

# Analysis of Voltage Collapse in a Power System Using Voltage Stability Indices

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## Abstract

Voltage collapse phenomenon and its incessant occurrence in a modern power system has of recent been a great concern to power system utilities. The aftermath of which could be very devastating and detrimental to the optimum operation of a power system. Thus, it becomes imperative to identify the most vulnerable bus to voltage collapse by power system operator to prevent outages and blackout that may result. Presented in this paper are techniques based on the use of voltage stability index (L-index) and voltage collapse proximity index (VCPI) to identify a voltage collapse bus. First, the basecase values of the L-index and VCPI for all the load nodes are evaluated. The system is then subjected to a gradual reactive power load change to determine the voltage collapse bus. The bus with the maximum value of L-index and VCPI is taken as the critical bus liable to voltage collapse. The effectiveness of the methodology is tested on the Southern Indian 10-bus and IEEE 14 bus power systems. Result obtained shows that the use of the techniques of L-index and VCPI in the identification of critical node in a power system could be of great benefit to the power system operation and planning.

**Keywords:** Voltage stability, Voltage collapse, L-Index, VCPI, power flow, power system

## 1. Introduction

Modern power systems are operating under stressed and heavy loaded conditions due to continuous increase in load demand, and various economic, environmental changes. Thus, with the increased loading of the power system, the issue of voltage collapse has attracted more attention and until now, maintaining voltage stability has become a growing concern for power utilities (Adebayo et al., 2017). Other factors such as insufficient reactive power resources, effect of Load Tap-changing transformers, load characteristics and among others may also contribute adversely to optimum operation of a power system. Hence, prediction of voltage collapse is indispensable in power system planning and secure operation (Telang and Bedekar 2015). The issue however, is, voltage collapse, whenever it occurs, may result in either partial or total blackout (Mobarak 2015). It is noteworthy that the incidence of this is global and not restricted only to developing nations, but to even nations with innovative and well developed electrical networks, for example, the blackout which transpired in 2003 in North America, and later in Europe, New Zealand, and so on (Rao and Sivanagaraju 2008, Zhang et al., 2010, Kaur et al., 2012). Consequently, the problems of voltage collapse have become a subject of importance and concern to the utilities (Canizares et al., 2001). Prior sensing of its possible occurrence will significantly help in reducing the problem; thus, the importance of technique for predicting its occurrence (Adebayo et al., 2015). A considerable number of techniques such as the use of P-V and Q-V curves, sensitivity analysis, continuation power flow, modal analysis of the reduced Jacobian matrix and so on, have been adopted in the literatures in a way to solve the problems of voltage collapse in power systems (Lee & Lee 1993, Anyanwu 2005, ZhihongJia & Jeyasurya 2000). The use of these techniques, however, is time consuming and laborious, especially for a large power system network as it involves conducting a repetitive power flow solution before a collapse point is detected. Recently, quite numbers of stability indices such as voltage collapse proximity index (VCPI), voltage stability index (L-index), Line stability index, Line stability factor (LPQ) and Fast Voltage stability index (FVSI) have been introduced and developed in the literatures to assess the condition of system voltage stability (Rao & Sivanagaraju 2008, Li et al., 2010, Kaur et al., 2012, Canizares et al., 2001, Yahia et al., 2015, Ajarapu & Lee). Yet, there are still issues with these indices as to which is more suitable and of significant for the identification of

voltage collapse bus in a power system. Although, the use of VCPI and L-Index seem more appropriate in the identification of voltage collapse bus in a power system. This present work is based on the identification of voltage collapse bus using VCPI and L-Index.

The remainder of the paper is organized as follows: section II gives the problem formulation of the indices under consideration while section III presents a brief description of the test case studies used. Result and discussion of the work is presented in section IV and section V concludes the work.

## 2. Problem Formulations

This section presents the mathematical formulations of both VCPI and L-Index.

### 2.1 Voltage Collapse Proximity Index (VCPI)

The detail mathematical derivations of the power flow based voltage collapse proximity index is reported in (Balamourougan *et al.*, 2004). This is formulated based on the power flow equation,

$$S_k = I_k^* * V_k \quad (1)$$

Where  $S_k$  represent the complex power at bus  $k$ ,  $I_k$  shows the current phasor at bus  $k$  and  $V_k$  is the voltage phasor at bus  $k$ . Equations (1) is further solved and separated into real and imaginary parts and thereafter, Newton Raphson iterative technique is used to determine the unknown, the VCPI of  $k$ th node is given as in (2) (Yorino *et al.*, 1992, Anthonwibi, & Musthafa 2010).

$$VCPI_k = \left| 1 - \frac{\sum_{\substack{m=1 \\ m \neq k}}^N V_m'}{V_k} \right| \quad (2)$$

where

$$V_m' = \frac{Y_{km}}{\sum_{\substack{j=1 \\ j \neq k}}^N Y_{kj}} V_m \quad (3)$$

$V_m$  represents the Phasor voltage at bus  $m$ ,  $V_k$  shows the phasor voltage at bus  $k$ ,  $Y_{km}$  represents the admittance between bus  $k$  and  $m$ ,  $Y_{kj}$  is the admittance between bus  $k$  and  $j$ ,  $k$  is the monitoring bus,  $m$  are other buses connected to bus and  $N$  is the bus set of the system. It follows that, the values of VCPI shows the nearness of the system to voltage collapse at a bus. This varies from 0 to 1. If the value is 0, the bus is considered stable and if the value of VCPI is close or equal to 1, the bus is unstable and liable to voltage collapse.

### 2.2 L- Index

A voltage stability based on the solution of load flow was developed by Kessel & Glavitsch (1986). This L-Index describe the stability of a complete system and is given as

$$L = \max \{L_K\} = \max_{k \in \alpha_L} \left| 1 - \frac{\sum_{i \in \alpha_G} F_{GL} V_L}{V_G} \right| \quad (4)$$

where  $\alpha_L$  and  $\alpha_G$  are the set of consumer and generator nodes respectively.  $L_k$  is a local indicator that determines the nodes from which voltage collapse may occur. The L-index ranges between 0 (no load) and 1 (voltage collapse).

## 3. Test Case Studies

The effectiveness of all the methodologies are tested on the Southern Indian 10-bus and IEEE 14 –bus power systems. The 10-bus grid system whose single line diagram is shown in Figure 1 consists of three (3) generating nodes, seven (7) load nodes and twelve transmission lines. On the other hand, the IEEE 14 bus test system comprises of five generator nodes and nine load nodes.

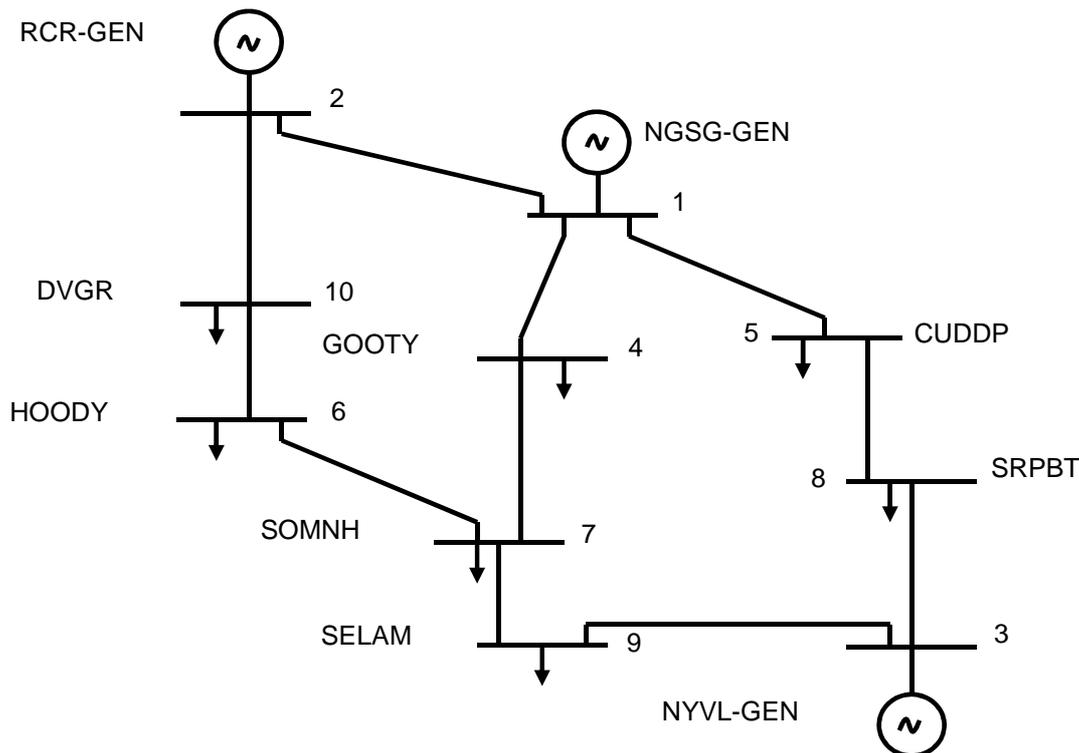


Figure 1: Single line diagram of a Southern Indian 10-bus power system

#### 4. Simulation Results And Discussion

Presented in this section are the simulation results and discussion of the indices employed for the identification of vulnerable bus in a power system. In all, MATLAB software package is used for the simulation results in this work.

##### 4.1 Results of VCPI, L-Index and Voltage magnitude

Power flow analysis of the the Southern Indian 10-bus practical test system is carried out. The system is then subjected to the contingency such as small reactive power load variation at the load buses 4, 5, 6, 7, 8, 9 and 10, one bus at a time. We first increase the reactive loads at the load bus 4 from the base case up to its maximum allowable load. Power flow solution is performed at each gradual increment in load variation for the chosen bus. This procedure is repeated for other load buses to determine the VCPI, L-Index and the voltage magnitude of each of the load buses. Similar procedures are also followed to determine the critical bus of the IEEE 14 bus test system. Results obtained are presented in the form of test cases I and II. Test cases I and II shows the result obtained for the VCPI, L-Index and voltage magnitude of the Southern Indian 10-bus and IEEE 14 bus power system respectively.

##### 4.2 Test Case I: The Southern Indian 10 bus test system

Tables 1 and 2 show the simulation results of the VCPI, L-Index and Voltage magnitude for the 10 bus power system.

Table 1: Results of VCPI for the 10-bus power system

Ranking Order	Load bus no.	Maximum Loadability Qmax (MVar)	VCPI (Basecase)	VCPI	Voltage Mag. (p.u)	Computational Time (secs.)
1 <sup>st</sup>	10	368	0.0630	<b>0.9722</b>	0.5532	
2 <sup>nd</sup>	4	448	0.0428	0.9512	0.5618	
3 <sup>rd</sup>	6	548	0.0133	0.8899	0.5624	
4 <sup>th</sup>	7	602	0.0325	0.7760	0.5881	97.096572
5 <sup>th</sup>	8	701	0.0518	0.6082	0.5908	
6 <sup>th</sup>	5	780	0.0660	0.4593	0.5971	
7 <sup>th</sup>	9	902	0.0674	0.2107	0.7706	

Table 2: Results of the L-Index for the 10-bus power system

Ranking Order	Load bus no.	Maximum Loadability Qmax (MVar)	L-Index (Basecase)	L-Index	Voltage Mag. (p.u)	Computational Time (secs.)
<b>1st</b>	<b>10</b>	<b>368</b>	<b>0.1463</b>	<b>0.9877</b>	0.5532	
2 <sup>nd</sup>	4	448	0.1373	0.9589	0.5618	
3 <sup>rd</sup>	6	548	0.1325	0.9073	0.5624	79.093424
4 <sup>th</sup>	7	602	0.1043	0.8740	0.5881	
5 <sup>th</sup>	8	701	0.1039	0.8565	0.5908	
6 <sup>th</sup>	5	780	0.0896	0.8023	0.5971	
7 <sup>th</sup>	9	902	0.0626	0.7883	0.7706	

5. Test Case II: The IEEE 14 bus test system

Tables 3 and 4 show the simulation results of the VCPI, L-Index and Voltage magnitude for the IEEE 14 bus power system.

Table 3: Results of the VCPI for the IEEE 14-bus power system

Rank	Load Bus	Maximum Loadability Qmax (MVar)	VCPI (Basecase)	VCPI	Voltage Mag. (p.u)	Computational time (sec)
9 <sup>th</sup>	6	550	0.0134	0.8171	0.6649	
7 <sup>th</sup>	7	320	0.0231	0.8687	0.6375	
8 <sup>th</sup>	8	400	0.0245	0.8355	0.6421	
5 <sup>th</sup>	9	220	0.0055	0.9050	0.6169	99.409127
2 <sup>nd</sup>	10	168	0.0131	1.0007	0.5700	
4 <sup>th</sup>	11	190	0.0043	0.9363	0.6091	
3 <sup>rd</sup>	12	170	0.0023	0.9704	0.5832	
6 <sup>th</sup>	13	280	0.0051	0.8903	0.6240	
1 <sup>st</sup>	14	110	0.0287	1.0289	0.5645	

Table 4: Results of L-Index for the IEEE 14-bus power system

Rank	Load Bus	Maximum Loadability Qmax (MVar)	L-Index (Basecase)	L- Index	Voltage Mag. (p.u)	Computational time (sec)
9 <sup>th</sup>	6	550	0.0484	0.6364	0.6649	
7 <sup>th</sup>	7	320	0.0414	0.8520	0.6375	
8 <sup>th</sup>	8	400	0.0480	0.7151	0.6421	
5 <sup>th</sup>	9	220	0.0736	0.8854	0.6169	95.902351
2 <sup>nd</sup>	10	168	0.0705	0.9936	0.5700	
4 <sup>th</sup>	11	190	0.0393	0.9568	0.6091	
3 <sup>rd</sup>	12	170	0.0223	0.9192	0.5832	
6 <sup>th</sup>	13	280	0.0336	0.8776	0.6240	
1 <sup>st</sup>	14	110	0.0810	1.0411	0.5645	

6. Discussion of results

The power flow analysis of the Southern Indian 10-bus of test case I was first carried out to determine the steady state operation of the system. The values of L-Index and VCPI for each load bus were then computed using equations (2) and (4) respectively. The voltage magnitudes of each of the load bus of the 10-bus practical power system were also calculated. We then subjected the 10-bus power system into the contingency of a gradual reactive power loads variation at load buses 4, 5, 6,7,8,9 and 10 from the base case up to their maximum loadability. The values of L-Index, VCPI and voltage magnitude at the maximum loadability of each load bus is determined. Based on the theoretical background afore-established on L-Index and VCPI, the load bus that has maximum value of these indices are noted and are considered as the critical bus of the system. Bus 10 of the Southern Indian 10-bus power system has the highest value. From Tables 1 and 2, bus 10 could be seen as the critical bus of the 10-bus power system having the highest values of L-Index (0.9877) and VCPI (0.9722) from

the basecase values of 0.1463 and 0.0630 respectively. This bus (10) also has the minimum permissible reactive power load of 368MVar compared with all other load buses of the Southern Indian 10-bus power system. Its basecase reactive power load is 25MVar. The value of voltage magnitude obtained at the maximum loadability of 368MVar is 0.5532 p.u. Thus, bus 10 is identified as the critical bus of the 10-bus power system liable to voltage collapse. The total computational time taken to attain this solution for both VCPI and L-Index methods are 97.096572seconds and 79.093424 seconds respectively.

Likewise, IEEE 14 bus test system is also considered. The reactive power load at each load buses 6, 7, 8, 9, 10, 11, 12, 13, and 14 is gradually varied until the maximum allowable load of the bus is reached. At this point, the power flow solution diverges. The results of the values of the voltage collapse proximity index (VCPI) obtained for the gradual reactive power load increase at the buses 6, 7, 8, 9, 10, 11, 12, 13, and 14 are as presented in Table 3. Bus 14 has the maximum value of VCPI (1.0289) from the basecase value of 0.0287, least maximum loadability (110MVar) and lowest value of voltage magnitude (0.5645p.u). Bus 14 is then ranked as the weakest bus of the IEEE 14 bus power system. Similar procedures were also followed to compute the values of L-Index for each load bus of the IEEE 14 bus power system. The initial values of the L-Index for each load bus are presented in Table 4. The weak bus is identified as bus 14 with the initial L-Index value of 0.0810. Notwithstanding, to find the voltage collapse bus, the test case system was subjected to a contingency of gradual reactive load variation at each load bus. After the whole scenario, load bus 14 was still identified as the most vulnerable bus of the IEEE 14-bus power system. This bus has the highest value of L-Index (1.0411), minimum allowable load of 110MVar and least voltage magnitude of 0.5645 p.u. It takes the total computational time of 95.902351 seconds to attain this solution.

## 7. CONCLUSION

This paper has demonstrated the effectiveness of using the approach based on voltage stability to analyze voltage collapse in electric power system. The effectiveness of the methodology is tested on the Southern Indian 10-bus and IEEE 14 bus power systems. To determine the bus liable to voltage collapse using the approaches of VCPI and L-Index, the system was subjected to contingency of gradual reactive power load increase at a particular load node and the variation continues until the minimum permissible load is reached at that node. Power flow solution is performed at each reactive power load variation to determine the value of VCPI and L-Index at each load bus. This continues repeatedly until power flow solutions fail to converge. The same procedure is repeated for other load buses. MATLAB software is used to carry out the simulations in this paper. Based on the computational time and consistency of the results obtained, simulations results obtained show the superiority of using the L-Index method in predicting the voltage collapse bus over the voltage collapse proximity index (VCPI) method. Notwithstanding, the use of L-index and VCPI in the identification of critical node in a power system could be of great benefit to the power system operation and planning

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