) their productivity and for the low heat rates areas, which are typical for refrigeration equipment, the refrigeration capacity will be slightly higher.
The standard finned tubes have much higher heat transfer coefficient. The even higher increase of is observed in the tubes with the partially closed volume. The great relative intensification at the low heat transfer rates \( q=1000-3000 \text{ Watt/m}^{2} \) is possible to observe, which is most characteristic for evaporators.
With the increasing pressure, the heat transfer rate increase, which is associated with the activation of larger number of the nucleation sites, and the degree of the influence of \( p \) on is different for the different surfaces. the greatest impact has been observed on the smooth surface, the smaller – in the partially closed volume (figure 6).
The results of the research allow to consider that with the transition of refrigeration equipment from R12 to the mixture of R22/142b the evaporators of refrigeration equipment with the smooth tubes almost keep former refrigeration capacity and for the intensification of the process in these heat exchangers it is recommended to use heat exchange tubes with partially closed volume. The usage of these tubes will allow to reduce metal consumption and dimensions of evaporators.
For the calculating of the heat transfer coefficient in the boiling mixture R22/142b in the evaporators with the different heat exchange surface the equations have been obtained from the research [4 ]:
- for a smooth surface:
\[ \alpha = 2.28 q^{0.7}p^{0.34} \] (26)
- for standard finning:
\[ \alpha = 7.07 q^{0.62}p^{0.19} \] (27)
- for the tubes with partially closed volume:
\[ \alpha = 13.04 q^{0.56}p^{0.13} \] (28)
Where the units of measurement for \( q-\text{Watt/m}^{2} \); for \( p-\text{mPas} \).
The obtained dependences reflect the different influence of the heat transfer rate and the pressure on the heat transfer using the different tubes.
The data of the dependence describes the experimental data with the measurement accuracy of ± 20%.
The comparative charts between the model and the experimental data are presented on figure 7.

Conclusions
1. The construction of the tubes with the partially closed volume which allows to increase the efficiency of the evaporator has been proposed.
2. The proposed mathematical model correctly reflects the impact of the heat transfer rate on the heat transfer.
3. The good coordination of the different calculations using the mathematical model and the experimental data has been taken out, indicating the correct formulation of the mathematical model.

FIG. 1. The principle diagram of the heat-exchange tube with partially closed volume.

FIG. 2. The bubble behavior in the partially Closed volume.
FIG. 3. Graphical dependence $\alpha(q)$.

FIG. 4. Dependence of the heat conductivity coefficient from the heat transfer rate at $p_o = 0.17\, MPas$ for the different heat transfer surface.

FIG. 5. Dependence of $\alpha/\alpha R12$ from the heat transfer rate at $t = -20\, ^\circ C$. 

FIG 6. Dependence of the heat transfer coefficient from the pressure at \( q = 3 \text{ kWatt/m}^3 \).

FIG 7. Graphical corresponding of the dependences \( \alpha(q) \) taken from the experiments and using the mathematical model, \( p_o = 0.17 \text{ mPas} \).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Boiling temperature, °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_{\text{Watt/mK}} )</td>
<td>-10</td>
</tr>
<tr>
<td>( d_m )</td>
<td>0.0977</td>
</tr>
<tr>
<td>( v_{m}/\text{sec} )</td>
<td>2.15 ( \times ) ( 10^5 )</td>
</tr>
<tr>
<td>( v''_{m}/\text{sec} )</td>
<td>0.763 ( \times ) ( 10^5 )</td>
</tr>
<tr>
<td>( \sigma_{\text{N/m}} )</td>
<td>13.3 ( \times ) ( 10^4 )</td>
</tr>
<tr>
<td>( \sigma_{\text{kHg}} )</td>
<td>212.2</td>
</tr>
<tr>
<td>( \Delta T, ^\circ \text{C} )</td>
<td>1</td>
</tr>
<tr>
<td>( p_{\text{mPas}} )</td>
<td>0.355</td>
</tr>
<tr>
<td>( \varepsilon, % )</td>
<td>0.01</td>
</tr>
<tr>
<td>( \delta_{m} )</td>
<td>2.303 ( \times ) ( 10^{-4} )</td>
</tr>
<tr>
<td>( \sigma_{\text{Watt/m}^2K} )</td>
<td>424.78</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>0.391</td>
</tr>
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References
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