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Investigation of the Dimension Identification Process of 12th Grade Students; Their Success, the Methods Used, and Mistakes

Nisa Tat[1*] Danyal Soybaş[2]
Math Education Department, Faculty of Education, Erciyes University, Kayseri, Turkey
* E-mail of the corresponding author: tatnisa6472@gmail.com

Abstract
This research examined 12th graders' dimension identification process regarding their success, methods, and mistakes using an explanatory mixed research model. The research was conducted with 150 high school students. The data were analyzed both quantitatively and qualitatively. Students' dimension identification success varied depending on whether the shape is closed, curvilinear, or planar, and it generally decreased from 3 dimensions to 0 dimensions. Students' dimension identification methods included measurable features of the shape (length, area, volume), making inferences regarding the shape being filled or empty, and taking the number of axes used in the coordinate system as a basis. The mistakes resulted from wrong reasoning related to the coordinate system, incorrect dimension identification based on the fallacies about the area of curvilinear or closed linear shapes and the volume of non-planar surfaces, and associating the number of visible faces with the shape's dimensions.
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1. Introduction

Dimension is a quantity that geometric shapes and everything tangible in the world we live in have. Although we frequently use this concept in our daily lives, it is difficult to understand and define. TDK dictionary defines it as "one of the three directions considered in the measurement of lines, surfaces or objects, length, width, and depth" (TDK, 2011). We encounter 1-dimensional, 2-dimensional, and 3-dimensional structures in the 3-dimensional world we live in. Menger (1943) suggests that all objects are 3-dimensional, but the surfaces of things such as paper and iron sheets give an idea about 2 dimensions, whereas a line drawn with wire, rope, or chalk gives an idea about 1 dimension. As it can be understood from the definition and examples, the concept of dimension is an abstract concept that covers the whole geometry course.

Ancient Greek philosophers also worked on the definition of the concept of dimension. One of these philosophers, Plato, stated that the point is dimensionless from an intuitive point of view (Urbanski, 2011, cited by Peker & Karakuş, 2015, p.193). In Republic Book VII, Plato stated that the plane expresses the surfaces, that the third dimension comes after the second dimension, and that the cube has a depth (Mandelbrot, 1983, cited by Peker and Karakuş, p.186). Euclid used the following expressions in his book "Elements," which he wrote in the period after Plato: the point has no parts; the line has no width, only length; the boundaries of a line are the points; the surface has length and width without thickness; the boundaries of a surface are the lines; a solid body has length, width, and depth; the boundaries of an object are the surfaces (Cited by Manin, 2006). One of the approaches examining the concept of dimension is related to the Cartesian system created by Descartes and consubstantiated with analytical geometry (Skordoulis et al., 2009). From the perspective of analytical geometry, dimension is expressed as the number of independent variables used to describe the position of an object (Elert, 2005, cited in Peker & Karakuş, 2015, p.187).

In the descriptions in which he addressed the concept of dimension analytically, Freudenthal (2002) used the expression "The line is formed by a moving point, the plane by a moving line, and the space by a moving plane" (as cited in Skordoulis et al., 2009). Parallel to this idea and Euclid's definition, Devlin (1994) stated that a straight line is 1-dimensional; if there is a move in the second direction perpendicular to this line, it is 2-dimensional, and if there is a move in the third direction perpendicular to the other two directions, it is 3-dimensional (As cited in Ural, 2011). Based on the dimension definitions mentioned above, the illustration in figure 1 was prepared.

Figure 1 Perception of dimension formed by moving point, line and plane

Freudenthal (1983) stated that the concept of dimension could be examined from three different perspectives: Euclidean geometry, analytical geometry, and topology (as cited in Skordoulis et al., 2009). The first two of these approaches have been described above. The other approach, topology, is outside school mathematics and deals with dimensions more abstractly.

In the descriptions addressing the concept of dimension from a topological point of view, Freudenthal (2002) suggested that an object is bounded by a surface and a surface by a line and a line by a point (as cited in Skordoulis et al., 2009). In the topological sense, dimension attempts to explain the interconnection of the points that form an object (Liebovitch, 1998, cited in Peker & Karakuş, 2015, p.191). According to Menger (1943), the dimension of a cluster can be understood by using the concepts of neighborhood and boundary. According to this idea, a cluster is n-dimensional if any point belonging to the cluster is located in the close neighborhood of the cluster and any of its intersections, whose boundaries are maximum n-1 dimensional.

On the other hand, Poincare tried to explain the concept of dimension with an inductive approach. According to this approach, if two points on a cluster can be separated from each other by removing a subset of n-1 dimension from the cluster, then this cluster is n-dimensional (Courant, Robbins, & Stewart, 1996, cited in Peker & Karakuş, 2015, p.193).

MoNE's (2004) 5th-grade mathematics curriculum includes the definition of the dimension and the properties of 1-dimensional, 2-dimensional and 3-dimensional geometric shapes and objects (Cited by Ural, 2011). However, the instruction of the dimension concept is not mentioned in the current primary and secondary education mathematics curriculum. Only the 7th-grade mathematics curriculum includes a gain, in which students are asked to draw 2D views of 3D objects. Although the concept of dimension occupies an important place in mathematics education, it is not included in textbooks much (Skordoulis, Vitsas, Dafermos & Koleza, 2009). Parallel to this situation, there has not been much research on the concept of dimension. The researches are as follows. Vitsas and Koleza (2000) asked mathematics students to identify the dimensions of a given geometric shape and explain the criteria they used to make this identification. As a result of the study, they reported that the coordinate system was one of the criteria used. The students could make more accurate dimension identification in the Euclidean plane than the Cartesian plane (Cited in Ural, 2011). Skordoulis et al. (2009) investigated whether university students' use of the coordinate plane while identifying a shape's dimensions causes errors. As a result of the research, they concluded that the coordinate system hinders understanding the concept of dimension both epistemologically and didactically. Ural (2011) conducted a study to measure the dimension identification skills of pre-service mathematics teachers. As a result of the research, pre-service teachers made mistakes due to misinterpretation of the coordinate system for dimension identification, wrong ideas about the concepts of width, length, height, and incorrect terminology in the geometry teaching process. The criteria pre-service teachers used in dimension identification were width, length, height, having an area and volume, being planar, and looking at the number of axes used to display in the coordinate system. Duatepe Paksu et al. (2012) researched the concept images of the pre-service classroom teachers for the concept of dimension. They found that approximately one-third of the pre-service teachers did not know the concept of dimension, and about half of them gave inconsistent answers. Only 15% of the pre-service teachers considered the properties such as volume, width, length, and height when deciding on the number of dimensions of geometric shapes. As a result, it has been observed that the pre-service teachers' dimension knowledge is insufficient, and they focus on different criteria such as the number of sides, the number of vertices, and the number of visible faces when deciding on the number of dimensions of a geometric shape. Tuluk (2014) conducted a study to learn the thoughts and methods used by pre-service primary school teachers to express their knowledge about point, line, surface, and space and saw that one of the ways used in the research was associating with dimension.

This research aims to measure the dimension identification success of 12th-grade students and reveal the methods used to identify the dimension and the mistakes made. For this purpose, the following sub-problems were addressed for 12th-graders.

1. How is the dimension identification success of 12th-grade students?
2. What are the methods that 12th-grade students use in dimension identification?
3. What are the mistakes made by 12th-grade students in dimension identification?

2. Method

In this study, which aims to measure the dimension identification skills of 12th-grade students, a mixed-method, which combines quantitative and qualitative methods, was used. The objective of this method is to provide a more detailed and more comprehensive understanding of the researched subject by using the superior aspects of quantitative and qualitative methods (Mills & Gay, 2016, cited by Alkan, Şimşek, & Erbil, 2019). In the explanatory mixed-method, a type of mixed-method, quantitative data is collected first. Then qualitative data is collected and used to explain and detail the results (Creswell & Plano Clark, 2011, cited by Fırat, Kabakçı Yurdakul, & Ersoy, 2014).

Study Group

This research was conducted with 12th-grade students studying mathematics in 4 high schools in the Bor district of Niğde province in the 2020-2021 academic year. A total of 150 students, 46 people from Bor Şehit Ramazan Konuş Science High School, 33 people from Bor Şehit Bora Çelik Anatolian High School, 39 people from Bor

Akın Gönen Anatolian High School, and 32 people from Bor Atatürk Anatolian High School, participated in the research. The high schools that constitute the sample have different achievement levels. In addition, both math&science and equal weight students of selected schools were included in the study. The reason for choosing schools with different achievement levels and including math&science and equal weight classes is to increase the diversity of data by using different perspectives regarding the situation to be researched. The maximum variation sampling method, one of the purposive samplings, was used in this study. The objective of this method is to maximize the diversity of students who will express their opinions on the researched subject (Yıldırım & Şimşek, 2006).

Data Collection Tools

The "test for the concepts of dimension and length-area-volume" was prepared by taking expert opinions to collect data.

In the first question of the test, students were given the drawing of 10 geometric shapes, one 0-dimensional, three 1-dimensional, three 2-dimensional, and three 3-dimensional, and they were asked to identify the dimensions of the given shapes and write the methods they used to identify these dimensions. It was aimed to determine students' dimension identification success, the methods they used, and the mistakes made by examining the findings of this question.

The remaining questions of the test addressed another research problem. However, the second and third questions of the test provided findings related to the third sub-problem of the study, "What are the mistakes made by 12th-grade students in dimension identification?". For this reason, they are also included in the study. In the second question of the test, the drawings of a line segment, curve segment, rectangle, and circle obtained using a rope of negligible thickness were given to the students, and they were asked to identify the dimensions of the shapes and their measurable properties (length, area, volume). In the third question, students were given the drawings of a rectangular region, polygonal region, and lateral region of the cylinder obtained using an A4 paper of negligible thickness; they were asked to identify the dimensions of the shapes and their measurable properties (length, area, volume).

Data Collection

After the data collection tool was prepared, necessary permission was obtained to administer it to the students. The school administration of the schools that participated in the study was met. According to the school's facilities, the students of math&science and equal weigh classes were gathered in the conference hall in two schools and the classroom in two schools. They were given 1 lesson hour, and the data were collected in 4 days (1 day at each school).

Data Analysis

After data collection, the questions were analyzed according to the sub-problems. First, the answers to the first question of the test, "Identify how many dimensions the shapes given below have," were analyzed regarding the first sub-problem. The answers given were analyzed for each dimension separately.

The next step addressed the second and third sub-problems. The answers given to the first question of the test were analyzed to determine the methods students used in identifying dimensions and the mistakes they made. Some students did not express any opinions, and some wrote irrelevant sentences. Therefore, only the data expressing students' opinions in line with the purpose were considered while analyzing this part. First of all, the correct answer given by the students regarding the shapes' dimensions and the most given wrong answer were determined, and these two situations were examined. Only for the sphere, both wrong answers were examined. The methods used by students to decide on a shape's dimension were tabulated. At this point, the methods used by those who correctly identified the shape's dimension and the methods used by those who were incorrect were tabulated separately. The methods used by those who incorrectly identified the dimensions were examined in detail, and students' mistakes in dimension identification were revealed and tabulated. The answers given to the second and third questions of the test were also analyzed. The mistakes made by the students in the dimension identification process were noted and added to the findings.

3. Results

Findings of the First Sub-Problem

In the data collection tool, the question of "Identify how many dimensions the shapes below have?" was asked to the students to answer the first sub-problem of the study, "How is the dimension identification success of 12th-grade students?". The answers were analyzed separately for each dimension.

Table 1: Responses for the dimension of the 0-dimensional point

	0-D	1-D	2-D	3-D	Other	NA
Point	24 (16%)	104 (69.4%)	9 (6%)	2 (1.3%)	2 (1.3%)	9 (6%)

The table shows the distribution of correct answers for the dimension of 0-dimensional point. Only 16% of the students could correctly identify the dimension of the point, and 69.4% thought of it as 1-dimensional. The highest number of blank answers occurred in this question.

Table 2: Responses for the dimension of the 1-dimensional shapes

	0-D	1-D	2-D	3-D	Other	NA
Square	0 (0%)	113 (75.3%)	36 (24%)	1 (0.7%)	0 (0%)	0 (0%)
Hexagon	0 (0%)	108 (72%)	39 (26%)	0 (0%)	1 (0.7%)	2 (1.3%)
Line Segment	5 (3.3%)	133 (88.7%)	10 (6.7%)	0 (0%)	0 (0%)	2 (1.3%)

The distribution of correct answers for 1-dimensional shapes is given in the table. The most frequent answer was 1-dimensional. After the correct answer, the most frequent incorrect answer was 2-dimensional. Students' ability to correctly identify the dimension of 1-dimensional shapes in closed form was lower than that of the line segment. However, in general, their success in identifying the dimension of 1-dimensional shapes is good.

Table 3: Responses for the dimension of the 2-dimensional shapes

	0-D	1-D	2-D	3-D	Other	NA
Quadratic region	0 (0%)	19 (12.6%)	124(82.7%)	5 (3.3%)	1 (0.7%)	1 (0.7%)
Circle	0 (0%)	27 (18%)	116(77.4%)	5 (3.3%)	0 (0%)	2 (1.3%)
Triangular region	0 (0%)	19 (12.7%)	124 (82.7)	6 (4%)	0 (0%)	1 (0.7%)

The table shows the distribution of the correct answers for 2-dimensional shapes. The ratio of those who correctly identified the dimensions of the shapes is close to each other and high. It can be said that students' success in identifying the dimension of 2-dimensional shapes is good.

Table 4: Responses for the dimension of the 3-dimensional shapes

	0-D	1-D	2-D	3-D	Other	NA
Rectangles Prism	0 (0%)	0 (0%)	7 (4.7%)	138 (92%)	5 (3.3%)	0 (0%)
Sphere	0 (0%)	17 (11.3%)	70 (46.7%)	63 (42%)	0 (0%)	0 (0%)
Cylinder	0 (0%)	0 (0%)	14 (9.3%)	135 (90%)	0 (0%)	1 (0.7%)

The table shows the distribution of the correct answers for 3-dimensional shapes. Two students stated that the rectangular prism is 4-dimensional, and three stated it has 6 dimensions. Overall, the correct identification rates of the rectangular prism's and cylinder's dimensions are high and close to each other. In contrast, the answers for the sphere dimension vary, and most students thought that the sphere is 2-dimensional (46.7%). No student thought that rectangular prism and cylinder could be 1-dimensional; on the other hand, 11.3% of the students thought that sphere was 1-dimensional. The success of accurately identifying the dimensions of the other two 3-dimensional shapes, except the sphere, is high.

Table 5: Success on dimension identification according to shapes

	Frequency	Percentage
Rectangles Prism	138	92%
Cylinder	135	90%
Line Segment	133	88.7%
Quadratic region	124	82.7%
Triangular region	124	82.7%
Circle	116	77.4%
Square	113	75.3%
Hexagon	108	72%
Sphere	63	42%
Point	24	16%

Students' dimension identification success for each shape is shown in the table. The first two most successfully identified shapes are 3-Dimensional, followed by the line segment, and 2-dimensional shapes. Correct identification rates of 2-dimensional shapes are pretty close to each other. The dimensions of 1-dimensional shapes were also identified correctly. Students failed to identify the dimension of the 0-dimensional point. The dimension identification success of the sphere was also behind the other 3-dimensional shapes.

Findings of the Second Sub-Problem

To address the second sub-problem of the study, "What are the methods that 12th-grade students use in dimension identification?" students were asked, "Write down how you decided on the dimensions of the geometric shapes, whose names and drawings are given below." The answers were analyzed separately for each dimension.

Table 6: Methods used by students who decided correctly on the dimension of 1-dimensional shapes

	Square	Hexagon	Line Segment
Being empty	22	21	0
Being composed of lines, edges, or being linear	21	16	26
Having only length	10	11	22
Having only one face or being visible from one side	19	16	16
Lack of area and volume	3	4	5
Lack of area	2	2	3
Lack of volume	4	1	1
Being in the page plane / monoplane / planar	4	5	1
Being expressed with a single axis in the coordinate system	0	0	4

The table shows the distribution of the methods used by the students who correctly decided on the dimension of 1-dimensional shapes. The students mostly used the following methods: being empty; being composed of lines and edges; having only length; having one face, or being visible from one side.

Table 7: Methods used by students who decided correctly on the dimension of 2-dimensional shapes

	Quadratic region	Circle	Triangular region
Having an area, having a length and area	39	32	33
Being filled	33	30	29
Being represented in the x-y plane in the coordinate system	9	6	10
Being planar, regional, surface	10	4	8
Being visible from one side or having one face	5	9	11
Being visible from two sides or having two faces	3	6	5
Having an area but no volume, being volumeless	2	5	3
Having a width and height	7	1	3
Having a base and height	0	2	3
Having a sense of reality	1	2	3

The table shows the distribution of the methods used by the students who correctly decided on the dimension of 2-dimensional shapes. They mostly mentioned the expression having an area and being filled, which has a similar meaning. Being planar, represented in the x-y plane, and visibility from one side or two sides are other methods used.

Table 8: Methods used by students who decided correctly on the dimension of 3-dimensional shapes

	Rectangles Prism	Sphere	Cylinder
Having a volume	36	16	39
Having more than one surface	25	0	14
Being seen from multiple angles / from all angles	8	3	11
Being represented in the x-y-z plane	13	5	9
Having a length, area, and volume	6	5	8
Having an area and volume	5	2	5
Being filled / fillable	3	7	6
Having a width, length, and height	6	1	5
Being tangible / concrete / realistic	8	2	9
Having a depth/thickness	4	1	2
Being formed by the combination of more than one surface/shape	3	1	3

The table shows the distribution of the methods used by the students who correctly answered the dimension of 3-dimensional shapes. The students decided on the shapes' dimensions regarding the volume. Some methods used to identify the dimensions are the number of visible faces, representation in the x-y-z plane, and tangibility.

Findings of the Third Sub-Problem

The answer to the third sub-problem of the research, "What are the mistakes made by 12th-grade students in dimension identification?" was sought. Students were asked, "Write down how you decided on the dimensions of the geometric shapes, whose names and drawings are given below." The methods used by the students who gave wrong answers were determined, and the mistakes made while identifying the dimension were revealed. In addition, students were given the drawings of a line segment, curve segment, rectangle, and circle obtained using a rope of negligible thickness and the drawings of a rectangular region, polygonal region, and lateral region of the cylinder drawings obtained using an A4 paper of negligible thickness. They were asked to identify the shapes' dimensions. The answers were examined, the methods of the students who made a wrong decision were tabulated, and the mistakes made by the students in dimension identification were determined.

Table 9: Mistakes made by students who think of 1-dimensional shapes as 2-dimensional

	Square	Hexagon	Line Segment
Being represented in the x-y plane in the coordinate system	9	8	3
Having a width and height	6	0	0
Having an area	5	6	0
Being planar	2	2	0
Having a base and height	0	3	0
Being linear	0	1	2

As can be seen in the table, the most significant factor in students' misconception was the coordinate system. The misconception that a square has a width and height and closed shapes have areas caused students to make mistakes in dimension identification.

Table 10: Responses for the dimension of the 1-dimensional shapes

	1-Dimensional	2-Dimensional	3-Dimensional
Line segment	133	11	3
Curve segment	117	21	3
Rectangle	76	62	5
Circle	79	51	12

Regarding the answers given to the dimensions of the line and curve segments, some students misperceived the curvilinear structure as a factor increasing the dimension. The number of students who described the circle as 3-Dimensional is higher than those describing the rectangle as 3-Dimensional. In addition, when the answers given for rectangle and circle are compared with the answers of line segments and curves, it was concluded that the closeness of shape is a factor that increases students' dimension perception.

Table 11: Mistakes made by students who think of 2-dimensional shapes as 1-dimensional

	Quadratic region	Circle	Triangular region
Being visible from one side or having one face	12	10	14
Being planar	1	3	1

Some students perceived the shapes on the plane as 1-dimensional even though they indicated an area. It can be said that these students made the mistake of associating being planar with 1- dimensionality.

Table 12: Mistakes made by students who think of 3-dimensional shapes as 2-dimensional

	Rectangles Prism	Sphere	Cylinder
Two faces can be seen	2	4	4
Having an area	0	15	0
Being filled	0	14	0
Being represented in the x-y plane in the coordinate system	0	4	0
Having edges and being filled	0	3	0
One face can be seen	0	3	0
Having no difference between it and the circle	0	2	0

The expression "Two faces can be seen," mentioned for all 3 shapes, is also used for 2-dimensional shapes, and it is thought to have the meaning of being planar or having two surfaces. Apart from this, all other methods represented the properties of 2-Dimensional shapes and were only mentioned for the sphere.

Table 13: Responses for the dimension of the 2-dimensional shapes

	1-Dimensional	2-Dimensional	3-Dimensional
Rectangular region	22	117	8
Polygonal region	19	104	12
The lateral region of the cylinder	3	24	118

In the table above, the responses given for 2-dimensional shapes were examined, and inferences were made about the mistakes made by the students who thought 2-dimensional shapes as 3-dimensional. The number of students who answered 3-dimensional for the polygonal region is 4 more than those of the rectangular region, indicating that some students associated the number of edges with dimension. The given three shapes only indicate surfaces, but the final shape being closed and not planar might have caused the 3-dimensional perception to be strengthened.

4. Discussion and Conclusion

According to the findings of the first sub-problem of the research, the following results were obtained. The rate of those who correctly identified the dimension of the 0-dimensional point was 16%. Students' tendency to leave blank and the most common answer being 1-dimensional (69.4%) suggest that the concept of 0-dimension is not known well. The ratio of correctly identifying the dimension of the line segment from 1-dimensional shapes is high (88.7%), the ratio of the square (75.3%) and the ratio of the hexagon (72%) are good. In 2-dimensional shapes, the correct answer rates for all three shapes are close to each other and good. The correct dimension identification rates for the quadratic region, circle, and triangular region are 82.7%, 77.4%, and 82.7%, respectively. The correct answer rates in 3-dimensional shapes are 92% for the rectangular prism and 90% for the cylinder, and the dimension identification success of these shapes is high. Regarding the other 3-dimensional shape, the sphere, 42% of the students answered 3-dimensional, whereas 46.7% answered 2-dimensional. The sphere is the most undecided shape among all shapes while identifying dimension.

The following results were obtained from the findings of the second sub-problem of the research. The students who correctly identified the dimension of the given shapes used the methods below.

•Expressions used by those correctly identified the dimension of the 0-dimensional point: lack of length; lack of height; lack of area; lack of volume; lack of a specific measure; lack of width and length.

• Expressions used by those correctly identified the dimension of 1-dimensional shapes: being empty; being composed of lines, edges, or being linear; having length only; having only one face or being visible from one side; lack of area and volume; being planar; being expressed with a single axis in the coordinate system.

• Expressions used by those correctly identified the dimension of 2-dimensional shapes: having an area; having a length and area; being represented in the x-y plane in the coordinate system; being planar; being visible from one side or having one face; being visible from two sides or having two faces; having an area but no volume; having a width and height; having a base and height; having a sense of reality.

• Expressions used by those correctly identified the dimension of 3-dimensional shapes: having a volume; having more than one surface; being seen from multiple angles / from all angles; being represented in the x-y-z plane, having a length, area, and volume; having an area and volume; being filled/fillable, having a width, length, and height; being tangible, concrete, realistic; having a depth or thickness; being formed by the combination of more than one surface/shape.

Consequently, when deciding a shape's dimension, students looked at the measurable properties (length, area, volume), the number of axes used in the coordinate system, being linear or planar. These methods used by the students in dimension identification are similar to the findings obtained by Ural (2011).

The following results were obtained for the third sub-problem of the research. The methods used by students who incorrectly identified the dimensions asked in the first question of the test were examined, and students' mistakes made in dimension identification were revealed.

• Most students who thought of 1-dimensional shapes as 2-dimensional gave this answer because the shape can be represented in the coordinate system. Vitsas and Koleza (2000) and Skordoulis et al. (2009) also reported that the coordinate system causes misconceptions in dimension identification. Students mainly used expressions related to the coordinate system for incorrectly identified dimensions in this research, and the mentioned studies support this. Similar to the misconception of the coordinate system, some students associated being planar with 2-dimensionality. Assuming that the square has a width and height and that square and hexagon have an area because they are closed shapes made students think that these shapes are 2-dimensional.

• Students who thought of 2-dimensional shapes as 1-dimensional explained it by seeing the shape from one side, seeing one face, and being planar. It can be said that some of the students associated being on a single plane, that is, being planar, with 1-dimensionality. Meanwhile, others associated the number of dimensions with the number of faces and answered as 1-dimensional.

• Students who thought of 3-dimensional shapes as 2-dimensional gave this answer because two faces of the object are visible, showing that some students based their dimension identification on the number of visible faces. Similarly, 2 students stated the rectangular prism as 4-dimensional, 3 as 6-dimensional. In the study conducted by Duatepe Paksu et al. (2012) to determine the concept images of pre-service classroom teachers for the concept of dimension, pre-service teachers focused on different criteria such as the number of edges, the number of vertices, and the number of visible faces in dimension identification. The criteria used by the students who incorrectly identified the dimension in this study are similar to the mentioned study's results. The object that caused the most mistakes was one of the 3-dimensional objects, the sphere. The answers of those who thought that the sphere is 2-dimensional showed that students likened the sphere to the circle and wrote the criteria used for 2-dimensional shapes.

• The answers given for the dimensions of line and curve segments were compared. The curvilinear structure was seen as a factor that increased dimension by students. When the answers given for the square and circle were compared with the answers of the line and curve segments, it was concluded that being a closed shape is also a factor that increases the dimension for students. The answers given for the rectangle and circle were rereviewed to confirm the effect of the curvilinear structure on dimension identification. The result strengthened the idea that the curvilinear structure is a factor that increases the dimension for students. Regarding the answers given to the rectangular region, polygonal region, and the lateral region of the cylinder, the correct dimension identification rates for the first two shapes were close to each other and high, while it was very low for the last shape. Most of the students thought that the lateral region of the cylinder was 3-Dimensional. Although all three given shapes only indicate surfaces, the 3-dimensional perception was higher only for the last shape, which may be attributed to the shape being closed and not planar.

Shapes of all dimensions can be expressed in a 3-dimensional plane. 0-dimensional, 1-dimensional, and 2-dimensional shapes can be displayed on the 2-dimensional plane. 1-dimensional and 0-dimensional shapes can be shown on a 1-dimensional line. This rule also applies to the coordinate system. Research findings show that most mistakes were due to the lack of knowledge or insufficient knowledge of the rule given above. Considering closed shapes in curved or polygonal form, which are 1-dimensional, as 2-dimensional because of being displayed on the plane supports this situation; considering the lateral region of the cylinder, which is 2-dimensional, as 3-dimensional because of being shown in space is another fact supporting it.

Dimension is a quantity possessed by all geometric shapes and objects. Knowing the dimension concept well will help students better recognize geometric shapes and objects. However, the knowledge of the dimension concept is not included in the mathematics curriculum. It is thought that the inclusion of the subject in the mathematics syllabus as achievement will play a role in increasing students' geometry achievement.

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