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The Teaching of Highlighting Mathematical Culture from the Perspective of HPM ——Take the Teaching for "Letters Represent Numbers" as an Example

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Abstract
"Compulsory Education Mathematics Curriculum Standards (2022 Edition)" clearly states that attention should be paid to mathematics culture, but at present, mathematics textbooks and classrooms still do not attach importance to mathematics culture, and mathematics culture is not really integrated into mathematics classroom teaching. For further exploring how to integrate mathematical culture into mathematical classroom teaching, the traditional teaching design of Letters Represent Numbers is transformed from the perspective of HPM by taking Letters Represent Numbers as a case, and the history of mathematics is used to highlight the mathematical culture embedded in mathematical knowledge, and based on this, ideas and suggestions are put forward for teaching to highlight mathematical culture from the perspective of HPM.
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It is clearly stated in "Mathematics Curriculum Standards for Compulsory Education (2022 Edition)" "paying attention to the frontier of mathematics development and mathematics culture, inheriting and carrying forward Chinese excellent traditional culture" in the nature of the curriculum, and it explicitly requires "creating reasonable situations and appropriately introducing mathematics culture" in the part of the test paper system[1]. Currently, many questions related to the mathematical culture have also emerged in the senior high school entrance examination questions, which further requires teachers to permeate mathematical culture while teaching mathematical knowledge. The 2022 edition of the curriculum standard is oriented to core literacy, which constructs the core literacy of mathematics disciplines in mandatory education. The most basic value of mathematics culture lies in improving mathematics literacy[1]. Currently, teachers have not paid much attention to math culture in teaching and have not integrated it into the math classroom. To enhance students' core literacy in mathematics and to respond to the requirements of the new curriculum standards, teachers need to integrate mathematical culture into the mathematical classroom, so that the mathematical classroom can be a classroom that teaches mathematical culture. Taking the first volume 3.1 of the seventh grade textbook of Jiangsu Science Edition as an example, the textbook draws a conclusion that the common letters in mathematics represent numbers by analogy from the life icon representing a certain meaning. It is concluded that the letters can represent any number, and then try, practice and exercise through the mathematical experiment of putting together squares, which does not reflect the mathematical culture contained in the Letters Represent Numbers. The section of "Letters Represent Numbers" contains rich mathematical culture. In case it is taught completely according to the textbook, it is difficult for students to understand and realize the necessity and significance of Letters Represent Numbers. Next, we design a teaching design of Letters Represent Numbers from the perspective of the history of mathematics and mathematics education (HPM), aiming at exploring a teaching approach and method to reveal the teaching of mathematics culture, and putting forward some thoughts and suggestions on the teaching of mathematics culture.

1.The Mathematical Cultural Connotation of "Letters Represent Numbers"

The Letters Represent Numbers contains rich mathematical and cultural connotations, i. e., the Letters Represent Numbers embodies the idea of mathematical symbol and embodies the concise and unified beauty of mathematics; the Letters Represent Numbers is the second abstract in the history of mathematics, which is a historical leap in the history of mathematics; the demand for Letters Represent Numbers has emerged from the birth of the arithmetic, which has gone through three stages of development, namely rhetorical algebra, reduced algebra, and symbol algebra, thus forming the now perfect system of Letters Represent Numbers. The more than 2,000 years of development of Letters Represent Numbers reflect the spirit of persistent and courageous innovation of mathematics, which plays a role of moral education in mathematics. The Tianyuan Technique by Li Ye, an ancient Chinese mathematician and Siyuan Technique by Zhu Shijie reflect the contribution of Chinese

mathematics to Letters Represent Numbers and reflect the excellent traditional Chinese culture, which is conducive to stimulating students' patriotism and national pride.

2 .Design of Mathematical Culture Teaching Process of "Letters Represent Numbers"

The following teaching design is designed based on the analysis of the mathematical cultural connotation of the above-mentioned Letters Represent Numbers and its mathematical cultural characteristics.

2.1 Create a context and introduce a topic

A question from the ancient Egyptian Rhind paper grass book is given.

Question 1: A quantity added to its $\frac{1}{7}$, is equal to 19. Seek this amount [4].

As students learn about equations in primary school, they naturally think of setting unknown structural equations to calculate this amount. So let this quantity be x, give the equation $x+\frac{1}{7}x=19$ according to the equal quantity relationship in the topic, and solve $x=16\frac{5}{8}$.

The teacher asked two questions after the students solved this problem.

Teacher: What does x here mean? Can I replace it with another letter[5]?

Student: x indicates an unknown number and can be replaced with other letters.

Problem creation through problems in the history of mathematics to stimulate students' desire for knowledge. The problems given are also problems that can be solved by students using primary school equation knowledge, which is in line with students' cognitive laws to realize the connection between primary school knowledge and junior high school knowledge. The two questions asked by the teacher are designed to make students aware that the letters can indicate the unknown and that the same unknown can be expressed by different letters.

2.2 Change Scenarios and Explore New Knowledge

Problem 2: A quantity is known plus its $\frac{1}{7}$ value. Seek this amount.

It is the key to the introduction of Letters Represent Numbers and the focus of this section. As students learn the equation in primary school, they know that the letter can indicate the unknown, so students will set this quantity as x, but $\frac{1}{7}$ is added to this quantity is known, but the topic does not clearly give the value of this quantity, and students will find that no equation can be presented, which leads to a cognitive conflict, and they are urgent to solve this problem. The teacher guided the student group communication, after which the group representative was presented with the ideas discussed in the group.

Teacher: Please ask the representatives of this group to talk about the results of your group discussion.

Student: What our team thought was that we would replace this quantity with a special value and add its $\frac{1}{7}$ value, and then we could list the equation and calculate the quantity.

Division: The idea discussed in your group is very good, and a special value is used to indicate this quantity plus its $\frac{1}{7}$, and then the equation can be constructed to solve this quantity In the 3rd century AD, it was the use of special values to solve such problems by the ancient Greek mathematician Diophantus. Does this method work out for the students?

Student: In case the special values are different, the results of the solution will be different, and this method will make the answer not unique.

Teacher: Ah, this student answered very well. The result of this method is special by taking special values. We can take any value, which means that the amount added to its $\frac{1}{7}$ value is arbitrary, which in our opinion is not strict. So do you have other ideas?

The ability of students to come up with special values to solve this problem shows that students have made significant progress in their ideas, and displaying their ideas is conducive to stimulating their enthusiasm, and they use their ideas to draw out Diophantus, penetrate the history of mathematics, and compare them with scientists. Students are also encouraged and gain a sense of achievement. Guide students to discuss the pros and

cons of this method, which they find to be unrigorous through their own thinking, prompting them to think further and pave the way for the known numbers to be expressed in letters.

Student: We are thinking of using letters to indicate it because the amount added to its $\frac{1}{7}$ is not told in the topic.

Teacher: Very well. What your group discussed in the group was using letters to indicate this quantity plus its $\frac{1}{7}$. Since this number is unknown, what can we indicate not only as the unknown, but also as well?

Student: Known numbers can also be indicated by letters.

Teacher: Very well, the letters can represent not only unknown but also known ones, because the known numbers are arbitrary, which means that the letters can represent any number. The mathematician Viète was the first to use letters to indicate known data, and students were very smart, achieving a historical breakthrough like the large mathematician Viète.

Naturally, the letters obtained through students' own thinking can indicate not only unknown but also known figures, and students are encouraged to gain a sense of accomplishment by comparing their writings with those of mathematical scientists.

2.3 Self-operation to Consolidate New Knowledge

Teacher: I'd appreciate it if students could solve problem 2 based on what we have just learned Letters Represent Numbers.

Consolidate students' knowledge of using letters to indicate known number through students' hands-on operation. In their first encounter, students will try to be diverse. On the basis of their own operation, teachers teach students some rules of using letters to indicate figures.

Teacher: The teacher saw that all the students can construct the equation, but there are many kinds of letters. Some students are capital letters and others are lowercase letters. Which one do the students think is more convenient [6].

Student: It is more convenient to write in lowercase letters, because they can be written in one stroke.

Teacher: We usually use lowercase letters to represent numbers for the convenience of writing. Mathematician Descartes was the first mathematician to use lowercase letters to represent numbers. The first letters in Descartes' alphabet (such as a , b , c) indicate known numbers, while the last letters (such as x , y , z) indicate unknowns. Students imitate Descartes and write the equation of question 2 [7].

Most of the students will write $x+\frac{1}{7}x=a$.

Students found through an independent operation that it is convenient to write in lowercase and experienced the mathematical idea of simplicity in mathematical pursuit. The rules of Descartes enable students to gain insight into the pursuit of unified thinking in mathematics, a rule that students will be more likely to accept through an introduction to the practice of mathematics through the history of mathematics.

2.4 Review the history and penetrate the culture

The teacher introduced the origin of the Algebra name and its mathematical history in the three stages of development.

We officially entered the field of Algebra from the beginning of the letter expression of number, which was translated by Li Shanlan, a mathematician in the Qing Dynasty, meaning Letters Represent Numbers. Algebra goes through three stages, namely rhetorical algebra, abbreviated algebra, and symbolic algebra. Rhetorical Algebra is to express an Algebra problem and its solution completely in words. Problems 1 and 2 we give belong to the category of rhetorical algebra. The complete use of text indicates that it is very tedious, which drives scientists to think about how to perform simple operations, and Letters Represent Numbers came into being. The symbol algebras refer to the unknown with letters, and the solution of our problem 1 belongs to the category of abbreviated algebra. Diophantus was the first mathematician to use letters to indicate unknown counts, but he was not aware that letters could represent any number and could only replace known numbers with special values. Viète was the first mathematician to use letters to indicate known counts, marking the entry of Algebra science into the symbolic Algebra stage. Our problem 2 solving process belongs to the symbolic algebra stage [4].

In the three stages in which the teacher has completed the introduction to algebra, students may wonder why they are all Western mathematical scientists and what contribution do we Chinese mathematical scientists make to Letters Represent Numbers. Finally, the teacher introduced the exploration of algebra by ancient mathematicians in China to the students: Li Ye, a mathematician in the Song and Yuan Dynasties, used "Tianyuan" to represent the unknown, which was called "Tianyuan Technique"; On the basis of Zhu Shijie's Li

Ye, he popularized the "Siyuan Technique", that is, expressing four different unknowns with heaven, earth, people and things [8].

Students are further aware of the necessity, process, and significance of the generation of Letters Represent Numbers by introducing the origin of Algebra names and their three stages of development. Via this introduction to the history of math, students will be able to further experience the mathematical culture of Letters Represent Numbers and deepen their understanding of this lesson. Finally, the teacher introduced the contribution of ancient Chinese mathematics to algebra, which is conducive to stimulating students' patriotism and fostering national integrity.

3. Reflections and Proposals

The above teaching of Letters Represent Numbers is a mathematical culture teaching with Letters Represent Numbers as the content. From the perspective of HPM, it is an attempt to include Letters Represent Numbers in the teaching of mathematical culture, and the mathematical knowledge of Letters Represent Numbers as part of the teaching of mathematical culture, so the teaching of mathematical knowledge permeates the culture at the same time. From the perspective of HPM, the teaching design integrates mathematical knowledge and culture, which avoids the neglect of mathematical culture in the teaching process. We should establish the concept of mathematics [3]education in the process of teaching that mathematics teaching is the teaching of mathematics culture, attach importance to mathematics culture, and explore effective ways to combine mathematics culture with mathematics knowledge.

Mathematical teaching from an HPM perspective is currently one of the effective methods to integrate mathematical knowledge with mathematical culture. The history of mathematics is an important part of mathematical culture. Practice shows that in mathematics teaching, the history of mathematics can reveal the harmony of knowledge, present the beauty of methods, create the joy of exploration, reach the help of capabilities, show the charm of culture, and realize the effect of moral education [9]. The history of mathematics is rich in mathematical culture. The teaching design from the perspective of HPM is to use the history of mathematics for mathematical teaching. The use of mathematical history teaching will inevitably combine the mathematical culture and mathematical knowledge embedded in mathematical knowledge. To reveal the mathematical culture, teachers should be more aware of the mathematical history and culture related to the mathematical knowledge in middle school. Then, they should adopt the situation of problems encountered by mathematical scientists in history to guide students to explore and solve them. The path that mathematical scientists have gone through can not only deepen the understanding of the knowledge, but also understand the reasons for the emergence of the mathematical knowledge and the process of the evolution of the knowledge, and feel the spirit of persistent and pioneering innovation of statisticians, which is the mathematical culture behind the mathematical knowledge.

Mathematical culture is an important part of human culture, and the essence of mathematical education is the education of mathematical culture. Mathematical knowledge will be lost after students leave school for many years, but mathematical cultures such as ideas and methods embedded in mathematical knowledge will affect students' emotions, attitudes and values. Teaching design that highlights mathematical culture from the perspective of HPM will enable students to understand the process of knowledge generation and development, and deepen their mastery and understanding of knowledge. Another, it is to promote the development of students' emotions, attitudes and values through the penetration of mathematical culture, and further stimulate students' interest in math. Hence, attention should be paid to exploring the mathematical culture from the history of mathematics that is conducive to mathematical education, and carrying out teaching design to highlight the mathematical culture through the HPM perspective to promote the teaching and learning of mathematics.

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