Examine Unsteady State of the Emergency Power System on Reactor Stability and Safety

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ABSTRACT
The research conducted safety margin test on some typical water-cooled reactor design (WCRD) models at an accident situation and at same time loss of emergency power supply occurred, secondly safety margin test was carried out on the thermal efficiency and thermal power output of the reactor when power supply failed and thirdly, safety margin test was perform on the reactor in relation to the high temperature effect within reactor core and the fuel temperature. The results of the statistical analysis on these types of nuclear reactor models reveals that the typical water-cooled reactor design (WCRD) models promises most stability under thermal efficiency of 45% and above. Meanwhile, at anything below 45% thermal efficiency the fuel element seems to be unstable in the reactor as the regression plot could not find it optimal. At this point the fuel temperature seems at maximum, the reactor agrees to be stable as the regression plot was at the best fit, that is the least square method finds its optimum when the sum, 5, of squared residuals became minimal. The safety margin prediction of 4.42% was validated for a typical WCRD model as an advantage over the current 5.1% challenging problem for plant engineers to predict the safety margin limit.

Keywords: water-cooled reactor design models accident, emergency power supply failure, high temperature effect, thermal efficiency and thermal power output, reactor stability and safety.

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Introduction
The emergency power systems of the nuclear plant are to provide backup power resources in a crisis or when regular power system fails. Researches have shown that system failure cases in nuclear reactor operation results from a variety of factors, including inadequate design, inadequate materials testing, and poor procedures and training [1]. Hence, System innovative technologies under consideration need safety hazards analyses process before testing or experimentation in other to avoid sudden failure. Malfunctioning of emergency power system of the nuclear plant could disconnect water pump to the heated up reactor core[2], there by leading to pressure built-up within the reactor core and this could degenerated to fatal accident[3]. Power supply restore could not be achieved hence there was a blast at the plant as identified in some reactor accidents [4]. In this work comparism of different test on water-cooled reactor design (WCRD) models with respect to failure or malfunction of emergency power system during operation or accident was carried out by testing for thermal efficiency and thermal power using regression analysis technique before conclusion. The purpose of this paper was to test power supply system failure and emergency power system failure on the stability and safety of power reactor.

Emergency Power System
In nuclear power plant an emergency power system is a standby generator which may include lighting, electric generators, fuel cells, uninterruptible power supplies and other apparatus. Emergency power systems can rely on generators, deep cycle batteries, flywheel energy storage or hydrogen fuel cells. Some homebrew emergency power systems use regular lead-acid car batteries.

The emergency power supplies of a nuclear power plant are built-up by several layers of redundancy, such as diesel generators, gas turbine generators and battery buffers. The battery backup provides uninterrupted coupling of the diesel/gas turbine units to the power supply network. If necessary, the emergency power supply allows the safe shut down of the nuclear reactor. Less important auxiliary systems such as, for example, heat tracing of pipelines are not supplied by these backups. The majority of the required power is used to supply the feed pumps in order to cool the reactor and remove the decay heat after a shut down.

Mains power can be lost due to downed lines, malfunctions at a sub-station, inclement weather, natural disaster, planned blackouts or in extreme cases a grid-wide failure. In modern buildings, most emergency power systems have been and are still based on generators. Usually, these generators are Diesel engine driven, although smaller buildings may use a gasoline engine driven generator and larger ones a gas turbine.
An emergency power supply system generally specifies power supply systems that are available at the unit in case of a loss of unit auxiliary power caused by a turbine or main generator trip. At some Nuclear Power Plants (NPPs), dedicated neighbouring power plant units are used as an alternative source of auxiliary power. Most NPPs also have dedicated off-site transmission lines to supply on-site reserve transformers as alternative sources of unit auxiliary power. Units usually have on-site emergency power sources, such as diesel generators, gas turbines and accumulator batteries, in case all off-site power sources are lost. Emergency power systems, called there Emergency Diesel Generators (EDGs), are a required feature in nuclear power plants. They are typically installed in sets of three. The EDG installation is designed to the same safety-grade requirements as the other safety systems in the plant. The next (upcoming) generation of nuclear power plants includes some designs with multiple independent banks of EDGs (as in the ABWRs).

Controlling the Emergency Power System

For a 208 VAC emergency supply system, a central battery system with automatic controls, which could be located in the power station building itself, is used to avoid long electric supply wires. This central battery system consists of lead-acid battery cell units to make up a 12 or 24 VDC system as well as stand-by cells, each with its own battery charging unit. Also needed are a voltage sensing unit capable of receiving 208 VAC and an automatic system that is able to signal to and activate the emergency supply circuit in case of failure of 208 VAC station supply.

List of the Unit Design Characteristics

The table 1 presents required measurement units of numerical parameters as entered in the Structure of Nuclear Power Plant Design Characteristics in the IAEA Power Reactor Information System (PRIS) database. In the right-hand column, validity check criteria are suggested.

Table 1. Unit Design Characteristics included in the PRIS Database

<table>
<thead>
<tr>
<th>System</th>
<th>Class</th>
<th>Unit of Measure</th>
<th>Validity Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Main generator</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rated active power</td>
<td>1</td>
<td>MW(e)</td>
<td>10 - 1600</td>
</tr>
<tr>
<td>Rated apparent power</td>
<td>1</td>
<td>MVA</td>
<td>10 - 1600</td>
</tr>
<tr>
<td>Output voltage</td>
<td>2</td>
<td>KV</td>
<td>10 - 50</td>
</tr>
<tr>
<td>Output frequency (multiple choice: 50, 60)</td>
<td>2</td>
<td>Hz</td>
<td></td>
</tr>
<tr>
<td><strong>Emergency power supply systems</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of alternative power sources from the neighbouring units (available per unit)</td>
<td>1</td>
<td></td>
<td>1 - 5</td>
</tr>
<tr>
<td>Number of alternative power sources from the transmission grid (standby transformers available per unit)</td>
<td>1</td>
<td></td>
<td>1 - 5</td>
</tr>
<tr>
<td>Number of on-site safety-related diesel generators (available per unit)</td>
<td>1</td>
<td></td>
<td>1 - 6</td>
</tr>
<tr>
<td>Number of on-site safety-related gas turbines (available per unit)</td>
<td>1</td>
<td></td>
<td>1 - 6</td>
</tr>
<tr>
<td>Number of on-site non-safety-related diesel generators (available per unit)</td>
<td>2</td>
<td></td>
<td>1 - 6</td>
</tr>
<tr>
<td>Number of on-site non-safety-related gas turbines (available per unit)</td>
<td>2</td>
<td></td>
<td>1 - 6</td>
</tr>
<tr>
<td>Other on-site emergency AC power sources</td>
<td>3</td>
<td>Text</td>
<td></td>
</tr>
<tr>
<td>Estimated time reserve of the batteries at full load</td>
<td>3</td>
<td>hours</td>
<td>1 - 5</td>
</tr>
<tr>
<td>Total installed capacity of the on-site emergency power sources per unit</td>
<td>3</td>
<td>MW</td>
<td>1 - 20</td>
</tr>
<tr>
<td>Total battery capacity (per vital power train)</td>
<td>3</td>
<td>Ah</td>
<td>1000 -50000</td>
</tr>
</tbody>
</table>

For MegaWatt of electricity(MWe), the power plant generates power at the rate of 1600 MW or 1600 million watts. Watts are a rate, a joule per second, or a watt-hour per hour. An average house uses electricity at the average rate of 1000 watts. Comparing, 8,900 kilowatt-hours of electricity each year is 8900 kw-hr x 1 day/24 hours x 1 year/365 days = 1.01 kw or 1000 watts, the same number. So if each house takes 1kw then 1600 MW will handle 1600000 houses.

MWe and MWt are units for measuring the output of a power plant. MWe means megawatts of electrical output, while MWt means megawatts of thermal output. For example, a nuclear power plant might use a fission reactor
to generate heat (thermal output) which creates steam to drive a turbine to generate electricity (electrical output). A reactor that generates 200 MWt (50 MWe), and another reactor that generates 800 MWt (200 MWe).

**KVA, Voltage**

kilovolt-ampere (KVA) refers to the product of the current and the voltage in an AC circuit. The little "k" in front means the number is in "thousands". For example, when someone says 13 kVA, they are saying 13,000 Volt-Amperes. For single phase connection, KVA can be mathematically derived from this equation (1) formula.

\[
KVA = \frac{Volts \times Amps}{1000} \tag{1}
\]

For three phase connection, KVA can be mathematically derived from this equation (2) formula

\[
KVA = \frac{Volts \times Amps \times \sqrt{3}}{1000} \tag{2}
\]

To measure the unknown voltage by two known quantities KVA and current applied to the equation (1) formulas. For single phase connection, voltage can be mathematically derived from this equation (3) formula

\[
Volts = \frac{KVA \times 1000}{Amps} \tag{3}
\]

For three phase connection, voltage can be mathematically derived from this equation (4) formula

\[
Volts = \frac{KVA \times 1000}{\sqrt{3} \times Amps} \tag{4}
\]

Current calculator is also used in electrical engineering to measure the unknown current by two known quantities KVA and voltage applied to the below formulas. For single phase connection, current can be mathematically derived from this equation (5) formula

\[
Amps = \frac{KVA \times 1000}{Volts} \tag{5}
\]

For three phase connection, current can be mathematically derived from this equation (6) formula

\[
Amps = \frac{KVA \times 1000}{\sqrt{3} \times Volts} \tag{6}
\]

**Electromotive force**, also called emf (denoted \( \mathcal{E} \) and measured in volts), is the voltage developed by any source of electrical energy such as a battery or dynamo. The word "force" in this case is not used to mean mechanical force, measured in Newtons, but a potential, or energy per unit of charge, measured in volts.

In electromagnetic induction, emf can be defined around a closed loop as the electromagnetic work that would be transferred to a unit of charge if it travels once around that loop. (While the charge travels around the loop, it can simultaneously lose the energy via resistance into thermal energy.) For a time-varying magnetic flux impinging a loop, the electric potential scalar field is not defined due to circulating electric vector field, but nevertheless an emf does work that can be measured as a virtual electric potential around that loop. Emf describes the work done per unit charge, and its unit is the volt.

In calculating electromotive force (emf), if we consider a perfect battery with no internal resistance, then its emf would be equal to the potential difference across the terminals. Real batteries have internal resistance, the terminal voltage is not equal to the emf for a battery. Consider a real battery like a perfect battery with a resistor in series (this resistor is the internal resistance). As we pass from the negative terminal to the positive, the potential increases by an amount \( E \) (the potential supplied by the perfect cell) and decreases by \( Ir \) (the voltage drop across the internal resistance). So the potential difference across the real cell would be given by:

\[
V = E - Ir \tag{7}
\]

where \( E \) is equal to the open circuit voltage (\( I = 0 \)).
Now, let's say there is a load resistor $R$ hooked up to the circuit. The load resistor might be a simple resistive circuit element, or it could be the resistance of an electrical device connected to the battery. The potential difference across the load resistor is:

$$V = IR \tag{8}$$

Using $V = E - Ir$ we get:

$$IR = E - Ir \tag{9}$$

and solving for emf we get:

$$E = IR + Ir \tag{10}$$

The internal resistance will need to be given in the question, or enough info must be given to calculate it (such as emf, current and load resistor).

An interesting discussion involves power. Solving for current we get:

$$I = E/(R + r) \tag{11}$$

Power is given by $P = (I^2)(R) = (E^2)(R)/(r+R)^2$

Analyzing this equation, we find that the power is at its maximum when $R = r$.

**Formal Definitions of Electromotive Force**

*Inside* a source of emf that is open-circuited, the conservative electrostatic field created by separation of charge exactly cancels the forces producing the emf. Thus, the emf has the same value but opposite sign as the integral of the electric field aligned with an internal path between two terminals $A$ and $B$ of a source of emf in open-circuit condition (the path is taken from the negative terminal to the positive terminal to yield a positive emf, indicating work done on the electrons moving in the circuit). Mathematically:

$$\mathcal{E} = - \int_A^B E_{cs} \cdot dl \tag{12}$$

where $E_{cs}$ is the conservative electrostatic field created by the charge separation associated with the emf, $dl$ is an element of the path from terminal $A$ to terminal $B$, and $\cdot$ denotes the vector dot product. This equation applies only to locations $A$ and $B$ that are terminals, and does not apply to paths between points $A$ and $B$ with portions outside the source of emf. This equation involves the electrostatic electric field due to charge separation $E_{cs}$ and does not involve (for example) any non-conservative component of electric field due to Faraday's law of induction.

In the case of a closed path in the presence of a varying magnetic field, the integral of the electric field around a closed loop may be nonzero; one common application of the concept of emf, known as "induced emf" is the voltage induced in a such a loop. The "induced emf" around a stationary closed path $C$ is:

$$\mathcal{E} = \oint_C E \cdot dl \tag{13}$$

where now $E$ is the entire electric field, conservative and non-conservative, and the integral is around an arbitrary but stationary closed curve $C$ through which there is a varying magnetic field. Note that the electrostatic field does not contribute to the net emf around a circuit because the electrostatic portion of the electric field is conservative (that is, the work done against the field around a closed path is zero).

This definition can be extended to arbitrary sources of emf and moving paths $C$:

$$\mathcal{E} = \oint_C [E + \mathbf{v} \times \mathbf{B}] \cdot dl + \frac{1}{q} \oint_C \text{effective chemical forces} \cdot dl$$

$$+ \frac{1}{q} \oint_C \text{effective thermal forces} \cdot dl \tag{14}$$

which is a conceptual equation mainly, because the determination of the "effective forces" is difficult.

**Electromotive Force in Thermodynamics**

When multiplied by an amount of charge $dZ$ the emf $\mathcal{E}$ yields a thermodynamic work term $\mathcal{E}dZ$ that is used in the formalism for the change in Gibbs free energy when charge is passed in a battery:

$$dG = -sdT + VdP + \mathcal{E}dZ \tag{15}$$
where $G$ is the Gibb's free energy, $S$ is the entropy, $V$ is the system volume, $P$ is its pressure and $T$ is its absolute temperature.

The combination $(\mathcal{E}, Z)$ is an example of a conjugate pair of variables. At constant pressure the above relationship produces a Maxwell relation that links the change in open cell voltage with temperature $T$ (a measurable quantity) to the change in entropy $S$ when charge is passed isothermally and isobarically. The latter is closely related to the reaction entropy of the electrochemical reaction that lends the battery its power. This Maxwell relation is:

$$
\left( \frac{\partial \mathcal{E}}{\partial T} \right)_Z = - \left( \frac{\partial S}{\partial Z} \right)_T
$$

(16)

If a mole of ions goes into solution (for example, in a Daniell cell, as discussed in equation (17)) the charge through the external circuit is:

$$
\Delta Z = -n_0 F_0
$$

(17)

where $n_0$ is the number of electrons/ion, and $F_0$ is the Faraday constant and the minus sign indicates discharge of the cell. Assuming constant pressure and volume, the thermodynamic properties of the cell are related strictly to the behaviour of its emf by:

$$
\Delta H = -n_0 F_0 \left( \mathcal{E} - T \frac{d\mathcal{E}}{dT} \right)
$$

(18)

where $\Delta H$ is the heat of reaction. The quantities on the right all are directly measurable.

When the nuclear fuel increases in temperature, the rapid motion of the atoms in the fuel causes an effect known as Doppler broadening. When thermal motion causes a particle to move towards the observer, the emitted radiation will be shifted to a higher frequency. Likewise, when the emitter moves away, the frequency will be lowered. For non-relativistic thermal velocities, the Doppler shift in frequency will be:

$$
f = f_0 \left( 1 + \frac{v}{c} \right)
$$

(19)

where $f$ is the observed frequency, $f_0$ is the rest frequency, $v$ is the velocity of the emitter towards the observer, and $c$ is the speed of light.

Since there is a distribution of speeds both toward and away from the observer in any volume element of the radiating body, the net effect will be to broaden the observed line.

If $P_v(v)dv$ is the fraction of particles with velocity component $v$ to $v + dv$ along a line of sight, then the corresponding distribution of the frequencies is:

$$
P_f(df) = P_v(v_f) \frac{dv}{df}df
$$

(20)

where

$$
v_f = c \left( \frac{f}{f_0} - 1 \right)
$$

(21)

is the velocity towards the observer corresponding to the shift of the rest frequency $f_0$ to $f$. 
therefore,

\[
P_f(f) df = \frac{c}{f_0} P_v \left( c \left( \frac{f}{f_0} - 1 \right) \right) df.
\]

\[\text{(22)}\]

We can also express the broadening in terms of the wavelength \(\lambda\). Recalling that in the non-relativistic limit

\[
\frac{\lambda - \lambda_0}{\lambda_0} \approx \frac{f - f_0}{f_0},
\]

we obtain

\[
P_\lambda(\lambda) d\lambda = \frac{c}{\lambda_0} P_v \left( c \left( 1 - \frac{\lambda}{\lambda_0} \right) \right) d\lambda
\]

\[\text{(23)}\]

In the case of the thermal Doppler broadening, the velocity distribution is given by the Maxwell distribution

\[
P_v(v) dv = \sqrt{\frac{m}{2\pi kT}} \exp \left( -\frac{mv^2}{2kT} \right) dv
\]

\[\text{(24)}\]

where, \(m\) is the mass of the emitting particle, \(T\) is the temperature and \(k\) is the Boltzmann constant.

Then,

\[
P_f(f) df = \left( \frac{c}{f_0} \right) \sqrt{\frac{m}{2\pi kT}} \exp \left( -\frac{mc^2 \left( \frac{f}{f_0} - 1 \right)^2}{2kT} \right) df
\]

\[\text{(25)}\]

We can simplify this expression as:

\[
P_f(f) df = \sqrt{\frac{mc^2}{2\pi kT f_0^2}} \exp \left( -\frac{mc^2 (f - f_0)^2}{2kT f_0^2} \right) df
\]

\[\text{(26)}\]

which we immediately recognize as a Gaussian profile with the standard deviation

\[
\sigma_f = \sqrt{\frac{kT}{mc^2 f_0}}
\]

\[\text{(27)}\]

and full width at half maximum (FWHM)

\[
\Delta f_{\text{FWHM}} = \sqrt{\frac{8kT \ln 2}{mc^2 f_0}}.
\]

\[\text{(28)}\]

The fuel then sees a wider range of relative neutron speeds. Uranium-238, which forms the bulk of the uranium in the reactor, is much more likely to absorb fast or epithermal neutrons at higher temperatures. This reduces the number of neutrons available to cause fission, and reduces the power of the reactor. Doppler broadening therefore creates a negative feedback because as fuel temperature increases, reactor power decreases. All reactors have reactivity feedback mechanisms, except some gas reactor such as pebble-bed reactor which is designed so that this effect is very strong and does not depend on any kind of machinery or moving parts.

**Mathematical Definition of Reliability**
The life of a system or a device under reliability study follows a sequence that results in an observable time to
failure. A new device is put into service, it functions acceptably for a period of time and then it fails to function satisfactorily. The observed time to failure is a value of the random variable $T$, which represents the lifetime of the device. $T$ takes its values in an interval of the real numbers, $R$, most often in the closed interval $[0, \infty)$. Since the lifetime of a device is represented by a random variable $T$, there is a probability distribution function (cdf) of $T$.

$$F_T(t) = P(T \leq t), \quad 0 \leq t.$$

$F_T(t)$ is usually called the unreliability at time $t$. It represents the probability of failure in the interval $[0,t]$. The probability of failure in the interval $(t_1,t_2]$ equals $F(t_2) - F(t_1)$.

**Definition:** The reliability function is:

$$R_T(t) = P(T > t) = 1 - F_T(t).$$

Thus, reliability is the probability of no failures in the interval $[0,1]$ or equivalently, the probability of failure after time $t$. Sometimes $T$ will take on only a countable number of values in $R$. This case, called the discrete case, occurs when $T$ is a number of cycles, for example, or when the failure time can occur at only discrete points. Most of the time, however, $T$ will be a continuous random variable and its distribution $F_T(t)$ will be a continuous distribution having a density $f_T(t)$.

**Reliability with Continuous Random Variables:**

Assume $T$ is a continuous random variable, taking values in open interval $(0, \infty)$ and with density function $f_T(t)$. The reliability function $R_T(t)$ is:

$$R_T(t) = \int_t^\infty f_T(x) \, dx = 1 - \int_0^t f_T(x) \, dx = 1 - F_T(t) \quad \ldots \ldots \quad (31)$$

where, $F_T(t) \geq 0$ and $\int_0^\infty f_T(x) \, dx = 1$

**Failure and Accident Analysis**

Several reports on the safety of generator these include “Emergency Diesel Generator Failure Review,”[5] “Power System Reliability Analysis with Distributed Generators”[6], “Accident analysis for nuclear power plants with graphite moderated boiling water RBMK reactors”[7] and “Design of Emergency Power Systems for Nuclear Power Plants,”[8] Others are “The unsteady state operation of chemical reactors”[9] and “Safety margins of operating reactors analysis of uncertainties and implications for decision making”[10]. These accidents may perhaps be as a result of design concept process of some of these reactors (which could involve novel technologies) that have inherent risk of failure in operation and were not well studied/understood. In avoiding such accidents the industry has been very successful. As in over 14,500 cumulative reactor-years of commercial operation in 32 countries, there have been only three major accidents to nuclear power plants – Fukushima, Chernobyl and Three Mile Island. As in other industries, the design and operation of nuclear power plants aims to reduce the likelihood of accidents, and avoid major human consequences when they occur.

However, recent study of the reactor fuel under accident conditions, reveal that after subjecting the fuel to extreme temperatures — far greater temperatures than it would experience during normal operation or postulated accident conditions — TRISO fuel is even more robust than expected. Specifically, the research revealed that at 1,800 degrees Celsius (more than 200 degrees Celsius greater than postulated accident conditions) most fission products remained inside the fuel particles, which each boast their own primary containment system.

**Methodology**

In this work, Ordinary Least Square (OLS) methodology, which is largely used in nuclear industry for modeling safety, is employed. Some related previous works on the application of regression analysis technique include: “Statistical Analysis of Reactor Pressure Vessel Fluence Calculation Benchmark Data Using Multiple Regression Techniques”[11], “Simplified modeling of a PWR reactor pressure vessel lower head failure in the case of a severe accident”[12]. Others are “Analyses of loads on reactor pressure vessel internals in a pressurized water reactor due to a loss-of-coolant accident considering fluid-structure interaction”[13], “Regression analysis of gross domestic product and its factors in Lithuania,”[14], “Investigating the Effect of Loss-of-Pressure-Control on the Stability of Water-Cooled Reactor Design Models,”[15]“Optimization of the Stability Margin for Nuclear Power Reactor Design

Objective of the Research
In this work comparism of different test on water-cooled reactor design (WCRD) models with respect to failure or malfunction of emergency power system during operation or accident was carried out by testing for thermal efficiency and thermal power using regression analysis technique before conclusion. The research aimed at demonstrating sufficient safety margins, for nuclear power plants. One objective of this research is to evaluate power system reliability analysis improvements with distributed generators while satisfying equipment power handling constraints. In this research, a computer algorithm involving pointers and linked list is developed to analyze the power system reliability. This algorithm needs to converge rapidly as it is to be used for systems containing thousands of components. So an efficient “object-oriented” computer software design and implementation is investigated. This algorithm is also used to explore the placement of distributed generators and how the different placements affect system reliability, which has not been done in previous research. This exploration makes possible the comparison of alternative system designs to discover systems yielding desired reliability properties.

In this paper, variation of power system reliability with the varying loads is also investigated. Other publications of distribution system reliability analysis associated with time varying loads have not been found.

The Research Motivation
The purpose of this work is to assist countries wishing to include nuclear energy for the generation of electricity, like Nigeria, to secure a reactor that is better and safe. Also, the studies intended to provide guidance in developing practical catalytic materials for power generation reactor and to help researchers make appropriate recommendation for Nigeria nuclear energy proposition as one of the solutions to Nigeria energy crisis. Moreover, the study is to provide a good, novel approach and method for multi-objective decision-making based on six dissimilar objectives attributes: evolving technology, effectiveness, efficiency, cost, safety and failure. Furthermore, this is to help Nigeria meet its international obligations to use nuclear technology for peaceful means. Finally, the achievement is to make worldwide contribution to knowledge.

Research Design/Approach
The design of emergency power supply plays significant role in the safety of the reactor as in the case of emergency it allows the safe shut down of the power reactor and prevent reactor meltdown during accident. Hence, in this work, a statistical analysis of a design input parameter of a typical reactor water-cooled reactor was investigated for safety under a failed emergency power supply. More specifically, the studies concentrated on technical factors that limit the achievement of regular power supply in various design of reactor emergency power supply, such as the mechanical interaction, malfunctioning, failure and the reactor thermal efficiency and thermal power. More also, the study examined the temperature of the fuel behaviour under reactor accident conditions. The Table 2 presents data input for safety margin against thermal power and thermal efficiency of some typical water-cooled reactor design model.

Table 2: Data input for thermal power and thermal efficiency of some typical water-cooled reactor design model.

<table>
<thead>
<tr>
<th>Nos. of trial (j)</th>
<th>Thermal Power (MW)</th>
<th>Thermal Power (MWe)</th>
<th>Thermal Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>100</td>
<td>30.00</td>
</tr>
<tr>
<td>2</td>
<td>210</td>
<td>105</td>
<td>31.00</td>
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<td>3</td>
<td>215</td>
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<td>4</td>
<td>218</td>
<td>110</td>
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<td>6</td>
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<td>11</td>
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<td>129</td>
<td>49.80</td>
</tr>
<tr>
<td>12</td>
<td>263</td>
<td>130</td>
<td>50.00</td>
</tr>
</tbody>
</table>
Table 3: Input data for fuel size and heat generated in a typical water-cooled reactor.

<table>
<thead>
<tr>
<th>Nos. of trial (j)</th>
<th>Fuel size in Mass (g)</th>
<th>Heat Generated °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.8</td>
<td>200</td>
</tr>
<tr>
<td>2</td>
<td>3.5</td>
<td>270</td>
</tr>
<tr>
<td>3</td>
<td>4.2</td>
<td>300</td>
</tr>
<tr>
<td>4</td>
<td>5.0</td>
<td>440</td>
</tr>
<tr>
<td>5</td>
<td>5.7</td>
<td>480</td>
</tr>
<tr>
<td>6</td>
<td>6.0</td>
<td>520</td>
</tr>
<tr>
<td>7</td>
<td>7.4</td>
<td>600</td>
</tr>
<tr>
<td>8</td>
<td>8.3</td>
<td>760</td>
</tr>
<tr>
<td>9</td>
<td>9.0</td>
<td>900</td>
</tr>
<tr>
<td>10</td>
<td>10.6</td>
<td>1050</td>
</tr>
<tr>
<td>11</td>
<td>11.0</td>
<td>1100</td>
</tr>
<tr>
<td>12</td>
<td>12.0</td>
<td>1200</td>
</tr>
</tbody>
</table>

RESULTS AND ANALYSES
1. Water-Cooled Reactor Design Model (WCRDM)

The result of the application of the linear regression analysis of the data in Tables 2 and 3 of a typical water-cooled reactor design model is presented as follows:

(i) Empirical Expression for Safety Factor, \( \hat{Y} \)

In examining the unsteady state of the emergency power system on reactor stability and safety during operation, the data obtained in Tables 2 and 3 which represents parameters for some typical water-cooled reactor design model was used in order to obtain the best fit for the model. The new conceptual fuel design for reactor operation could optimize the performance of this type of water-cooled reactor design model. The linear regression model equation to be solved is given by:

\[
\hat{Y} = B_0 + B_1X_j + e_j
\]

where,

- \( B_0 \) is an intercept,
- \( B_1 \) is the slope,
- \( X_j \) is the rate of increase in fuel volume,
- \( e_j \) = error or residual, \( j = 1, 2, 3, \ldots, k \) and \( k \) is the last term.

Empirical Expression for Safety Factor, \( \hat{Y} \) for Normal Pressure Reading

The model empirical expression is the equation of the straight line relating heat in the reactor and the volume of fuel in the reactor as a measure of safety factor estimated as:

\[
\hat{Y} = -49.6924 + 0.7664X_j + e_j
\]

- The equation (33) is the estimated model or predicted where,

\( \hat{Y} = \) Dependent Variable, Intercept = -49.6924,
Slope = 0.7664, \( X = \) Independent Variable,
\( e = \) error or residual, \( j = 1, 2, 3, \ldots, 12 \) and 12 is the last term of trial.

The Figure 1 shows the linear regression plot section on thermal efficiency and thermal power

(ii) Linear Regression Plot on the relationship between thermal efficiency and thermal power
Figure 1: Thermal efficiency and Thermal power

(iii) F-test Result

Table 4: Summary of F-test Statistical Data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent Variable</td>
<td>Y</td>
</tr>
<tr>
<td>Independent Variable</td>
<td>X</td>
</tr>
<tr>
<td>Intercept ( B_0 )</td>
<td>-49.6924</td>
</tr>
<tr>
<td>Slope ( B_1 )</td>
<td>0.7664</td>
</tr>
<tr>
<td>R-Squared</td>
<td>0.9135</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.9558</td>
</tr>
<tr>
<td>Mean Square Error (MSE)</td>
<td>5.275179 x 10^{-2}</td>
</tr>
<tr>
<td>Coefficient of Variation</td>
<td>0.0591</td>
</tr>
<tr>
<td>Square Root of MSE</td>
<td>2.296776</td>
</tr>
</tbody>
</table>

Table 5: Descriptive Statistics Section

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Dependent</th>
<th>Independent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>Thermal efficiency</td>
<td>Thermal power</td>
</tr>
<tr>
<td>Count</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Mean</td>
<td>38.8917</td>
<td>115.5833</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>7.4476</td>
<td>9.3879</td>
</tr>
<tr>
<td>Minimum</td>
<td>30.0000</td>
<td>100.0000</td>
</tr>
<tr>
<td>Maximum</td>
<td>50.0000</td>
<td>130.0000</td>
</tr>
</tbody>
</table>

The Table 6 is the regression estimation section results that show the least-squares estimates of the intercept and slope followed by the corresponding standard errors, confidence intervals, and hypothesis tests. These results are based on several assumptions that are validated before they are used.
Table 6: Regression Estimation Section

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Intercept B(0)</th>
<th>Slope B(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression Coefficients</td>
<td>-49.6924</td>
<td>0.7664</td>
</tr>
<tr>
<td>Lower 95% Confidence Limit</td>
<td>-68.9510</td>
<td>0.6003</td>
</tr>
<tr>
<td>Upper 95% Confidence Limit</td>
<td>-30.4339</td>
<td>0.9325</td>
</tr>
<tr>
<td>Standard Error</td>
<td>8.6433</td>
<td>0.0746</td>
</tr>
<tr>
<td>Standardized Coefficient</td>
<td>0.0000</td>
<td>0.9558</td>
</tr>
<tr>
<td>T-Value</td>
<td>-5.7492</td>
<td>10.2791</td>
</tr>
<tr>
<td>Prob Level (T-Test)</td>
<td>0.0002</td>
<td>0.0000</td>
</tr>
<tr>
<td>Reject H0 (Alpha = 0.0500)</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Power (Alpha = 0.0500)</td>
<td>0.9993</td>
<td>1.0000</td>
</tr>
<tr>
<td>Regression of Y on X</td>
<td>-49.6924</td>
<td>0.7664</td>
</tr>
<tr>
<td>Inverse Regression from X on Y</td>
<td>-58.0763</td>
<td>0.8389</td>
</tr>
<tr>
<td>Orthogonal Regression of Y and X</td>
<td>-52.8638</td>
<td>0.7938</td>
</tr>
</tbody>
</table>

In Table 7 the analysis of variance shows that the F-Ratio testing whether the slope is zero, the degrees of freedom, and the mean square error. The mean square error, which estimates the variance of the residuals, was used extensively in the calculation of hypothesis tests and confidence intervals.

Table 7: Analysis of Variance Section

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Squares</th>
<th>F-Ratio</th>
<th>Prob Level</th>
<th>Power(5%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>1</td>
<td>18150.74</td>
<td>18150.74</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope</td>
<td>1</td>
<td>557.3774</td>
<td>557.3774</td>
<td>105.6604</td>
<td>0.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>Error</td>
<td>10</td>
<td>52.75179</td>
<td>5.275179 X10^2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adj. Total</td>
<td>11</td>
<td>610.1292</td>
<td>55.46629</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>12</td>
<td>18760.87</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In Table 8 Anderson Darling method confirms the rejection of H₀ at 20% level of significance but all of the above methods agreed that H₀ Should not be rejected at 5% level of significance. Hence the normality assumption is satisfied as one of the assumptions of the Linear Regression Analysis is that the variance of the error variable δ² has to be constant.

Table 8: Tests of Assumptions Section

<table>
<thead>
<tr>
<th>Assumption/Test Residuals follow Normal Distribution?</th>
<th>Test Value</th>
<th>Prob Level</th>
<th>Is the Assumption Reasonable at the 20% or 0.2000 Level of Significance?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shapiro Wilk</td>
<td>0.8901</td>
<td>0.169812</td>
<td>No</td>
</tr>
<tr>
<td>Anderson Darling</td>
<td>0.5842</td>
<td>0.128324</td>
<td>No</td>
</tr>
<tr>
<td>D'Agostino Skewness</td>
<td>1.0600</td>
<td>0.289166</td>
<td>Yes</td>
</tr>
<tr>
<td>D'Agostino Kurtosis</td>
<td>-0.5545</td>
<td>0.579233</td>
<td>Yes</td>
</tr>
<tr>
<td>D'Agostino Omnibus</td>
<td>1.4310</td>
<td>0.488954</td>
<td>Yes</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Constant Residual Variance?</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Modified Levene Test</td>
<td>0.3515</td>
<td>0.569628</td>
<td>Yes</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Relationship is a Straight Line?</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Lack of Linear Fit F(0, 0) Test</td>
<td>0.0000</td>
<td>0.000000</td>
<td>No</td>
</tr>
</tbody>
</table>

Notes:
A 'Yes' means there is not enough evidence to make this assumption seem unreasonable.
A 'No' means that the assumption is not reasonable
(iv) Residual Plots Section

The plot section is used as further check on the validity of the model to satisfy all the assumptions of the linear regression analysis.

Amir D. Aczel (2002, P528) have stated that the normality assumption can be checked by the use of plot of errors against the predicted values of the dependent variable against each of the independent variable and against time (the order of selection of the data points) and on a probability scale.

The diagnostic plot for linear regression analysis is a scatter plot of the prediction errors or residuals against predicted values and is used to decide whether there is any problem in the data at hand Siegel F (2002, p.578).

The Figure 2 is for the plot of errors against the order to selection of the data points (e = 1, 2, ..., 12). Although the order of selection was not used as a variable in the model, the plot reveal whether order of selection of the data points should have been included as one of the variables in our regression model. This plot shows no particular pattern in the error as the period increases or decreases and the residuals appear to be randomly distributed about their mean zero, indicating independence. The residuals are randomly distributed with no pattern and with equal variance as volume of fuel increases.

Note:
1. Residual = original value for heat (Y) minors predicted value for heat, Ŷ
2. Count = the design number (design 1, 2, 3, ..., 12)

Figure 2: Residuals of Heat (°C) versus Fuel (g)

Figure 3 shows the histogram of residuals of error ($e_t$) and this is nearly skewed to the right but the software used indicated that the plot is normal.
While Figure 4 is the result on plot graph of experimental errors. The residuals are perfectly normally distributed as most of the error terms align themselves along the diagonal straight line with some error terms outside the arc above and below the diagonal line. This further indicates that the estimated model is valid.

2. Summary/Conclusion
In summary this paper examined the possibilities to derive and implement a method for safety assessment based on regression analysis techniques. The research conducted safety margin test on some typical water-cooled reactor design (WCRD) models at an accident situation and at same time loss of emergency power supply occurred, secondly safety margin test was carried out on the thermal efficiency and thermal power output of the reactor when power supply failed and thirdly, safety margin test was perform on the reactor in relation to the high temperature effect within reactor core and the fuel temperature. The results of the statistical analysis on these types of nuclear reactor models reveals that the typical water-cooled reactor design (WCRD) models promises most stability under thermal efficiency of 45% and above. Meanwhile, at anything below 45% thermal efficiency the fuel element seems to be unstable in the reactor as the regression plot could not find it optimal.
The research implication is that the WCRD models could be significantly most stable at thermal efficiency of 45% and above. Secondly, the safety margin prediction of up to 4.42% has been validated for reactor design models on water-cooled reactor regarding the design dimension of graphite moderated reactor core parameter, core temperature and fuel temperature. The research effort served as an advantage over the current 5.1% challenging problem for plant engineers to predict the safety margin limit. According to Xianxun Yuan (2007, P49) in “Stochastic Modeling of Deterioration in Nuclear Power Plants Components” a challenging problem of plant engineers is to predict the end of life of a system safety margin up to 5.1% validation.

The current design limits for various reactors safety in a nuclear power plant, defined by the relative increase and decrease in the parametric range at a chosen operating point from its original value, varies from station to station. However, the finding in the work would suggest that the design of the plant should ensure that operating reactor core are made up of large graphite core in order to minimize core melting in an extreme high temperature condition which can damaged the reactor.

It is suggested that the WCRD models “should allow for thermal efficiency of 45% and above in their construction and possibly provision for extra or an in-built automatic emergency power supply (EPS) in the design features to ensure safe operation of nuclear reactor”.

If emergency power supply technology solution must be addressed properly then the following areas of applicable EPS technology needs to be well study these include power system reliability analysis improvements with distributed generators while satisfying equipment power handling constraints. An efficient “object-oriented” computer software design and implementation needs employ for investigation. Dynamic and seismic analysis; safety and reliability; and verification and qualification of analysis with relevant software.

Thermodynamically speaking, the design of the plant should ensure thermal efficiency of 45% during operation for safety purpose. The discoveries shall provide a good, novel approach and method for multi-objective decision-making based on seven dissimilar objectives attributes: materials selection, evolving technology, effectiveness, efficiency, cost, safety and failure. The implication of this research effort to Nigeria’s nuclear power project drive.

It is therefore recommended that for countries wishing to include nuclear energy for the generation of electricity, like Nigeria, the design input parameters of the selected nuclear reactor should undergo test and analysis using this method for optimization and choice.

Acknowledgments
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References